摘要

本研究旨在發展一套實驗室之試驗設備，以探討土石流及因地震引發之崩塌滑動。這些現象存在於台灣本島地區，因台灣地勢處於陡坡又強烈雨型與地震頻繁之地理位置。這些現象有一些極重要的力學機制，即在飽和顆粒力學，如何由固態過渡成為液態。由於因為實驗儀器及設備的普遍不足，使得在物理觀念的瞭解及數值模擬的進行，受到極大的阻礙。

本研究即針對上述的研究動機提出下列的研究方法：(1)設計及建造一套土石流循環水槽；(2)建立反射因子映象，以探討土石流內部水與沙顆粒之間的變形及混和；(3)建立三維數值顆粒顯像流速儀，以測量水與土沙顆粒之運動狀態。透過這些實驗設備及數據的量取，將有助於土石流及土石滑動之微力學之物理機制及土石流基本流場之瞭解。

我們將應用這套系統(此系統將安置於台大水工所新設置之現代流體力學實驗室內)，來研究下列三大重要主題：(1)因地震引發之邊坡破壞及土石流；(2)土石流之基本物理機制；(3)土壌液化與土石流之不穩定之現象。吾人相信這套最新的設備，將可以達到上述三項目標。同時，透過高解析度的量測系統，將可量到土石流顆粒之間的內部變形，並能將這些量得之實驗數據，用作理論及數值計算之回饋，也可以將這些數據提供給其他同好研究工作者做其他研究之用。相信透過本研究計畫，吾人對台灣地區土石流發生之原因及防治對策，將可更上一層樓。

關鍵詞
土石流；因地震引發之崩塌滑動；土壌液化；固體—流體之微力學；反射因子映象；三維數值顆粒顯像流速儀。
English summary
The present report documents a three-year research project aimed at developing novel experimental methods for the laboratory study of debris flows and flowslides. Transparent access to the micromechanics of liquid-granular flows is sought through a combination of three components: 1) liquid and granular media with special optical properties allowing unhindered 3D imaging; 2) a tilting flume equipped with two recirculation circuits for the liquid and granular phases; 3) digital imaging algorithms allowing acquisition and analysis of the 3D particle flow field inside the bulk. Key results from the project include: 1) the successful preparation of transparent liquid-granular mixtures with small visible cores at the centre of the granular particles, achieved using Refractive Index Matching (RIM) and laser marking; 2) the demonstration that stereo imaging techniques can reliably track the 3D positions and motions of large sets of such marked particles, using high-resolution video and Particle Tracking Velocimetry (PTV) algorithms; 3) the design, implementation and testing of a novel double recirculation flume in which rapid downslope surges of granular-liquid mixtures can be made stationary and characterised in detail.
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1. Background and objectives

Subject to heavy rainfall and frequent earthquakes, the upland areas of Taiwan are exposed to high risks of debris surges and flowslides. A variety of applied research efforts are currently directed at mitigating the hazards due to these flows. Many fundamental aspects of their behaviour, however, remain poorly understood. One major reason lies in the difficulty of probing experimentally the flow behaviour of high-concentration granular-liquid mixtures. Intrusive instruments perturb or are perturbed by the flow, while grain occlusion precludes optical access to the flow interior.

The present research seeks to address this challenge by developing novel experimental methods. The methods sought feature three components: 1) special liquid-granular materials allowing optical access and unhindered 3D imaging; 2) new laboratory apparatus allowing rapid phenomena to be made stationary and probed at leisure; 3) post-processing methods allowing insights to be gained from the large sets of data typically acquired by Particle Tracking Velocimetry (PTV) methods; 4) novel imaging algorithms allowing the capture of 3D positions and motions of dense sets of particles in liquid-granular dispersions.

Documented in the present report, the techniques developed to meet these requirements rely on the following principles: 1) the use of Refractive Index Matching to achieve transparency of liquid-granular mixtures, and the laser marking of particle cores to allow their tracking; 2) the assembly of a fluidization cell for pilot tests, and the development of a novel tilting flume equipped with two recirculation circuits for the liquid and granular phases; 3) statistical flow field analysis methods, highlighting correlations of granular motions in space and time; 4) robust stereo imaging methods based on the simultaneous matching of particle positions between views and tracking of particle positions in time.

In the next sections, we first describe the “hardware” part of the research project: the development of new experimental materials and laboratory apparatus. We then proceed to describe the “software” part: the development of novel flow field analysis and stereo imaging algorithms.
2. New experimental materials and apparatus

The laboratory methods developed involve three components: 1) the preparation of a novel particle-liquid system using Refractive Index Matching (RIM) and laser marking methods; 2) the assembly of a fluidization cell apparatus for the conduct of pilot tests; 3) the design and development of a novel double recirculation flume to study rapid downslope flows of granular-liquid mixtures.

2.1. Refractive index matching and laser marking

A major objective of the overall research programme was to identify a combination of liquid and solid materials that would allow observation of rapid motions inside a dense 3D liquid-granular mixture. This goal was attained through a combination of refractive index matching (RIM) and laser marking techniques. The combination of these two techniques yields a transparent dispersion of particles marked with a small but highly visible core suitable for identification and tracking.

After consideration of various alternatives and testing on small samples, a solid-liquid combination proposed recently by Haam et al. (2000) was selected as RIM system. Solid particles consist of small 7 mm spheres of poly-methyl metacrylate (PMMA), a transparent material more commonly known as acrylic or perspex. The corresponding index-matched liquid is 1-Methyl-4 - (1-methylethyl) - benzene, an organic liquid traded under the name para-cymene. It has a low viscosity suitable for high Reynolds number experiments. Acrylic particles of excellent optical qualities are available commercially in Taiwan, while para-cymene can be imported at reasonable cost. Neither material presents excessive flammability or health risks. The characteristics of both media are summarised in Table 1.

<table>
<thead>
<tr>
<th>property / material</th>
<th>para-cymene</th>
<th>PMMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>specific gravity</td>
<td>0.855</td>
<td>1.19</td>
</tr>
<tr>
<td>viscosity [ Pa s ]</td>
<td>$1.023 \times 10^{-3}$</td>
<td>n.a.</td>
</tr>
<tr>
<td>refractive index</td>
<td>1.489</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Table 1. Summary of the properties of the selected liquid and solid media.

Thanks to the close match between the refractive indices of these two media, a high degree of transparency is achieved for their mixture, eliminating to a large extent the light ray distortion occurring across the boundaries of the spheres. This allows unhindered optical penetration even for high concentrations of particles. This is illustrated on Fig. 1. The density ratio between the two phases also falls in a range which is well-suited for laboratory investigations of gravity-driven liquid-granular flows.
Fig. 1. Refractive index matching. Left: transparent particles visible in air become invisible when bathed in the index-matched liquid; right: a grid is seen undistorted through the matched liquid-solid mixture. Faint outlines of the solid spheres can be detected upon closer look at the photo.

The refractive index of the liquid varies with temperature. As illustrated in Fig. 2, the matching with the acrylic solid spheres is optimal only within a rather narrow range of temperatures close to 15°C. Since this is lower than typical ambient temperatures, refrigeration equipment must be used to cool the liquid before and/or during the experiments. With adequate temperature control, excellent transparency can be achieved.

Fig. 2. RIM dependence on temperature. Left: the transparency is better at temperature of 15°C; right: the outline of PMMA spheres is stronger at temperature of 25°C.

Transparency of course is not an end in itself, and our ultimate purpose is to be able to track individual particles located inside the flowing bulk. To achieve this, particles must be made visible again. Previous RIM researchers (e.g. Cui and Adrian, 1997; Haam et al. 2000) have all solved this problem by coating the outside of a small subsample of particles with a
chemical dye or a metal plating. As a result, only this small subset (typically 1-5% of all the particles) can be seen and tracked on digital images, leading to sparse velocity measurements. This limits the resolution of the measured flow field and precludes capture of the interacting motions of neighboring grains.

After exploring some coating and inclusion options, we were able to solve the problem in a novel way thanks to a recently developed technology in use in Taiwan's crystal ornament industry. The technique called internal laser etching is used to carve 3D patterns of visible dots inside transparent blocks of lead crystal. Thanks to the cooperation of a local crystal company, we were able to apply this technique to acrylic spheres, and mark a small visible core at the centre of the transparent particles (Fig. 3). The key advantage of this technique is that a particle can be seen without occluding other particles along a given line of sight. As a result, rather than a small subset, all particles can be marked and used for velocity tracking.

![Fig. 3. Left photo: visible core laser-etched at the centre of an acrylic sphere. Right photo: flasks of unmarked (left) and laser-marked (right) acrylic spheres bathed in para-cymene.](image)

To process a large number of particles, the marking procedure had to be made efficient. High-precision machining was used to prepare a pair of identical perforated plates each of which can hold ~ 400 particles. While one plate is being loaded with particles, the other is mounted on a computer-controlled x-y-z carriage which sequentially moves particles below the laser beam. In ~ 8 hours of laser bench time, it was possible to etch 20,000 particles in this fashion. The process is illustrated in Figure 4. The laser-marking approach does not seem to have been used before in RIM experiments, and should allow us to obtain high resolution measurements of the motions of interacting grains.
Fig. 4. Laser-marking process. Top: sequential laser-marking of the particles placed on the carriage-mounted plate; bottom: close up of the laser lens and carriage during the laser-marking sequence.

After successful testing of the techniques on small samples liquid and solid materials, a batch of 20,000 acrylic particles and two drums of 200 liters of para-cymene were obtained. Laser marking of the large number of particles was performed, and the materials have been successfully prepared for use in the fluidisation and recirculation tests.
2.2. Fluidisation cell apparatus

In a second stage of the project, a fluidisation cell apparatus was assembled to conduct pilot tests of the novel particle-liquid system described above. The aim of these pilot tests was threefold: to demonstrate the feasibility of refractive index matching RIM measurements; to troubleshoot materials and methods to be used later in the large scale flume; to obtain a first set of reference measurements in homogeneous flow conditions.

Figure 5. Fluidisation cell apparatus. From left to right: a) transparent-walled device; b) schematic; c) close-up of the inner cylindrical working section. Temperature control is achieved here with the cooling coil seen on the right.

Through these pilot tests, the combination para-cymene / acrylic was confirmed as an excellent choice of transparent liquid-granular mixture. Temperature control was achieved by plunging a cooling circuit in the liquid bath (see Fig. 5c), or by pre-cooling the liquid in a refrigeration unit. The marking of the particle cores using laser etching, described above, was also found to yield excellent results. The best viewing conditions are found to be obtained using oblique backlighting, with stereo views angled at around 10 degrees. Video images obtained in such conditions yield sharply contrasted black cores on a light background. The cores are thus easily identifiable on the digital images, while at the same time being small enough to minimise occlusion effects. An example of a digital frame is shown on Fig. 6. The corresponding tracking measurements are discussed infra in the present report.
The tests have also highlighted various technological issues which require special care. First, wall, sealant, and piping materials must be compatible with the para-cymene liquid. The set of compatible materials was found to include acrylic, nylon, PVC, glass, stainless steel, and resin, but to exclude common sealants and plastics used for water flow experiments. Secondly, conditions of uniform liquid inflow are best approximated using a combination of expansion chamber and filtration layer of heavy particles. These results have helped guide the development of the larger scale flume apparatus discussed in the next section.
2.3. Double recirculation flume for the study of downslope granular-liquid flows

The third major component of the project was the design and development of a novel double recirculation flume to study rapid downslope flows of granular-liquid mixtures. The originality of the new apparatus is its ability to independently recirculate the liquid and granular phases involved in the flow. A further constraint is that all materials used for the flume have to be compatible with the special RIM liquid. This aims to insure that the flume will be suitable for the acquisition of 3D measurements using the transparent materials.

Figure 7 shows the conceptual design proposed initially to achieve this purpose, while Figure 8 shows the revised design adopted after a more comprehensive study. Instead of performing the double recirculation using two conveyer belts, the new design uses one conveyer belt to recirculate the solid phase, and an external pumping circuit to recirculate the liquid.

Figure 7. Initial design for the proposed double recirculation flume (principle sketch): 1) glass-walled open-channel; 2) internal conveyer belt; 3) external conveyer belt; 4) imaging sensor and lighting; 5) typical viewing window.
Figure 8. Revised design incorporating technological and economical constraints, in which the double recirculation is achieved by combining an internal conveyer belt with an external pumping system.

A first version of the flume apparatus was assembled according to this design. A view of the flume installation is shown on Figure 9. Key components include the adjustable slope, the high-speed conveyer belt for granular recirculation, and the independent pumping circuit for liquid recirculation. All three components of the device are controlled using an electronic panel. Operating parameters can be set precisely and varied over a wide range, in accordance with the design.
While the overall design was successfully implemented, difficult mechanical problems were encountered with the conveyor belt. The purpose of this belt is to entrain granular material upstream along the floor of the flume. Driven by a variable speed motor, it forms a continuous loop around the apparatus (see Fig. 9). As this belt is in direct contact with the liquid-granular mixture, its material must be compatible with the para-cymene liquid. This led to the choice of a flexible, stainless steel perforated belt entrained by solid rollers with matched pins. The original design called for a series of rollers of 10 cm diameter placed at the four corners of the flume. To form a continuous loop, the two ends of the belt must be carefully welded together.

The scheme was implemented and found to work fine, but only for a limited number of cycles: repeated bending of the joint as the belt curves around the rollers rapidly leads to fatigue failure (see Fig. 10). Improved endurance is obtained by subjecting the welded joint to an annealing process prior to belt installation, however joint failure nevertheless occurs after a number of cycles of the order of 1,000 to 10,000. This has forced us to reconsider the design of the belt circuit: instead of 4 small rollers at the four corners of the flume, the revised design features two large wheels of 70 cm diameter at both ends of the flume. The enlarged radius is crucial to minimize bending of the belt and was calculated to extend the belt lifetime by at least 2 orders of magnitude.
Figure 10. Fatigue failure of the welded joint of the stainless steel belt. The revised design currently being implemented replaces the small rollers at the flume corners by two wheels of large diameter.

Figure 11. Replacement of the small rollers by large wheels to reduce belt fatigue problems.

Testing at the workshop.

Figure 12. New version of the double recirculation flume installed in our laboratory.

The flume has now undergone a number of tests and is fully operational.
The revised double recirculation flume is shown on Figures 11 and 12. Thanks to the wheel replacement, fatigue problems have been solved. The flume was found to perform well for a number of tests, and is now fully operational. A first set of experiments has been carried out, aimed at identifying the flow regimes that can be obtained under different conditions. One flow pattern of particular interest for debris flow studies is shown on Fig. 13. Both surges and uniform flow regimes are expected to be targets of great interest for further studies.

Figure 13. Granular-liquid surges observed in the double recirculation flume. The flume is inclined and the belt operated at large speeds to make the downslope surge stationary with respect to the laboratory frame of reference. Top: overall view of the surge. Bottom: close-up of the front, where three-phase flow is observed (air, liquid and grains).
3. New imaging measurement techniques

To complement the novel materials and apparatus described in the previous section, a second major part of the work undertaken during the three-year project consists in developing new imaging measurement techniques. These techniques fall in two different categories: 1) statistical methods used to extract information from the very large datasets typically generated by PTV measurements. 2) 3D imaging algorithms aimed at reconstructing the 3D positions and motions of large sets of particles from stereo images.

3.1. Statistical analysis of Particle Tracking Velocimetry (PTV) data

Particle Tracking Velocimetry (PTV) measurements typically yield very large sets of data. To handle such data sets, statistical flow analysis techniques have been developed and refined in the course of the project. While the new experimental devices described above were still under development, the analysis techniques were first applied to footage obtained using existing equipment. This was done in cooperation with two other universities: 1) a detailed analysis of steady uniform dry granular flows was performed in cooperation with the National Central University (Professor H.-T. Chou); 2) a comprehensive analysis of steady uniform granular-liquid flows was conducted in cooperation with the Università degli Studi di Trento (Professor A. Armanini). Two manuscripts based on this work have been submitted for publication and are included in appendix.

Fig. 14. Liquid-granular flow imaged through a flume sidewall. Left: side view image (mean flow is from left to right); right: convected trajectories (i.e. measured grain trajectories from which the mean flow component has been subtracted).

We have developed statistical tools for two different purposes: first, to isolate physical signals from random experimental errors; and secondly to identify correlated motions in fluctuating flow fields. As an illustration of what statistical techniques can help achieve, Fig. 14 shows a debris flow run imaged in 2D through a flume sidewall, from which detailed particle trajectories were extracted (this run is taken from a previous study done in cooperation with the Università degli Studi di Trento, Italy). A spatially correlated flow pattern could be
extracted from these 2D measurements, and is shown on Fig. 15. Coherent motion components can thus be successfully extracted from a seemingly random granular flow field. A variety of other results obtained by statistical post-processing of PTV data are documented in the two manuscripts included in appendix.

Fig. 15. Correlated flow pattern averaged around a single particle, as sampled inside the fluctuating granular ensemble of Fig. 14. Each unit of the spatial grid correspond to one grain diameter.

3.2. Three-dimensional Particle Tracking Velocimetry (PTV) measurements

Three-dimensional particle tracking algorithms were developed and refined in the course of the project, and tested on the fluidisation cell experiments. Image acquisition is performed using a single camera and a system of mirrors. The acquired images correspond to two distinct viewpoints, which have to be calibrated independently. The calibration target used for this purpose is also shown on Fig. 16. The illumination is provided by backlighting, using spots aimed at translucent acrylic board placed behind the fluidisation cell.

Figure 16. Imaging configuration (left) and calibration target (right).
Once images have been acquired in this fashion, 3D particle positioning and tracking can be performed. A first set of methods for the 3D positioning and tracking of particles viewed on stereoscopic image pairs was developed and tested in cooperation with the Université catholique de Louvain (Belgium) and Università degli Studi di Trento (Italy). The algorithms are described in detail in the Appendix and we will not repeat the description here. Illustrating the kind of results obtained, Fig. 17 shows the fluctuating 3D trajectories of an ensemble of opaque particles viewed through a transparent wall. Due to particle occlusion, here the depth of field is limited to the vicinity of the sidewall.

![Fig. 17. Near-wall trajectories of a fluidised dispersion of opaque grain bathed in water: (a) top view; (b) oblique view; (c) frontal view through the sidewall. Measurements obtained using the stereo matching and tracking methods described in Appendix.](image)

The same methods were applied to the fluidization cell experiments. Thanks to the RIM materials and laser marking technique, of course, particles inside the 3D dispersion can now be tracked, rather than only those close to the sidewall. When only 10 marked particles were introduced in a fluidisation of 1,000 transparent particles, the methods generated good results (see Fig. 18). However, when the full set of 1,000 particles were laser-marked, two problems were encountered. First, the temporal and spatial resolution of the video camera was no longer sufficient to resolve the motions of all the particles. Secondly, the imaging algorithms themselves were not sufficiently robust, yielding poor tracking statistics.
Figure 18. 3D particle tracking of 10 marked particles in a fluidization of 1,000 transparent spheres bathed in a liquid of identical refractive index.

To solve these problems, new footage was acquired using a high-resolution video camera on loan from the Taiwan representative of the camera maker. Images acquired using this camera are shown on Fig. 19. To take advantage of this higher resolution and higher frame rate, new algorithms were developed. Instead of positioning particles in 3D first, and then tracking them, the new methods combine the 3D positioning and tracking in a single step. This is aided by a preliminary step, shown on Figure 20, in which particles are first tracked in the 2D image plane.

Figure 19. Sample high-resolution video image: the particle cores show up as sharp black speckles on a light background, and are identifiable in both left and right windows of the stereo image.
Results using this higher resolution and improved algorithms were found to be excellent. Instead of being able to track only a small subset of particles (10), the new methods are able to reliably position and track a majority of the 1,000 particles of the fluidization. Position results and the mean circulation pattern averaged from the tracking results are shown on Fig. 21. This pattern would be very difficult to measure by any other means.

Figure 21. Fluidisation of 1,000 marked particles. Left: matched particle positions; right: mean particle circulation in a meridian plane. The particle diameters in the left plot are the true diameters of the solid particles, which is much larger than the diameter of their visible cores.
The results obtained demonstrate the unique potential of the methods developed in the project. By combining Refractive Index Matching, laser marking, and stereo imaging techniques, it is demonstrated for the first time that dense 3D systems of particles bathed in a liquid can be tracked, unveiling the details of the rapid motions inside the dispersion. We are currently working on getting these results ready for a paper submission.
Goals attained

From the point of view of method development, the goals of the research project have been fully attained:
1) Matched liquid and solid materials allowing optical access to the interior of high-concentration granular dispersions have been successfully identified and prepared.
2) A novel double recirculation flume was successfully designed, assembled and tested.
3) Post-processing techniques for the detailed statistical analysis of large flow field data sets have been derived and implemented.
4) Robust algorithms for the 3D tracking of particle positions and motions in systems of many particles have been developed and tested.

A highlight of the present work is the successful tracking of a majority of particles from a dense fluidisation of 1,000 marked transparent particles in a liquid of matched refractive index. To the best of our knowledge, tracking results of this kind had never been obtained before.

From the point of view of method application, however, the original goals of the present research project have only been partially attained. It was initially hoped that the various techniques described above could be jointly applied, in order to obtain detailed 3D measurements of liquid-granular surges in the double recirculation flume. This unfortunately turned out to be unrealistic within the three-year time frame. Each component was successfully developed, but the integration of the various components was not achieved in time. A major factor was the late date at which the double recirculation flume became operational (in the third year of the project). This was due to necessary modifications in the design: first, to insure that all flume materials could withstand the RIM liquid; secondly, to resolve fatigue problems by installing large wheels instead of small rollers.

As a result, the imaging techniques could not be applied to surges observed in the recirculation flume (it was not yet online). Instead, other applications were used for testing of the imaging algorithms: 1) downslope flows observed in existing devices at other universities (through cooperation with the National Central University, Taiwan, and the Università degli studi di Trento, Italy) were used to test and exploit the statistical flow field analysis methods; 2) fluidisation cell tests performed at our own laboratory were used to test the RIM techniques and 3D stereo imaging algorithms.

Addressing this limitation, an integrated exploitation of the methods developed in the present project is currently in progress, thanks to continued support by the NSC.
Appendixes

Four papers (two published and two tentatively accepted for publication) document the most mature aspects of the techniques underlying the present research effort. They are included as appendixes to the present report.


Voronoï imaging methods for the measurement of granular flows

H. Capart, D. L. Young, Y. Zech

Abstract A set of digital imaging methods derived from the Voronoï diagram is proposed and tested on various liquid–granular flow applications. The methods include a novel pattern-based particle-tracking algorithm, as well as estimators of the three-dimensional granular concentration from two-dimensional images. The proposed algorithms are able to resolve individual grain motions even for rapid shear flows involving dense, fluctuating granular ensembles. Full automation is achieved, allowing the derivation of accurate statistics from large sets of individual measurements, as well as the construction of complete sets of grain trajectories. Results are presented for different applications: homogeneous fluidization, steady uniform debris flow, and unsteady debris surges.

1 Introduction
Flows of granular materials are notable for the diversity of their behaviours and their involvement in a wide range of geophysical and industrial processes. While much about them can be learned from computational simulations (Campbell 1990; Kalthoff et al. 1997), there is a corresponding need for detailed experimental measurements. A variety of techniques are available for multiphase flow measurements (Bachalo 1994; Chaouki et al. 1997), and those applied more specifically to granular flows include force and impact measurements (Savage and McKeown 1983; Zenit et al. 1997), acoustic probes (Bennett and Best 1995), tracked transmitters (Dave et al. 1999), and magnetic resonance imaging (Nakagawa et al. 1993). The present work focuses on digital imaging techniques applied to the analysis of monocular image sequences. These are typically acquired by filtering a flow free-surface from above or by imaging the flow through a transparent wall. As granular flows are characterized by high solid fractions, opacity of the material generally prevents optical penetration beyond a few grain diameters. In many instances, this does not prevent one from making meaningful measurements, and it is not necessary to resort to sophisticated techniques such as magnetic resonance imaging or refractive-index matching (Cui and Adrian 1997) to penetrate the flow interior. One would therefore wish to resort to the powerful imaging techniques of experimental fluid mechanics (Adrian 1991) to provide nonintrusive, whole-field coverage of the flow kinematics of the visible grains.

Granular flows are, however, characterized by three features that pose problems to imaging velocimetry algorithms: dense dispersions of grains, fluctuating motions produced by interparticle contacts, and sharp flow gradients on the scale of only a few grain diameters. The simplest methods of particle-tracking velocimetry (PTV) rely on minimum displacement matching (Guler et al. 1999) and fail when the interframe displacement becomes significant with respect to the mean particle interdistance. This limit is quickly reached for dense, rapid granular flows. More sophisticated particle-tracking methods involving trajectory-based matching (Sethi and Jain 1987) are easily offset by the uncorrelated granular velocity fluctuations. The techniques of particle imaging velocimetry (PIV) are robust with respect to large displacements of dense dispersions (Willert and Gharib 1991) but have difficulties in dealing with intense shear (Huang et al. 1993). Finally, the direct-correlation (Fujita et al. 1998) or spatial filtering methods (Uddin et al. 1998) do not resolve individual grain motions, an information which is most important if one seeks to test microstructural theories.

Similar difficulties are encountered for concentration estimation. When grains become closely packed, occlusion

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The fluidization cell tests were performed at the Laboratoire du Génie Civil, Université catholique de Louvain, Belgium. The uniform debris flow experiments were conducted by L. Guarino at the Hydraulics Laboratory of the Civil and Environmental Engineering Department of the Università degli Studi di Trento, Italy. H. C. benefited there from the hospitality and support of professors A. Armanini and L. Fraccarollo. The dam-break measurements were obtained with the help of Y.H. Liu at the Hydrotech Research Institute, National Taiwan University, Taiwan, and with support of a travel grant from the Fonds National de la Recherche Scientifique, Belgium.
effects hamper the correspondence between two-dimensional observations of the grain number density and their three-dimensional volumetric concentration. Suggestions have been made for such a correspondence (Capart et al. 1997), but have not been tested in detail. Proposals based on lumiance measurements (Louge and Jenkins 1997; Kanda et al. 1999), on the other hand, require case-by-case calibration and a careful control of illumination, which is difficult to obtain even in laboratory conditions.

Various strategies have been adopted by researchers to cope with these difficulties. A first possibility is to diminish tracking problems by restricting the investigations to dilute dispersions or slow deformation rates, and to avoid concentration estimation problems by dealing with two-dimensional dispersions of spheres or disks moving between two closely spaced plates (Elliott et al. 1998; Wildman et al. 1999). Another widely used approach is to track only a small proportion of coloured “tracer” particles included in the dispersion (Natarajan et al. 1995; Liu et al. 1997; Hsiau and Jang 1998), leading to a corresponding loss of resolution. Finally, a third strategy consists in performing the tracking manually or with manual supervision (Drake 1991; Capart and Young 1998), limiting the number of measurements to the bounds of human patience.

In the present work, we develop and test imaging methods that are aimed at addressing the above shortcomings. Both for velocimetry and concentration, the methods derive from a pattern-based principle. The idea is that the local pattern formed by neighbouring grains will remain stable over a certain time, even for a rapidly deforming, fluctuating granular phase, and that it can therefore serve as a match template. Furthermore, it can also be used to characterize the local degree of packing of the dispersion. The specific tool chosen to describe local patterns is the Voronoi diagram. Various properties of the Voronoi diagram endow the approach with advantages over other pattern-based methods (Haynes and Turner 1992; Song et al. 1996, 1999; Ruan et al. 1999), and this will be discussed in detail in the presentation of the algorithms that forms the first part of the paper. In the second part, the methods are tested on selected liquid-granular flows, including homogeneous fluidization, steady uniform channel flow, and dam-break induced debris surges.

2 Principle and algorithms

Consider a sequence of images depicting the flow of an ensemble of grains. Rather than directly correlating windows of pixel values, the present work deals with grain images by first abstracting them into point-like particles. The granular ensemble is thus reduced to a set of feature-points corresponding to grain centroids that are dispersed in the image plane. Particle identification can be achieved using a variety of segmentation or filtering methods, and the particular algorithm used for the applications is described in Appendix 1. After this operation, the analysis of the motions and patterns of sets of point-like particles constitutes the general object of the Voronoi methods.

2.1 Voronoi construction and properties

Let a dispersion of $n$ feature-points $P_i$ occupy positions $r_i = (x_i, y_i) \ (i = 1, \ldots, n)$ in the two-dimensional plane (Fig. 1). The Voronoi construction designates the tiling of the plane into $n$ polygonal regions (or “cells”) such that each polygon $V_i$ encompasses the region of the plane that is nearest to $P_i$ than to any other feature-point. Feature-points characterized by Voronoi cells sharing an edge are termed “natural neighbours” of each other. The graph that connects natural neighbours further defines a second tesselation, dual to the Voronoi diagram: the Delaunay triangulation, composed of triangles $D_j$.

The two constructions present many remarkable properties that have led to recent applications in a variety of fields including cell biology (Marcelpoil and Usson 1992), computational mechanics (Braun and Sambridge 1995), astronomy (Bernardeau and van de Weygaert 1996), and molecular hydrodynamics (Español 1998). A general overview of the Voronoi diagram, its properties, and applications is given in Okabe et al. (1992), and a mathematical introduction in Preparata and Shamos (1985). In the present context, the Voronoi and Delaunay diagrams are of interest primarily because they provide useful local structures, or “tokens”, which can be exploited for pattern characterization and matching (Ahuja 1982). The most obvious among these structures are the Voronoi cells and Delaunay triangles themselves, with properties such as area and perimeter.

Structures of a second type, most useful for pattern-matching, define local “neighbourhoods”. The triplets of feature-points that are vertices to a common Delaunay triangle provide one such local neighbourhood (Song et al. 1999). Another structure of this type, key to the present work, is the Voronoi 1-star (Fig. 2). The first vertex star (or simply 1-star) $S_i$ of feature-point $P_i$ is defined as the set of its natural neighbours, including itself (Senechal 1995). The 1-star can be visualized as a “star of spokes” originating at feature-point $P_i$. (Likewise, the 2-star of feature-point $P_i$ can be defined as the set of feature-points that are natural neighbours to the 1-star of feature-point $P_i$, and so on). As shown in detail in the present work, the Voronoi 1-star constitutes a very useful token for pattern-matching in a particle-tracking context. A comparison with other neighbourhood definitions is provided in Sect. 2.2.

Fig. 1. Voronoi diagram (— —) and its dual, the Delaunay triangulation (· · ·), constructed on a random dispersion of feature-points $P_i$ (o)
The Voronoï and Delaunay tessellations are characterized by the following desirable properties:

1) Geometric properties. First, except for unlikely degenerate cases, the construction is unique for a given set of feature-points. Secondly, the construction is local, i.e., the position of remote points does not affect the local structure of the diagram (Sibson 1981). The tokens described above can thus be construed as reliable descriptors of the local point pattern. Thirdly, the construction is adaptive with respect to variations in the local density of points. In particular, natural neighbours of a feature-point tend to “surround” it evenly, regardless of a possible density gradient in one direction or another (Ahuja 1982).

2) Kinematic properties. The Voronoï diagram is stable to continuous deformation. This means that if the feature-points move along continuous trajectories, then the shape of the Voronoï cell will also evolve gradually, and neighbourhood relations will change one by one (except for unlikely degenerate cases). If the trajectories of the feature-points are known, this can be used to continuously update the Voronoï diagram rather than reconstruct it from scratch at discrete times (Albers et al. 1998). This property makes the Voronoï 1-star a particularly attractive token for flowing dispersions of particles, as branches of the star can be expected to deform gradually, except for isolated topological events that will affect one branch of the star at a time. Delaunay triangles are not so attractive in this respect, since they are affected much more severely by these “swap” events (Fig. 3). The Voronoï diagram also remains stable in the case of addition or suppression of a feature-point, as in the case of particle occlusion.

3) Computational properties. The construction presents a number of desirable computational properties. First, algorithms are available that can construct the Voronoï diagram in $O(n \log n)$ operations (where $n$ is, as before, the number of feature-points). The particular algorithm used in the present work is based on Fortune’s plane sweep method (Fortune 1987). Secondly, once the construction is obtained, a nearest-neighbour search (useful for matching operations) can be performed in $O(\log n)$ operations (Preparata and Shamos 1985). Finally, local reconstruction of the Voronoï diagram (after removal of a spurious particle image, for example) can be performed in $O(1)$ operations. These features make it possible to devise efficient implementations of the algorithms detailed in Sect. 2.2.

2.2
The matching problem

Consider now a set of moving particles, with positions sampled on two image frames acquired at instants $t_1$ and $t_2$. Supposing that particle images are indistinguishable from each other (particles are of the same size, colour, etc.), the information available is limited to the two sets of feature-point positions $r_{i,1}$ ($i = 1, \ldots, n_1$) and $r_{j,2}$ ($j = 1, \ldots, n_2$). Based on this information, the objective is to match particles between the two frames. Formally, one seeks a pairing $\{i(k), j(k)\}$ associating particle image $P_i$ on frame 1 and particle image $P_j$ on frame 2 to one and the same physical particle $k$. Particle velocity vectors can then be obtained from:

$$v_k = \frac{r_{j(k),2} - r_{i(k),1}}{t_2 - t_1}$$  \hspace{1cm} (1)

If the time interval $\Delta t = t_2 - t_1$ is small enough, then a reasonable algorithm is to associate to each feature-point $P_{i,1}$ of frame 1 the nearest feature-point of frame 2. This is the “minimum displacement” algorithm (Jähne 1995; Guler et al. 1999), which can be written formally

$$\text{match}(P_{i,1}) = \min_{P_{j,2}} (\text{dist}(P_{i,1}, P_{j,2})) .$$  \hspace{1cm} (2)

Statement (2) is to be interpreted as follows: for a given point $P_{i,1}$ on frame 1, the best match among points $P_{j,2}$ on frame 2 is chosen as the one that minimizes the “point-distance” $\text{dist}(P_{i,1}, P_{j,2}) = ((x_i - x_j)^2 + (y_i - y_j)^2)^{1/2}$, i.e., the
standard Euclidean distance. Although such a general notation is not necessary at this stage, it will be useful in the remainder of the presentation as other distances are to be introduced.

To make the “minimum displacement” algorithm slightly more robust and avoid multiple matches to one and the same feature-point, a refinement is to set up the following optimization problem: find the global pairing \( \{i(k), j(k)\} \) that minimizes the objective function

\[
\min_{n_1, n_2} \sum_{k=1} \text{dist}_P(P_{i(k),1}, P_{j(k),2})
\]

(3)

that is, a sum over all selected pairs of the distances between matched particles \( P_{i(k),1} \) and \( P_{j(k),2} \). This is a standard bijective graph optimization problem, which is difficult (and computationally expensive) to solve thoroughly for large numbers of points. An approximate solution is, however, easily found using the Vogel algorithm. The algorithm consists in considering for each particle image the best match and the second best match, then constructing a reasonable global optimum by picking particle pairs in the order of maximum difference between first and second best choices.

The simple match algorithm sketched above presents two shortcomings. First, from a computational point of view, it is unnecessarily expensive by requiring the tabulation of the distance between each feature-point of frame 1 and every feature-point of frame 2, however far apart from each other. It would thus be desirable to narrow down the match candidates using some suitable criterion. Secondly, a more serious problem is that the algorithm fails as soon as the interframe displacement of particles \( \Delta r \) is not small with respect to the mean particle interdistance \( \delta r \). In that case, nearest points from two successive images are likely to correspond to two different physical particles, and a “goodness-of-match” criterion more robust than the minimum displacement is required. In Capart (2000), it is shown that for the case of a rigidly-moving random dispersion of points, one must have \( \Delta r / \delta r < 0.35 \) for minimum displacement matches to be reliable at the 90% confidence level. When the minimization improvement (3) is used, the condition relaxes to \( \Delta r / \delta r < 0.7 \), still quite a stringent constraint in the case of rapidly moving, dense granular dispersions.

To address these problems, a first possibility is to use information from the particle’s previous trajectory. Provided the past trajectory is known, a predictor step can be used to estimate the likely position of a particle on the next frame, and restrict the match candidates to those encountered in a limited search window around this position. Path regularity can then be used as an indicator of goodness-of-match (Sethi and Jain 1987; Jain et al. 1995). A refinement consists in solving a complex multi-frame optimization problem. Such methods, used for instance for the radar monitoring of air traffic (Brookner 1998), are successful for a number of PTV applications (Malik et al. 1993; Ushijima and Tanaka 1996) where particles are sparse and their trajectories well-behaved. However, when particle density is high and velocity fluctuations are important, these methods are highly unstable. This was experienced by the authors in their first attempts at imaging granular flows (Capart et al. 1997; Capart and Young 1998), with the consequence that close manual supervision was necessary to obtain reasonable results.

A second possibility is to use spatial rather than temporal information, and focus on pattern regularity rather than path regularity. Provided that particle neighbourhoods can be defined, these neighbourhoods can be used, on the one hand, to restrict the match candidates, and on the other hand, to extract point patterns that can be compared from one image to the next. These two operations can be respectively referred to as the “screening” and “selection” processes. Various ways of defining neighbourhoods have been developed (a review is given in Ahuja 1982) and applied to particle tracking applications. With reference to Fig. 4a–b, a common definition is to take a circular window of radius \( R \) around feature-point \( P_i \) of image 1, and consider as match candidates the feature-points of image 2 belonging to this window (or \( R \)-neighbourhood). A second circular window (possibly with a different radius \( r \)) can then be used to define a template of neighbours (or \( r \)-star) of particle image \( P_i \), which can be compared with similar templates associated with the various match candidates on frame 2 (Fig. 4a–b). Such an approach has been used by Haynes and Turner (1992),

\[ \text{Fig. 4a–d. Screening (left) and selection (right) operations performed with circular windows (above) and Voronoi 1-stars (below): a a window of radius } R \text{ (} \rightarrow \text{) is used to select the match candidates and a window of radius } r \text{ (} \cdot \cdot \cdot \text{) defines the template to be matched; b goodness-of-match is evaluated by examining the area overlap between disks of radius } \rho \text{ centred on each feature-point of the template; c the 1-star } S_{0,2} \text{ (} \rightarrow \text{) of the minimum-displacement match } P_{0,2} \text{ is used to select the match candidates, and the point’s } 1\text{-star } S_{1,1} \text{ (} \cdot \cdot \cdot \text{) defines the template; d goodness-of-match between stars } S_{1,1} \text{ and } S_{1,2} \text{ is estimated by looking at interpoint distances according to the Hausdorff-type measure of (5). Window-based algorithms (a) and (b) require the definition of three parameters } R, r, \rho \text{, whereas Voronoi methods (c) and (d) are non-parametric and are free to adapt to variations in the local state of the point dispersion.} \]
Lloyd et al. (1992), and Ruan et al. (1999), choosing a particle area-overlap criterion for goodness-of-match between the templates (Fig. 4b). A similar approach could be devised with neighbourhoods (and stars) defined in terms of the $K$ ($k$) nearest neighbours (defining $K$-neighbourhoods and $k$-stars). However, both these approaches have the drawback that the radii or numbers of nearest-neighbours considered must be defined a priori, and cannot adapt to variations or local gradients in point density (Ahuja 1982). The data structures necessary for constructing and handling the neighbour lists, which take the same form as the Verlet lists of molecular dynamics (Frenkel and Smit 1996), are also rather heavy to manipulate in comparison with a structure such as the Voronoi diagram.

Recently, Song et al. (1996, 1999) proposed to use the triangles of the Delaunay tesselation as match tokens. The present work proposes to resort to the Voronoi 1-star both to define a search neighbourhood and to provide a match template.

2.3 Voronoi match algorithm
With reference to Fig. 4c–d, the proposed match algorithm involves the Voronoi construction for both screening and selection operations. It consists first in finding on frame 2 the minimum-displacement match $P_{0,2}$ corresponding to feature-point $P_{1,1}$ of image 1. The match candidates are selected as the feature-points on image 2 that belong to the Voronoi 1-star $S_{0,2}$ of point $P_{0,2}$. The Voronoi 1-star of each of the candidates is then compared with the 1-star of feature-point $P_{1,1}$ to evaluate the goodness-of-match. Formally, the best match of point $P_{1,1}$ is thus

$$\text{match}(P_{1,1}) = \min_{P_{2,1} \in S_{0,2}} \left( \text{dist}_S(S_{1,1}, S_{1,2}) \right)$$

(4)

where $S_i$ designates the Voronoi 1-star of point $P_i$ and $\text{dist}_S(S_i, S_j)$ represents a suitably defined “star-distance” reflecting the degree of discrepancy between the patterns formed by two stars $S_i$ and $S_j$. For such a function, it is proposed to choose the median of the distances between the star extremities once the star centres have been made to coincide (Fig. 4d). Formally, this can be written

$$\text{dist}_S(S_{1,1}, S_{1,2}) = \text{median}_{k_1=1...m_1} \left[ \text{min}_{k_2=1...m_2} \left| (r_{k1,1} - r_{1,0,1}) - (r_{k2,2} - r_{2,0,2}) \right| \right]$$

(5)

where vertex $k_1 = 0$ (resp. $k_2 = 0$) is the centre of star $S_1$ (resp. $S_2$), vertices $k_1 = 1...m_1$ (resp. $k_2 = 1...m_2$) are the extremities of star $S_1$ (resp. $S_2$), and $|r| = (x^2 + y^2)^{1/2}$ is the usual Euclidean norm. Expression (5) features three successive stages: 1) a translation making the star centres coincide, allowing their shapes to be compared in a common frame of reference; 2) an inner loop based on the minimum norm, whereby for each extremity $r_{k1,1}$ of star $S_1$, the nearest extremity of star $S_2$ is found, and their distance from each other is added to a list; and 3) an outer loop based on the median norm, whereby the median value of the list is adopted as an overall measure of the discrepancy between the two stars $S_1$ and $S_2$. The above definition allows comparison between the shapes of any two stars, without requiring that they have the same number of branches. It corresponds to the Hausdorff distance (e.g., Goodrich et al. 1999), except for the use of the median rather than the maximum in the outer norm. The choice of the median is made in order to increase the robustness of the comparison: even the correctly matched stars (from a physical point of view) are expected to differ significantly at some of their extremities (when topological swap events result from flow deformation, when particle occlusion occurs, when particles reach the boundaries of the domain, etc.). Note that the star-distance defined above is “directed”, i.e., in general, $\text{dist}_S(S_{1,1}, S_{2,2}) \neq \text{dist}_S(S_{2,2}, S_{1,1})$. An undirected equivalent is easily defined as $1/2[\text{dist}_S(S_{1,1}, S_{2,2}) + \text{dist}_S(S_{2,2}, S_{1,1})]$, but this was not found necessary in the present context.

Rather than simply setting $j(k) = \text{match}(i(k))$, an optimization problem can again be set up to improve the matching and avoid multiple pairings with one and the same particle. This can be performed once again by seeking the global matching $\{i(k), j(k)\}$ that minimizes the objective function

$$\min_{(n_{i,k}, n_{j,k})} \sum_{k=1}^{n_{i,k}} \text{dist}_S(S_{i(k),1}, S_{j(k),2})$$

(6)

where an arbitrarily large distance value is attributed to pairings involving match candidates that the screening process has not retained (i.e., such that $P_{j,k} \notin S_{0,2}$). Approximate solutions to the optimization problem can again be obtained using the Vogel algorithm. It is shown in Capart (2000) that, for a rigidly moving random dispersion of points, screening the match candidates on the basis of their membership in the 1-star of the minimum displacement-neighbour leads to a limitation in range of $\Delta r/\delta r < 1.5$ at the 90% confidence level. This is more than twice the range of the optimized minimum displacement algorithm, but may still be too restrictive in certain situations. In such cases, the Voronoi algorithm can be generalized to consider the Voronoi 2-star or higher-order stars of the feature-points. This was, however, found unnecessary in the applications examined in Sect. 3.

Qualitatively, the Voronoi match algorithm has the effect of pairing the neighbour feature-points that are characterized by similar Voronoi cells on successive images. For the granular flows examined, it has been verified that Voronoi cells tend to remain sufficiently stable along trajectories to allow the procedure to be successful. A close-up view of Voronoi diagrams corresponding to actual granular flow images illustrates this point in Fig. 5. More applications and results are detailed in Sect. 3.

2.4 Natural-neighbour spatial filtering
The algorithm above evidently does not succeed in matching all particle images correctly. It is thus desirable to have an automatic filtering procedure to remove mismatches from the set. In the next paragraph, a procedure for performing such a filtering based on multiple-image trajectories is presented. Relying on information derived from a single pair of image frames, it is also possible to devise spatial filtering procedures. Various proposals exist for this purpose. Most of them involve comparing each
velocity vector with a local average obtained from neighbouring feature-point velocities. An interesting alternative (Jähne 1995; Song et al. 1999) consists in exploiting flow invariants associated with polygonal shapes: an example is the area of Delaunay triangles constructed on passive particle tracers, which should be conserved in a divergence-free two-dimensional flow. In the present work, we are interested in the flow of a granular phase that is not locally divergence free (the granular dispersion is compressible), and we adopt the first approach. The construction of local averages, however, can again be based on the Voronoi construction.

The adopted approach resorts to the so-called "natural-neighbour weights", introduced by Sibson (1981) for interpolation purposes. The idea consists in subdividing each Voronoi tile \( V_i \) into \( m \) subtiles \( W_k \), corresponding to the intersections of \( V_i \) with the Voronoi polygons obtained if feature-point \( P_i \) is removed from the set (Fig. 6). Weights can then be defined as

\[
\lambda_k = \frac{\text{area}(W_k)}{\text{area}(V_i)}
\]

(7)

These weights present various remarkable properties outlined in Sibson (1981). In particular, they can be used to derive \( C^2 \) interpolants (Sibson 1981), which have been used in a particle-tracking context by Lloyd et al. (1995). By construction, the weights \( \lambda_k \) sum to 1 and reflect the proximity of point \( P_i \) with each of the extremities of its associated 1-star (Fig. 6). It is thus possible to define the “star-averaged” velocity associated with feature-point \( P_i \), as the weighted sum

\[
v_i^* = \sum_{k=1}^{m} \lambda_k v_k
\]

(8)

where indices \( k = 1, m \) refer to the \( m \) extremities of the 1-star of \( P_i \), and the \( v_k \) are obtained from (1) once the matching has been performed. The above-defined average does not involve the velocity \( v_i \) at the star-centre, yet it presents the remarkable property of yielding \( v_i^* = v_i \) if the velocities are governed by first-order variations (i.e., constant gradients) around feature-point \( P_i \) (Sibson 1981).

One can then define a second-order spatial difference operator as

\[
\Delta^2 v_i = \frac{v_i^* - v_i}{\text{area}(V_i)}
\]

(9)

where \( V_i \) is the Voronoi cell associated with point \( i \).

Expression (9) can be verified to reduce to the classical finite difference Laplacian if the local tessellation happens to be an orthogonal grid. A robust spatial filtering procedure can then be devised by imposing \( |\Delta^2 v_i| < \text{tol} \) (where tol is a user-defined tolerance), and removing one by one the outliers by starting with those that create the maximum local “curvature”. Implementation of the subtiling algorithm is complicated, but reasonably efficient (Sibson 1981). Once the subtiling is obtained, however, it can also be used for other purposes, for example to interpolate velocities onto a regular grid (as in Lloyd et al. 1995).

2.5 Trajectory reconstruction and temporal filtering

If a sequence of many images is recorded, displacement data obtained from each pair of frames according to the above algorithms can, of course, be concatenated and edited by using the more complete temporal information. Once successive match correspondences are known, pure concatenation of trajectory segments is trivial. Parallel to this operation, however, trajectories can be checked and edited using split-merge operations. The principles involved are similar to those of path-coherence based particle tracking (e.g. Ushijima and Tanaka 1996). The important difference is that these operations can now be performed on the basis of a pattern-based “skeleton” of trajectories rather than from scratch.

Temporal filtering can be performed efficiently using the 4-point stencils shown in Fig. 7a. A subtrajectory \( T_{i-1}, T_{i+1} \) composed of points \( P_{i-1}, P_{i+2} \) located at positions \( r_{i-1}, r_{i+2} \) for successive times \( t_{i-1}, t_{i+2} \) is considered. Focusing on the central link \( T_i \) of the subtrajectory, a robust local measure of path coherence is given by the “trajectory-distance”

\[
\text{dist}_T(P_i, P_{i+1}) = \min(\Delta_-, \Delta_+) = \min(|r_i^- - r_i|, |r_{i+1}^- - r_{i+1}^-|)
\]

(10)

where \( r_i^- = 2r_{i+1} - r_{i+2} \) and \( r_i^+ = 2r_i - r_{i-1} \) are respectively backward and forward extrapolations based on the surrounding segments. One can again filter out likely mismatches by imposing \( \text{dist}_T(P_i, P_{i+1}) < \text{tol} \). The motive for using the minimum norm is illustrated in Fig. 7b: to avoid
Fig. 7a–b. Trajectory filtering: a four-point stencil used to evaluate the local path coherence; see text for details. b by resorting to the minimum norm in (10), link \( T_3 \) can be discarded without discarding link \( T_2 \)
discarding correct links, only those presenting excess discrepancies with respect to both the forward and backward extrapolations are filtered out.

Once trajectories are “split” at their weak links, one can attempt “merge” operations for trajectory segments that appear to fit together well according to the same criterion. An elegant way to perform the “split” and “merge” operations in one single sweep consists in recasting the problem as one of finding for each pair of frames the global match \( \{i(k), j(k)\} \) that minimizes the objective function

\[
\min_{i,j} \sum_{k=1}^{K} \text{dist}(P_{i(k), j(k)})
\]

and keeping only the matches that satisfy a given tolerance. By starting from the “trajectory skeleton” obtained by the Voronoi matching process, trajectory-based improvements can be made in substeps that do not require the handling of more than two frames at a time. In this way, one avoids the algorithmic complexity involved in keeping track of various multiframe trajectories.

In a granular flow context, trajectory information can be quite useful, for instance, to visualize the granular motions or to extract autocorrelation statistics. These are of theoretical importance in granular flows, for instance, to estimate collisional frequencies or compare self-diffusion statistics with those of computational simulations.

2.6 Granular concentration estimation

Because they tile the plane without gaps, the Voronoi cells and Delaunay triangles can be used to provide a local measure of the planar density \( \eta \) of feature-points observed in an image. One only needs to take the reciprocal of the areas, which corresponds to 1 particle in the case of a Voronoi cell, and 1/2 particle in the case of a Delaunay triangle. When dealing with two-dimensional granular dispersions (for instance, disks or spheres constrained to move between two closely spaced walls), this constitutes an immediate estimate of concentration. In contrast, when three-dimensional dispersions of grains are imaged through a sidewall, the estimation of volumetric concentration \( \phi \) from two-dimensional images is a nontrivial task. Various authors (Louve and Jenkins 1997; Kanda et al. 1999) propose to use luminance information for this purpose. Even in laboratory conditions, however, illumination conditions are difficult to control precisely. In the present work, it is therefore sought to use exclusively the information associated with particle positions, which is much less dependent on lighting.

Situations are different for the dilute and dense cases. In the dilute limit, for instance, in the case of sparse dispersions of particles restricted to a thin sheet (e.g., a laser light sheet), one can relate the 3D concentration to the 2D density of visible particles by assuming that their positions obey a Poisson process (Adrian 1991). A short analysis sketched in Appendix 2 yields:

\[
\eta = \frac{4}{\pi d^2} \left[ 1 - \exp \left(-\frac{3\phi \Delta z}{d} \right) \right]
\]

where \( \eta \) is the number of visible particle centroids by unit image surface, \( \phi \) is the volumetric solid concentration, and where it was assumed that spherical particles of diameter \( d \) are contained in a viewing volume of thickness \( \Delta z \). From (12) it is immediately apparent that as soon as the ratio \( \phi \Delta z/d \) grows, occlusion effects cause the surface density \( \eta \) to lose its sensitivity to changes in \( \phi \).

Fortunately, when the dispersion becomes dense, excluded volume effects intervene, and the particle positions are no longer governed by a random Poisson process. When the mean particle interdistance becomes of the order of the diameter, neighbouring particles are forced to organize with respect to each other in a type of glassy state (Allen and Thomas 1999). This creates short-range correlations between grain positions and opens a possibility of trying to link solid concentration with local descriptors of particle configurations. Various approaches can be used to construct such descriptors (for a review and comparison, see Wallet and Dussert 1998). The Voronoi diagram provides once again a possible tool, assessed by Wallet and Dussert (1998) to be one of the best from the point of view of discrimination power and stability (the best results for their tests were, however, obtained with minimal spanning tree approaches). In the present work, three Voronoi-based indicators are tested.

The first is an estimator for the point density \( \eta \):

\[
\eta_i = \frac{1}{\text{area}(V_i)}
\]

where \( V_i \) is the Voronoi cell enclosing point \( i \). The second is the “roundness factor”, which provides a descriptor of the shape of Voronoi polygons:

\[
\xi_i = \frac{4\pi \text{area}(V_i)}{[\text{perimeter}(V_i)]^2}
\]

Finally, a third indicator reflects the local Voronoi “area disorder” and is defined as:

\[
\chi_i = \frac{1}{1 + \sigma/\mu}
\]
where

\[ \sigma^2 = \sum_{k=1}^{m} \lambda_k [\text{area}(V_k + W_k) - \mu]^2 \]  

(16)

and

\[ \mu = \sum_{k=1}^{m} \lambda_k \text{area}(V_k + W_k) \]  

(17)

where indices \( k = 1 \ldots m \) designate the \( m \) natural neighbours of point \( i \), and where the weights \( \lambda_k \) and polygons \( V_k \) and \( W_k \) are defined as in (7) (see also Fig. 6). The “area disorder” estimator can be interpreted as the normalized variance of the areas of the “petals” of the Voronoi “flower” shown in Fig. 8. The estimator is a local version of the global area disorder defined by Marcelpoil and Usson (1992). The weighted averages are chosen in analogy with the treatment of mixture densities in Duda and Hart (1973). Note that in contrast with (13), estimators (14) and (15) are dimensionless and scale-invariant.

For the three indicators above, relationships with concentration \( \phi \) are sought in the following power-law form:

\[ \frac{\phi}{\phi_{\text{rcp}}} = \left( \frac{\eta}{\eta_{\text{rcp}}} \right)^x \]  

(18a)

\[ \frac{\phi}{\phi_{\text{rcp}}} = \left( \frac{\xi - \xi_0}{\xi_{\text{rcp}} - \xi_0} \right)^\beta \]  

(18b)

\[ \frac{\phi}{\phi_{\text{rcp}}} = \left( \frac{\lambda - \lambda_0}{\lambda_{\text{rcp}} - \lambda_0} \right)^\gamma \]  

(18c)

where indices “rcp” and “0” designate the state of random close packing and the dilute state, respectively; \( \phi_{\text{rcp}} \approx 0.64 \) for spheres (Allen and Thomas 1999); \( \xi_0 \approx 0.72 \) and \( \lambda_0 \approx 0.80 \) are obtained from Monte-Carlo simulations (as they concern the dilute state, the values do not vary with particle shape); random close-packing values \( \eta_{\text{rcp}} \), \( \xi_{\text{rcp}} \), and \( \lambda_{\text{rcp}} \) as well as exponents \( x \), \( \beta \), and \( \gamma \) remain to be determined from calibration tests.

3 Applications and results

The algorithms above were tested on a number of liquid-granular flow experiments, which also motivated and guided the developments. In the present report, three applications are selected as testing grounds for the algorithms: (i) homogeneous fluidization tests; (ii) steady uniform debris flow; (iii) transient debris surges. These contrasted applications make it possible to highlight different features of the proposed methods.

3.1 Homogeneous fluidization

In order to test the concentration estimators, it is desirable to obtain states of homogeneous dispersion for various values of the solid fraction. A most convenient way of achieving this is to set up fluidization cell experiments. These consist of subjecting a static array of grains to an upward fluid flux, causing the granular assembly to expand until the voidage is such that the interphase drag balances the submerged weight of the granular phase. At some stage, when concentrations become lower than the random loose-packing concentration \( \phi_{\text{rcp}} \approx 0.55 \), the grains become mobile with respect to each other and undergo disordered fluctuating motions. Provided one can avoid the regions of instabilities (Batchelor 1988), a statistically homogeneous state is obtained in which the grains randomly explore a variety of local configurations. Such conditions are the ideal ones to test and calibrate pattern-based concentration estimators.

In the present work, tests were performed with light spherical grains fluidized by a water current. The grains (artificial pearls) have a diameter \( d = 6.1 \) mm and a relative density \( \rho/\rho_w = 1.048 \), where \( \rho \) is the density of the granular material and \( \rho_w \) is the density of water. The cylindrical fluidization cell has a height of 25 cm and a diameter of 10 cm. The lower part of the cell is filled with a 5-cm-deep layer of small lead spheres in order to diffuse the incoming water current. Finally, the cylinder is fitted with a plane rectangular observation window of dimensions 5 by 10 cm allowing one to film the dispersion without optical distortion. Varying the upward water velocity in a range of 1 to 3 cm/s enables observations of fluidized granular dispersions having concentrations between \( \phi = 0.2 \) and \( \phi = 0.55 \). The relationship between concentration and fluidizing flux was verified to correspond reasonably well with the Richardson–Zaki empirical relation (Richardson and Zaki 1954). As the granular motions are rather slow in this case, tracking of the particles constitutes a simple matter, and was carried out only to verify that the state of the dispersion was close to statistically homogenous. The measured kinetic energy of the velocity fluctuations was found to agree well with the relationship proposed by Batchelor (1988) for homogeneous fluidization conditions. These verification data are presented in Capart (2000).

The main purpose of the fluidization experiments is to test the pattern-based concentration indicators. Once particle centroids are located with the algorithms of Appendix 1, the Voronoi diagram can be constructed on these feature-points, and estimators (13)–(15) obtained for each of the Voronoi cells. To prevent edge effects from biasing the statistics, the criterion of Kenkel et al. (1989; see Okabe et al. 1992) is used to discard Voronoi cells close to the image boundaries. It consists in excluding...
from consideration all cells for which a circle centred at any vertex and passing through the cell feature-point and two of its natural neighbours intersects the boundary (Fig. 9). In Fig. 10, the normalized indicators (18) are plotted against the normalized concentration. Each data point represents an average over 20 frames and over all the visible particles (about 100 particles per image). Power-law fits are seen to well approximate the data trends. The calibrated constants take values \( n_{\text{rep}} = 1.40/d^2 \) (where \( d \) is the particle diameter); \( \zeta_{\text{eap}} = 0.84 \); \( n_{\text{rep}} = 0.92; \alpha = 6.0; \beta = \gamma = 3.5 \). The particle number indicator (13) is seen to present the least scatter, but the sensitivity is rather low. In fact, the sensitivity to particle number is much lower than was anticipated by Capart et al. (1997) on the basis of a geometrical reasoning. The two pattern-based indicators (14) and (15), on the other hand, present a higher degree of scatter, but with better sensitivity. Overall, the results suggest that the indicators can be used to measure solid concentrations, under the conditions that (i) averages can be performed over a large amount of individual data points; (ii) the granular dispersion behaves locally in a random fashion close to the conditions of the fluidization tests; (iii) calibration can be performed for a given material in lighting conditions similar to those of the measurements. One case where these conditions are met is presented in Sect. 3.2.

3.2 Steady uniform debris flow

To investigate the vertical structure of free-surface liquid-granular flows, it is of particular interest to be able to materialize steady uniform flow conditions. A recirculating flume of novel design was developed for this purpose by scientists of the Università degli Studi di Trento, Italy (Armanini et al. 2000). With reference to Fig. 11, the flume is composed of two main components: a tilting glass-walled channel, 6 m in length and narrowed to 20 cm in width, in which free-surface flows are observed; a conveyor belt, connected to the channel by chute guides, which recirculates both water and sediment. During a run, except for minor losses, the set-up forms a closed loop in which a determined volume of water and granular material circulates. The global flow in the channel is governed by the volumes of water and sediment and by the channel angle, which can be varied from 0 to 25 degrees. The apparatus is used to study steady uniform flows of various degrees of maturity in rigid-bed and loose-bed conditions (Armanini et al. 2000).

To test the Voronoi imaging methods, one of the runs analysed by Capart et al. (2000) is examined here in detail. It consists of a rapid high-concentration debris flow in contact with the rigid flume bottom. The sediment material is composed of roughly identical cylinder-shaped PVC granules having the following characteristics: diameter = 3.2 mm; height = 2.8 mm (hence an equivalent spherical diameter \( d = 3.5 \) mm); relative density \( \rho / \rho_{\text{w}} = 1.540 \). The flow conditions were the following: slope = 3 degrees; total discharge (water + sediment) = 12.4 l/s; delivered sediment concentration (sampled at the outlet of the flume) \( \phi = 0.49 \). The image sequence was acquired by filming the flow through the flume sidewall, using a CCD camera with an image resolution of 320 \( \times \) 280 pixels at a frame rate of 500 Hz.

Figure 12 presents typical particle tracking results obtained using the Voronoi methods. One original image of the sequence is shown in Fig. 12a. The flow is about 3 cm in depth and is heavily loaded with PVC particles. Voronoi matching results for two subsequent frames are shown in Fig. 12b. To improve the matching results, one additional criterion was imposed for the screening of match candidates, and consists in an elliptic window constraining the allowable interframe displacements (Fig. 13). With reference to Fig. 13, the following ellipse parameters were adopted for the run under consideration: \( u_0 = 2 \) mm; \( v_0 = 0; w_1 = 3 \) mm; \( w_2 = 2 \) mm. The constraint is applied uniformly to the whole field, and exploits information known a priori about the flow to reduce the number of candidates with respect to the more general Voronoi 1-star screening criterion. For the selection

---

**Fig. 9.** Criterion of Kenkel et al. (1989) for the identification of Voronoi cells subject to boundary effects (after Okabe et al. 1992). Because the circle (---) intersects the boundary, feature-point \( P_i \) (and its associated templates) is excluded from consideration.

---

**Fig. 10a, b.** Relationship between the volumetric grain concentrations and the Voronoi-based estimators: (○) point density \( n_i \); (□) roundness \( \zeta_i \); (○) area disorder \( \gamma_i \); (---) power law fits (the roundness and area disorder estimators are approximately fitted by a single curve). *Inset:* images of the dispersions for a \( \phi \approx 0.3; \) b \( \phi \approx 0.5 \).
operation, the Voronoï algorithm is used without changes. In Fig. 12b, some displacements are seen to be likely mismatches. Fig. 12c shows the displacement field after the application of the natural-neighbour spatial filtering procedure of Sect. 2.4. The procedure is successful at removing the most irregular displacement vectors, without pruning too many correct ones. Finally, Fig. 12d shows trajectories reconstructed from 20 successive frames, after application of the temporal “split and merge” filtering of Sect. 2.5.

Granular trajectories offer a vivid view of the flow behaviour. The lowermost part of Fig. 12d exhibits irregular trajectories characteristic of saltating motions (Lee and Hsu 1994) interrupted by collisions with the rough bed and with other grains. Trajectories in the upper part, on the other hand, present smaller fluctuations. These features are characteristic of the rapid (collisional) granular flow regime (Campbell 1990). Such dense trajectories, obtained by tracking all visible grains in an intensely sheared fluctuating ensemble, have not been obtained before from experiments. They demonstrate both the challenging features of granular flows and the robustness of the Voronoï methods.

The Voronoï methods function in a fully automated way. It is thus possible to apply the steps demonstrated in Fig. 12 to a full sequence of images. This is necessary if one seeks to derive accurate statistics for vertical flow profiles. From numerical simulations, Louge and Jenkins (1997) found that a sample size of a minimum of 10,000 measurements was necessary for this purpose, which is far beyond what can be achieved using manual or even semi-
automated procedures. For the flow under consideration, a sequence of 500 frames was analysed, producing around 50,000 individual measurements for velocity and concentration. The good performance of the Voronoi procedure is reflected by the vector yields obtained in this case: on average, 104 particles are identified on each frame, of which 94 are matched to particles on the next frame, with 92 acceptable velocity vectors remaining after temporal filtering. The percentage of successful matches is therefore around 92/104 = 88%.

The resulting averaged vertical profiles, obtained by subdividing the flow into 20 horizontal layers, are presented in Fig. 14. Profiles are shown for the mean longitudinal velocity \(<u>\), the mean concentration \(\phi\) derived from averages of the point density, roundness, and area disorder indicators \(\eta\), \(\xi\) and \(\gamma\) and for the granular temperature \(\theta\). The latter is defined in analogy with the kinetic theory of gases and constitutes a measure of the strength of the particle velocity fluctuations (Jenkins and Hanes 1998). It is estimated here as

\[
\theta = \frac{1}{3} \langle v' \cdot v' \rangle = \frac{1}{2} \left( \langle u'^2 \rangle + \langle v'^2 \rangle \right)
\]  

(19)

where \(v' = v - \langle v \rangle\) is the fluctuation velocity vector, and \(u' = u - \langle u \rangle\) and \(v' = v - \langle v \rangle\) are its components along directions parallel and normal to the bed, respectively. The brackets \(\langle \rangle\) denote an average performed over time and over all particles located within each horizontal layer. The estimated root-mean-square (rms) error of particle position is of the order of 0.25 pixel, or 2% of the diameter of the particles. At mid-depth of the flow illustrated in Fig. 14, this leads to an expected rms error of \(\epsilon = 0.05\) m/s for the individual velocity measurements. The relative errors in individual measurements of velocity \(v\) and granular temperature \(\theta\) can then be estimated as:

\[
e_v = \frac{\epsilon}{\langle u \rangle} = 3\%; \quad e_\theta = \frac{\epsilon^2}{\langle v' \cdot v' \rangle} = 20\% .
\]  

(20)

Thus a small relative rms error is obtained for the velocities, while a greater but still reasonable relative error affects the granular temperatures. Errors in velocities cancel out when the mean velocity profile \(\langle u \rangle\) is estimated from the large set of measurements. By contrast, errors in the granular temperature \(\theta\) do not average out upon aggregation of a large measurement sample. Thus even small errors in particle position (only 2% of the particle diameter) are enough to significantly perturb granular temperature measurements. Because error noise is added in, the profile of Fig. 14c is likely to overestimate by about 20% the actual strength of the velocity fluctuations. More information about PTV errors is given in Veber et al. (1997).

In the collisional regime, the randomly fluctuating dispersion of grains is expected to be in a state similar to the fluidized state of Sect. 3.1. Hence the concentration indicators tested in the fluidization experiments should respond correctly to changes in the volumetric concentration. This is not the case for all granular shear flows. For the slower and denser frictional flow regime, in particular, the local arrangement of grains can be governed by a flow-dependent microstructure, leading to a breakdown of the pattern-based estimates. In such cases, it is probably necessary to resort to other measurement principles (stero imaging, radiometric density meters, or other techniques).

Since the PVC granules are cylinder-shaped and respond differently to illumination, some of the calibration parameters involved in the concentration relations had to be obtained anew. A value for the random close-packing concentration \(\phi_{rcp} = 0.69\) was obtained by measuring the concentration of a static, close-packed assembly of the PVC grains. Values for parameters \(\eta_{rcp}\), \(\xi_{rcp}\) and \(\zeta_{rcp}\) were further recalibrated by comparing sidewall and outlet measurements of the delivered concentration \(\phi\), defined as the ratio of depth-integrated granular discharge over the total depth-integrated discharge. Calibration was performed by requiring that the sidewall integrated \(\phi\) equal to the bulk \(\phi\) sampled by trapping outgoing volumes of water and grains at the flume outlet, i.e.,

\[
\bar{\phi} = \frac{\int_{y_0}^{y_{0}+h} \phi(u) dy}{\int_{y_0}^{y_{0}+h} (u) dy} \frac{\text{volume of grains}}{\text{total volume}}_{\text{outlet}} ,
\]  

(21)

in which \(y_0\) is the elevation of the rigid bed and \(h\) is the flow depth. The values obtained by this procedure are \(\eta_{rcp} = 0.60/d_1^2; \xi_{rcp} = 0.835\) and \(\zeta_{rcp} = 0.915\). All other parameters in (18) are left unchanged. With these recalibrated values, the three concentration profiles in Fig. 16b are seen to be in approximate agreement with each other. Recalibrated values of \(\xi_{rcp}\) and \(\zeta_{rcp}\) are rather close to the values derived from the fluidization tests, whereas \(\eta_{rcp}\) changes drastically. Thus the pattern-based indicators (roundness and area disorder) appear to present a reasonable degree of stability with respect to particle shape and illumination. In contrast, the point-density estimator \(\eta\) is found to be highly shape- and/or illumination-dependent. Its use is therefore not recommended unless one can perform well-controlled calibration tests in the same conditions as the experiments of interest.

![Fig. 14a–c. Averaged vertical profiles for the steady uniform debris flow experiments: a longitudinal velocity \(<u>\); b granular concentration \(\phi\) as derived from (—) point density, (- - -) roundness, and (—) area disorder indicators; c granular temperature \(\theta\)]
The profiles of Fig. 14 further highlight the characteristics of collisional flow. They are similar to profiles obtained for rigid bed conditions in computational simulations for dry, two-dimensional dispersions of disks (see the illustrative example in Campbell 1990), even though we are dealing here with a three-dimensional dispersion of particles embedded in water. Thus the description of Campbell (1990) applies here with few changes: the mean velocity presents its steepest gradient at the bottom; concentration is low near the bottom, reaches a maximum towards the centre, and tails off again near the free surface; the granular temperature is large near the bottom (where a large temperature generation results from the steep velocity gradient) and decreases towards the free surface. The quantitative information gathered from these profiles complements the qualitative picture provided by the trajectories of Fig. 12d: a dense upper layer of grains flows rapidly over a relatively dilute layer of highly fluctuating grains close to the bottom, which supports the submerged weight of the upper layer through collisional contacts. For more details concerning the physical analysis of such measurements, the reader is referred to Guarino (1998), Armanini et al. (2000), and Capart et al. (2000).

3.3 Dam-break-induced debris waves

Transient liquid–granular flows constitute a challenging application for experimental techniques. Such flows are of importance in geophysical fluid mechanics, for instance, in mountain catchments where debris flows usually take the form of highly unsteady debris surges (Takahashi 1991). Dam-break waves constitute one of the possible triggering mechanisms for debris flows (Costa and Schuster 1988; Capart et al. 2001), and are relatively simple to set up in idealized laboratory conditions (Capart and Young 1998). It is this problem which first motivated the development of the present Voronoi imaging methods.

The flow configuration considered is shown in Fig. 15. In this small-scale laboratory set-up, clear water is retained behind a sluice gate (representing an idealized “dam”) in a 10 cm wide rectangular channel. A sudden channel enlargement occurs 20 cm downstream of the gate, where the flume width goes abruptly from 10 cm to 20 cm. A horizontal layer of loose granular material, 5 cm in depth, fills the channel bottom both upstream and downstream. Upstream of the gate, the water stage rises to a depth of 10 cm above the horizontal granular bed, whereas the downstream water stage coincides with the bed surface. The granular phase is thus saturated with water throughout the domain. The granular material is composed, as in Sect. 3.1, of artificial pearls having a diameter $d = 6.1$ mm and a relative density $\rho_\text{gr} = 1.048$. A dam-break flow analogue is obtained by impulsively raising the sluice gate (within 50 ms), releasing a water wave that entrains granular material from the loose bed. The flow is filmed from the side and from above using a CCD camera operating at a resolution of $256 \times 256$ pixels and frequencies of 100 Hz (side view) and 125 Hz (top view). Lighting is provided by two 500 W projectors oriented at angles of 45° with respect to the camera axis. More details about the experimental procedure are provided in Capart and Young (1998).

We first focus on the initial stages of the flow, before the wave reaches the channel enlargement, which correspond to the prismatic channel conditions treated in Capart and Young (1998). For these initial stages, the flow is essentially two-dimensional (it is uniform across the width), and can be entirely characterized by sidewall observations. In Capart and Young (1998), granular velocity fields for this flow were presented at selected times, and were obtained through a painstaking manually supervised procedure. The Voronoi methods now allow the tracking to be performed automatically, with the same accuracy and much less effort. In particular, tracking can be performed over many more images in order to reconstitute the long time trajectories of the grains. This is shown in Fig. 16. Figure 16a presents an original image frame taken at a time of 0.17 s after the dam-break. Figure 16b shows the corresponding displacement field derived from the last two images, whereas Fig. 16c presents the same data after natural-neighbour spatial filtering. The results of Fig. 16c

Fig. 15. Dam-break wave over granular bed experimental set-up installed at the Hydrotech Research Institute of the National Taiwan University. Shown are the initial conditions before removal of the sluice gate.

Fig. 16a–d. Side-view analysis for the dam-break experiments: a original image frame at time $t = 0.17$ s; b displacement field derived from two successive frames (displacement vectors are shown true scale); c displacement field after application of natural-neighbour spatial filtering; d grain trajectories reconstructed from 18 successive frames (with temporal filtering).
compare quite favourably with the manually obtained fields of Capart and Young (1998). Finally, Fig. 16d is a plot of the granular trajectories over the first 18 frames of the sequence following the dam-break. The flow field is characterized by rapid deformation and complex trajectories are obtained. Downstream particles surge upwards with the wave, and are replaced by other particles flowing in from upstream (hence their trajectories cross in the course of the transient flow). This new plot would have been very difficult to obtain without robust, fully automated procedures.

Beyond the enlargement, the two-dimensional debris wave expands into a three dimensional surge overrunning the granular bed. The wave can then be followed by filming from above, and tracking the visible grains of the flow free surface. This is shown in Fig. 17, where reconstructed granular trajectories are plotted for selected instants. The surge head is characterized by a sharp wavefront, or “erosional bore”, which radiates from the enlargement and impulsively sets in motion a layer of bed material. The wavefront evolves from the debris snout shown in cross-section on Fig. 16. Seen from above, it creates a zone of sharp velocity gradients that the Voronoi methods are seen to handle rather well. For physical descriptions and numerical simulations of such debris surges, the reader is referred to Capart and Young (1998) and Capart et al. (2001).

4 Conclusions
In the present work, the properties of the Voronoi diagram have been exploited to derive a family of methods for the imaging analysis of particulate flows. The methods include a novel matching algorithm for particle tracking velocimetry and a set of indicators for the three-dimensional granular concentration. They also comprise procedures for spatial and temporal filtering, based respectively on pattern and path coherence. The techniques have been successfully tested on various liquid–granular flow applications. Specifically, the Voronoi methods were used to derive granular velocities, concentrations, temperatures, and trajectories. They were able to capture rapidly sheared, densely packed dispersions of fluctuating grains in a fully automated way. This makes it possible to obtain long time trajectories and statistically significant measurement samples from experiments, which could otherwise only be obtained from computational simulations. While the methods are presently limited to rather idealized situations with well-sorted artificial grains, work is presently in progress to extend them to more complex granular materials. To obtain three-dimensional measurements, stereoscopic extensions are also contemplated.

Appendix 1: Particle-positioning algorithm
For the application of Voronoi imaging techniques to dense granular flows, it is desirable to obtain particle positions (and above all, displacements) accurate to a small fraction of the grain diameter. Starting with images for which a particle diameter typically spans between 5 and 20 pixels, the following algorithm has been found to yield good results, producing an estimated root-mean-square error of the order of 0.25 pixel.

The images considered feature white, nearly spherical particles, which contrast well against the darker surrounding fluid. With reference to Fig. 18, the first step of the positioning algorithm is to convolute the images with a Mexican hat filter. Let \( g(x,y) \) denote an image in the form of a grey-level map, and \( G = g_{ij} \) the corresponding grey-level matrix of size \( n_x \times n_y \). A smoothing operator is first applied in the form of a binomial filter of order \( m = 2r \) (Jähne 1995):

\[
g' = (B^{(m)}_y (B^{(m)}_x G)^T)^T
\]

(22)

where \( B^{(m)}_x \) (resp. \( B^{(m)}_y \)) is a band diagonal matrix of size \( n_x \times n_x \) (resp. \( n_y \times n_y \)) and bandwidth \( m+1 = 2r+1 \), having non-zero terms

\[
b_{i,i+k} = \frac{m!}{2^m (r-k)!(r+k)!} (-r, \ldots, r)
\]

(23)

The size \( m \) of the binomial filter is chosen as the even integer closest to \( 1/3 \ d^2 \), where \( d \) is the pixel diameter of the particles. The following Laplace operator (Jähne 1995) is then applied:

\[
g'' = g' - (B^{(2)}_y (B^{(2)}_x G)^T)^T
\]

(24)

Fig. 17a–d. Top-view analysis for the dam-break experiments: a original image frame at time \( t = 0.35 \); b granular trajectories tracked until \( t = 0.19 \); c \( t = 0.27 \); d \( t = 0.35 \). Dimensions are given by the flume width, equal to 10 cm before the enlargement, and 20 cm beyond.

Fig. 18a–d. Particle-positioning algorithm: a close-up of an original image frame; b after binomial smoothing operator; c after Laplace operator; d obtained particle positions.
where $B_1^{(2)}$ and $B_2^{(2)}$ are defined as in (22). The specific form of Laplacian (24) is chosen to reduce the anisotropy induced by the orientation of the pixel array (Jähne 1995). The conjunction of the two operators is equivalent to a convolution of the original image with a Mexican hat filter. It has the net effect of highlighting grain centroids, characterized by a radially symmetric pattern of brightness at the centre and darkness one radius away. The discrete positions of particle centroids are then obtained as local maxima $g''_{l}(k,j,k)$ of the filtered image $G''$. Finally, the positions are interpolated to subpixel accuracy according to

$$x_k = x_{i(k)} + \frac{1}{g''_{l}(k,j,k)} \frac{g''_{l}(k-1,j,k) - g''_{l}(k+1,j,k)}{\Delta x}$$

$$y_k = y_{j(k)} + \frac{1}{g''_{l}(k,j,k)} \frac{g''_{l}(k,j-1) - g''_{l}(k,j+1)}{\Delta y}$$

(25a,b)

obtained by fitting parabolas to the grey levels along the x- and y-directions.

Appendix 2: Visibility of individual particles in a random dispersion

In the dilute case, reasoning in terms of Poisson dispersions can be used to derive a relationship between volumetric concentration $\phi$ and the feature-point density $\eta$ observed on two-dimensional images. With reference to Fig. 19, consider a set of spherical particles of diameter $d$, randomly dispersed inside a sheet of thickness $\Delta z$. If the dispersion is dilute, one can consider in first approximation that the particle positions are uncorrelated and governed by a homogeneous Poisson process of intensity $n = \phi d^3/16$. Assuming that a particle is seen if its centre is not hidden from view by another particle, the probability that a particle is visible depends on its distance $z$ from the side wall and is given by

$$\text{prob} \left[ \text{no particle inside cylinder of volume} \frac{\pi d^2 z}{2} \right] = \exp \left( -n \frac{\pi d^2 z}{4} \right)$$

(26)

Since the distance to the wall $z$ of a particle picked at random is uniformly distributed over interval $[0, \Delta z]$, one obtains by integration the result (12), i.e.,

$$\eta = \frac{4}{\pi d^2} \left[ 1 - \exp \left( -\frac{3 \phi \Delta z}{d} \right) \right]$$

(27)

where $\eta$ is the expected density of feature-points (i.e., the number of feature-points per unit image surface) observed through the side wall. The reasoning above is rather idealized (illumination is taken for granted and the simplest possible projection onto the image plane is assumed). The important point is that, in any hypothesis, the sensitivity of $\eta$ with $\phi$ rapidly decays with a growth in the ratio $\phi \Delta z/d$. In particular, a direct proportional relation between $\eta$ and $\phi$ is only obtained in the very dilute state, far from the concentrations encountered in the granular flow applications of the present work.

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Three-dimensional Voronoï imaging methods for the measurement of near-wall particulate flows

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Abstract A set of stereoscopic imaging techniques is proposed for the measurement of rapidly flowing dispersions of opaque particles observed near a transparent wall. The methods exploit projective geometry and the Voronoï diagram. They rely on purely geometrical principles to reconstruct 3D particle positions, concentrations, and velocities. The methods are able to handle position and motion ambiguities, as well as particle-occlusion effects, difficulties that are common in the case of dense dispersions of many identical particles. Fluidization cell experiments allow validation of the concentration estimates. A mature debris-flow experimental run is then chosen to test the particle-tracking algorithm. The Voronoï stereo methods are found to perform well in both cases, and to present significant advantages over monocular imaging measurements.

1 Introduction
Flows of disperse phases are involved in a wide variety of situations of scientific and engineering interest. These include liquid-entrained gas bubbles, aerosols, dry granular flows, fluidized beds of particles, and liquid–saturated particulate currents. In many of these situations, the individual elements (bubbles or grains) can be approximated as rigid bodies undergoing distinct motions, and the dynamic system can be abstracted into an evolving configuration of particle positions (Campbell 1990; Zhang and Prosperetti 1994; Kang et al. 1997). The system behavior in the dense limit is often of particular interest, featuring active particles interacting with each other, and adopting flow-induced preferential arrangements (Fortes et al. 1987; Savage and Dai 1993; Rouyer et al. 2000). The present work addresses some of the experimental challenges associated with such flows through the development of novel stereo imaging techniques.

We consider the following typical measurement setup (Fig. 1). The flow of interest involves a dense ensemble of identical opaque particles immersed in a transparent fluid and imaged by twin cameras through a transparent plane (e.g., flume side-wall). Because of the high concentration of particles, optical penetration within the bulk is limited to a depth of a few grain diameters. Measurements sought include the 3D positions of the visible grains, the in-plane (i.e., parallel to the image plane), and out-of-plane (normal to the image plane) particle velocities, and the volumetric particle concentration. Two main problems must be faced in deriving such measurements: (1) particle-pairing ambiguities, which hamper both stereoscopic matching and velocimetric tracking; this is especially true for dense, rapidly sheared dispersions of identical particles undergoing irregular motions; (2) particle occlusion, which biases estimates of the volumetric particle concentration; in the dense case, the apparent concentration of visible particles is not equivalent to the actual concentration of physical particles.

In some instances, it is possible to circumvent these difficulties. Problems caused by matching and tracking ambiguities can be diminished by focusing only on the positions and motions of a subset of marked particles or tracers. Problems caused by occlusion, on the other hand, can be bypassed by using refractive index matching techniques (Cui and Adrian 1997) or full volumetric scanning as in nuclear magnetic resonance imaging (Phillips et al. 1992; Nakagawa et al. 1993; Seymour et al. 2000). A number of other non-intrusive methods have been used for the dynamic characterization of opaque solid–liquid flows, including positron emission particle tracking (Fairhurst et al. 2001; Wildman et al. 2001), and diffusing wave spectroscopy (Menon and Durian 1997). These techniques, however, are not always feasible and present their own drawbacks. The use of subsets of marked
particles, as in PEPT for example, can yield good Lagrangian statistics for the motion, but misses the spatial correlation and particle arrangement effects that act on the scale of a few particle diameters. Refractive index matching, in addition, is only possible for liquid–solid mixtures of very special optical properties. The approach of the present paper is to stick with more widely available test materials and digital cameras, and to deal with the ambiguity and occlusion problems in the context of stereo imaging methods.

Stereo imaging techniques applied to dilute dispersions (passive tracers seeding a fluid flow) have been described in the review of Adrian (1991) and in the studies of Malik et al. (1993), Ushijima and Tanaka (1996), and Virant and Dracos (1997). A method to obtain concentration estimates from statistical distributions associated with stereo imaging techniques is described in Murai et al. (2001). Monocular imaging methods, on the other hand, have been applied to dense flows by various experimenters. Most of these works have been restricted to 2D analogues, e.g., a monolayer of disks or spheres sandwiched between two parallel plates (Drake 1991; Azanza et al. 1999; Wildman et al. 1999), or to the imaging of a subset of marked particles (Natarajan et al. 1995; Hryciw et al. 1997; Hsiau and Jang 1998). In Capart et al. (2002), monocular imaging methods were proposed for the measurement of the discrete kinematics of full sets of visible grains in a dense, rapidly sheared dispersion imaged near the wall. The issues associated with stereo imaging of dense 3D dispersions do not seem to have been specifically addressed before.

The approach adopted follows the work of Capart (2000) and Capart et al. (2002). The core of the proposed methods consists in exploiting the special properties of Voronoï diagrams (Ahuja 1982; Okabe et al. 1992). These define spatial tessellations of the 2D plane or 3D space into cells centered around individual feature points (see Sect. 3). After abstraction of digital images into discrete sets of point-like particle positions, the Voronoi diagram is used for the three main steps of the analysis: (1) stereoscopic matching of particles; (2) estimation of volumetric concentration; (3) pattern-based particle tracking. Presentation of these various methods constitutes the central section of the present paper. This is preceded by an outline of the 2D particle-identification technique and generic 3D ray tracing concepts, which are needed as basic tools. The final part of the paper is devoted to the application of the methods to two particular flows in water. The first case, a homogeneous fluidized bed of light particles, is of particular interest for the validation of concentration estimation methods. The second case is a mature debris flow of PVC particles featuring steep concentration and velocity gradients, and constitutes a challenging test for the particle-tracking techniques. Monocular imaging results for similar flows were presented in Capart et al. (2002), while some preliminary stereo results were described in Douxchamps et al. (2000) and Spinewine et al. (2000).

2 Particle identification and ray geometry

This first section outlines the basic methods required to pinpoint particles on 2D images and to reconstruct their positions in 3D space. While these basic methods are not new, their presentation sets the stage for the more original Voronoï developments of the next section. The following notations are adopted throughout the paper: upper-case letters denote 2D imaging quantities measured in pixels and ticks (intervals of time separating successive images); lower-case letters denote 3D world quantities measured in meters and seconds. In both cases, vectors are indicated in bold face.

2.1 Particle identification

The first step of the analysis consists in the localization of particle centroids on individual images. For each instant at which synchronized images are acquired under two viewpoints A and B, one seeks to identify sets of particle positions \( \{ R_i^{(A)} \} \) and \( \{ R_i^{(B)} \} \) on the two digital frames. Particle images show up as white blobs of a certain size against a dark background (see Fig. 2a). An image neighborhood associated with a particle can be approximated by a Gaussian gray-level function centered on the particle centroid, with a diameter \( D \) that scales with the pixel diameter of the particle. Such bell-like regions are highlighted by convoluting the image with a Laplacian-of-Gaussian (Mexican hat) filter of width \( D \) (Jähne 1995), as shown in Fig. 2b and c. Local brightness maxima of the highlighted images are then identified iteratively using a “dish-clearing” algorithm: a global maximum is found, then a Gaussian bell of diameter \( D \) is subtracted from the neighborhood gray-level values; a new global maximum is found, and so on. The position of each maximum is finally refined to subpixel accuracy by fitting a second-degree interpolation surface around the discrete pixel position. The resulting set of particle centers is shown in Fig. 2d. The expected rms accuracy on the X and Y image coordinates obtained in this fashion is of the order of 0.25 pixel.
that then specify a matrix $A$ (Faugeras 1999). For each viewpoint $A$, one can tracing and matching operations (Trucco and Verri formation process and facilitates tremendously the ray projection from a virtual camera focal point $A$ onto the image plane

Define $A_x, A_y, A_z$ and $b(A)$ as the world coordinates of point $P$ associated with the camera viewpoint $A$ (Fig. 3), and $R^A = [X^A, Y^A, z]^T$ as the 2D image coordinates of point $P$ associated with the camera viewpoint $A$ (point $P'$ in left image plane $\Phi$). The transformation is obtained by modeling the image formation as a central projection from a virtual camera focal point $A$ onto the image plane $\Phi$ (Tsai 1987; Jain et al. 1995). Conserving the alignment of points, this projective transformation constitutes a reasonable approximation of the image formation process and facilitates tremendously the ray tracing and matching operations (Trucco and Verri 1998; Faugeras 1999). For each viewpoint $A$, one can then specify a matrix $[A(A)]$ and a vector $b(A)$ such that

$$z \begin{bmatrix} x(A) \\ y(A) \\ 1 \end{bmatrix} = \begin{bmatrix} A(A) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + b(A)$$

(1)

where $z$ is a scalar parameter best interpreted in the context of Eq. (5) below. Matrix $[A(A)]$ and vector $b(A)$, on the other hand, must be calibrated from a set of points $P_k$ for which both the world coordinates $[x_k, y_k, z_k]^T$ and the image coordinates $[x_k, y_k]^T$ are known. Eliminating parameter $z$ from (1), one obtains for each calibration point $P_k$ two linear equations in the unknown coefficients $a_i^{(A)}$ and $b_j^{(A)}$ of $[A(A)]$ and $b(A)$:

$$x_k a_{11} + y_k a_{12} + z_k a_{13} - x_k x_k a_{31} + y_k y_k a_{32}$$
$$z_k x_k a_{33} + b_1^{(A)} - x_k^2 a_{33} = 0,$$

(2)

$$x_k a_{21} + y_k a_{22} + z_k a_{23} - x_k y_k a_{31} + y_k y_k a_{32}$$
$$z_k y_k a_{33} + b_2^{(A)} - y_k^2 a_{33} = 0.$$  

(3)

A minimum of six calibration points $P_k$ (12 equations) are thus required to determine the 12 unknown coefficients. Since the system is homogeneous, one needs to append one more equation, e.g.,

$$b_1^{(A)} = 1,$$

(4)

to avoid the trivial solution with all zero coefficients. The system is further over-determined if more than six calibration points are used, hence a least-square procedure is needed to obtain an optimal solution. In practice, it is recommended to use a large number of calibration points with positions distributed evenly inside the viewing volume.

Once calibrated, Eq. (1) can be used to obtain image coordinates from known world coordinates. Conversely, a point $P$ having its projection $P'$ under viewpoint $A$ is known to belong to a ray $AP$ (or $AP'$ as seen in Fig. 3) defined by parametric equation

$$r_{AP}(x) = r_A + x s_{AP},$$

(5)

where $x$ is a free parameter, and vectors $r_A$ and $s_{AP}$ are given by

$$r_A = -[A(A)]^{-1}b(A),$$

(6)

$$s_{AP} = [A(A)]^{-1} \begin{bmatrix} X_P \\ Y_P \\ 1 \end{bmatrix}. $$

(7)
The vector \( r_A \) is the position of the focal point A of the projection, and lies at the intersection of all rays associated with viewpoint A. It is obtained by setting \( x \) to zero in (1). The vector \( s_{AP} \), on the other hand, specifies the direction of the particular ray which pierces the image plane at \( P' \). It is obtained as the difference \( r_{P'} - r_A \) or by setting \( x \) to unity in (1).

When the imaged scene is immersed in a liquid and seen from the outside through a transparent wall, the projection is altered by refraction effects. In the simple case of a liquid-bathed scene observed through a plane wall, and under some restrictions relative to imaging configuration, however, the affine character of the projection can be preserved to a very good approximation, provided that the calibration step is performed in refraction conditions identical to those of the actual experiments (calibration target immersed in the fluid). This can be verified through a detailed analysis of ray refraction, and has been checked empirically to lead to negligible errors in De Backer (2001).

2.3 Ray intersection and epipolar constraint

Consider now a physical particle \( P \) having unknown position \( r_p \) in 3D space. Let projections of the particle center on the left (focal point A) and right (focal point B) views have known image positions \( R_{AP}^{(A)} \) (\( P' \) on plane \( \Phi \)) and \( R_{BP}^{(B)} \) (\( P'' \) on plane \( \Psi \)). Referring to (5), the corresponding rays are given by parametric equations

\[
r_{AP}(x) = r_A + x s_{AP},
\]

\[
r_{BP}(\beta) = r_B + \beta s_{BP},
\]

where \( x \) and \( \beta \) are the two free parameters. The 3D position of the particle can now be retrieved by finding the intersection of the two rays \( AP' \) and \( BP'' \) (Fig. 3). Due to imperfections of the imaging process, this intersection cannot be exact and instead we look for the point of closest encounter between the two rays. Let \( M \) and \( N \) be the points on both rays separated by the smallest inter-distance. Their positions

\[
r_M = r_A + 2 z_M s_{AP},
\]

\[
r_N = r_B + \beta_N s_{BP}
\]

are found by solving the following linear system in the two unknown parameters \( z_M \) and \( \beta_N \):

\[
\begin{pmatrix}
s_{AP}^T s_{AP} & -s_{AP}^T s_{BP}
s_{BP}^T s_{AP} & -s_{BP}^T s_{BP}
\end{pmatrix}
\begin{pmatrix}
z_M \\
\beta_N
\end{pmatrix} = \begin{pmatrix}
(r_B - r_A)^T s_{AP} \\
(r_B - r_A)^T s_{BP}
\end{pmatrix}.
\]

The midpoint of the segment joining \( M \) and \( N \) constitutes an approximation of the true particle position \( P \). The length \( \ell \) of the segment, on the other hand, measures the distance of closest encounter between the two rays and provides an estimate of the quality of the approximation. They are given by

\[
r_p = \frac{1}{2} [r_M + r_N];
\]

\[
\ell = \sqrt{(r_N - r_M)^T (r_N - r_M)}.
\]

Suppose now that the only available information about a particle \( P \) is the position \( R_{AP}^{(A)} \) of its image \( P' \) on the left view (plane \( \Phi \)). Can anything be said about projection \( R_{BP}^{(B)} \) of the same physical point on the right view (plane \( \Psi \))? Physical point \( P \) is known to belong to the plane \( AP' \) containing both focal points \( A \) and \( B \), and image \( P' \) on the left view. Its parametric equation is

\[
r(x,\beta) = r_A + x s_{AP} + \beta (r_B - r_A)
\]

where \( x \) and \( \beta \) are again two free parameters. This plane \( AP'B \) is called the epipolar plane (see Fig. 3). Its intersection with the right image plane is a straight line \( \epsilon \), called the epipolar line, which constitutes the locus of all possible projections, in the right image plane, of a physical point having projection \( R_{AP}^{(A)} \) in the left image plane (\( P' \) on \( \Phi \)). Parametric equation \( R^{(B)}(\Gamma) \) of the epipolar line is easily found by projecting two points of the left ray \( AP' \) on plane \( \Psi \), via (1).

The true projection \( R_{BP}^{(B)} \) (\( P'' \) on \( \Psi \)) is known to lie along this line, or, due to inevitable imperfections of the imaging process, to fall somewhere close to the epipolar line, i.e.,

\[
R_{BP}^{(B)} \approx R^{(B)}(\Gamma),
\]

for a certain unknown value of parameter \( \Gamma \). This restriction is known as the epipolar constraint, and can be used advantageously to guide the search for a correspondence between two stereoscopic views. It will be exploited below to accelerate the stereoscopic matching step.
3 Voronoï Imaging Methods

With reference to Fig. 4a, consider a set of feature points \( \{ \Phi_i \} \) placed at positions \( \{ R_i \} \) in the 2D plane or \( \{ r_i \} \) in 3D space. The Voronoï diagram is a geometrical construction that divides the space into a set of polytopes, or cells, surrounding each feature point. Each Voronoï cell \( \gamma_i \) (in 2D) or \( \gamma_i \) (in 3D) encompasses the region that lies closer to \( R_i \) (respectively \( r_i \)) than to any other feature point of the set. The construction presents many useful properties, discussed in a general context in Okabe et al. (1992). In the present imaging context, three main properties are of interest:

1. The diagram organizes the dispersion of points into a network of neighborhood relations that can be used to speed up nearest-neighbor queries (Preparata and Shamos 1985). This will prove very useful to accelerate the stereoscopic matching step, which involves a repeated search for points of the image plane located in the vicinity of a given epipolar line.

2. The diagram defines a natural partition of space into non-overlapping regions, each containing a single particle. The inverse area or volume of the cell thus defines a local concentration estimate at the smallest possible scale. This can be used to sample statistical distributions in a finer and more robust way than by binning observations into an arbitrary partition (Bernardeau and van de Weygaert 1996). The property is exploited below to estimate particle concentrations.

3. The Voronoï diagram defines local patterns of neighboring feature points, which can be used as match templates (Ahuja 1982; Song et al. 1999). For moving dispersions of points, these patterns remain stable over a certain duration of time, a property exploited for 2D particle tracking by Capart et al. (2002). The present work extends this idea to 3D.

Stereoscopic matching, volumetric sampling, and particle-tracking operations are now outlined in three separate subsections.

3.1 Stereoscopic matching

3.1.1 Basic procedure

For a single particle seen on two stereo views, the 3D particle position can easily be retrieved using the approximate ray intersection procedure of Sect. 2.3. To perform this operation when the imaged scene features a large number of particles, it is necessary first to solve the correspondence problem: find which particle image on one view corresponds to which one on the other. As required when all particles are identical, it can be solved on the basis of purely geometrical information by minimizing ray intersection discrepancies.

Consider two stereo images of a large number of identical particles (Fig. 1). From the particle-identification step, sets of 2D particle positions \( \{ R_i^{(A)} \} \) and \( \{ R_i^{(B)} \} \) have been located on the left and right views, respectively. Using the inverse projective transformation of Sect. 2.2, 3D rays can then be traced back through each of these image points. Ray bundles \( \{ r_i^{(A)}(x) \} \) and \( \{ r_i^{(B)}(\beta) \} \) are thus obtained for the two views, and solving the correspondence problem amounts to finding a pairing \( i(j) \) subject to the constraint that no more than one ray of one view can be paired with any one ray of the other (some rays can be left out, however, as a result of partial occlusion effects and differing viewing prisms). Once pairing \( i(j) \) has been formed, the set of measured 3D positions \( \{ r_i \} \) is easily obtained from (13).

The pairing itself can be found by an optimization procedure, minimizing the objective function

\[
\sum_i \ell_{ij(i)}
\]

(17)

where the distance of closest encounter \( \ell_{ij} \) given by (14) is taken as a “goodness-of-match” between any two rays \( r_i(x) \) and \( r_j(\beta) \). This constitutes a standard bipartite graph optimization problem (e.g., Kim and Kak 1991), which is difficult (and computationally expensive) to solve thoroughly for large numbers of points. An approximate solution can however be found using the Vogel algorithm. It consists in considering for each ray the best match and the second best match, then constructing a reasonable global optimum by picking ray pairs in the order of maximum difference between first and second best choices.

3.1.2 Voronoï epipolar screening

For large numbers of particles, the procedure above becomes prohibitively expensive in terms of computational time and memory allocation. This is because it requires the computation of discrepancies \( \ell_{ij} \) for all possible pairings of rays, and the handling of a full matrix in the Vogel optimization procedure. To limit those expenses, it is advantageous to conduct a first screening of possible match candidates by resorting to the epipolar constraint (16). This can be done very efficiently using the 2D Voronoï diagram.

With reference to Fig. 5a, consider a particle \( i \) viewed on the left image at position \( R_i^{(A)} \). In the right image plane (Fig. 5b), let us construct the 2D Voronoï diagram \( \gamma_i^{(B)} \) on the set of particle positions \( \{ R_j^{(B)} \} \). The projection \( R_j^{(B)} \) of particle \( i \) in the right image is known to lie in the vicinity of epipolar line \( R_i^{(B)}(\Gamma) \). A simple way to screen match candidates is then to retain only those particle images \( j \) that have their Voronoï cells \( \gamma_j^{(B)} \) pierced by the epipolar line (see Fig. 5b, where 12 such candidates are marked). If particles are expected to lie within a limited depth range \( y_{min} \rightarrow y_{max} \) (where \( y \) is the normal to sidewall coordinate), parametric line \( R_i^{(B)}(\Gamma) \) becomes a line segment, and the set of match candidates can further be restricted (to the five candidates shown in black in the example of Fig. 5a, b).

If \( N \) denotes the total number of visible particles, the procedure reduces considerably the number of candidates...
to be considered from \( N \) to \( \sim \sqrt{N} \) (screening along a line instead of the full area). The 2D Voronoi diagram must be computed only once at a cost of \( \sim N \log N \) operations (Okabe et al. 1992). Epipolar line piercing then requires only a nearest-neighbor search and neighbor-by-neighbor propagation within the Voronoi diagram, for a combined cost of \( \sim \sqrt{N} \). Overall, the Voronoi epipolar screening reduces the cost of stereo matching from \( \sim N^2 \) to \( \sim N^{3/2} \). For the experiments described in Sect. 4, this amounts to a 10–30-times speed-up over the brute-force approach.

Figure 5c and d show the reconstructed 3D dispersion of particles resulting from application of the complete stereo matching procedure. Starting from sets of image positions \( \{ r_k^{(A)} \} \) and \( \{ r_k^{(B)} \} \), the procedure is seen to be successful at pinpointing the 3D positions \( \{ r_k \} \) of most of the visible particles. An indication of the robustness of the approach is that partial occlusion relationships (visible by eye on the image but not exploited in the analysis) are well conserved by the reconstruction. It is also seen in Fig. 5d that, for such a densely packed dispersion, occlusion effects prevent the retrieval of particle positions beyond a depth of a few grain diameters. This issue is addressed again in the next subsection.

3.2 Volumetric sampling

3.2.1 Binning and Voronoi sampling estimates of volumetric concentration

Supposing one can pinpoint a full set of 3D particle positions \( \{ r_k \} \) inside the viewing region, this set can be exploited to yield further spatial information about the granular dispersion. The quantity of most interest is the local volumetric concentration \( \phi(r) \). The simplest way of obtaining estimates for \( \phi \) is to subdivide the viewing volume into an arbitrary spatial partition \( \{ W_j \} \), then to count the number of particle centroids \( n_j \) falling into each box \( W_j \) of the partition. The local concentration in each box is then estimated as:

\[
\hat{\phi}_j = \frac{n_j V_p}{\text{volume}(W_j)}
\]

where \( V_p = \frac{1}{2} \pi d^3 \) is the volume of a single solid particle. This is the so-called “binning” procedure, widely used to estimate statistical distributions.

Instead of defining an arbitrary partition, an alternative approach makes use of the 3D Voronoi diagram. By construction, the Voronoi diagram partitions the region into a set of cells \( \{ V_k \} \), each one containing a single particle. Local concentrations can then be estimated as:

\[
\hat{\phi}_k = \frac{V_p}{\text{volume}(V_k)}
\]

Advantages of the approach are that Voronoi partition \( \{ V_k \} \) is more natural than arbitrary partition \( \{ W_j \} \), and that concentrations are sampled at the locations \( \{ r_k \} \) where particle velocities are defined. To apply such estimates in the case of a dispersion bounded by a plane side-wall, it is convenient to construct an extended Voronoi diagram on the basis of the set \( \{ r_k \} \) complemented by set \( \{ r_k' \} \) of its mirror images on the other side of the wall (refer to Fig. 8a, discussed more in details in Sect. 3.2.3). This guarantees that near-wall cells will be bounded by facets belonging to the plane boundary, hence allowing concentration estimate (19) to take the wall into account.

3.2.2 Occlusion effects

Unfortunately, for a dense dispersion of opaque grains, the set of measured particle positions \( \{ r_k \} \) picked up by the stereo procedure is not the full physical set \( \{ r_k \} \), but a subset of this including only the particles which are visible under both camera viewpoints. Occlusion effects intervene, whereby particles in the front hide from view particles in the back. To get a feel for how occlusion affects concentration estimates, it is useful to analyze first a simple model, introduced by Capart et al. (2002) in a monocular context and extended here to stereo acquisition.

Let us assume a 3D dispersion of spherical grains of diameter \( d \), characterized by constant volumetric concentration \( \phi \) in the near-wall region. Disregarding excluded volume effects due to impenetrability of the solid particles and side-wall, one may assume that the
Particle centroids are distributed inside the 3D volume according to a homogeneous Poisson process, with an average number of particles per unit volume equal to $\mu_P = \frac{\rho}{V}$. Considering the idealized stereo imaging configuration shown in Fig. 6a, a particle centroid will be visible on both stereo views only if no other particle center invades the two viewing cylinders shown as hatched surfaces. Given the depth of the particle $y_p$ and the viewing angle $\psi$ under which it is seen by each camera, the hatched volume can be approximated by

$$V(\theta_p) = \frac{\pi \delta p^2}{4} \left( y_p + \frac{y_p \tan \psi}{d} \right) \quad \text{if } y_p \leq \frac{d}{2 \tan \psi},$$

$$V(\theta_p) = \frac{3 \pi \delta p^2}{16 \tan \psi} + \frac{\pi \delta p^2}{2} \left( y_p - \frac{d}{2 \tan \psi} \right) \quad \text{if } y_p \geq \frac{d}{2 \tan \psi},$$

in which angle $\psi$ is assumed to be small, and where the complex sub-volume of intersection of the two cylinders is only roughly estimated. The probability that a particle is viewed under both viewpoints is then given by Poisson distribution (e.g., Adrian 1991; Okabe et al. 1992)

$$P[0 \text{ particle in } V] = \frac{(\mu_P V)^0}{0!} \exp(-\mu_P V) = \exp(-\mu_P V).$$

As expected, the two concentrations are equivalent right next to the side-wall ($y=0$). Away from the side-wall, the concentration of visible particle drops as a result of the rising probability of occlusion.

Figure 7 compares the predictions of (23) with measurements $\hat{\phi}_j(y_j)$ obtained from actual fluidization cell experiments. The measured functions were obtained by binning actual stereo measurements into partition $\{W_j\}$ given by

$$W_j \equiv \left\{ y_j - \frac{1}{2} \delta y < y \leq y_j + \frac{1}{2} \delta y \right\},$$

i.e., layers of small thickness $\delta y$ parallel to the side-wall, then applying (18). The measured profiles of Fig. 7b exhibit clear humps and troughs. This is not an artefact, but a consequence of excluded volume effects (neglected in the simple occlusion analysis). Physical particle centers are

$$\tilde{\phi}_p(y) = \phi P[0 \text{ particle in } V(y)] = \phi \exp \left(-\phi \frac{V(y)}{V_p} \right).$$

The average volumetric concentration of visible particles $\tilde{\phi}$ is thus only a fraction (22) of the actual one $\phi$ and can be expressed as the following function of depth $y$:

$$\hat{\phi}(y) = \phi P[0 \text{ particle in } V(y)] = \phi \exp \left(-\phi \frac{V(y)}{V_p} \right).$$

As expected, the two concentrations are equivalent right next to the side-wall ($y=0$). Away from the side-wall, the concentration of visible particle drops as a result of the rising probability of occlusion.

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i.e., layers of small thickness $\delta y$ parallel to the side-wall, then applying (18). The measured profiles of Fig. 7b exhibit clear humps and troughs. This is not an artefact, but a consequence of excluded volume effects (neglected in the simple occlusion analysis). Physical particle centers are
located preferentially at depths corresponding to certain multiples of particle diameter $d$, in a crystalline-like order induced by the side-wall boundary. These effects are especially strong for the denser packing ($\phi \approx 0.57$). The decay curves, which underlie these quasi-periodic variations, on the other hand, show the effect of occlusion. This exponential-like decay is seen in Fig. 7 to be reasonably well captured by the simple occlusion model introduced above. In particular, it is observed on both sets of results that away from the side-wall, the concentration of visible particles may actually be higher for the case of lower concentration ($\phi \approx 0.26$) than for the densely packed case ($\phi \approx 0.57$). This is entirely a result of occlusion, and explains why one must be careful in trying to derive actual concentrations $\phi$ from measured concentrations of visible grains $\phi$.

Another signal that caution must be exercised comes from examining the consequences of the simple Poisson occlusion model outlined above in the case of monocular imaging (Fig. 6b). In that case, the viewing region, which must be free of occluding particles, simplifies to a cylinder, and the surface particle density $\tilde{\eta}$ obtained by projecting the images of all the visible particles onto the side-wall (i.e., as seen by the monocular camera) is given by (Capart et al. 2002)

$$\tilde{\eta} = \frac{4}{\pi dl^2}. \quad (25)$$

The key feature of (25) is that the resulting estimate $\tilde{\eta}$ is independent of the actual particle concentration $\phi$. The result is surprising because it is very tempting to assume that one can estimate grain concentrations by simply counting the surface density of particles seen on monocular images, then resorting to some ad hoc calibration to convert $\tilde{\eta}$ into $\phi$. This obvious estimate, however, is geometrically completely insensitive to $\phi$. Only because of attenuation in illumination with depth does some sensitivity appear, making the measurements hinge upon illumination conditions, which are usually not well controlled in the laboratory. Such questionable estimates are nevertheless found in the literature including, we must confess, some of the authors’ earlier work (Capart et al. 1997).

### 3.2.3 Surface Voronoi sampling of the near-wall concentration

Because of occlusion, one should not expect to be able to measure concentrations inside the bulk of the opaque disperse phase. What should be possible, however, on the basis of the stereo position results $\{\tilde{r}_i\}$, is to estimate the near-wall volumetric concentration

$$\hat{\phi}_0 \approx \phi(y \approx 0). \quad (26)$$

As seen in Fig. 7, occlusion effects are so severe for large concentrations that some care must be exercised even to attain this more modest objective. One possibility examined in a preliminary work (Spinewine et al. 2000) is to resort to estimate

$$\tilde{\phi}_1 = \frac{V_p}{\text{volume}(1V_i)} \quad (27)$$

where the brackets $\langle \rangle$ denote a spatial average, and $n_0$ is the number of near-wall particles picked up within area $A$, i.e., the number of visible particles which have a Voronoi cell sharing a face with the wall within that area.

Stereo-scopic surface estimate (29) differs from monocular surface estimate (25) in one fundamental regard. In the monocular case (25), all visible particles are counted regardless of the depth. In the stereoscopic case (29), by contrast, only the near-wall particles (defined in the Voronoi sense) are counted. The stereo near-wall estimate is therefore biased neither by occlusion effects (which

![Fig. 8a, b. Volumetric sampling: a mirror image construction of a Voronoi diagram in which the plane side-wall is embedded; b incomplete near-wall Voronoi cell closures resulting from occlusion of back row particles. (Note: whereas 3D diagrams are actually used in the experiments, 2D analogues are shown here for illustration purposes)](image_url)
affect the volumes of the 3D Voronoi cells) nor by visible particles located away from the wall (which affect the monocular estimate).

3.2.4 Monte Carlo estimation of the stereological coefficient

A final step is required, which is to convert surface estimate \( \langle \eta_0 \rangle \) into volumetric estimate \( \phi_p \). This relationship can be predicted a priori if the 3D distribution of particle centers within the measurement volume is assumed to derive from a homogenous Poisson process, defining once again \( \mu_p \) as the average number of particles per unit volume. In that case, it follows from dimensional considerations that the relationship between \( \eta_0 \) and \( \mu_p \) must be of the form

\[
\langle \eta_0 \rangle = \chi \mu_p^{2/3}
\]  

(30)

where \( \langle \eta_0 \rangle \) is the expected surface density of near-wall particles and \( \chi \) is a non-dimensional constant: the so-called stereological coefficient.

While it is not impossible that an exact value for constant \( \chi \) could be derived theoretically, this is beyond our level of expertise. We thus resorted to Monte Carlo simulations (e.g., Ross 1990) to obtain an approximate value. Repeated simulation runs were conducted, each run involving the following steps:

1. With the aid of a random number generator, a 3D Poisson process of given intensity \( \mu_p \) is simulated within a box of finite size;
2. The resulting dispersion of points is reflected on the other side of one of the box walls (= the side-wall);
3. The 3D Voronoi diagram is computed and the near-wall cells identified;
4. The surface density \( \eta_0 \) is estimated by counting near-wall particles or averaging the areas of the cell faces \( \mathcal{A}_\mathcal{F}(\mathcal{F}) \) coinciding with the mirror plane.

For step 4, estimates are sampled in a central region only in order to avoid boundary effects. Except for step 1, the procedure closely emulates the one applied to the actual stereo measurements. A batch of 1,000 Monte Carlo runs was carried out, each run involving \( \sim 2,000 \) particles. The result of these computations is \( \chi = 0.92 \pm 0.01 \), where the bounds correspond to a 95% confidence interval (see Ross 1990).

Recalling that \( \mu_p = \phi V_p^{-1} \), relation (30) implies

\[
\phi_p = V_p \left( \frac{\langle \eta_0 \rangle}{\chi} \right)^{3/2},
\]  

(31)

which is our estimate for the near-wall volumetric concentration \( \phi_p \) in terms of the averaged wall-sampled surface number density \( \langle \eta_0 \rangle \). Strictly speaking, relation (31) with constant \( \chi = 0.92 \) should be expected to hold only in the dilute limit. For dense dispersions, excluded volume effects will make the distribution of particle centers differ from the results of a Poisson process. As verified by the experiments of Sect. 4.1, it will turn out nevertheless to constitute a reasonable approximation over the entire range of concentrations. Such a favorable situation is encountered in other stereology problems (Lorz 1990; Okabe et al. 1992). Estimate (31) is remarkable in that it derives from purely geometrical considerations. Up to moderate concentrations, it requires no ad hoc calibration. This contrasts with the much less favorable situation faced when trying to extract concentration estimates from monocular images (Capart et al. 2002).

3.3 Pattern-based particle tracking

The third operation that exploits the properties of the Voronoi diagram is the particle-tracking step. Sets of 3D particle positions at successive times are first acquired by repeated application of stereo matching to each frame of a movie sequence. Let \( \{ t_{i,m} \} \) and \( \{ t_{i,m+1} \} \) be two such sets of particle positions sampled at successive times \( t_m \) and \( t_{m+1} = t_m + \Delta t \). Particle velocities \( v_i(t_{i,m+1/2}) \) can be estimated by expression

\[
v_{i,m+1/2} = \frac{r_{i,(m+1)} - r_{i,m}}{\Delta t},
\]  

(32)

provided one can first “connect the dots”, and establish a pairing \( f(i) \) between positions \( r_{i,m} \) and \( r_{i,(m+1)} \) belonging to one and the same physical particle. When dealing with a moving dispersion of many identical particles, the main problem consists in establishing this correspondence: finding which particle on one snapshot corresponds to which one on the next. The particle-tracking problem can thus be seen as a time-domain variant of the stereo matching problem addressed previously.

For dilute particle dispersions or slow motion, the correspondence problem can easily be solved simply by pairing together the particles on one frame and the next, which are nearest to each other (see, e.g., Güler et al. 1999). For dense dispersions or rapid motion, however, legitimate pairing candidates may travel further away and the minimum displacement criterion breaks down. An alternative approach derives from the following observation: while individual particles are identical to each other, the local arrangements that they form with their neighbors are unique and may be preserved by the flow long enough to serve as a basis for tracking. Particle pairing can then be performed based on pattern similarity.

Following Capart et al. (2002), we again resort to the Voronoi diagram to implement such pattern-based tracking. Nearby particles are paired according to the geometrical similarity of their Voronoi cells. This similarity is estimated as follows: “stars” are first constructed by connecting a given particle center to its Voronoi neighbors (i.e., the particles with which it shares a cell face), as illustrated in Fig. 4b and c for both the 2D and 3D cases. The stars belonging to two pairing candidates can then be compared for goodness-of-fit by making their centers coincide and measuring the distances between the star extremities. Once these “goodness-of-fit” indicators are available for all possible pair candidates, a global optimum problem can be defined and solved in the same fashion as in the stereo matching case (see Sect. 3.1 and Eq. 17).

For two dimensions, the overall method is illustrated in Fig. 9 for a plane granular flow. While graphical representation is harder in 3D, the algorithms themselves generalize straightforwardly to the third dimension. The
reader is referred to Capart et al. (2002) for a detailed presentation and comparison with alternative approaches. Three-dimensional applications and results are detailed further below.

4 Liquid–granular flow applications

The Voronoï imaging methods presented in the last two sections are now applied to two different cases of immersed particulate flows, each aimed at highlighting particular aspects of the techniques. Fluidization cell experiments are chosen as a first test case, ideally suited for validation of concentration estimates. The steady uniform flow of a water–granular mixture down an inclined open channel was selected as a second test case. Featuring strong gradients in both solid fraction and granular velocities, this second case constitutes a challenging application for both velocity and concentration measurements.

4.1 Fluidization cell experiments

4.1.1 Principle and set-up

The proposed imaging techniques are now applied to fluidization cell tests. The principle of the tests is as follows. Subject to an ascending water current, a layer of loosely packed grains expands into a fluidized suspension. The concentration adapts to the water flux until the mean drag balances the submerged weight of the grains. In this fluidized state, the suspended particles undergo weakly correlated fluctuating motions, exploring a variety of spatial arrangements. Most important in the present context, the average concentration of particles is spatially uniform throughout the fluidized layer. Different concentrations can be obtained simply by tuning the fluid flow. Beyond their intrinsic interest, such homogeneous states of known concentration are ideal for testing concentration measurement methods.

The device used for the experiments is presented in Fig. 10. The cylindrical fluidization cell has a height of 25 cm and an inner diameter of 10 cm. A 5 cm deep layer of small lead spheres is placed at the bottom of the cell to diffuse the incoming water current and provide uniform fluid velocity throughout the cross-section. Above this heavy static layer comes the granular bed to be fluidized. It is composed of light spheres (artificial pearls) of relative density \( \rho_s/\rho_w = 1.048 \) and diameter \( D = 6.1 \) mm. To allow visualization without optical distortion, a plane rectangular observation window of dimensions 5x10 cm is fitted to the cell wall.

The evolving granular dispersion is imaged by two synchronized CCD cameras placed in a stereoscopic arrangement. The sensors, each 256x256 pixels in size, are positioned approximately 35 cm away from the cell, and mounted \( \pm 25 \) cm apart from each other. The angle between the two viewpoints is thus around 20°. Both lenses have a focal distance of 16 mm. Particle diameters span around 10 pixels. The resulting positioning error on particle centroids is estimated to be of the order of 0.2 mm.

The viewpoint calibration procedure sketched in Sect. 2.2 is carried out by imaging an "open-book"-shaped dihedron placed within the cell filled with water. A total of 56 calibrations points is used for the least-square derivation of matrices \( A^{(A)}, A^{(B)} \) and vectors \( b^{(A)}, b^{(B)} \) needed to determine the left (A) and right (B) viewpoints. The world coordinate system \( (x, y, z) \) is as follows: the \( x–z \) plane coincides with the cell wall \( (x \) taken horizontally), and \( y \) represents the out-of-plane horizontal coordinate (i.e., depth inside the cell).

Two independent series of tests were carried out under slightly different camera and lighting conditions. For each series, measurements were performed for a number of experimental runs corresponding to different solid concentrations (see Table 1). The Richardson–Zaki empirical correlation (Richardson and Zaki 1954) was found to describe well the relation between fluidization velocity and concentration.

In order to derive granular velocities and concentrations, the sets of simultaneous images are first preprocessed to
pinpoint particle positions in the two image planes. Stereoscopic ray matching and intersection is then used to retrieve the 3D positions of the grains. These procedures were described and illustrated in Sect. 2. We now focus on the concentration estimation process.

### 4.1.2 Concentration estimation

For every fluidization velocity, the height of the fluidized layer furnishes a direct and fairly accurate measure of the bulk concentration, which can serve as validation of the imaging estimates. The latter are extracted from uncorrelated snapshots acquired at a very low frame rate. Since the number of particles seen on each image is limited, this is needed to sample a variety of particle configurations and obtain good statistics by averaging concentration measurements over a number of images (16 and 30 images per run were used, respectively for the first and second series of tests).

Figure 11a presents a plot of the bulk concentration versus its imaging estimate. The latter is obtained by positioning the particle centers, constructing their 3D Voronoï diagram, and sampling the surface density of near-wall particles using estimate (29). Coefficient $\gamma = 0.92$ derived from the Monte Carlo simulations is then used in (31) to convert surface number density $\hat{g}_0$ into volumetric concentration $\hat{\phi}_0$. In Fig. 11a, the imaging results are seen to be close to the line of perfect agreement up to moderate concentrations ($\phi < 0.4$). For higher concentrations, the theoretically based imaging estimate $\hat{\phi}_0$ begins to underestimate the actual concentration $\phi$. This departure from perfect agreement most likely arises due to excluded volume effects, which induce a quasi-crystalline arrangement of the grains at dense packing. The Poisson process assumption adopted to derive relation (31) and compute the value of stereological coefficient $\gamma$ then breaks down.

Remarkably, however, the discrepancies are not very severe. Without any adjustment, the predicted relation remains accurate to 10% over the entire range. Furthermore, the data sets obtained from the two independent series of tests are very consistent with each other. Based on the fluidization data, an empirically adjusted estimate can be proposed in the form

$$\hat{\phi}^* = \frac{1}{\kappa} \tanh^{-1} (\kappa \phi)$$

where $\kappa$ is a dimensionless coefficient and value $\kappa = 1.2$ provides a good fit to the measurements. The functional form of (33), involving an inverse tangent, reduces to (31) in the dilute limit, and introduces a significant correction only at high concentrations ($\phi > 0.4$) for which the Poisson process assumption breaks down. The data are plotted again in Fig. 11b based on this empirically adjusted relation. The apparent robustness and sensitivity of the stereoscopic imaging estimate contrasts with the much less favorable properties of the monocular imaging indicators examined in Capart et al. (2002).

### 4.1.3 Three-dimensional particle motions

The evolution of the particle arrangement in time can further be characterized by following the 3D motions of the imaged particles using the procedures of Sect. 3.3. In this application, the movement is relatively slow and the tracking procedure easy to carry out. A more challenging application for the tracking component of the algorithms is

---

**Table 1. Test conditions for the fluidization experiments**

<table>
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<tr>
<th>First series of tests</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Fluidization velocity (cm/s)</td>
<td>1.97</td>
<td>1.05</td>
<td>2.46</td>
<td>1.78</td>
<td>2.20</td>
<td>2.54</td>
<td>0.93</td>
<td>0.72</td>
<td>1.18</td>
<td>1.84</td>
<td>2.95</td>
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<tr>
<td>Bulk concentration (%)</td>
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<td>52.1</td>
<td>30.7</td>
<td>40.5</td>
<td>33.6</td>
<td>29.2</td>
<td>55.7</td>
<td>59.0</td>
<td>43.5</td>
<td>38.3</td>
<td>26.6</td>
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<tr>
<td>Fluidization velocity (cm/s)</td>
<td>0.90</td>
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<td>1.26</td>
<td>1.70</td>
<td>1.96</td>
<td>2.36</td>
<td>2.57</td>
<td>1.08</td>
<td>1.87</td>
<td>0.95</td>
<td>1.48</td>
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<tr>
<td>Bulk concentration (%)</td>
<td>55.7</td>
<td>50.7</td>
<td>47.2</td>
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<td>37.1</td>
<td>32.4</td>
<td>29.3</td>
<td>51.4</td>
<td>38.4</td>
<td>54.3</td>
<td>43.6</td>
</tr>
</tbody>
</table>

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Fig. 11a, b. Granular concentration: bulk measurements vs. imaging estimate. ○ first series of tests; △ second series of tests; – line of perfect agreement. a Imaging estimate $\phi_0$ (31); b empirically adjusted estimate $\phi^*_0$ (33) with $\kappa = 1.2$. 

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the rapidly sheared example presented in the next section. The key component here is the stereoscopic positioning step. Reliable 3D positions must be measured in order to record continuous trajectories. To damp out high-frequency noise due to the finite sensor resolution, particle displacements are averaged over nine successive frames (with a frame rate of 200 fps, this corresponds to an interval of 0.04 s, which was checked to be much less than the average time between collisions) to yield velocity and trajectory measurements. Sample particle paths obtained for a fluidization concentration \( \phi = 0.38 \) are presented in Fig. 12. The fluctuating motions of neighboring grains are influenced by each other on a scale of a few particle diameters. This appears typical of hydrodynamic effects whereby conjugate motions of the embedding fluid transmit the influence of particle motions over a certain distance.

Mean squared velocity fluctuations along the three spatial directions \( x, y, z \) are plotted in Fig. 13 for different fluidization concentrations \( \phi \). The observed velocity fluctuations are greater at lower concentrations, for which particles have more freedom to move around. Lower concentrations also correspond to a faster fluidization flux and greater transfer of kinetic energy to the fluctuating motions. Vertical fluctuations \( \langle w^2 \rangle \) are found to be slightly larger than the horizontal fluctuations \( \langle u^2 \rangle \) and \( \langle v^2 \rangle \). Deviations from isotropy are not very large, however. A significant observation is that, despite the presence of the side-wall, the in-plane and out-of-plane horizontal fluctuations have similar magnitudes.

### 4.2 Uniform debris-flow experiments

#### 4.2.1 Principle and set-up

The second test case is an open-channel flow of a highly concentrated liquid–granular mixture imaged through the side-wall. The flow is obtained in a recirculatory flume developed at the University of Trento, Italy, for the study of torrential sediment transport and debris-flow processes (Armanini et al. 2000). Shown in Fig. 14, the device features a glass-walled flume of adjustable slope (from 0 up to 23 degrees) having the following dimensions: length=6 m; width=20 cm; wall height=40 cm. A fast conveyer belt is used to recirculate material collected at the flume outlet. This scheme achieves steady, longitudinally uniform flow conditions within the flume.

The solid grains used for the tests are PVC particles having a cylindrical shape and the following dimensions: diameter=3.2 mm, height=2.8 mm; equivalent spherical diameter=3.5 mm. The volumetric concentration of such grains is around \( \phi_{\text{rep}} \approx 0.69 \). The material density is \( \rho_s = 1,540 \text{ kg/m}^3 \). Water is again used as entraining fluid. The specific flow chosen for the present testing purposes is a mature debris-flow case for which the transport layer, many grains thick, fills the entire flow depth (i.e., there is no grain-free water layer in the upper part of the flow). This case was selected because it combines a wide range of concentrations, rapid velocities, and intense shear, all desirable features to test the applicability and limits of the technique. Operating conditions for the run are: bottom slope angle=7.2°; total discharge=14.5 l/s; \( \phi \) = delivered solid concentration=ratio of granular discharge to water discharge=49%. The latter two parameters are derived from bulk measurements performed at the end of the run by diverting the outlet flow to a trap.

To resolve individual grains in a flowing layer of such thickness, a reasonably high image resolution is needed. For flow speeds of the order of 1 m/s, a high image acquisition frequency and an adjustable shutter are further necessary to reliably sample the grain motions and observe blur-free particles. For this purpose a high speed CCD camera was operated at a resolution of 480x420 pixels, a frequency of 250 fps, and an exposure time shuttered down to 1/500 s. Because of the availability of only one camera with these characteristics, stereo viewing requires a
special device composed of four mirrors arranged as shown in Fig. 14. The camera is positioned at a distance of about 1.5 m from the side-wall of the flume. The mirror set-up is then interposed midway between the camera and the flume.

4.2.2 Experimental results

Measurements obtained by application of the 3D Voronoï imaging techniques described in the present paper are shown in Figs. 15 and 16. Figure 15 displays typical trajectories of individual grains reconstituted using the stereoscopic matching and velocimetry tracking algorithms. The results shown have been filtered over three successive frames (0.008 s.) to suppress high-frequency noise. Results in the $x$–$z$ plane show the higher velocities attained by the grains in the upper part of the flow layer. Results in the $y$–$z$ plane show that trajectories can be reconstituted farther away from the side-wall in the upper part of the flow. This is because concentration is lower there and occlusion effects are less severe. Despite the high concentration and the irregular nature of the granular motions, long granular trajectories are successfully reconstructed by the proposed methods.

Figure 16 shows vertical profiles for the mean velocities, fluctuation velocities, and solid concentration averaged from a sequence of 512 images. In Fig. 16a, the mean velocities in the vertical and normal to wall directions are seen to be close to zero. The mean longitudinal velocity, on the other hand, varies from zero in the static bed layer to maximum speed at the free surface. An approximately constant shear rate $\partial u/\partial z$ is obtained in the upper part of the flow. Fluctuating motions keep a roughly constant magnitude in this upper part, as shown quantitatively in Fig. 16b and qualitatively in Fig. 15. There is a slight peak in the fluctuations at the free surface. This appears to be due to a corner effect involving intermittent (stick-slip) motion of partly emerged grains, interacting with the side-wall because of surface tension.

In the lowermost part, the bed is virtually motionless and fluctuations should decrease to zero. Residual values there represent noise due to inaccuracies in particle positioning. Small errors in position, uncorrelated from one
frame to the next, translate into apparent velocity fluctuations. This effect is small for the in-plane components of the mean squared velocity fluctuations \( \langle u'^2 \rangle \) and \( \langle w'^2 \rangle \).

Near the bed, a much higher magnitude is observed for the normal to wall component \( \langle v'^2 \rangle \). Not unexpectedly, this indicates that the stereo measurements of 3D positions achieve a lower accuracy in the depth-wise direction than in directions more closely parallel to the camera image planes. The resulting noise is seen in Fig. 16b to be significant. Encouragingly, however, it does not drown out the physical signal.

Figure 16c shows the measured concentration profile obtained using adjusted estimate (33). The measured solid concentration exhibits a maximum value in the static lower layer and decreases to a minimum near the free surface. Again, corner effects perturb the measurements right at the free surface. A slightly jagged profile is observed in the lower part of the flow. This occurs because the bed is virtually motionless there: the particle configuration does not change over time hence averaging over many frames does not improve the statistics for the concentration measurements. Overall, the shape of the profile is quite regular and features reasonable concentration values. Values close to static packing are observed in the motionless bed. The following integration over depth further yields an imaging estimate for delivered concentration \( \bar{\phi} \):

\[
\bar{\phi} = \frac{\int_{z_0}^{z_0+h} \phi \langle u' \rangle \, dz}{\int_{z_0}^{z_0+h} \langle u' \rangle \, dz},
\]

in which \( z_0 \) is the elevation of the rigid bed and \( h \) is the flow depth. The resulting value is \( \phi = 0.46 \), comparable to the value of \( \phi = 0.49 \) derived from bulk measurements at the outlet. Even if the side-wall measurements were absolutely accurate, the two values would not be expected to perfectly coincide because the flow departs somewhat from uniformity in the transverse direction. Nonetheless, similar magnitudes should be obtained, as is the case with the present stereoscopic methods. Significantly, such agreement is obtained without any adjustment to the stereological relation derived from the fluidization tests, itself very close to the theoretically predicted relation. This was far from the case for the monocular imaging methods examined in Capart et al. (2002), for which concentration estimates had to be recalibrated in an ad hoc fashion when going from one type of experiments to another.

5 Conclusions

Three-dimensional imaging techniques were developed for the measurement of near-wall particulate flows. The methods include special matching and tracking algorithms, which exploit the properties of projective geometry and Voronoi diagrams to reconstruct 3D particle positions and velocities. A novel estimate based on the surface density of near-wall particles was also proposed to measure volumetric solid concentrations. Because they can handle occlusion effects and resolve position and motion ambiguities, the methods are particularly well suited for applications involving highly concentrated, rapidly sheared dispersions of many identical particles. Fluidization cell tests and an open-channel solid–liquid flow experiment were used to demonstrate the potential of the proposed techniques, as well as point out some of their limitations. It is hoped that they will prove valuable tools for the measurement of particulate flows of scientific and industrial interest.

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Granular motions and patterns in slow uniform flows

 driven by an inclined conveyor belt

T. H. Perng¹, H. Capart², H. T. Chou³

Abstract

The present experimental study examines the behaviour of slow granular flows, focusing on the details of particle motions and patterns over the depth of a sheared layer. A conveyor belt circuit enclosed in an inclined flume is used to generate steady uniform open-channel flows of dry granules. Particle positions near the transparent sidewall are extracted from video sequences. The Voronoi diagram is then used to characterise the patterns formed by neighbouring grains and to assist particle tracking over successive frames. This allows a qualitative visualisation of the internal structure of the flowing layer, as well as a quantitative characterisation of the distribution of mean velocity, velocity fluctuations, and density of lattice defects over depth. We examine in particular how the macroscopic and microscopic features of the flow respond to different conveyor belt speeds.

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1

Introduction

Dry granular flows are encountered in a variety of geophysical phenomena and industrial processes. Apart from applications, they are of interest as a striking example of complex system, for which complicated behaviours arise from the interaction of simple constituents. One of the key complications is the existence of two distinct flow regimes. In rapid granular flows, particles interact through short-lived collisions and the stresses vary roughly as the square of the shear rate. Slow granular flows, by contrast, are characterised by long-lasting, frictional contacts between grains, and exhibit stress magnitudes which are independent of the rate of deformation. How do microscopic features at the granular scale affect the macroscopic flow behaviour? For rapid granular flows, convincing answers to this question have emerged from theoretical, computational and experimental studies [1,2]. Likewise, an understanding of what controls the transition between frictional and collisional behaviour is being reached [3,4]. For the slow granular flow regime itself, by contrast, a coherent picture is yet to be assembled. While theoretical approaches have been proposed [5,6], wide gaps remain to be filled using empirical information.

We document in the present paper some detailed experimental observations which we hope can be useful in this regard. They pertain to slow free surface flows of spherical particles examined in an inclined conveyor belt channel. Devices of similar design have been used by previous researchers to study liquid-granular debris flows [7,8]. The apparatus is found in the present work to be useful for the study of dry granular flows as well. In particular, conditions very close to steady uniform rectilinear flow are obtained. These conditions are more difficult to approximate in rotating drums [9], annular shear cells [10], or non-recirculating chutes [11], which are the devices most commonly used to study granular shear flows.
To characterise the parallel shear flows, we rely on video images of the grains moving near the sidewall of the channel. Thus a limitation of the present experiments is that while the granular flow is three-dimensional (as opposed to a two-dimensional system of particles held between parallel plates), visual access is restricted to the near-wall grains. To analyse the patterns and motions of the visible grains, standard image analysis techniques are complemented by special tools based on the Voronoï diagram. This geometrical construction has been found useful by a number of investigators seeking to analyse experimental or computational systems of interacting particles [12,13,14,15,16]. Here we use the Voronoï diagram for two different tasks: first, to characterise patterns of neighbouring grains on still images, and secondly to assist velocimetric particle tracking over a sequence of frames.

The paper is organised as follows. Sections 2 and 3 describe the experimental apparatus and imaging techniques, respectively. Section 4 provides a qualitative picture of the internal structure observed in a typical run. Velocity and defect density profiles are then presented and discussed in sections 5 and 6. Finally conclusions are drawn in section 7.

2

Laboratory apparatus and experimental procedure

Experiments were conducted at the Environmental Fluid Mechanics Laboratory (EFML) of the Department of Civil Engineering, National Central University. The laboratory device adopted for the present research is schematised on Figure 1. A conveyor belt circuit is enclosed in a glass-walled channel of rectangular cross section and adjustable slope. The rubber conveyor belt is mounted on four fixed corner rollers, with a fifth roller used to tune the belt tension. The lower left roller is driven by a variable speed electric motor. The conveyor belt slides along the channel floor in the central compartment of the channel. The vertical plates bounding this compartment left and right have small gaps at the bottom
allowing passage of the belt. The inside dimensions of the central compartment are: length = 150 cm; width = 12 cm; height of the glass sidewalls = 35 cm. The entire device can be inclined at angles going from 0 to 30 degrees. The conveyor belt has transversal grooves of trapezoidal shape (trapeze height = 2.2 mm, base length = 9.2 mm and top length = 5.8 mm), as shown on Figure 1 (inset).

The granules used for the experiments are white monosized plastic beads of spherical shape having the following properties: diameter $d = 5.85$ mm; density = 1.915 g/cm$^3$; angle of repose = 22.5 degrees. In the present work, all experiments were conducted with the same volume of particles placed evenly in the central compartment, rising to a depth of approximately 12 cm. Experiments are conducted by driving the conveyor belt and observing the resulting granular flow. Confined within the compartment, the granular material is driven upslope by the moving conveyor belt and pulled downslope by gravity. Preliminary experiments conducted at various inclinations and belt speeds indicated that the granules tend to heap at the upper end of the channel for low inclinations and at the lower end of the channel for higher inclinations. There is a unique inclination at which the depth of the deforming layer is approximately uniform over the whole length of the channel. This occurs for an inclination of 15.6 degrees. This angle is not measurably affected by the belt velocity at least up to speeds of 20 cm/s. All experiments described in the present paper were obtained at this precise inclination where the flow is approximately steady uniform.

To characterise the flows, digital video footage is acquired using a TRV-30 Sony DV camera. The camera is positioned to capture a side view of the granular layer close the channel mid-length, where the motion is most nearly parallel. The viewing window is inclined with the channel and covers the full depth of the granular layer. Illumination is provided by two spotlights oriented at approximately 45 degrees. The camera operates at a frequency of 30 Hz.
in interlaced mode (odd and even rows correspond to staggered time instants), with an image resolution of 480 rows by 720 columns. Before processing, images are split into their odd and even rows in order to obtain instantaneous images at twice the frame rate (60 Hz) but half the vertical resolution. For each experimental test, the granular flow runs for more than a minute at the chosen conveyor belt speed before footage is acquired over a duration of 6 seconds.

3

Image analysis

We seek to analyse the patterns and motions of near-wall particles in the vertical \((x, y)\) plane (see Figure 1). Basic imaging algorithms (see reference [14] for details) are first used to automatically extract particle positions from the images (Figure 2a). De-interlaced frames are convoluted with a Laplacian-of-Gaussian filter. Particle centres are then identified as brightness maxima of the filtered images. Sub-pixel accuracy is obtained by fitting a quadratic surface to the neighbourhood of each maximum and taking the position of the surface peak as estimate of the particle position on the image. Calibrated scale factors are finally used to convert to physical units. This yields sets of coordinate pairs \((x_i^{(m)}, y_i^{(m)})\), where a given pair denotes the position of the \(i\)-th particle identified on the \(m\)-th frame of the sequence.

The two subsequent steps both resort to the Voronoï diagram. For a given frame \(m\), the Voronoï tiling assigns to each particle \(i\) a polygonal region of the plane (its Voronoï cell) in which points lie closer to particle centre \((x_i, y_i)\) than to any other particle centre in the set. In the present work, we use the Voronoï diagram for two different purposes (Figure 2b-c). The first purpose is to characterise the local arrangement of near-wall grains on the basis of the number of sides of individual Voronoï cells [12]. Regions where particles are arrayed in a crystal-like hexagonal packing feature 6-sided polygons only, while more disordered regions
include polygons with different side counts, 5 or 7 sides being the most frequent (see Figure 2b). Such polygons appear in the neighbourhood of voids or dislocations in an otherwise hexagonal array, or in regions where particle arrangements are completely irregular. Departures from crystalline packing can thus be quantified by introducing a local “defect density”, defined as the ratio \( \nu_\delta \) of the number of \( n \)-sided Voronoï cells \( n \neq 6 \) to the total number of Voronoï cells in a given region of the plane. To avoid edge effects, Voronoï cells located along the boundaries of the imaged domain are excluded from consideration according to the criterion of reference [12]

The second task for which we resort to the Voronoï diagram is the velocimetric tracking of particles. Once sets of particle positions \( (x_i^{(m)}, y_i^{(m)}) \) and \( (x_j^{(m+1)}, y_j^{(m+1)}) \) have been identified on two successive images \( m \) and \( m + 1 \), the problem is to establish the one-to-one correspondence \( j(i) \) linking two successive sightings of each physical particle. If this correspondence can be obtained, particle velocities \( (u_i^{(m)}, v_i^{(m)}) \) can be approximated by [17]

\[
\begin{align*}
  u_i^{(m)} &= \frac{x_j^{(m+1)} - x_i^{(m)}}{\Delta t}, \\
  v_i^{(m)} &= \frac{y_j^{(m+1)} - y_i^{(m)}}{\Delta t}.
\end{align*}
\]

In the present experiments, the particles are identical but move relatively slowly hence a reasonable correspondence \( j(i) \) can be obtained by simply assuming that the next position of a given particle is the one closest to its previous position. This is the so-called minimum displacement algorithm [18]. To make this approach more robust, however, we also exploit point-pattern information extracted from the Voronoï diagram: two particle positions are considered good matches if the associated displacement is small and if their corresponding Voronoï cells are similar in shape, according to the criterion detailed in reference [14]. This is illustrated on Figure 2c.

4

Structural features
Before examining velocity and defect distributions in a quantitative fashion, it is useful to take a qualitative look at a typical experimental run. Figures 3 and 4 show short-exposure and long-exposure views, respectively. The figures depict a number of structural features similar to those documented in detail by Drake [11] in experiments with granular monolayers held between vertical parallel plates.

Panels a and b of Figure 3 document typical instantaneous arrangements of near-wall particles during the slow shear flow. Throughout the layer, particles are densely packed and interact with their neighbours through enduring contacts rather than short-lived collisional encounters. Only at the very top do some particles occasionally tumble downslope, bouncing rapidly along the free surface. Comparison of panels a and b illustrates the relationship between granular patterns and the number of sides of their Voronoï cells. Side counts differing from 6 are clearly associated with localised defects or disordered regions. Conversely, 6-sided cells do not imply a perfect tiling of regular hexagons: distorted lattices also qualify, and regions where patterns are most irregular nonetheless yield a certain proportion of 6-sided cells.

Contrasted patterns are observed in the lower and upper parts of the flow. The lower part features disordered arrangements, as grains must adjust to the geometrically rough boundary of the conveyor belt. The trapezoidal grooves of the belt are periodic, but their dimensions do not match those of the grains, hence a disrupting effect felt up to some 5-10 diameters above the belt. In the upper part of the flow, on the other hand, one observes coherent blocks where particles are arrayed with crystal-like regularity. Distinct blocks are separated from each other in two different ways. They may join across clean fault lines oriented parallel to the bed or at angles of ±60 degrees. Alternatively, they may be separated by irregular interstitial zones where defects tend to cluster.
Complementing the pattern information, panel c of Figure 3 shows the corresponding velocity field. The velocities shown are derived from individual particle displacements measured over 5 successive frames, and no spatial filtering or averaging has been applied. Superimposed upon the overall shear deformation, coherent granular motions are observed, with vortex-like structures apparent at mid-depth. Comparison with panels a and b indicates that these correspond to rotations of the crystal-like blocks. Zones across which velocities change directions further coincide with the boundaries between blocks. The flow field is approximately divergence-free, indicating that the compact granular flow behaves like an incompressible phase. No slip is observed at the base of the flow: grain velocities at the base are equal to the speed of the conveyor belt (14.7 cm/s for this run).

Granular motions are further documented on Figure 4. Panel a is an artificial long exposure image obtained by merging together a sequence of 5 video frames. The image conveys an intuitive sense of how the flow deforms and corroborates the more precise information of Figure 3c. Panels b and c of Figure 4 are obtained by tracking particles over 50 successive frames, and indicate how particles move over a longer time interval (0.83 sec). Particle trajectories are plotted in two different ways. Panel b shows the raw trajectories \((x(t), y(t))\), while panel c shows convected trajectories \((x'(t), y'(t))\) obtained by subtracting a time integral of the local mean flow velocity from the raw trajectories. This amounts to observing individual particle paths in a frame of reference moving with the local mean flow, as suggested in reference [19].

Both sets of trajectories further highlight the internal structure of the slow granular flow. On panel 4b, contrasted pictures are again obtained for the lower and upper parts of the flowing layer. In the lower part, trajectories tend to distribute evenly over depth due to the more irregular arrangement of the corresponding grains. In the upper part, by contrast, trajectories
cluster into horizontal threads, much like lanes of a highway along which cars follow one another, changing lanes only occasionally. Distinct horizontal rows of particles slide one on top of the other to accommodate the overall shear deformation. On panel 4c, the convected trajectories highlight the block structure of the flow. In a given block, the motion histories of individual particles exhibit long range correlations, tracing orbits which are strikingly similar in shape. On the other hand different blocks exhibit dissimilar fluctuation velocity signatures. The motions of distinct blocks are not synchronised, and they move intermittently with respect to each other. These kinematical features closely match the “grain-layer gliding” and “block gliding” mechanisms described by Drake [11]. Layering of particles in organised rows has also been observed in computations of dense granular flows in the collisional regime [20], and in liquid-granular flow experiments [21].

5

Velocity profiles

The imaging procedures outlined above yield large sets of data, of the order of 250,000 velocity vectors and Voronoï cell side counts for each experimental run. In order to reduce these data to profiles over depth, the flow layer is subdivided into non-overlapping bins of constant thickness into which measurements are distributed. The bin thickness $\Delta y$ corresponds approximately to one grain diameter. The average longitudinal velocity $\bar{u}(y_k)$ within the $k$-th bin is given by [10]

$$\bar{u}(y_k) = \frac{\sum_i \sum_m I_i^{(m)}(k)u_i^{(m)}}{\sum_i \sum_m I_i^{(m)}(k)},$$

(2)

where $y_k$ is the height of the centre of the $k$-th bin and $I_i^{(m)}(k)$ is an indicator function defined by
\[
\begin{cases}
I_i^{(m)}(k) = 1 & \text{if } y_k - \Delta y / 2 \leq y_i^{(m)} < y_k + \Delta y / 2 \\
I_i^{(m)}(k) = 0 & \text{otherwise.}
\end{cases}
\] (3)

Once bin-averaged velocities \( \bar{u}_k \) have been estimated, fluctuation velocities are taken as

\[
u_i^{(m)} = u_i^{(m)} - \bar{u}(y_i^{(m)}),
\] (4)

where \( \bar{u}(y_i^{(m)}) \) is obtained at the actual instantaneous position of the particle \( y_i^{(m)} \) by linearly interpolating from profile \( \bar{u}(y_k) \). Root-mean-squared (rms) velocity fluctuations in each bin are then given by

\[
\sqrt{\nu'^2}(y_k) = \sqrt{\frac{\sum_m \sum_i I_i^{(m)}(k) u_i^{(m)2}}{\sum_m \sum_i I_i^{(m)}(k)}}
\] (5)

Depth-profiles of fluctuation velocities in the \( y \)-direction \( \sqrt{\nu'^2}(y_k) \) are obtained likewise.

Profiles were obtained for a series of 14 experimental runs conducted at belt speeds ranging from 4 to 18 cm/s. Figures 5 and 6 show the corresponding results for the mean and fluctuation velocities. On panel 5a, the mean longitudinal velocity profiles \( \bar{u}(y) \) are plotted in dimensional units. On panel 5b, the same data are plotted in terms of dimensionless ratios \( y / h \) and \( \bar{u} / u_0 \), where \( h = 12 \) cm is the approximate depth of the granular layer and \( u_0 \) is the speed of the conveyor belt (shown for each run as a filled symbol on panel 5a). Mean granular velocities at the base of the layer are practically equal to the belt speed, confirming previous observations that the no slip condition applies to the geometrically rough boundary.

Integration of the mean velocity profiles over depth yields values close to zero, as expected for a steady granular flow confined within rigid boundaries at the left and right sides of the channel. Grains move up along the bottom, driven by the belt, and flow back down along the surface. The profiles obtained for various belt speeds are highly similar in shape. For any
given profile, the shear rate $\dot{\gamma} = |\partial \bar{u} / \partial y|$ peaks at the free surface, and decreases to a local minimum at mid-depth. The shear rate increases again in the lower half of the layer before decreasing in the immediate vicinity of the belt. The low shear rates observed at mid-depth most likely reflect the influence of friction along the sidewall. In that zone, mean velocities close to zero allow particles to undergo stick-slip motions, intermittently adhering to or sliding along the stationary wall. Where the mean velocity is further away from zero, by contrast, particles continuously slide along the wall.

As documented on panel 5b, the most striking feature of the flow is the way the profiles very nearly collapse together when the mean granular velocity is scaled with respect to the belt speed. This implies that the mean flow velocity throughout the layer is controlled by the speed of the conveyor belt. In other words, the self-similar profiles are kinematically constrained by the speed imposed at the lower boundary. The fact that steady uniform flow can be obtained for various belt speeds further implies rate-independent granular stresses, a hallmark of the slow frictional regime. In steady uniform flow, the shear and normal stresses at any given depth must balance the weight of the overlying granular layer, and are thus determined by the inclination of the channel and the packing density of the particles. Both these parameters are held virtually constant in the present experiments, hence the stress levels at a given depth must be nearly identical for all runs. What varies is the rate of deformation, controlled by the speed of the conveyor belt as it increases from 4 to 18 cm/s. Thus stresses are not affected even when multiplying the deformation rate by a factor of 4.

Fluctuation velocities are displayed on Figure 6. Panel 6a shows profiles of normal-to-bed rms fluctuation velocities $\sqrt{v'^2}(y)$ in dimensional units. Panel 6b plots the same data in terms of dimensionless ratios $y/h$ and $\sqrt{v'^2}/u_0$, where again the layer depth $h$ and
conveyor belt speed $u_0$ are chosen as normalising variables. Finally, panel 6c compares normal-to-bed rms fluctuation velocities $\sqrt{v'^2}$ against the corresponding longitudinal fluctuations $\sqrt{u'^2}$ (taken at the same depth). The fluctuation velocity profiles exhibit various features which are similar to those observed earlier for the mean velocity profiles. First, profiles for different runs are highly similar in shape (panel 6a). Qualitatively, the fluctuation velocity evolves over depth in much the same way as the shear rate. For any given profile, the fluctuation velocity $\sqrt{v'^2}$ peaks at the free surface, decreases to a local minimum at mid-depth, then increases again in the lower half of the layer before decreasing in the immediate vicinity of the belt.

When scaled by the corresponding conveyor belt speeds, the fluctuation velocity measurements are also observed to approximately collapse together (panel 6b). The points do not align as closely as the mean velocity data (Figure 5), but this may be due to limitations of the measurements rather than a physical effect. Comparatively larger quotients $\sqrt{v'^2} / u_0$ are recorded for the slower flows, but these are also those for which one expects a larger noise-to-signal ratio (errors on particle positions registering as spurious extra fluctuations). On panel 6c, the fluctuation velocities in the two perpendicular directions $\sqrt{u'^2}$ and $\sqrt{v'^2}$ are seen to have roughly the same strength. A slight anisotropy is recorded, with longitudinal fluctuations $\sqrt{u'^2}$ stronger than normal-to-bed fluctuations $\sqrt{v'^2}$ by some 15%. This feature is shared with simulations of collisional granular flows [22], although the rate-independence of the present observations puts them squarely in the frictional regime.
Defect distribution

A characterisation of the distribution of packing defects can be sought based on the same subdivision of the flowing layer into non-overlapping bins. Particle centres falling into each bin have the number of sides $n$ of their Voronoï cells counted. Cells for which $n \neq 6$ are further recorded as lattice defects and contribute to the defect density of the binning sublayer $\nu_\delta(y_i)$. Here we define the defect density as the ratio $\nu_\delta$ of the number of $n$-sided Voronoï cells ($n \neq 6$) to the total number of Voronoï cells centred in a given sublayer.

Before examining depth profiles of the defect density, it is useful to take a more detailed look at the probability distribution $P(n)$ of the number of cell sides in separate sublayers of a typical run. Figure 7 shows such side count statistics for three different sublayers of the run documented already in Figures 3 and 4 (for conveyor belt velocity $u_o = 14.7$ cm/s). To facilitate interpretation, panel 7a shows the side count distribution $P(n)$ associated with Voronoï diagrams derived from completely random dispersions of points in the plane. It was obtained by Le Caer and Ho [23] (see also [24]) using Monte-Carlo simulations of a Poisson process. Even for such random dispersions, 6-sided cells are the most likely. The corresponding proportion is only $P(n = 6) \approx 0.2946$, however, and counts of $n = 4, 5, 7$ and 8 are also frequent.

In the granular flow experiments, the side count distributions are significantly narrower. The distribution closest to the idealised Poisson process distribution occurs in the lowermost sublayers of the flow, near the conveyor belt where the belt geometry strongly disrupts the particle packing (panel 7b). Even there, however, the spread around $n = 6$ is much narrower, with few cells having counts smaller than 5 or greater than 7. This indicates that the
randomness of the granular arrangement there is limited, constrained as it is by excluded volume effects (monosized particles which cannot interpenetrate). In sublayers located at the flow mid-depth (panel 7c), the arrangement is much more regular. More than 80% of the cells are 6-sided, with the remainder evenly split between 5-sided and 7-sided cells. This reflects the tendency of defects to occur in pairs of 7-sided and 5-sided cells, as shown on Figure 3b where such cells are respectively denoted by symbols (+) and (−). The situation is similar in the uppermost sub-layers of the flow, close to the free surface (panel 7d), but the proportion of defects \((n \neq 6)\) there is greater than at mid-depth.

Depth profiles of the defect density \(\nu_\delta(y_k)\) for the 14 different experimental runs are shown on Figure 8 (hollow symbols, where the different shapes correspond to separate runs using the same convention as in Figure 5). As measured by indicator \(\nu_\delta\), defects are observed to be most prevalent at the bottom and at the top of the flowing layer. Conversely, the most regular granular arrangements (low defect density \(\nu_\delta\)) are encountered in the middle of the layer. For any given run, a relatively regular C-shaped curve is obtained for the distribution \(\nu_\delta(y_k)\) of the defect density over depth. This quantifies and corroborates the qualitative trends observed earlier.

On Figure 9, the minimum value of the defect density for each run \(\nu_\delta^{(\min)}\) (obtained close to the layer mid-depth) is plotted against the corresponding conveyor belt speed \(u_0\). Values for different belt speed are seen to present substantial scatter around a mean of approximately 0.17. Beyond that no clear trend emerges which would allow one to surmise either a greater or lesser prevalence of defects as the belt speed rises, indicating that the defect density is not strongly dependent on the mean shear rate. Alternatively, the duration over which profiles are averaged may not be sufficient for convergence towards the long term mean, obscuring a
relationship which may nonetheless be present. Since the velocity profiles do converge over the same time period (6 s), the fact that regression to the mean is not observed for the defect density profiles suggests that two time scales may be present. Velocities would appear to adjust on a fast time scale, while geometrical disruptions of the particle packing may take a long time to evolve.

To clarify the above picture, a separate series of tests was conducted to measure the defect density profile $\nu_s(y_k)$ for granular layers at rest. For each test, steady uniform flow is first driven by the conveyor belt for at least one minute. The belt is then stopped, granular motion ceases completely, and a digital image of the static layer is acquired. In total, 32 still frames were acquired in this fashion. The defect density profile obtained by averaging the corresponding data is plotted as well on Figure 8. In the upper part of the flow, the static profile (filled disks) differs significantly from the dynamic profiles (hollow symbols). This indicates that even if no strong dependence is observed with respect to the rate of deformation, the granular assembly is nonetheless sensitive to being at rest or undergoing shear. While the static and dynamic data diverge in the upper part of the flow, they differ much less in the lower part. This is very likely to reflect the effect of the geometrically rough boundary, which disrupts the granular packing in the lowermost sublayers regardless of whether the grains are moving or not.

Seeking to separate this static boundary disruption from dynamic features possibly generated by the shear flow, we fit an exponential curve to the lower part of the static data (see Figure 8) and subtract the resulting profile from the dynamic data. The resulting defect density profiles are shown on Figure 10. Upon subtraction of the static fit, the defect density now monotonously decreases from a maximum at the free surface to a minimum at the bottom.
Inspired loosely from the void propagation description of reference [25], we can formulate a simple model which roughly accounts for the shape of the defect density profile. Assume that the sheared layer is subdivided into discrete rows of particles numbered (from bottom to top) \( k = 1, \cdots, N \) where \( N = 20 \approx h/d \) and \( d \) is the particle diameter. The number density of defects in each row is \( \nu_\delta[k] \) and is assumed to evolve according to the following non-dimensional, semi-discrete evolution equation

\[
\frac{\partial \nu_\delta[k]}{\partial \tau} = J[k - \frac{1}{2}] - J[k + \frac{1}{2}] + C
\]  

(6)

where \( \tau = t\dot{\gamma} \) is a dimensionless time set by the shear rate \( \dot{\gamma} = |\partial u / \partial z| \) (assumed uniform over the depth), \( J[k + \frac{1}{2}] \) denotes a rate of transfer of defects from row \( k \) to row \( k + 1 \), and \( C \) is the rate at which defects are generated (assumed constant throughout the layer). The transfer rate \( J \) is assumed to take the form

\[
J[k + \frac{1}{2}] = A \nu_\delta[k] + B(\nu_\delta[k] - \nu_\delta[k + 1])
\]

where the first component is the rate at which defects migrate upwards (e.g. when a vacancy in a lower layer is filled by a grain from an upper layer, hence an upwards migration of the vacancy) and the second component is a diffusive flux. This flux formulation can be seen as a discrete analogue of the Rouse theory of suspended sediment transport [26]. Parameters \( A \) and \( B \) are again taken to be constant. Boundary conditions can finally be set as

\[
J[\frac{1}{2}] = 0, \quad \nu_\delta[N] = 0.7054
\]

(7)

i.e. no flux at the rigid bottom and maximum randomness (the value \( \nu_\delta = 1 - P(n=6) \) corresponding to a Poisson point process) at the free surface. At steady state, \( \partial \nu_\delta[k] / \partial \tau = 0 \) and the resulting ordinary difference equation (ODE) can be solved (see e.g. [27]) to obtain

\[
\nu_\delta[k] = \frac{B}{A} + \frac{C}{A} k + \left( 0.7054 - \frac{B}{A} - \frac{C}{A} N \right) \left( \frac{B}{A+B} \right)^{N-k}.
\]

(8)

The steady profiles depend solely on the two ratios \( B/A \) and \( C/A \), and a reasonable fit to the measured data is obtained by choosing \( B/A = 2 \) and \( C/A = 0.01 \) (see Figure 10). The
quality of the fit deteriorates significantly if any component of the above model is neglected.

This very rough model suggests that, once static effects have been discounted, the distribution of defects results from a dynamic balance between three processes: the downwards diffusion of defects starting from the more random free surface, the upwards migration of defects as a result of their buoyancy (the pull of gravity favors packings of greater density and regularity), and the ongoing generation of defects inside the bulk due to shear. In the above model, the shear rate influences the time scale on which these mechanisms act but not the shape of the equilibrium reached at steady state. It is clear that the model falls far short of accounting for the rich internal structure documented earlier (Figures 3 and 4), in particular the organisation of the grains into blocks. Nevertheless, it appears to account for some basic features of the defect distribution over depth.

7

Conclusions

Before concluding, important limitations of the present work must again be acknowledged. First and foremost, the present observations of 3D flow are restricted to 2D projections of granular patterns and motions visible near the channel sidewall, monitored over a limited time. Secondly, particle patterns were probed using but a single indicator of packing regularity. Finally, a limited range of flow conditions was explored, excluding for instance variations in depth of the granular layer or belt speeds fast enough to reach collisional flow conditions.

Various tentative conclusions can nonetheless be drawn. First, the measurements of mean velocities underscore the high degree of rate-independence which constitutes the hallmark of the frictional regime. Profiles obtained over a range of conveyor belt speeds are strikingly self-similar, and collapse together when scaled with respect to the belt speed. Although the
scatter is wider, velocity fluctuations are found to roughly scale with the belt speed as well. Variations of the belt speed further have no clear influence on the density of lattice defects. This suggests that the microscopic features of the flow, just like the macroscopic mean velocities, exhibit a certain degree of invariance with respect to the rate of deformation.

Overall, the picture which emerges from these observations is that the motions and arrangements of particles in slow granular flows are essentially a matter of kinematics. Velocities scale with the belt speed, acting as a kinematical boundary condition, but the driving speed otherwise appears to have little influence on the character of the flow. Or, to state this more carefully, whatever influence may be present is not strong enough to be clearly resolved by the measurements.

As a second tentative conclusion, observations of the distribution of lattice defects over depth indicate that two types of effects can be distinguished: 1) static disruption by the geometrically rough boundary which affects nearby grains regardless of whether they are moving or not; 2) a dynamic balance of ordering and disordering mechanisms which intervenes when the granular layer is undergoing shear.

While the above observations may provide some useful indications, even a cursory look at Figures 3 and 4 makes it clear that they are far from accounting for the rich internal structure of slow granular flows. This structure includes a dual organisation of particles into longitudinal chains and extended blocks, inducing long range correlations of both particle motions and arrangements. Intermittency in the evolution of these features is another defining characteristic difficult to convey in still images but clearly apparent when looking at the videos. Further probing of slow granular flows is clearly needed if one hopes to clarify the precise mechanisms underlying such phenomena.
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References


**Figure legends**

Figure 1. Inclined channel with internal conveyor belt circuit used for the experiments.

Figure 2. Image analysis: **a** estimated particle positions; **b** voronoi cells with counts of the number of edges of each cell; **c** particle displacements (arrows) extracted by matching voronoi cell shapes between one image (thin lines) and the next (thick lines).

Figure 3. Short-exposure snapshots: **a** single video frame with marked particle positions (∗); **b** n-sided cells of the Voronoï diagram with indications of defects (+ for \( n > 6 \) and − for \( n < 6 \)); **c** velocity vectors based on displacements over 5 successive frames.

Figure 4. Long-exposure snapshots: **a** superposed sequence of 5 successive frames; **b** particle trajectories over 50 successive video frames; **c** ‘convected’ particle trajectories (upon subtraction of the local mean flow component) over 50 successive video frames.

Figure 5. Measured mean velocities \( \bar{u}(y) \) for different conveyor belt speeds: **a** profile data (hollow symbols) in dimensional units, with belt speeds shown as filled symbol at the base of the layer; **b** data scaled with respect to depth \( h \) and belt speed \( u_0 \).

Figure 6. Fluctuation velocities: **a** depth profile of normal-to-bed rms fluctuation velocity \( \sqrt{\nu^2(y)} \); **b** data scaled with respect to depth \( h \) and belt speed \( u_0 \); **c** comparison of longitudinal and normal-to-bed rms fluctuation velocities \( \sqrt{\nu^2} \) and \( \sqrt{\nu^2} \).

Figure 7. Histograms of the number of sides \( n \) of Voronoï cells: **a** computed for a Poisson random point process [23]; **b** measured at the base of the flowing layer (\( y/h = 0.1 \)); **c** at mid-depth (\( y/h = 0.5 \)); **d** near the top (\( y/h = 0.9 \)), for a belt speed of \( u_0 = 14.7 \) cm/s.

Figure 8. Depth profiles of the defect density \( \nu_\delta(y) \) for granular layers under shear (hollow symbols) and at rest (filled circles and exponential fit to the lower half of the data).

Figure 9. Measured minimum defect density \( \nu_\delta^{(\text{min})} \) for different belt speeds \( u_0 \).

Figure 10. Depth profiles of the defect density after subtraction of the static fit, \( \nu_\delta - \nu_\delta(u_0 = 0) \) (hollow symbols) and rough OΔE model fitted to the data (curve).
Figures

Figure 1

Figure 2
Figure 3
Figure 5
Figure 6
Rheological stratification in experimental free-surface flows of granular-liquid mixtures

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Abstract

Laboratory experiments are conducted to study the rheological behaviour of high-concentration granular-liquid mixtures. Steady uniform free-surface flows are obtained using a recirculating flume. Cases in which a loose deposit forms underneath the flow are contrasted with runs for which basal shear occurs along the flume bottom. The granular motions are observed through the channel sidewall, and analysed with recently developed Voronoï imaging methods. Depth profiles of mean velocity, solid concentration, and granular temperature are obtained, and complemented by stress estimates based on force balance considerations. These measurements are used to probe variations in rheological behaviour over depth, and to clarify the role of the granular temperature. The flows are found to evolve a stratified structure. Distinct sub-layers are characterised by either frictional or collisional behaviour, and transitions between one and the other occur at values of the Stokes number which suggest that viscous effects intervene. The observed frictional behaviour is consistent with shear cell tests conducted at very low shear rates. On the other hand, the collisional data corroborate both the Bagnold description and the more recent kinetic theories of granular flows, provided that one accounts for the inertia of the interstitial liquid.
1. Introduction

Gravity-driven flows of high-concentration granular-liquid mixtures intervene in a wide variety of geomorphological and industrial processes. Within the scope of civil engineering, natural flow examples include sediment-laden currents, debris surges and soil flowslides, while technological applications include the handling of dredging slurries and fresh concrete. A key feature of these flows is the highly inhomogeneous rheological behaviour of the granular-liquid mixture.

A typical situation, examined in depth in the present paper, is illustrated on Fig. 1. Here a moving layer of grains and liquid flows over a static layer of loose material. The two layers are composed of identical solid and liquid constituents, yet respond differently to shearing action due to contrasts in solid concentration and normal stress conditions. In other situations, the same material may undergo repeated transitions between states of flow and no-flow. For instance, a debris flow slurry may freeze into place before being remobilized by further debris surges. This constitutes a crucial issue in the description of many earth surface processes. How does the rheological behaviour of granular-liquid flows vary over depth? What determines the transitions from one flow state to another?

The present work seeks to address this issue by carrying out detailed measurements of simplified laboratory experiments. The experiments involve steady uniform free-surface flows of pellet-water mixtures in a recirculating flume apparatus. As in the pioneering experiments of Bagnold (1954), attention is restricted to simply sheared dispersions of identical solid grains in a Newtonian liquid. Whereas Bagnold conducted his experiments with neutrally buoyant solid grains, however, our experiments involve pellets which are denser than water. While Bagnold sought to avoid gravity effects, our aim is precisely to examine the inhomogeneous structure induced when dense grains are dispersed in a lighter liquid.
The restriction to simply sheared dispersions of identical particles in a Newtonian liquid represents a strong idealization. Field cases commonly involve heterogeneous particles in a non-Newtonian fluid. Naturally occurring debris flows, for instance, are often composed of poorly sorted clasts embedded in a mud matrix. Volumetric deformations of the granular phase may also play an important role, due to their effect on the liquid pore-pressure (Iverson, 1997). Despite their practical significance, these aspects will not be addressed here.

Granular-liquid flows remain challenging even without such complications. Responding to applied stresses, a granular phase composed of identical particles may behave alternatively like a solid, a liquid or a gas (Jaeger et al. 1996; Goldhirsch 2003). The coupled motions of the interstitial liquid may further strongly affect the properties of the deforming mixture (Koch & Hill 2001). Despite the simplified character of the present experiments, their interpretation will thus raise a number of fundamental questions. How do discrete grains interact with each other and with the continuous embedding fluid? How do these interactions lead to distinct rheological behaviours? How do these distinct behaviours co-exist within inhomogeneous flows?

In recent years, such questions have been addressed in some depth for two related physical systems: dry granular flows, and bubbly flows. These share many of the features of granular-liquid flows, but take the density ratio of the disperse and continuous phases to distinct limits: $\rho_d / \rho_c >> 1$ for dry granular flows, and $\rho_d / \rho_c << 1$ for bubbly flows. In these cases, simplifications arise because either the inertia or the vorticity of the continuous phase can be neglected (Koch & Hill 2001).

For both these systems, theoretical approaches based on the kinetic theory of dense gases
(Chapman & Cowling 1971) have been successfully applied to rapid flows dominated by collisional interactions. For dry granular flows, milestones include the works of Ogawa (1978), Savage and Jeffrey (1981) and Jenkins and Savage (1983). More recently, the effects of an interstitial gas phase have also been addressed (e.g. Sangani et al. 1996). For bubbly flows, on the other hand, recent developments include the studies of Russo and Smereka (1996), and Kang et al. (1997).

For both dry granular flows and bubbly flows, theoretical results have been corroborated and extended by dynamical simulations (Campbell & Brennen 1985; Walton & Braun 1986; Smereka 1993; Sangani & Didwania 1993). Computational experiments have further highlighted possible transitions in behaviour and the development of structure in inhomogeneous flows (Zhang & Campbell 1992; Savage & Dai 1993; Smereka 1993; Sangani & Didwania 1993; Zhang & Rauenzahn 1997; Aharonov & Sparks 1999).

A number of laboratory studies have provided additional information regarding dry granular flows. Most measurements of individual particle motions have been obtained for two-dimensional analogues such as monolayers of particles held between parallel plates (Drake 1991; Elliott et al. 1998; Azanza et al. 1999). Yet detailed experimental results have also been gathered for 3D flows. These include measurements of the local particle motions in the vicinity of a sidewall (Ahn et al. 1991; Natarajan et al. 1995; Hsiau & Jang 1998), and observations of global flow behaviour in wide channels (Louge & Keast 2001; Ancey 2002).

Studies of collisional flows owe much to the pioneering work of Bagnold (1954), who showed that stresses in rapidly sheared granular dispersions exhibit a quadratic dependence on the deformation rate. As mentioned earlier, the work of Bagnold (1954) was based on experiments with neutrally buoyant particles in water, for which the ratio $\rho_d / \rho_e$ was
precisely unity. Paradoxically, ensuing efforts have been less successful at addressing liquid-granular flows \( \left( \rho_d / \rho_c \sim 1 \right) \) than at treating dry grains and bubbles. Despite Bagnold’s early breakthrough, it has proven difficult to probe granular-liquid mixtures at a level of detail comparable to the dry granular and bubbly flow studies cited above.

Direct computations of granular-liquid flows, for instance, remain very difficult to conduct. Impressive recent efforts include the fluidization studies of Kalthoff \textit{et al.} (1997), and Pan \textit{et al.} (2002), based on coupling the granular dynamics together with the Navier-Stokes equations for the interstitial fluid in the evolving domain. For shear flows, such detailed computations have not yet appeared. Discrete particle simulations have been conducted, but rely on simplified models of inter-particle and particle-fluid interactions (Yeganeh \textit{et al.} 2000; Drake & Calantoni 2001).


Previous experimental studies have examined solid-liquid flows in annular shear cells (Bagnold 1954; Savage & McKeown 1983), closed ducts (Sumer \textit{et al.} 1996; Bakhtiary & Asano 1998), non-recirculatory chutes (Aragon 1995), or rotating drums (Courrech du Pont \textit{et al.} 2003). Here a novel recirculatory flume introduced by Armanini \textit{et al.} (2000) is used to obtain rectilinear open-channel flows in steady uniform conditions. The special flume permits flows with and without loose deposit at the base. Most importantly, the parallel flow
conditions allow local stresses within the bulk mixture to be estimated based on simple force balance considerations.

Recently developed digital imaging methods are then used to obtain a detailed characterisation of the local flow kinematics, as seen through the flume side wall. In contrast with previous attempts at applying particle-tracking techniques to dense granular flows (Natarajan et al. 1995; Hsiau & Jang 1998; Azanza et al. 1999), the methods are able to automatically resolve individual grain motions without being limited to sparse tracer particles or two-dimensional dispersions. This is achieved through the use of a robust pattern-matching algorithm based on the Voronoï diagram (Okabe et al. 1992; Capart et al. 2002). In addition to mean velocity and solid concentration profiles, the methods yield detailed granular temperature data (a measure of the strength of velocity fluctuations) averaged from samples of up to $10^5$ individual measurements per run.

Thanks to the flow configuration and imaging methods used, a comprehensive characterisation of the flow structure can be derived for different regimes. The data set documented in the present paper includes 16 different runs. For each of these runs, the measurements obtained include detailed depth profiles of mean velocity, solid concentration, and velocity fluctuations, complemented by estimates of shear and normal stresses. These data will be interpreted in light of available theories and compared with corresponding information regarding dry and bubbly flows. The ultimate aim of the data analysis will be to construct a global interpretation of the rheological structure of the observed flows.

The paper is organised as follows. Section 2 documents in detail the apparatus, materials and methods used for the experiments, as well the range of conditions covered by the tests. Section 3 is then devoted to the presentation of the measurements and section 4 to their
rheological interpretation. The concluding section, finally, aims to provide an overall synthesis.

2. Experimental conditions and methods

2.1. Laboratory flume and granular material properties

Figure 2 shows the special laboratory flume developed at the Hydraulics Laboratory of the Università degli Studi di Trento (Armanini et al. 2000) for the study of steady uniform granular-liquid flows. The apparatus is composed of two main components: the first is a glass-walled open channel, in which downslope flows are observed; the second is an external conveyor belt, connected to the channel by chute guides, which recirculates upslope both water and sediment. The channel can be tilted at angles going from 0° to 25°. It has a length of 6 m and a width of 40 cm, narrowed down to 20 cm for the present study by placement of a glass partition. The channel floor is artificially roughened by glued coarse sand. Imaging measurements are made 2 m upstream of the channel outlet, by filming the flow through the sidewalls and from the top. To view the basal flow condition at the centre of the cross-section, a small port-hole is pierced through the opaque channel floor. The external conveyor system has a length of 8.40 m, and is equipped with a 50 cm wide V-shaped flexible belt that can be operated at speeds of up to 5 m/s. The belt is aligned with the flume and tilted slightly more, in order for the material to be conveniently collected and refed. A hopper is placed at the flume outlet to collect water and grains. The system permits steady flows at discharges of up to 20 l/s, without limitations on the solids content.

During an experimental run, the set-up forms a closed loop in which a determined volume of water and granular material circulates. Provided it is sufficiently high, the speed of the conveyor belt does not noticeably influence the flow inside the channel. For the present experiments, fast belt speeds of 5 m/s and 2.5 m/s were adopted for test runs 5-11 and 38-106,
respectively (see Table 2). The throughput is then controlled solely by the volumes of water and sediment inside the system. When these parameters are kept constant (and aside from certain regimes which exhibit a pulsating behaviour, see infra), the recirculating flow settles to a steady state within a matter of minutes. Camera and lighting adjustments can then be made, and video footage acquired. At the end of a run, the channel outflow is diverted to a trap during a short timed interval (of 5 to 10 s) in order to independently measure the water and granular discharges.

Cylinder-shaped extruded PVC pellets were selected as granular material for the present tests. The particles have an equivalent spherical diameter $d = 3.7$ mm and a specific gravity $s = \rho_s / \rho_w = 1.54$. The material is lighter than natural sediment materials ($s \approx 2.65$), hence more readily entrained by water, facilitating observation. The white colour of the PVC pellets contrasts well with darker surrounding fluid, and is ideal for the automated imaging measurements presented in the next section. For quick characterisation of the flow deformation, the white particles were seeded with a small proportion of black particles (1%). These black particles facilitate eye observation, and can be manually tracked on video images to get a preliminary idea of the mean velocity profile across the depth. The sparse black grains are ignored by the automated algorithms, which instead capture the motions of the dense dispersion of white grains.

Various characterisation tests were performed to determine the properties of the individual particles and their bulk assembly. These tests and their results are summarised in Table 1 and Figures 3-4. Properties of special interest for the ensuing analysis include permeability, capillary rise, and critical angle of friction. Note that due to their relatively large size and greater elasticity, the PVC pellets differ significantly from the natural sediment grains handled
in standard geotechnical tests, and the corresponding procedures had to be adapted. For instance, typical methods of sample preparation involving controlled material compaction with a hammer could not be applied because of elastic rebound.

The simple shear tests illustrate both the usefulness of geotechnical characterisation and the need for caution. These tests were performed on our behalf by Dr. J.-F. Vanden Berghe, using a direct simple shear apparatus developed by the Norwegian Geotechnical Institute. The apparatus allows the shearing of a soil under conditions of constant volume and uniform shear strain throughout the sample. The specimen is confined within a wire reinforced rubber membrane which prevents radial deformation. Horizontal deformation is forced by translating the base relative to the fixed specimen top at the very slow speed of 0.03 mm/min, and an actuator continuously adjusts the normal force to prevent vertical deformation. The material is saturated with water, maintained at atmospheric pressure throughout the test. Shear and effective normal stresses are measured during the deformation. Details of the procedure and typical results for natural materials are documented in Dyvik et al. (1987), and Vanden Berghe et al. (2001).

Stress path results for the PVC material at different void ratios $e = (1-c_s)/c_s$ are plotted on Fig. 3. For dense sand of a given void ratio, stress paths typically converge onto a proportional path (a straight line passing through the origin) rising up to the so-called steady state point where stresses remain constant under continuing deformation (Castro & Poulos 1977). Tests for different void ratios yield separate steady state points which fall onto a unique straight line, and the inclination of this line defines the critical angle of friction (see e.g. Bauer 1996). For the PVC material, curved stress paths are obtained rather than straight lines. Nonetheless, the void ratio-dependent steady state points (reached after a shear strain of the order of 50 %) do approximately collapse onto a straight line, from which a critical friction angle $\theta_c \approx 31^\circ$ can
be deduced. This result is very useful because steady simple shear approximates closely the local deformation conditions in the flume experiments. This will allow comparison of the steady state stress ratios of the very slow direct shear tests with those of the channel flows. Some degree of caution will however be necessary when invoking the shear cell tests to interpret the flume experiments. Due to the small sample size, the smallest stresses that could be applied and resolved in the simple shear apparatus are roughly 100 times larger than those experienced by the material during the free-surface flow.

2.2. Voronoï imaging measurements

Flows are imaged through the transparent flume sidewall using a Redlake high-speed camera. At the chosen video rate of 250 frames per second, the digital images have a resolution of 420 × 480 pixels × 256 levels of grey. To prevent motion-induced blur, the camera shutter is set at 1/2500 s. For runs with a very thick moving layer, the top and bottom halves of the flow (with 20 % overlap) are filmed separately one after the other in quick succession. Finally a mirror can be placed above the channel to acquire top view images of the flow free surface. The image analysis is performed entirely off-line.

The motions and arrangements of visible particles are captured using the imaging methods of Capart et al. (2002), developed specifically for granular flows. The digital frames are first convoluted with a Laplacian-of-Gaussian filter (Jähne 1995). To highlight particle images, the filter width is set equal to the pixel diameter of the grains. Particles positions are then obtained by pinpointing local brightness maxima to subpixel accuracy. Tracking of the particles on successive frames is performed using a pattern-based matching principle based on the Voronoï tesselation (Okabe et al. 1992; Capart et al. 2002). The method is illustrated on Fig. 5: Voronoï diagrams are constructed on the sets of particle positions identified on separate frames; particles on one frame and the next are then paired based on the shape
similarity of their Voronoï cells; velocities are finally derived from the corresponding
displacements using the straightforward approximation

\[ \text{\( v_{i}^{(n+1) - v_{i}^{(n)} \)} \]

\[ \Delta t \]

where \( r_{i}^{(n)} = (x_{i}^{(n)}, y_{i}^{(n)}) \) is the 2D position of the \( i \)-th particle at time \( t^{(n)} \), \( v_{i}^{(n)} = (u_{i}^{(n)}, v_{i}^{(n)}) \) is the corresponding in-plane velocity, and the \( x \) and \( y \) axes are oriented in directions parallel and perpendicular to the mean flow direction, respectively (the 2D images of the near-wall flow do not allow measurement of the out-of-plane velocity component \( w_{i}^{(n)} \)). Transformation from pixel to physical coordinates is easily performed based on a calibrated scale factor and a rotation adjustment.

Estimate (1) is subject to three sources of error which may significantly affect the velocity fluctuation statistics: (i) pair mismatches arising from incorrect tracking can generate spurious outliers; (ii) the limited accuracy of particle positioning induces some artificial noise; (iii) the finite time resolution \( \Delta t \) damps high-frequency physical fluctuations. To deal with the first source of error, post-processing steps are used to eliminate mismatch outliers on the basis of trajectory and neighbourhood regularity (see Capart et al. 2002). To avoid statistical bias due to excessive pruning, the rejection criteria are set to be no stricter than twice the interquartile interval of the corresponding velocity distribution, and affect less than 5% of the data. Errors of type (ii) and (iii), by contrast, cannot be eliminated directly. Their effect on the flow statistics, however, can be estimated and compensated for. The procedures developed for this purpose are presented in the appendix.

Sample velocity measurements obtained with the above techniques are shown on Fig. 6. The results illustrate the various features which make particle tracking difficult in rapid granular flows. First, large interframe displacements (shown true to scale) relative to interparticle
distances are observed in the upper part of the flow (Fig. 6b). In such conditions, the simplest tracking algorithms based on minimum displacement (e.g. Guler et al. 1999) break down. The rather irregular motions documented on Fig. 6c,d also thwart more sophisticated PTV methods involving trajectory-based matching (e.g. Sethi & Jain 1987). Intense shear (up to ~40 s$^{-1}$ in this example) is further observed through most of the flow layer, a condition known to pose problems to conventional PIV methods (Huang et al. 1993). An attempt (Cenedese 2000) to apply PIV to the present flows failed for shear rates higher than 25 s$^{-1}$.

The Voronoi methods, by contrast, perform well throughout. Vector yields are excellent. On average for the run shown, 367 particles per frame are identified, 92 % of which are successfully tracked to the next frame.

The 2D images are also used to estimate the 3D volumetric granular concentration $c_s$ near the wall. As explained in Capart et al. (2002), the point density of visible particles per unit image surface is a poor indicator of 3D concentration, due to the prevalence of occlusion effects. Instead, characteristics of the local particle arrangement can be exploited using the Voronoi diagram built on the particle centres. As the solid concentration rises, excluded volume effects tend to organise a dispersion of identical particles into an increasingly ordered assembly. A measure of this ordering is furnished by the roundness of the Voronoi polygons (see Fig. 5b), defined as the ratio $\xi = 4\pi A / P^2$ where $A$ is the polygon area and $P$ its perimeter. On average, the solid concentration was found by Capart et al. (2002) to vary with roundness according to the following normalised power-law relation:

$$\frac{c_s}{c_s^{(rcp)}} = \left( \frac{\xi - \xi^{(min)}}{\xi^{(rcp)} - \xi^{(min)}} \right)^b$$

(2)

where $c_s$ = solid concentration, i.e. the volumetric fraction of solid material in the mixture; superscripts (rcp) and (min) designate the state of random close packing and the dilute state, respectively. As $c_s \to 0$, particle positions become uncorrelated and the corresponding
roundness value $\xi^{(\text{min})} = 0.73$ can be evaluated by Monte-Carlo simulations of a Poisson point process (see Okabe et al. 1992). The other parameters were calibrated by Capart et al. (2002) on the basis of fluidisation cell tests, yielding values $\xi^{(\text{rep})} = 0.84$ at $c_s^{(\text{rep})} = 0.64$ for the state of random close packing, and $b = 3.5$ for the power-law exponent. Derived from fluidisation cell experiments, relation (2) applies to well-agitated granular dispersions. It may thus be expected to hold for dilute shear flows. When particles are densely packed, however, effects other than volume exclusion can contribute to the ordering of nearby grains and bias the roundness estimator. The particle shape may for instance play a role at high concentrations. This can be seen in the lower part of Fig. 6a, where the cylindrical particles are seen to orient their axes preferentially in the direction perpendicular to the flow. As such effects can lead to unreasonably high concentration values when the roundness estimate is used, a cap must be enforced at the maximum concentration $c_s^{(\text{max})} = 0.69$ evaluated from compaction tests (see Table 1).

In order to ascertain their range of validity, Larcher (2003) compared concentration measurements obtained using the above monoscopic technique with stereoscopic imaging measurements (Spinewine et al. 2003). For steady uniform granular-liquid flows similar to those of the present paper, the results obtained using the two techniques are in good agreement (difference less than 0.05 between the two estimates) for concentrations up to $c_s = 0.55$. As expected, however, the results diverge beyond this limit and only the stereo technique yields reasonable values for higher concentrations. Developed more recently and more complicated to apply, the stereo technique was not used for the present measurements, based exclusively on monoscopic images. As a result, the validity of the concentration measurements presented below is restricted to dilute and moderate concentrations ($c_s < 0.55$). Measurements in the high concentration range $0.55 < c_s < 0.7$ are documented for
completeness, but a gray band will be used to highlight their uncertainty on the corresponding figures.

The typical data set for a single experimental run includes on the order of 100,000 velocity vectors (see Fig. 7) and cell roundness measurements. This is one order of magnitude higher than the 10,000 measurements identified by Louge and Jenkins (1997) as the minimum size needed to obtain statistically meaningful vertical profiles of granular temperature. Such large sample sizes are similar to those generated by computer simulations, and the same statistical techniques (e.g. Allen & Tildesley 1987; Rapaport 1995) are needed to extract useful information.

Binning is used to extract vertical profiles of velocity and concentration. The flow depth is subdivided into non-overlapping horizontal slices of thickness \( \delta = 1 \) to 2 particle diameters, into which measurements are distributed to obtain local averages (see Fig. 7). The average horizontal velocity \( \bar{u}_k \) within the \( k \)-th bin, for instance, is given by

\[
\bar{u}_k = \bar{u}(y_k) = \frac{\sum \sum \sum I_i^{(n)}(k) u_i^{(n)}}{\sum \sum II^{(n)}(k)},
\]

where \( y_k \) is the height of the centre of the \( k \)-th bin and \( I_i^{(n)}(k) \) is an indicator function defined by

\[
I_i^{(n)}(k) = \begin{cases} 
1 & \text{if } y_k - \delta / 2 \leq y_i^{(n)} < y_k + \delta / 2 \\
0 & \text{otherwise.}
\end{cases}
\]

For the volumetric concentration, bin averages \( \bar{\xi}_k \) are first obtained from roundness values \( \xi_i^{(n)} \), then transformed into solid concentration values \( c_s \) using relation (2).

Since some of the runs present a certain degree of unsteadiness (see infra), we also estimated
coarse-grained mean velocities $\bar{u}_k^{(l)}$, averaged over blocks of 10 successive image frames rather than the entire image sequence. Once these mean velocities are obtained, fluctuation velocities are estimated from

$$u_i^{(n)} = u_i^{(a)} - \bar{u}_i^{(a)},$$

where $\bar{u}_i^{(a)}$ is the mean velocity component taken at location $y_i^{(a)}$ and time $t_i^{(a)}$. Rather than simply using the average $\bar{u}_k^{(l)}$ of the corresponding bin $k$ and block $l$, we estimate the $\bar{u}_i^{(a)}$ by bilinear interpolation between the $\bar{u}_k^{(l)}$. Interpolation in space is especially important because the product of the mean flow gradient by the bin thickness is of the same order of magnitude as the velocity fluctuations. Simply subtracting the local bin average would thus add a spurious gradient contribution to the actual fluctuations.

In a manner analogous to (3), one could now estimate the mean-squared velocity fluctuations from

$$\overline{u'^2}(y_k) = \frac{\sum \sum I_i^{(n)}(k)u_i^{(n+2)}}{\sum \sum I_i^{(n)}(k)}$$

(see for example Hsiau & Shieh 1999). This estimate is however sensitive to measurement errors due to the limited accuracy and finite frequency at which particle positions are sampled. Based on the full velocity autocovariance function $\langle u_i^{(n)}u_j^{(n-k)} \rangle$, we show in appendix how it is possible to compensate both errors and obtain a more reliable estimate. The mean-squared velocity fluctuations reported hereafter are all obtained using this novel correction procedure.

2.3. Flow regimes

Experimental runs can first be subdivided into solid bed and loose bed runs. Flows over solid bottom occur when the granular phase deforms at finite shear rates down to the fixed floor of
the flume. Flows over loose beds, by contrast, occur when a static deposit forms underneath
the moving granular-liquid layer (see Figure 0). Inside this loose granular deposit, shear rates
decrease to zero before reaching the flume floor. The thickness of the deposit is not
predetermined, but adjusts to the flow through erosion or accretion until a state of dynamic
equilibrium is reached (Egashira et al. 2001). Intermediate conditions can also be observed, in
which the flowing layer is in contact with the solid floor in the central portion of the flume (as
seen through the port-hole) while static deposits line the corners between the floor and
sidewalls. Ancey (2002) documented a similar situation for dry granular flows. To avoid
unnecessary complications, test runs in which this hybrid situation was encountered are not
included in the present data set.

The macroscopic behaviour of the experimental flows can be described using three
non-dimensional parameters: (i) the inclination angle of the channel \( \beta \); (ii) the volumetric
transport concentration \( C_s = \frac{Q_s}{Q} \) where \( Q_s \) is the solid discharge and \( Q \) the total
discharge (grains + water); (iii) the Froude number

\[
Fr = \frac{U}{\sqrt{gH}},
\]

where \( H \) is the thickness of the flow layer (not including the static deposit), \( U \) is the
depth-averaged velocity of the flow, and \( g \) is the gravitational acceleration. The transport
concentration can be measured in two different ways: first, as the ratio

\[
C_s^{(bulk)} = \frac{Q_s^{(bulk)}}{Q^{(bulk)}}
\]

of solid to total discharge measured in bulk at the flume outlet; secondly, by integrating the sidewall imaging measurements over depth according to

\[
C_s^{(imag)} = \frac{q_s^{(imag)}}{q^{(imag)}} = \frac{\int_{y'}^{(max)} c_s \bar{u} dy}{\int_{y'}^{(max)} (c_s + c_w) \bar{u} dy},
\]

where \( y'^{(min)} \) and \( y'^{(max)} \) are lower and upper limits of the imaging window, \( q_s \) and \( q \) are
discharges per unit width, \( c_w \) is the water content \( (= 1 - c_s \) wherever the granular phase is saturated with water). Here the mean velocity of water is assumed to coincide with the mean velocity \( \bar{u} \) of the grains. Based on the permeability measurements of Fig. 4, the specific discharge due to relative motion between the water and the grains was computed for all runs. It was checked to amount to no more than 10% of the corresponding total discharge.

For solid bed runs, the depth \( H \) can be measured manually as the height of the flow free surface above the flume bottom, then the mean velocity estimated from \( U = \frac{Q^{(\text{bulk})}}{BH} \) where \( B \) is the channel width (20 cm). For loose bed runs, the thickness of the flow layer is more difficult to estimate since the boundary between moving and motionless grains is not sharply defined (see e.g. Fig. 6). Precise definitions of \( H \) and \( U \) can nonetheless be introduced based on depth-averaged momentum and kinetic energy:

\[
\left( C_s \rho_s + (1 - C_s) \rho_w \right) U H = \Pi = \int_{y^{(\text{min})}}^{y^{(\text{max})}} \left( C_s \rho_s + (1 - C_s) \rho_w \right) \bar{u} dy, \tag{9}
\]

\[
\left( C_s \rho_s + (1 - C_s) \rho_w \right) \frac{H U^2}{2} = K = \int_{y^{(\text{min})}}^{y^{(\text{max})}} \left( C_s \rho_s + (1 - C_s) \rho_w \right) \frac{\bar{u}^2}{2} dy, \tag{10}
\]

where \( \Pi \) and \( K \) are the depth-integrated momentum and kinetic energy per unit bed surface, and \( \rho_s \) and \( \rho_w \) are the densities of the granular material and water, respectively. In all three definitions (8), (9) and (10), the integration limits \( y^{(\text{min})} \) and \( y^{(\text{max})} \) do not need to correspond to any physically meaningful boundary. In effect, material contributions are weighted according to their participation in the flow motion, as measured by the local mean velocity \( \bar{u} \). Estimates (9) and (10) are inspired from analogous treatments of boundary layers (e.g. Liggett 1994) and turbidity currents (Parker 2000). When detailed imaging measurements are available, they are also applicable to the solid bed runs.

The above macroscopic parameters can now be used to contrast the different flow regimes. A plot of Froude number \( Fr \) versus channel inclination \( \beta \) is first given on Fig. 8. Very
different trends are observed depending on the state of motion along the floor of the flume. For the solid bed runs, the Froude number grows with increasing channel inclination. This is in accordance with intuitive expectations for open-channel flows. For the loose bed runs, by contrast, the Froude number decreases as the channel becomes steeper! This apparent paradox can be understood by examining the role of the granular concentration. In the solid bed case, the inclination is assigned and the transport concentration (adjusted by altering the solid-liquid ratio inside the flume) affects only moderately the flow behaviour. As shown in Fig. 8, flows with a high transport concentration (in the range $C_s = 0.53 - 0.55$) are somewhat slower than flows with a low transport concentration (in the range $C_s = 0.51 - 0.53$), due to the contribution of the granular phase to the flow resistance. The impact of concentration on the Froude number is limited, however, because the flow depth is constrained by the solid bottom.

The situation in the loose bed case is quite different. First, the slope is no longer predetermined, but becomes coupled with the transport concentration. The flowing layer must find an equilibrium with the underlying static bed, reached when the flow neither erodes nor accretes the loose deposit. For a given transport concentration, there is a unique inclination at which this uniform, equilibrated state is obtained. Figure 9 shows the resulting relationship between slope and transport concentration: overall, higher solid to liquid ratios are needed to achieve balance when the inclination steepens. The influence of these higher solid concentrations on the flow rheology leads to slower flows. Furthermore, the thickness of the flowing layer also adjusts to the flow conditions: thicker flowing layers are obtained for steeper inclinations. The slower, thicker flows obtained on steeper slopes are thus characterised by significantly reduced Froude numbers, as illustrated on Fig. 8.

The loose bed data plotted on Figures 8 and 9 exhibit a rather sharp break of trend at a
channel inclination of around 8 degrees. This corresponds to a transition of the flow from oversaturated to undersaturated conditions. Below an inclination of 8 degrees, water more than saturates the entire granular layer: a clear water layer flows over a fluid-driven sheet of granular material supported by contacts with the stagnant bed. This regime is called *immature flow* in the debris flow literature (Takahashi 1991), and *sheet-flow* in the context of coastal sediment transport (see e.g. Jenkins & Hanes 1998). Above 8 degrees, by contrast, water does not saturate the whole moving layer: a partially emerged assembly of grains translates rigidly over a liquid-granular shear layer. This is the so-called *plug flow* regime. The intermediate state, or *mature flow* regime, features a saturated moving layer, composed throughout of a mixture of liquid and grains. The transport concentration at which full saturation occurs lies around $C_s = 0.5$.

As schematised on Fig. 10, four main regimes of interest can thus be distinguished: (a) loose bed, immature sheet-flow; (b) loose bed, mature flow; (c) loose bed, plug flow; and (d) solid bed flow. A more complete typology would include hybrid flow states featuring mixed loose-solid bed conditions across the flume width, and would further differentiate basal shear flows according to the degree of saturation. In the present work, we do not pursue such refinements and concentrate our attention on the micromechanics of loose bed flows. The ensuing analysis will thus focus on loose bed cases (a) to (c), with the solid bed case (d) examined for comparison purposes. Over solid beds, we restrict our attention to flows which are close to full saturation. Out of a catalogue of more than 100 experiments, 4 runs of each type are chosen for detailed imaging analysis. These 16 runs and their properties are listed in Table 2.

**2.4. Steady uniform, planar flow approximation**

Throughout our physical interpretation of the imaging measurements, we will assume that the
observed mean flow field is steady in time, uniform in the longitudinal direction, and homogeneous in the transverse direction. Variations along the direction normal to the bed will be the only ones considered, greatly simplifying the picture. In particular, the stress state within the flow can then be deduced from simple equilibrium considerations. Here we present various checks that were made to verify and qualify this idealisation.

As illustrated on Fig. 7b, the steady uniform character of the flow can be ascertained by examining mean velocity profiles averaged over separate time intervals (here 10 successive intervals of 1/10 the total footage duration for run 085). The longitudinal velocity profiles fluctuate within a certain range, but indicate no systematic drift that would indicate a sustained acceleration or deceleration of the flow. Plotted on the same Fig. 7b, normal-to-bed velocity profiles fluctuate likewise around their long term average, but the latter is very close to zero. If the average flow was not longitudinally uniform, converging or diverging streamlines would lead to non-zero mean velocities normal to the bed. Similar profiles are obtained for the other runs, although some of them feature stronger fluctuations.

To document the strength and nature of these fluctuations, Figure 11 presents time histories of the observed kinetic energy for representative flows of each of the four types. Here the specific kinetic energy is defined as

$$E(t^{(n)}) = \frac{1}{2} \sum \left( u_i^{(n)2} + v_j^{(n)2} \right)$$

i.e. the sum of the kinetic energies of the particles observed inside the imaging window at any given time $t^{(n)}$, divided by particle mass. This indicator was proposed by Campbell (1989) to verify steady state in the context of granular dynamics simulations. It is verified that, aside from some fluctuations, the specific kinetic energy remains steady over the measurement time frame. Some low frequency pulsations are nonetheless observed for the solid bed runs.
Among all the tests, run no. 46 exhibited the strongest fluctuations (see Fig. 11a). This run also made clear the source of the unsteady behaviour: roll wave instability of the steep flows. In-phase variations of velocity and depth are a characteristic signature of roll waves. As shown on Fig. 12, this feature shows up particularly vividly on a plot of specific kinetic energy $E$ versus specific potential energy $P$ of the observed particles, where the latter is estimated at any given time $t^{(n)}$ from

$$P(t^{(n)}) = g \sum_i y_i^{(n)}.$$ 

(12)

Even for the most extreme case, however, the pulsations are moderate and superposed onto a recognisable steady state. This is possibly because the channel is too short for roll wave coarsening to develop (Chang et al. 2000). Similar free surface instabilities have been observed for chute flows of dry granular materials (Prasad et al. 2000; Louge & Keast 2001).

For flows over loose beds (Fig. 11b), steady state is closely approximated, except for some pulsations observed for the immature debris flow runs. Froude numbers greater than 3 for run 46 and greater than 1 for run 85 lie within the range of possible occurrence of rollwaves and antidunes, respectively.

The assumption that the flow is homogeneous in the transverse direction is most subject to caution. Three different means are used to ascertain departures from this assumption: 1) measurements of free surface velocities across the channel width; 2) observations of the state of motion at the bottom boundary through the flume floor porthole; 3) comparison of depth-integrated side-wall measurements with bulk volumetric measurements made at the flume outlet. Measurements of free surface velocities imaged from the top yield nearly flat velocity profiles across the width, indicating a high degree of transverse uniformity. This is illustrated on Fig. 11a-b for run 41, and similar results were obtained for earlier runs over both solid and loose beds. It would be tempting to assume that this high degree of transverse
uniformity recorded at the free surface applies as well to the flow interior, but this is not the case. Bottom observations through the flume floor porthole show that the state of motion at the flume centreline can be quite different from the motion close to the sidewall. For cases in which hybrid loose-solid bed conditions occurred, a static layer having a thickness of up to 6 cm could be observed at the sidewall while the lowermost grains at the centre of the channel were still moving (as seen through the bottom porthole). Thus corner effects appear to endow the flow with a significant three-dimensional character at the base of the moving layer. Similar features have been documented by Spence and Guymer (1997) and Ancey (2002).

A further clear indication of transverse non-uniformity is provided by the discharge measurements. The total volumetric discharge $Q^{(\text{bulk})}$ measured at the outlet provides a reference value. On the other hand, an indirect estimate can be obtained by extrapolating sidewall imaging measurements to the whole flume width $B$. This estimated discharge is given by $Q^{(\text{imag})} = q^{(\text{imag})} B$, where $q^{(\text{imag})}$ is the depth-integrated flux defined in eq. (8). Figure 14 plots the measured values of $Q^{(\text{imag})}$ and $Q^{(\text{bulk})}$ against each other. For transversely uniform flows, one would expect $Q^{(\text{imag})} \approx Q^{(\text{bulk})}$. Instead, the sidewall imaging estimate $Q^{(\text{imag})}$ is found to underestimate the total discharge $Q^{(\text{bulk})}$ by some 30 %. This represents a much greater discrepancy than would be anticipated based on free-surface velocity data alone, which underlines that departures from uniformity become more severe towards the bottom.

Although measurements of the longitudinal velocity inside the flow are not available, it is possible to reconstruct a hypothetical velocity distribution which fits the above observations. The paraboloid surface illustrated on Fig. 12c was tuned to roughly match the observed pattern of motion along the top, side and bottom in a typical loose bed run. For this reconstruction as in the experiments, the apparent sidewall discharge is 30 % lower than the true total discharge. To check that the planar flow approximation remains reasonable despite
such deviations from transverse uniformity, force balance considerations will be required. This will be addressed in the next section.

Even if sidewall measurements underestimate the overall flow rate, they may still reliably capture local flow properties such as the solid concentration and the granular temperature. The sidewall estimates of transport concentration, for instance, are in relatively good agreement with bulk outlet measurements, as shown on Fig. 14b. The exceptions are the dense plug flow runs (data circled on Fig. 14b), but the large discrepancies for these data are likely due to the measurement technique (the pattern-based concentration estimate breaks down in the dense limit) rather than to sidewall effects. Using computer simulations, Louge and Jenkins (1997) checked that the granular temperature is not strongly affected by the presence of a smooth sidewall. At least for rapid flow conditions, a good correspondence between near wall and interior flow states should thus be obtained.

3. Measured profiles and stress estimation

3.1. Depth profiles of velocities and concentration

Depth profiles of longitudinal velocity $\bar{u}$, shear rate $\dot{\gamma} = \partial \bar{u} / \partial y$, root-mean-squared fluctuation velocity $T^{1/2} = \sqrt{\frac{1}{2} \left( u'^2 + v'^2 \right)}$, and granular concentration $c_s$ are plotted on Figures 15 to 18 for the four different flow regimes. For three-dimensional granular flows, the granular temperature $T$ is defined as $T = \frac{1}{2} \left( u'^2 + v'^2 + w'^2 \right)$, however the third component of velocity $w$ is not measured in the present experiments, and the rms fluctuation velocity is given instead as the square root of $T = \frac{1}{2} \left( u'^2 + v'^2 \right)$. The plots are made dimensionless using a combination of depth-averaged and viscous scales. First, depth-averaged quantities $H$ and $U$ are used to normalise the depth axis $y$ and the mean velocity $\bar{u}$. Secondly, to obtain
dimensionless descriptors of the shear rate and fluctuation velocity, we define the two Stokes numbers

\[ St = \frac{1}{18} \frac{\rho_s}{\rho_w} \frac{d^2 \dot{\gamma}}{\nu_w} \quad \text{and} \quad St' = \frac{1}{18} \frac{\rho_s}{\rho_w} \frac{dT^{1/2}}{\nu_w} \]  

(13a,b)

where \( \nu_w \) is the kinematic viscosity of the liquid, and \( d \) the particle size. Let a particle move at velocity \( V \) with respect to the surrounding liquid. The Stokes number represents the ratio of two time scales: the time scale \( \frac{1}{18} \frac{\rho_s d^2}{\mu_w} \) of deceleration due to Stokes’ viscous drag force \( 3\pi \mu_w dV \), and the time \( d/V \) needed for a particle to move a distance of one diameter through the liquid. Expressions (13a) and (13b) result from choosing as representative velocity \( V \) either the relative velocity \( d\dot{\gamma} \) between adjacent sheared rows of particles, or the fluctuation velocity \( T^{1/2} \).

On Figures 15-18, the data points for a specific run are given a unique symbol shape, and are plotted as filled or hollow symbols according to a criterion which will be introduced at a further stage of the analysis. A precise legend establishing the correspondence between runs and symbols is included in Table 2. The choice of dimensionless scales can be motivated at this stage. The mean velocity profiles are normalised using depth-averaged variables in order to highlight the high degree of macroscopic similarity between different test runs corresponding to the same flow type. As shown on panel (a) of each of the Figures 15-18, the mean velocity profiles collapse onto well-defined curves for each of the flow regimes. The solid concentration, shear rate, and granular temperature data (see panels (b)-(d) of Figures 15-18), by contrast, are not normalised by their depth-averaged values. These measurements
are plotted in a way which reflects the local rheology of the flow rather than its global features.

In the present experiments, all runs are conducted with the same granular material and liquid, hence the only evolving parameters in dimensionless Stokes numbers (13a) and (13b) are $\dot{\gamma}$ and $T^{1/2}$. The dimensionless scales are thus not strictly necessary to compare measurements within the present dataset. They are used here to facilitate comparisons with data obtained by other investigators for different solid particles and liquids. To highlight the fact that both dimensionless and dimensional representations are equivalent for variables $\dot{\gamma}$ and $T^{1/2}$, scales are given in dimensional units as well on panels (c) and (d) of Figures 15-18.

The various profiles bring out the characteristics of each flow regime. For the sheet-flow runs (Fig. 15), the mean velocity profiles (panel a) are convex, with maximum shear rates (panel c) observed in a sub-layer close to the free-surface. The solid concentration (panel b) evolves monotonously over depth, with maximum values in the closely packed bed and very small values close to the free surface. The concentration profiles are close to linear in the flowing region, in accordance with the observations of Shook et al. (1982). Unlike the kinetic theory computations of Jenkins and Hanes (1998) and Hsu et al. (2003), the present results do not exhibit any conspicuous shoulder region of constant concentration within the flowing layer. Finally the fluctuation velocity profiles of panel (d) present an interesting region of nearly constant values spanning much of the flow depth, with decaying values in the vicinity of the bed. Such a region of constant fluctuation velocity at the top of the sheet-flow layer was
predicted by the computations of Jenkins and Hanes (1998; see also Hsu et al. 2003).

For the mature flow runs (Fig. 16), the maximum shear rate occurs at mid-depth, where a change of concavity of the mean velocity profile occurs (see Fig. 16a,c). The velocity profile is close to linear over much of the shear layer depth, tapering off at the top and bottom. Comparison of panels (c) and (d) of Fig. 16 shows that the shear rate and fluctuation velocity profiles are similar to each other over much of the flow depth. The most significant departure from similarity occurs at the top of the flow, where the shear rate approaches zero but the fluctuation velocity reaches a new maximum. A similar inflection near the free surface was observed by Silbert et al. (2001) in their simulations of dry granular flows on a rough incline, without any lateral boundary. In the present case, however, this effect seems to be associated with stick-slip motion of partially emerged, but wet particles adhering intermittently to the flume sidewall. Images filmed from the side show these motions clearly. On the other hand, footage filmed from above indicates that the free surface fluctuation velocities are much lower away from the side wall. The high fluctuation velocity measured here at the top thus appears to be an artefact due to the lateral boundary.

As seen on Fig. 17, the plug flow runs feature a mid-depth shear layer comprised between two nearly rigid blocks of densely packed grains. The plug layer at the top translates at constant velocity, while the loose bed stays motionless at the bottom (see Fig. 17a). The corresponding shear rate and fluctuation velocity profiles (Fig. 17c,d) have a Gaussian-like shape with
values reaching a maximum in the middle of the shear layer, and decreasing to zero in both the overlying plug and underlying bed. The shear rate and fluctuation velocity curves are seen to present a very high degree of similarity. On panel 17b, pattern-based solid concentration measurements are seen to break down for the dense plug flow runs. This is indicated by the flat profiles capped at the maximum admissible value $c_{s}^{(\text{max})} = 0.69$.

For the above three regimes, the flows find an equilibrium with the erodible static bed. The lowermost parts of the curves of Figures 15-17 (panels a, b, and c) are very similar: they asymptotically connect to a state of no motion at the bottom. As shown on Fig. 18, profiles for the solid bed flows are quite different. The maximum shear rates are observed at the bottom of the layer, close to the flume floor, rather than at the top or at mid-depth (Fig. 18a,c). Solid concentrations are minimum close to bottom (Fig. 18b). Finally, the fluctuation velocity (and hence the granular temperature) is maximum at the base of the flowing layer (Fig. 18d). Very different from the loose bed results of Fig. 15-17, the present solid bed measurements are similar to computed profiles derived from discrete particle simulations of dry granular flows (Campbell 1990, Silbert et al. 2001).

For further observations regarding the global behaviour of the different flow regimes, the reader is referred to Armanini et al. (2000) and Larcher (2002). Our purpose for the remainder of the present analysis is to probe the flow behaviour on the local scale. Specifically, our aim is to examine the rheology of the flow: relate the local stress state to the local concentration
and motions of the liquid-bathed solid grains. Preliminary findings from this effort were reported in Capart et al. (2000).

3.2. Estimation of the granular stresses

Assuming that the flows are steady, uniform, and planar, granular stresses can be determined based on force balance in directions normal and parallel to the bed. In the present section, the procedure adopted to estimate the stresses is first explained. The procedure does not take into account non-equilibrium pore pressures or friction along the side walls. These effects will then be examined to check that their influence on the results is limited.

In order to determine the granular stresses, both the solid and liquid concentrations must be known over the flow depth. The imaging measurements yield solid concentration profiles $c_s(y)$. The water concentration profiles $c_w(y)$ are then estimated as follows. Below the saturation line $y_w$ (identified by visual inspection of the video images), the water content is simply $c_w = 1 - c_s$. Above the saturation line, on the other hand, it is set to the value $c_w = S_r = 0.08$ of the specific retention (see Table 1). Assuming that the flow is planar and steady uniform, the shear stress $\tau(y)$ at a given level $y$ balances the longitudinal component of the overlying weight according to

$$\tau(y) = \int_y^{y_{\text{max}}} \left( c_s(y) \rho_s + c_w(y) \rho_w \right) g \sin \beta \, dy,$$

where $\beta$ is the inclination of the flow with respect to the horizontal. Likewise the total normal stress $\sigma(y)$ at level $y$ balances the normal component of the overlying weight.
\[ \sigma(y) = \int_y^{y_{(\text{max})}} \left( c_s(y) \rho_s + c_w(y) \rho_y \right) g \cos \beta \, dy. \] (15)

To obtain the effective normal stress representing the contribution of granular contacts, one must then subtract the pore water pressure

\[ \sigma' = \sigma - p_w. \] (16)

When the entire granular layer is submerged and saturated with water, the pore water pressure is simply \( p_w = \rho_w g (y_w - y) \cos \beta \), where \( y_w \) is the water surface position. The effective normal stress is then

\[ \sigma' = \int_y^{y_w} c_s(y) (\rho_s - \rho_w) g \cos \beta \, dy. \] (17)

which balances the submerged weight of the overlying grains. On the other hand, when the top of the granular layer de-saturates and partly emerges above the water level \( y_w \), the pore pressure at level \( y < y_w \) becomes

\[ p_w = \rho_w g (y_w - y) \cos \beta - \rho_w g h_c. \] (18)

where \( h_c = \) height of capillary rise. Upon partial de-saturation, the full weight of the emerged grains, the retained water, and the weight of the capillary water must be supported by granular contacts underneath (see e.g. Das 1990). Capillary rise was observed for a partially emerged plug flow run by connecting at position \( x_p \) a piezometric tube of large diameter to the sidewall of the flume. The height of capillary rise \( h_c \) was then measured during flow as the difference in elevation \( z_w(x_p) - z_p \) between the saturation line and the water level in the tube. The height obtained in this fashion was found to lie between 5 and 6 mm, corresponding to the lower end of the range measured in static tests (see Table 1). A value \( h_c = 5 \) mm is retained for the present calculations.
Two important assumptions made in the above analysis need to be critically examined. First, pore pressures \( p_w \) are assumed to be hydrostatic. Non-equilibrium pore pressures associated with volumetric expansion or contraction of the granular phase are thus neglected. These effects are known to be important for unsteady debris surges in which the granular matrix has a low permeability (Iverson 1997; Major & Iverson 1999). They are however expected to be very limited in the present experiments involving steady uniform flow of coarse grains and a low-viscosity liquid. A time scale for pore-pressure relaxation can be estimated as (Spence & Guymer 1997)

\[
    t_{\text{relax}} = \frac{\rho_w}{\rho_s - \rho_w} \frac{c_b - \overline{c}_s}{c_b \overline{c}_s} \frac{H}{k}
\]

(19)

where \( H \) is the depth of the flowing liquid-granular layer, \( k \) and \( \overline{c}_s \) are respectively the permeability and average solid concentration in this flowing layer, and \( c_b \) is the solid concentration in the resedimented static bed. A corresponding length scale is given by

\[
    \ell_{\text{relax}} = U t_{\text{relax}} \quad \text{where } U \text{ is the mean velocity of the flowing layer. For the present experiments, representative values are (see Tables 1 and 2)} \quad \rho_s / \rho_w = 1.54, \quad \overline{c}_s = 0.5, \quad k = 7 \text{ cm/s}, \quad c_b = 0.6, \quad H = 8 \text{ cm and } U = 30 \text{ cm/s}. \text{ Pore pressure relaxation effects can therefore be expected to act on scales of the order of } t_{\text{relax}} = 0.6 \text{ s and } \ell_{\text{relax}} = 20 \text{ cm, whereas our experimental flows are allowed to develop on time scales of a few minutes and length scales of a few meters. This appears to be more than enough for pore pressure to equilibrate over the depth and attain the hydrostatic values corresponding to steady uniform conditions.}
A second important assumption is that forces due to the sidewalls can be neglected in the longitudinal force balance. For dry granular flows over self-forming stationary heaps in channels of finite width, Taberlet et al. (2003) found that lateral friction effects could play a strong role. Those experiments, however, concern thin channels for which the width $W$ is small in proportion to both the grain diameter $d$ ($W/d \sim 10$) and flow depth $H$ ($W/H = 2/3$ to 2).

In the present experiments, the width to diameter ratio $W/d$ is around 70, hence arching effects should be much weaker. Runs classified as sheet-flow and mature are also characterised by higher aspect ratios $W/H = 3$ to 10. For these runs, it appears reasonable to neglect the effect of lateral friction compared to the influence, say, of the liquid content, which plays a key role in our experiments.

This can be checked by looking at the macroscopic force balance for these runs (Takahashi 1991; Armanini et al. 2000). Consider a flowing layer of grains over a loose slope of inclination $\beta$, fully saturated with water, having thickness $H$ and average solid concentration $\bar{c}_s$ assumed to coincide with the transport concentration $C_s$. At the base of the layer, frictional conditions apply, and we expect a Coulomb-Terzaghi relationship between shear stress and effective normal stress

$$\tau = \tan \theta_c \sigma'$$

where $\theta_c = 31^\circ$ is the critical friction angle derived from the simple shear tests (see Table 1).

On the other hand, assuming steady uniform conditions, shear and normal effective stresses can be estimated by balancing forces applied to the flowing layer. Neglecting friction along
lateral walls, the forces involved are the pull of gravity and stresses along the base. Hydrostatic pore pressures are also assumed. The resulting expressions are

\[ \tau = \left[ \rho_w + (\rho_s - \rho_w)C_s \right] gH \sin \beta \]  
(21)

\[ \sigma' = (\rho_s - \rho_w)C_s gH \cos \beta \]  
(22)

Substitution into the Coulomb-Terzaghi expression yields a relationship between the inclination \( \beta \) and the transport concentration \( C_s \):

\[ \tan \beta = \frac{(\rho_s - \rho_w)C_s}{\rho_w + (\rho_s - \rho_w)C_s} \tan \theta_s. \]  
(23)

The resulting curve is plotted on Fig. 9 against the measured data. Good agreement is found up to an inclination \( \beta = 8 \) degrees. This is the slope beyond which a plug flow regime is observed, and the relationship no longer applies because part of the moving granular layer desaturates. At least for the sheet-flow and mature runs, a macroscopic force balance conducted by neglecting side wall and non-equilibrium pore pressure contributions is thus found to be consistent with the observations.

For the plug flows, the ratio \( W/H \) drops to 2 and below. Also, partial desaturation of the granular layer leads to a corresponding change in the nature of the contacts between the grains and the walls. For these two reasons, a more significant influence of the sidewalls on the force balance can be expected. A quantitative estimate of this influence could be obtained if measurements were available for various flume widths (as in Taberlet et al. 2003). Since such experiments were not performed, even for the plug flow runs we prefer to neglect sidewall effects altogether.
4. Rheological interpretation

Subject to certain limitations highlighted above, measurements of mean velocity, fluctuation velocity, and solid concentration are obtained for the present experiments, and complemented by estimates of shear and normal stresses. The uniform flow flume thus provides sufficient information to serve as a rheometer. The present section now seeks to exploit this information to construct an interpretation of the flow structure.

4.1. Transition between frictional and collisional behaviours

The relationship between stress and shear rate will first be examined in light of the ideas proposed by Bagnold (1954). To describe the stresses induced by shearing a liquid-saturated granular dispersion, Bagnold introduced a non-dimensional shear rate defined as

\[ Ba = \frac{\rho_d \lambda^{1/2} d^2}{\mu_w} \gamma, \]

subsequently called the Bagnold number by Hill (1966). Aside from the absent numerical factor \(1/18\), the Bagnold number \(Ba\) differs from the Stokes number \(St\) defined earlier only by the presence of a non-dimensional factor \(\lambda\) called the linear concentration. This factor was introduced by Bagnold to account for the effect of solid concentration \(c_s\) and is defined as

\[ \lambda(c_s) = \frac{1}{(c_{s0}/c_s)^{1/3} - 1}, \]

where \(c_{s0} = 0.74\) is the solid concentration corresponding to a crystalline close packing of
equal spheres. He further introduced non-dimensional shear and effective normal stresses

\[
\frac{\tau}{\mu_s^\lambda/(\rho_s d^2)} \quad \text{and} \quad \frac{\sigma'}{\mu_s^\lambda/(\rho_s d^2)}. \tag{26a,b}
\]

The term *dispersive stress* was used by Bagnold to refer to the rate-dependent collisional normal stress observed in his experiments. Here we retain the more general term *effective normal stress* used in soil mechanics to describe the normal stress associated with particle interactions regardless of the nature of these interactions. On Figures 19-21, we use these quantities to characterise the relationship between stresses and shear rate observed in the present experiments. Figures 19, 20, and 21 respectively show the dimensionless shear stress, dimensionless normal stress, and stress ratio \( \tau / \sigma' \) over the full range of Bagnold numbers \( Ba \) covered by the flow profiles of Figures 15-18.

Based on his experiments performed with neutrally buoyant wax particles bathed in either water or a glycerine-water-alcohol mixture, Bagnold identified two different rheological behaviours: a macro-viscous regime (for \( Ba < 40 \)) characterised by a linear relation \( \tau \propto \dot{\gamma}^1 \) between stress and shear rate, and a grain-inertia regime (for \( Ba > 450 \)) where stresses vary as the square of the shear rate \( \tau \propto \dot{\gamma}^2 \). On the log-log plots of Fig. 19 and 20, these relationships correspond to straight lines with inclinations 1 and 2, respectively. For the present experiments performed with particles heavier than the surrounding liquid \( (\rho_s / \rho_w = 1.57) \), the measured data suggest a different picture. For large Bagnold numbers \( (Ba > 1000) \), the data are indeed consistent with a quadratic relation \( \tau \propto \dot{\gamma}^2 \) between shear stress and shear rate, corresponding to Bagnold’s inertial regime. However for lower Bagnold
numbers (slower rates of deformation), the switch which occurs is not to a linear relationship, but to a band of roughly zero slope implying rate-independent shear stresses $\tau \propto \dot{\gamma}^0$.

Turning our attention to the stress ratio $\tau/\sigma'$ (Fig. 21), Bagnold suggested constant values $\tau/\sigma' \approx 0.75$ and $\tau/\sigma' \approx 0.32$ applicable to the macro-viscous and grain-inertia regimes, respectively. Here again a different behaviour is recorded in the present experiments. For slow deformation, the data do cluster around a roughly constant stress ratio. The level of this stress ratio further appears reasonably well approximated by the value $\tan 31^0 = 0.6$ applicable at steady state to the very slow deformation rates of the simple shear tests (see Table 1). This correspondence is observed despite the fact that the stresses applied in the simple shear tests have a much greater magnitude than those experienced in the channel.

For larger Bagnold numbers, however, stress ratios do not lock onto another constant value. While both the shear and normal stress vary roughly quadratically with the shear rate, their precise ratio is neither constant nor dependent solely on the shear rate. Rather, measurements for differing flow conditions diverge from each other in a way that indicates that an important degree of freedom is missed. For dry granular flows, kinetic theories hold that the extra variable which is involved is the granular temperature, a measure of the intensity of velocity fluctuations. These fluctuations are likewise expected to play a role when rapidly sheared grains are bathed in a liquid. Thus our rapid shear data are consistent with a collision-dominated inertial regime, but the observed behaviour is not quite as simple as
Bagnold’s description would imply.

To summarise, for slow shear ($Ba < 1000$) the data present all the characteristics of Coulomb-type frictional behaviour: shear and normal stresses proportional to each other and independent of the rate of shear. For rapid deformation, on the other hand, the stresses scale quadratically with the shear rate in accordance with expectations for a collision-dominated inertial regime. Putting the pieces together, the data thus indicate a transition from frictional to inertial behaviour instead of the transition from macro-viscous to inertial behaviour identified by Bagnold. A sharp contrast between frictional and collisional behaviours was similarly observed by Aharonov and Sparks (1999) in their numerical simulations of sheared granular assemblies.

This frictional-collisional picture presents one puzzling feature. Nowhere does one observe the scaling $\tau \propto \mu_w \dot{\gamma}$ expected of a Newtonian-like macro-viscous fluid. In the frictional state, the shear and normal stresses relate to each other rather than to the rate of deformation. In the collisional state, on the other hand, the water viscosity $\mu_w$ drops out from the quadratic scaling

$$\frac{\sigma'}{\mu_w^2 \lambda / (\rho_s d^2)} \propto Ba^2 = \left( \frac{\rho_s \lambda^{1/2} d^2}{\mu_w} \right)^2 \dot{\gamma}^2.$$

Hence for both frictional and collisional states, the viscosity of the liquid plays no apparent role in controlling the stresses. And yet the transition from frictional to collisional behaviour appears well-defined on Bagnold plots constructed on the basis of viscosity scalings.
Specifically, the transition seems to occur around a Bagnold number $Ba^* \approx 1000$. This means that while the viscosity of the pore liquid controls neither the frictional nor the collisional behaviour, it may determine the point at which the sheared mixture switches from one behaviour to the other. In other words the frictional-collisional transition could be controlled by viscous effects through a mechanism that remains to be elucidated.

At this point, we have not yet exploited the information provided by the direct measurements of granular temperature. We will now use these data to try to refine this emerging picture of the flow rheology.

4.2. Relationship between shear rate and fluctuation velocity

Figure 22 documents the observed relationship between shear rate $\dot{\gamma} = \partial \bar{u} / \partial y$ and square root of the granular temperature $T^{1/2}$. Both quantities are plotted in their dimensionless Stokes number forms $St$ and $St'$ defined in equations (13a,b). An alternative way to compare the mean rate of shear $\dot{\gamma} = \partial \bar{u} / \partial y$ with the intensity of the granular velocity fluctuations is to form the ratio

$$R = \frac{d\dot{\gamma}}{T^{1/2}} = \frac{St}{St'},$$

(28)

a dimensionless parameter introduced by Savage and Jeffrey (1981). As discussed in Savage (1998) and Louge (2003), the parameter $R$ is typically found to be of order unity in computer simulations of sheared granular media. Savage (2002, personal communication) even suggests that a constant value of $R \approx 0.8$ may characterise granular behaviour over a wide range of
conditions, including rapid collisional flow and slow frictional deformation.

In the present experiments, shear rate and fluctuation velocity are indeed found to be approximately proportional to each other. Constant values of the Savage-Jeffrey parameter $R$ plot as straight lines on Figure 22, and the data cluster in a sector that goes from $R = 1$ to $R = 2.5$. However the measured Stokes numbers $St$ and $St'$ do not lie on a straight line over their entire range of variation. Stokes numbers corresponding to slow deformation (roughly $St < 10$) evolve roughly in the one-to-one ratio

$$ R = \frac{St}{St'} \approx 1. $$

Yet beyond values of $St \approx St' \approx 5 \div 10$, the data deviate from this straight line and eventually reach ratios as high as $R = 2.5$ for rapid shear rates. Here again, the data suggest a transition dependent upon the rate of shear. The Stokes number $St^* \approx 7.5$ at which the transition occurs corresponds to a shear rate $\dot{\gamma}^* \approx 6 \text{ s}^{-1}$ and to a fluctuation velocity $(T^{1/2})^* \approx 2.4 \text{ cm/s}$. This value of the Stokes number also matches the transitional Bagnold number $Ba^* \approx 1000$ identified earlier, if one adopts for the linear concentration $\lambda$ the value $\lambda^{(\text{max})} \approx 40$ associated with a compact granular medium ($c_s = c_s^{(\text{max})} = 0.69$).

On both sides of the transitional Stokes number $St^* \approx 7.5$, the data trends are consistent with frictional and collisional behaviours, respectively. Under slow deformations, frictional stresses are transmitted through enduring contacts among solid particles. As they stay in close contact, neighbouring particles must undergo coordinated motions. Under such kinematic
constraints, fluctuating and mean shear components of the granular flow must relate to each other. It is thus not surprising to find that in the limit of slow deformation, grain motions are characterised by a constant Savage-Jeffrey parameter. For rapid collisional flow, by contrast, particles interact through short-lived contacts. The fluctuation energy and shear rate remain coupled through energy constraints, as the kinetic energy dissipated by random motions must be derived from the work of the mean shear. However the random motions of the particles are only loosely constrained by the motions of their neighbours, hence the fluctuation velocity and the shear rate must not remain locked in strict proportion to each other.

There are two other reasons to believe that Stokes numbers in the vicinity of \( St^* \approx 7.5 \) have a special significance. The first reason is microscopic, and has to do with the effect of the ambient fluid on collisions between individual grains. Various studies (Davis et al. 1986; Joseph et al. 2001; Gondret et al. 2002) have shown that particles immersed in a fluid bounce off each other in a way that is governed by the value of Stokes number

\[
St_{\text{coll}} = \frac{1}{18} \frac{\rho_i}{\rho_w} \frac{d(2U_i)}{v_w}
\]

where \( U_i \) is the particle speed before impact, measured relative to the collision plane (either the plane tangent to two particles at their point of contact or a rigid wall in the case of particle-wall collisions). At very high Stokes numbers (\( St_{\text{coll}} > 1000 \)), particles rebound with a velocity \( U_r \) characterised by a coefficient of restitution \( e = U_r / U_i \) close to the value applicable to dry collisions. For lower Stokes numbers, by contrast, the coefficient of restitution decreases under the influence of the viscous ambient fluid. Most dramatically,
when the Stokes number falls below a critical value of about $St_{\text{crit}} \approx 10$, the elastic energy stored upon contact is entirely dissipated in the fluid and the coefficient of restitution drops to zero. Particles no longer exhibit any rebound.

Drawn from experiments involving collisions of individual particles with a vertical wall (Joseph et al. 2001) or horizontal floor (Gondret et al. 2002), the above observations are consistent with the elastohydrodynamic analysis of Davis et al. (1986) and have also been found recently to account for a change in avalanche regime in rotating drum experiments (Courrech du Pont et al. 2003). They suggest a compelling microscopic interpretation of the change in behaviour observed in the present experiments in the vicinity of $St^* \approx 7.5$. Under rapid shear, particles are able to rebound upon colliding and thus interact through short-lived contacts. Under slower deformations, on the other hand, the shear-induced fluctuating motions are not strong enough for particles to rebound upon impact and take off from their enduring contacts with neighbouring grains. Under the influence of the viscous ambient fluid, therefore, inelastic collapse occurs at the transition.

The second reason to attach a special significance to the transitional Stokes number $St^* \approx 7.5$ lies in the observed macroscopic behaviour of the downslope flows. As discussed earlier, uniform flows over loose beds increase their solid-to-liquid ratio and decrease their velocity as flow inclinations become steeper. However, the experiments indicate that there is a clear limit to how slow the steep flows can become. This is documented on Figure 23, which plots
maximum shear rates against flow inclination. The maximum shear rate \( \dot{\gamma}_{\text{max}} = (\partial \dot{u} / \partial y)_{\text{max}} \)
observed for a given run is defined as the maximum value taken by the shear rate over the entire depth of the debris flow. It is made dimensionless by introducing the Stokes number

\[
St_{\text{max}} = \frac{1}{18} \frac{\rho_s d^2 \dot{\gamma}_{\text{max}}}{\rho_w v_w} \quad (31)
\]

Rather than decreasing indefinitely as flow inclinations get steeper, the maximum shear rate is found to reach an asymptote

\[
(St_{\text{max}})_{\text{min}} \approx 5 \approx St^* \quad (32)
\]

where \( St_{\text{max}} \) levels off to values close to the transitional Stokes number \( St^* \approx 7.5 \). The transitional Stokes number thus appears to set a \textit{minimax} constraint on the downslope flow, determining a minimum value that must be attained by the maximum shear rate.

This suggests that for the slowest flows, the maximum shear rate \( \dot{\gamma}_{\text{max}} \) (observed for the plug flows at the core of their mid-depth shear layer) does not decrease below the minimum level necessary for viscous and/or collisional effects to retain control on the overall flow rate. If the whole layer was to deform at a rate so slow that only frictional behaviour occurred, then there would be no constraint on the flow rate due to the rate-independent character of the frictional rheology. By contrast, the emergence of a rate-dependent control anywhere in the flow (in this case in the middle of the shear layer) acts as an internal boundary condition which sets the overall flow rate.
4.3. Rheological structure of the loose bed flows

Based on the above observations, the full set of measurements corresponding to the 16 detailed test runs documented on Figures 15-18 were tentatively split into two subsets. Data for which \( St' < St^* \) form the first subset, and are assumed to represent predominantly frictional behaviour. Data for which \( St' > St^* \), on the other hand, are taken to represent predominantly collisional behaviour. On the various figures, data classified as frictional or collisional in this fashion are plotted as filled or hollow symbols, respectively. Admittedly, the choice of a precise cut-off at the value \( St^* \approx 7.5 \) is somewhat arbitrary. Nonetheless, the resulting binary scheme accounts rather well for the distinct trends documented on Figures 19 and 22.

The classification can now be used to examine the distribution of rheological behaviour over depth for the various loose bed flow regimes. When mapped back to the original profile data of Figures 15-18, the frictional-collisional classification organises the flow into separate sub-layers governed predominantly by one mechanism or the other. For the immature flow regime (Fig. 15), water-entrained grains are sheared sufficiently strongly to be characterised by collisional behaviour over most of the depth of the flowing layer. Frictional behaviour is relegated to a thin boundary layer separating the collisional sheet from the static underlying loose bed. This is consistent with the description of sheet-flow sediment transport proposed by Jenkins and Hanes (1998) and analysed recently in more detail by Hsu et al. (2003). At the other end of the range (Fig. 17), the plug-flow regime features relatively slow deformations
and frictional behaviour over the entire depth, except for the core of the shear layer which lies close to the transition point between frictional and collisional behaviours. In between, the mature regime (Fig. 16) exhibits a composite structure in which a collisional layer of finite thickness is sandwiched between two frictional layers. Granular-liquid flows over loose bed can thus be described as rheologically stratified flows, responding to changes of inclination and liquidity by modifying the extent of their frictional and collisional sub-layers.

4.4. Collisional stress functions

A final objective of the present rheological analysis is to examine more closely the collisional behaviour of the granular-liquid mixture. In accordance with our binary classification, this examination is restricted to measurements for which \( St' > St^* \), the transitional Stokes number ascertained earlier to mark the approximate onset of collisional behaviour. Specifically, the aim of this subsection is to compare measurements in that range with both the original relations of Bagnold and the more recent predictions of kinetic theories.

The relationships between stresses and shear rate proposed by Bagnold (1954) on the basis of his neutral suspension experiments can be written

\[
\sigma' = 0.042 \cos \alpha \lambda^2 \rho_s d^2 \dot{\gamma}^2 \\
\tau = 0.042 \sin \alpha \lambda^2 \rho_s d^2 \dot{\gamma}^2
\]

where \( \tan \alpha = 0.32 \) and \( \lambda(e_s) \) is the function of granular concentration defined in eq. (25). The corresponding stresses predicted by kinetic theories of collisional granular flows involve
the granular temperature $T$ in addition to the shear rate $\dot{\gamma}$. Derived in analogy with the kinetic theory of dense gases (Chapman & Cowling 1970), the relationships can be written (Jenkins & Hanes 1998)

\[ \sigma' = (1 + 4c_s g_0) c_s \rho_s T \]  
\[ \tau = \frac{8c_s g_0 E}{5\pi^{1/2}} c_s \rho_s d T^{1/2} \dot{\gamma} \]  
where
\[ E = 1 + \frac{\pi}{12} \left( 1 + \frac{5}{8c_s g_0} \right)^2. \]

In the above relations, the function
\[ g_0(c_s) = \frac{2 - c_s}{2(1 - c_s)^3} \]  
describes the dependence of the rate of collisions between particles on the granular concentration $c_s$, and was derived by Carnahan and Starling (1969) from an approximate virial expansion.

The description of Bagnold and kinetic theory are similar in various regards. One can check first that the functional form of Bagnold’s equations (33)-(34) is retrieved by assuming a constant value of the Savage-Jeffrey parameter $R = d \dot{\gamma} / T^{1/2}$ in relations (35)-(36). Through functions $\lambda(c_s)$ and $g_0(c_s)$, furthermore, both relations account for the highly nonlinear influence of the granular concentration $c_s$. A difference is that while the Bagnold relations are semi-empirical in nature, the kinetic relations follow from theory through a rigorous approximation procedure and involve no adjustable empirical constants. Their predictions have been validated to a great extent by 3D computational simulations (Campbell 1989).
Predictions for 2D granular flows (monolayers of disks or spheres held between parallel plates) were further found to agree well with detailed experiments (Azanza et al. 1999). As he was the first to propose that granular shear and normal stresses could be quadratic in the shear rate, Bagnold himself can of course be credited for some of the achievements of the kinetic theories.

Despite this success, the applicability of either or both of these descriptions to liquid-granular flows of the type examined in the present study is not a settled issue. First, Bagnold’s 1954 experiments were performed with neutrally buoyant spheres in water ($\rho_s = \rho_w$) instead of particles denser than the ambient fluid. In addition, a recent re-analysis of the experimental device and results of Bagnold (Hunt et al. 2002) has cast some doubts on his conclusions. Kinetic theories of collisional granular flows, on the other hand, were originally derived for dry granular flows. While it was argued by Jenkins and Hanes (1998) that relations (35) and (36) could be applied to granular-liquid flows as well, this has not to date been checked against detailed measurements of granular motions.

Such comparisons are presented on Figures 24-27. To compare data with the Bagnold stress functions (33)-(34), the ratios $\frac{\sigma'}{[\rho_s d^2 \dot{\gamma}^2]}$ and $\frac{\tau}{[\rho_s d^2 \dot{\gamma}^2]}$ are plotted against the granular concentration $c_s$ on Figures 24 and 25. To compare with the kinetic functions (35)-(36), corresponding plots of the ratios $\frac{\sigma'}{[c_s \rho_s T]}$ and $\frac{\tau}{[c_s \rho_s d \dot{\gamma} T^{1/2}]}$ are provided on Figures 26 and 27. The Bagnold and kinetic theory descriptions are both seen to account
reasonably well for the qualitative data trends. The data from the different flow regimes collapse around curves similar to the Bagnold and kinetic theory stress functions. However the level of quantitative agreement varies considerably from one plot to another.

As seen on Fig. 24, the present data closely match the Bagnold normal stress function (33) across the whole range of concentrations $c_s$. For the shear stress function (34) plotted on Fig. 25, however, the agreement is much less satisfactory: the shear stress ratio $\tau / [\rho_s d^2 \dot{\gamma}^2]$ is strongly underestimated at lower values of concentration. The kinetic stress functions plotted as solid lines on Figures 26 and 27, on the other hand, underestimate both the normal and shear stresses.

Since computational and experimental data for dry granular flow have been found to confirm the predictions of kinetic theories, we tentatively attribute their discrepancies with the present experiments to the presence of a dense interstitial liquid. More specifically, we surmise that added mass effects are largely responsible for the underestimation of the normal and shear stresses documented on Figures 26 and 27. For particles to move about within the two-phase mixture, conjugate motions of the surrounding water are required. In particular, abrupt changes in granular velocities associated with inter-particle impacts must be associated with pressure impulses and sudden accelerations of the pore water. The resulting dynamic coupling between the particles and the liquid can be described as an added mass contributing to particle inertia.
Such added mass effects play a key role in bubbly flows (Russo & Smereka 1996; Kang et al. 1997), where the apparent inertia of the massless bubbles is entirely due to the embedding liquid. While not quite as prevalent, they should likewise be expected to play a role in liquid-granular flows, where the density of the interstitial fluid is of the same order as the density of the particles. This was verified by Zenit et al. (1997, 1998), who documented experimentally the water pressure impulses induced by submerged collisions of particles with each other and with walls.

A quantitative estimate of the added mass effect in the present flows is derived in the following way. First, the added mass associated with random particle motions is estimated in the way suggested by Zuber (1964; see Batchelor 1988). It is assumed that a single particle accelerates in an environment constrained by neighbouring motionless particles, with the particle packing dependent on the concentration $c_s$ (as in the experiments of Mahgerefteh & Khodaverdian 1996). This constraining effect is then approximated by a rigid spherical shell of volume $(1/c_s)\pi d^3/6$ centred on the particle and surrounding its associated region of freely moving liquid. The added mass associated with the motion of a particle in such a shell is given by (Lamb 1932, pp. 124-125)

$$\rho_a = \rho_w \frac{1 + 2c_s}{2(1 - c_s)}. \quad (39)$$

The expression reduces to the well-known result $\rho_a = \frac{1}{\tau} \rho_w$ in the dilute limit $c_s \to 0$. However as the concentration rises and the interstitial liquid is forced to flow within
increasingly tight boundaries, the added mass can take much higher values. The reasoning above is undoubtedly rough, as surrounding particles are not in fact motionless. Not only are they endowed with their own independent motions, they will also respond to any sudden change (e.g. due to a collision) in the motions of their neighbours (Sangani & Didwania 1993). The expression (39) is thus to be taken as indicative only, and is likely to provide an upper bound rather than a precise estimate of the added mass in fluctuating solid-liquid flows.

The influence of this added mass can then be incorporated into the stress functions in the way proposed by Kang et al. (1997) for dense bubble suspensions. The density $\rho_s$ of the particle itself is supplemented by the added mass density $\rho_a$ in the stress relations (35)-(36), i.e. we rewrite the normal and shear stress functions as

$$\sigma' = (1 + 4c_s g_0) c_s (\rho_s + \rho_a) T,$$

$$\tau = \frac{8c_s g_0 E}{5 \pi^{1/2}} c_s (\rho_s + \rho_a) dT^{1/2} \dot{\gamma},$$

where the dependence of the added mass on concentration $\rho_a(c_s)$ is given by (39). The resulting relations are plotted as dashed lines on Figures 26 and 27. For both the normal and shear stress functions, the added mass correction improves substantially the agreement of kinetic theory with the data. Note that while a number assumptions had to be made, this improved agreement is obtained without any tuning parameter.

The present data permit an assessment of both the Bagnold and kinetic descriptions of granular-liquid flows. The normal stress function of Bagnold is found to agree well with the
measurements. By contrast, Bagnold’s shear stress function deviates substantially from the data. This stems from the fact that the present collisional flow measurements do not exhibit the constant stress ratio $\tau/\sigma'$ assumed by Bagnold. While not consistent with Bagnold’s analysis, this feature is nonetheless compatible with Bagnold’s own measurements. In their recent re-assessment, Hunt et al. (2002) point out that the stress ratio in Bagnold’s experiments was not constant but varied by as much as a factor of 3. A similar range of variation is observed in the present experiments (see Fig. 21). The kinetic description yields a better match. Upon correcting for added mass, the normal and shear stress functions of kinetic theory are in relatively good agreement with our collisional measurements.

In their re-assessment, Hunt et al. (2002) suggest that Bagnold’s 1954 experimental findings could in fact be due to flaws in the design of his annular rheometer, rather than reflect actual collisional interactions between grains. In particular, they show that the shear stress dependence on the shear rate measured by Bagnold could just as well have been produced by a Newtonian fluid with a concentration-dependent viscosity undergoing boundary layer flow along the end plates of the annulus.

While we cannot rule out the alternative mechanism suggested by Hunt et al., the present experiments do provide some support for Bagnold’s original conclusions regarding the inertial regime. Our measurements of liquid-granular flow in a rectilinear flume exhibit behaviours close to those observed by Bagnold in his annular rheometer and attributed by him to
collisional effects. Most strikingly, the normal stress relation proposed by Bagnold is consistent with both our measurements and with the corresponding predictions of kinetic theories of collisional granular flow. Since the alternative mechanism of Hunt et al. can account for the shear stresses but not for the normal stresses observed by Bagnold, we feel that Bagnold’s original interpretation in terms of granular collisions remains compelling.

The collision frequency constitutes one further quantity measured in the present experiments which can be compared with the predictions of kinetic theory. It is obtained as a by-product of the procedure used to estimate the granular temperature presented in Appendix 1. Based on the kinetic theory of dense gases (Chapman & Cowling 1970), an estimate for the exponential decay time $\tau_0$ of the velocity autocovariance function is given by (Savage & Dai 1993):

$$\frac{1}{\tau_0} = \frac{8(1+e)}{\pi^{1/2}} c_s g_0 \frac{T^{1/2}}{d}$$

Expression (42) is similar to, but not identical with, the collision frequency given by

$$\omega_{\text{coll}} = \frac{24}{\pi^{1/2}} c_s g_0(c_v) \frac{T^{1/2}}{d}$$

(43)

due to the fact that particles retain some memory of their previous velocity after a collision event (Chapman & Cowling 1970). For the case of elastic particles ($e = 1$), equations (42) and (43) indicate that the exponential decay time of the velocity autocorrelation is 3/2 times longer than the mean time interval between collisions $\omega_{\text{coll}}^{-1}$. The correlation decay time $\tau_0$ is also related to the self-diffusion coefficient $D$ through the relation
\[ D = t_0 T \] 

where \( T \) is the granular temperature. Studies of self-diffusion in dry granular flows have been conducted by Savage and Dai (1993), Campbell (1997), Hsiau and Shieh (1999), and Wildman et al. (1999).

Figure 28 compares the collision frequency function (43) with the measured autocorrelation decay times \( t_0 \) derived from the observed particle trajectories (see Appendix 1). The normalised collision frequency \( \omega_{\text{coll}} d / T^{1/2} \) is plotted against the solid concentration \( c_s \), where the theoretical \( \omega_{\text{coll}} \) is given by relation (43) and the measured \( \omega_{\text{coll}} \) is approximated by

\[ \omega_{\text{coll}} = \frac{3}{2} t_0^{-1} \]  

i.e. the expression appropriate for perfectly elastic collisions. On Fig. 28, the collisional data from the different experimental runs are seen to collapse convincingly together. However the theoretical collision frequency (43) matches the inverse correlation time only for low solid concentrations \( c_s \). For higher granular concentrations, the observed fluctuation velocities remain correlated longer (by up to one order of magnitude) than predicted. Thus agreement of theory and experiment for the collisional stresses does not guarantee that all features of collisional flow behaviour are well described by kinetic theory.
5. Conclusions

Using a recirculating flume and imaging techniques, the present work has sought to characterise in detail the vertical structure of steady uniform granular-liquid flows. For four different flow regimes (loose bed immature flow, mature flow and plug flow, and solid bed flow), depth profiles were obtained for the mean velocity, solid concentration, shear and normal stresses, and granular temperature. Based on these data, it was found possible to probe the local rheological behaviour of the flow as well as examine variations in behaviour over the flow depth. A rather painstaking inductive process was necessary to make sense of the experimental measurements, yet the overall picture which emerges is surprisingly coherent.

Gravity-driven granular-liquid flows are found to organise themselves into distinct sub-layers dominated by either frictional or collisional behaviour. Frictional behaviour is characterised by 1) slow deformations; 2) a nearly constant ratio of mean shear rate to fluctuation velocity for which the Savage-Jeffrey parameter is close to unity ($R \approx 1$); 3) rate-independent shear and normal stresses which are proportional to each other. Collisional behaviour, on the other hand, is characterised by 1) rapid deformations; 2) a slightly variable ratio of mean shear rate to fluctuation velocity (from $R \approx 1$ up to $R \approx 2.5$); 3) quadratic dependence of the shear and normal stresses on the shear rate, with a variable ratio of shear to normal stress. From both a kinematic and kinetic point of view, therefore, the two behaviours exhibit distinct signatures.

The transition between frictional and collisional behaviour appears controlled by viscous effects. First, changes in the local flow rheology are observed to occur within a narrow range of the Stokes number, around $St = 5 \div 10$. Secondly, this Stokes number threshold also acts as a macroscopic constraint on the downslope flows: it defines the minimum value that must be attained by the maximum rate of shear of the slowest flows. Finally the same Stokes number range can be readily interpreted from a microscopic perspective: it corresponds to the
threshold below which particles colliding in a viscous liquid cease to bounce off each other, as reported in various recent studies. Thus the available evidence conspires to suggest this surprising paradox: no viscous or macroviscous flow behaviour is observed alongside frictional and collisional behaviours, but the frictional-collisional transition seems controlled by viscosity.

Looking at the observed local rheology in greater detail, the present frictional measurements are found to be consistent with simple shear experiments conducted at the very low rates of shear typical of geotechnical tests. The collisional measurements, on the other hand, corroborate both the Bagnold description and the more recent kinetic theories of granular flows, even if the agreement requires some qualifications. In the present experiments, the shear and normal stresses do not exhibit a constant ratio, which runs counter to Bagnold’s description but agrees with Bagnold’s own measurements. Overall, however, our results tend to confirm rather than undermine Bagnold’s pioneering work. Turning to the kinetic theories, it was found possible to achieve quantitative agreement with the present measurements only by accounting for the inertia of the interstitial liquid. This is achieved by incorporating in the stress functions an added mass which varies with the solid concentration as if each particle moves about within a liquid-filled rigid shell.

To account for the present observations, it was found unnecessary to invoke a hypothetical frictional-collisional regime which would blend at the microscale features associated with both frictional and collisional behaviours (as in the proposals of Johnson & Jackson 1987, or Savage 1998). Rather, our results suggest that the local behaviour is either frictional or collisional, and that the granular-liquid medium can switch between the two behaviours much like ice can melt into water. The behaviour shift would thus be analogous with a thermodynamic phase transition (as in the views proposed by Jenkins & Askari 1991, or
The gravity-driven flows observed in the present experiments appear rather sharply stratified into separate frictional and collisional sub-layers. Hence the term frictional-collisional regime would apply only in the macroscopic sense of flows exhibiting the two behaviours at different locations over their depth, without implying hybrid behaviour at the local level.

To be sure, the interpretation summarised above is open to argument and bound to be revised as experimental and theoretical work proceeds. Alternative interpretations of the present data are possible, and we have retained in the present paper only the explanations which we find most compelling. To compensate, we have tried to document the present experiments as completely as we could, and hope that this will allow the reader to come to his or her own conclusions.

The experiments and data are also subject to a number of significant limitations. Foremost among them is probably the two-dimensional nature of the sidewall measurements. The present 2D measurements are bound to reflect only partially the 3D conditions near the sidewall, and doomed to miss the 3D features of the flow inside the bulk. One specific limitation in this regard concerns the solid concentration measurements, for which the 2D pattern-based estimate breaks down in the dense limit. Because of this, the data gathered in the present work are not suited to examine the role of solid concentration in the frictional regime.

To address these limitations, efforts are under way to obtain three-dimensional measurements. Preliminary results in this direction are encouraging (see Larcher 2002; Spinewine et al. 2003). However the technical difficulties involved mean that the acquisition of a data set comparable in scope to the present one lies some distance into the future. Further
improvements would also include direct measurements of stresses and pore pressures, to confirm and refine the force balance estimates used in the present work. Finally, experiments with liquids of different viscosities would be desirable to test the effect of viscosity on the flow behaviour in general and on the frictional-collisional transition in particular. New experimental efforts are definitely needed to further the understanding of granular-liquid flows.

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Appendix: Autocovariance-based estimation of the granular temperature

Velocity measurements obtained by particle-tracking velocimetry (PTV) are prone to errors due to limited spatial accuracy and finite sampling frequency. For the mean velocity components, these errors average out and can be made arbitrarily small provided a large ensemble of data is used. This is not the case, however, for the root-mean-square velocity fluctuations or, in the case of granular flows, the granular temperature. On the one hand, spurious contributions to the apparent fluctuation velocity arise due to measurement noise. On the other hand, attenuation of the apparent fluctuation velocity occurs due to the smoothing effect of finite sampling frequency. The present section outlines a procedure to correct for these effects when estimating the granular temperature from PTV measurements. The procedure is based on an examination of the Lagrangian velocity autocovariance functions.

Error model

To simplify the analysis, it is assumed that tracking mismatches have been previously filtered out. This is achieved in practice using the pattern and path-based methods detailed in Capart et al. (2002). Particle images observed on two successive frames can thus be unambiguously paired and associated to one and the same physical particle. We further assume that the particle is tracked over many successive frames, and restrict our attention to a single velocity component. The PTV estimate of the $x$-wise velocity history of the particle is given by (e.g. Adrian 1991)

$$\hat{u}^{(n)} = \frac{\hat{x}^{(n+1)} - \hat{x}^{(n)}}{t^{(n+1)} - t^{(n)}},$$  \hspace{1cm} (A 1)

where $\hat{x}^{(n)}$ is the measured position of the particle sampled at regularly spaced instants $t^{(n)} = n\Delta t$. To devise a simple error model, one can first assume that the measured position $\hat{x}^{(n)}$ is equal to the true particle position $x^{(n)} = x(t^{(n)})$ to which is added a random measurement error, i.e.
\[ \hat{x}^{(n)} = x^{(n)} + s \epsilon^{(n)} \] (A 2)

where \( s \) is the standard deviation of the measurement error on position, related to the pixel resolution of the images, and the \( \epsilon^{(n)} \) are random variables of zero mean and unit standard deviation. We assume that successive position errors \( \epsilon^{(n)} \) are uncorrelated, i.e.

\[ \langle \epsilon^{(m)} \epsilon^{(n)} \rangle = 0 \quad m \neq n \] (A 3)

where the brackets denote an average carried out over an ensemble of trajectories. Substituting \( A2 \) into \( A1 \) and invoking relation \( x(t) = \int u(t) dt \), the following relationship is obtained between the measured velocity history \( \hat{u}^{(n)} \) and the true velocity history \( u(t) \) of a particle

\[ \hat{u}^{(n)} = \bar{u}^{(n)} + \frac{s}{\Delta t} \left( \epsilon^{(n+1)} - \epsilon^{(n)} \right), \] (A 4)

where

\[ \bar{u}^{(n)} = \frac{1}{\Delta t} \int_{t^{(n)}}^{t^{(n+1)}} u(t) dt \] (A 5)

represents the average of the unresolved particle velocity over time interval \( \Delta t \). The above relationship constitutes an observation equation (Honerkamp 1998) for the PTV measurement process, i.e. an idealised description of the connection between the true signal and the observed one. Expression \( A4 \) accounts for two discrepancies between the measured velocity and the actual velocity. The first discrepancy is due to the discrete sampling of particle positions, which amounts to averaging the velocity over interval \( \Delta t \) and damps out high-frequency velocity fluctuations. The second discrepancy, expressed by the second term on the right hand side, is a corrupting noise due to random position error. One can immediately observe that

\[ \langle \hat{u} \rangle = \langle \bar{u} \rangle = \langle u \rangle = \bar{u} \] (A 6)

i.e. neither the averaging nor the zero-mean noise on the position induce any bias on the average velocity. For the fluctuation components defined by

\[ u'(t) = u(t) - \bar{u} , \quad \tilde{u}'^{(n)} = \tilde{u}^{(n)} - \bar{u} , \quad \hat{u}'^{(n)} = \hat{u}^{(n)} - \bar{u} , \] (A 7)
however, the situation is not so favourable. Their mean squared values are not equivalent, i.e.

\[
\langle \hat{u}^2 \rangle \neq \langle \hat{u}^2 \rangle \neq \langle u^2 \rangle .
\]  

(A 8)

**Incidence on the velocity autocovariance function**

Let us take a slightly more general view and examine the autocovariance functions (or acf in shorthand)

\[
\langle u'(t)u'(t + \hat{\alpha}) \rangle, \quad \langle \hat{u}'^{(n)}\hat{u}'^{(n+k)} \rangle, \quad \text{and} \quad \langle \hat{u}'^{(n)}\hat{u}'^{(n+k)} \rangle,
\]  

(A 9)

where \(\hat{\alpha}\) is arbitrary but \(k\) is restricted to integer values. The averaged acf \(\langle \hat{u}'^{(n)}\hat{u}'^{(n+k)} \rangle\) is related to the true acf \(\langle u'(t)u'(t + \hat{\alpha}) \rangle\) by the exact relation

\[
\langle \hat{u}'^{(n)}\hat{u}'^{(n+k)} \rangle = 1/(\Delta t)^2 \left( \int_0^{\Delta t} \int_0^{\Delta t} dt' dt'' \langle u'(t' + k\Delta t)u'(t'') \rangle + \int_0^{\Delta t} \int_0^{\Delta t} dt' dt'' \langle u'(t' + k\Delta t)u'(t'') \rangle \right)
\]  

(A 10)

in which \(t'\) and \(t''\) are variables of integration. Equation (A10) derives (after some developments) from definition (A5). It generalises a relation proposed by Batchelor (1949) and exploited by Hansen and McDonald (1986) and Campbell (1997) to analyse self-diffusion processes. The generalised relation (A10) and its usefulness in the context of PTV error analysis do not seem to have been reported before.

On the other hand, the measured acf \(\langle \hat{u}'^{(n)}\hat{u}'^{(n+k)} \rangle\) (including the presence of noise) is related to the averaged acf \(\langle \hat{u}'^{(n)}\hat{u}'^{(n+k)} \rangle\) by relation

\[
\langle \hat{u}'^{(n)}\hat{u}'^{(n+k)} \rangle = \begin{cases} 
\langle \hat{u}'^{(n)}\hat{u}'^{(n+k)} \rangle + \frac{2s^2}{(\Delta t)^2} & \text{if } k = 0, \\
\langle \hat{u}'^{(n)}\hat{u}'^{(n+k)} \rangle - \frac{s^2}{(\Delta t)^2} & \text{if } |k| = 1, \\
\langle \hat{u}'^{(n)}\hat{u}'^{(n+k)} \rangle & \text{if } |k| > 1.
\end{cases}
\]  

(A 11)
which follows immediately from (A2) and the assumption that errors on successive particle positions are uncorrelated. An important consequence of this assumption is that the noise does not affect the acf beyond the first two points \( k = 0 \) and \( k = 1 \).

### Signal model

To exploit the observations above, it is useful to adopt a simple model for the physical signal as well. Theoretical arguments by Hansen & McDonald (1986), computer simulations by Savage & Dai (1993) and by Campbell (1997) and experiments with vibro-fluidised disks by Wildman et al. (1999) indicate that for granular flows the short time behaviour of the physical velocity auto-correlation function should be well approximated by an exponential curve, i.e.

\[
\langle u'(t) u'(t + \delta t) \rangle = \langle u'^2 \rangle \exp(-|\delta t|/t_0), 
\]

where the decay constant \( t_0 \) is related to the average time between successive particle collisions (Savage & Dai 1993). Substituting (A12) into (A10) and carrying out the integrations yields the explicit result

\[
\left\{ \frac{\langle \tilde{u}'^{(n)} \tilde{u}'^{(n+k)} \rangle}{\langle u'(t) u'(t + k\Delta t) \rangle} \right\} = \begin{cases} 
2t_0 \frac{\Delta t}{\Delta t} \left( 1 + \frac{t_0}{\Delta t} \left[ \exp\left( -\frac{\Delta t}{t_0} \right) - 1 \right] \right) & \text{if } k = 0, \\
\frac{t_0^2}{(\Delta t)^2} \left[ \exp\left( -\frac{\Delta t}{t_0} \right) + \exp\left( \frac{\Delta t}{t_0} \right) - 2 \right] & \text{if } |k| > 0,
\end{cases}
\]

(A 13)

The effect of finite sampling frequency on the signal is two-fold: first, the variance (corresponding to \( k = 0 \)) is attenuated by a factor \( f(\Delta t/t_0) < 1 \); conversely, the tail of the autocovariance function (\( |k| > 0 \)) is amplified by a factor \( g(\Delta t/t_0) > 1 \). As the function \( g \) is independent of \( k \), furthermore, the tail of the observed acf retains an exponential shape governed by the same decay time \( t_0 \) as the physical acf. Under the assumptions of the simple model adopted above, it is therefore concluded that the tail of the observed autocovariance function \( \langle \tilde{u}'^{(n)} \tilde{u}'^{(n+k)} \rangle \) \( (k > 1) \) is affected only in a very limited way by the two sources of
error which corrupt the variance $\langle \hat{u}^2 \rangle$.

Corrected estimate of the granular temperature

The adopted correction procedure derives immediately from the observations above. Instead of choosing direct estimate $\langle \hat{u}^2 \rangle$ for the mean square velocity fluctuations, the idea is to exploit the less corrupted information contained in the tail of the observed autocovariance function $\langle \hat{u}^{(n)}(k)\hat{u}^{(n+k)} \rangle$ ($k > 1$). An exponential curve is fitted to this tail, keeping only the points $2 \leq k \leq m$, where $m$ is a time cut-off chosen to avoid the deviations from exponential and the poorer statistics of the long-time tail. The fit yields an estimate for the decay time $t_0$, as well as a revised zero-intercept $\langle u^2 \rangle_{fit}$ for the acf. This latter estimate is affected by the amplification factor $g(\Delta t / t_0)$ given in (A13), which is easily accounted for by letting

$$\langle u'^2 \rangle_{corr} = \frac{\langle u'^2 \rangle_{fit}}{g(\Delta t / t_0)}$$

which is our corrected estimate for the mean square velocity fluctuations. Depending on whether the effect of noise or attenuation is prevalent, this value may be either lower or higher than the direct estimate $\langle \hat{u}^2 \rangle$. In practice, we apply the above procedure jointly to the two components of the velocity vector $u = (u, v)$ by considering the covariance $\langle u'(t) \cdot u'(t + \Delta t) \rangle$. This allows us to derive a corrected estimate of the granular temperature

$$T = \frac{1}{2} \langle u' \cdot u' \rangle = \frac{1}{3} \langle u'^2 + v'^2 \rangle$$

(A 15)

and a single estimate of the correlation decay time $t_0$.

As a further useful by-product, the procedure also yields an estimate for the rms error on particle position $s$. We get from (A13)

$$\langle \tilde{u}^2 \rangle = f(\Delta t / t_0) \langle u'^2 \rangle_{corr}.$$  

(A 16)
From (A11), on the other hand,

\[ s = \frac{\Delta t}{\sqrt{2}} \left( \langle \hat{u}'^2 \rangle - \langle \hat{u}'^2 \rangle \right)^{1/2} \]  

which allows an \textit{a posteriori} evaluation of the magnitude of the incurred position error.

\textit{Monte-Carlo tests}

The theory and procedure presented above were checked using Monte-Carlo simulations. To synthesise an artificial ‘physical’ signal, a simple model derived from the Enskog theory of uncorrelated binary collisions is adopted (see Hansen and McDonald 1986). In this model, a single particle collides with other particles at random time intervals of mean duration \( t_0 \). Particle velocities are constant between collisions, and each collision is assumed to completely decorrelate the velocity. Thus velocities on separate intervals are taken to be independent Gaussian variables of constant mean and standard deviation. For this process, the exponential velocity \textit{acf} (A12) is exact with decay time equal to \( t_0 \). Realisations of this process are straightforward to generate using typical Monte-Carlo methods (see e.g. Ross 1990). The ‘true’ random particle trajectories can be integrated exactly, and positions sampled at regular time intervals to simulate PTV measurements. Uncorrelated Gaussian noise of zero mean and constant standard deviation can then be added to the data to reproduce the random position errors.

‘True’ and observed fluctuation velocities \( u'(t) \) and \( \hat{u}' \) generated in this fashion are plotted on panel (a) Fig. 29. The corresponding auto-covariance functions (acf) are shown on panel (b). Finally panel (c) compares raw rms values and corrected acf estimates with the true fluctuation velocity \( T^{1/2} \), for various signal-to-noise ratios \( T^{1/2} \Delta t / s \). The raw root-mean-square average \( \langle \hat{u}'^2 \rangle^{1/2} \) (black dots) overestimates the fluctuation velocity when noise dominates, and underestimates the true value when attenuation dominates. In both cases,
the corrected estimate (A14) based on the Lagrangian acf (white squares) succeeds in recovering values closer to the true fluctuation velocity. All fluctuation velocity and granular temperature measurements reported in the present work were obtained using this procedure.

Sample results

Application of the autocovariance-based approach to actual measurements is illustrated on Fig. 30. Panel (a) shows actual acf results measured at mid-depth for experimental run 85. Overall, the observed shape of the autocovariance function is consistent with the assumptions of the present analysis. There is however one significant difference: errors on particle position include a periodic perturbation at a frequency of 50 Hz in addition to uncorrelated random noise. This shows up on the autocovariance plot of panel (b) as high-frequency saw teeth. By filming a bed of particles at rest under conditions identical to those of the experiments, it was checked that this noise is due to a faint flickering of the halogen lamps used to illuminate the flows. Since this noise occurs at sufficiently high frequency, however, it does not affect the low frequency acf fit used to estimate the granular temperature. Using relation (A17) the root-mean-squared error on particle position was estimated to be of the order of 0.2 pixel in the present experiments. This is comparable to the error magnitude reported by Veber et al. (1997) in their analysis of PTV accuracy.

Panel (b) of Fig. 30 shows the fluctuation velocity profile $T^{1/2}$ of run 85. The values of fluctuation velocity $T^{1/2}$ obtained using the direct rms estimate and those obtained using the acf-based correction procedure are plotted as solid and dashed curves, respectively. The correction procedure is applied separately to each of the 20 averaging bins shown earlier on Fig. 7. The noise component is seen to be the predominant source of error, causing the direct root-mean-squared average to overestimate the fluctuation velocity. In relative terms, this effect is most severe in the lowermost part of the flow, where the granular motion is slow.
Below a depth \( y \approx 9 \) cm, the grains are actually motionless, as indicated by the mean velocity profile of the same run plotted on Fig. 7. The direct estimate nonetheless yields a finite value of the fluctuation velocity. By contrast, the corrected estimate successfully filters out the noise component. Error bars on Fig. 30b represent the statistical error (± 2 standard deviations) associated with the estimates. This statistical error is evaluated by coarse-graining (see e.g. Rapaport 1995): quantities are estimated on 10 separate data subsets rather than the full data set, then the standard deviation of the 10 resulting numbers is divided by \( \sqrt{10} \) to approximate the standard deviation of their average. Due to the large data set acquired in the present experiments, the statistical errors are small. In this case they are also rather poor indicators of the actual measurement errors. The small error bars around the raw rms values at the bottom of the flow, for instance, indicate only that noise is measured very precisely, not that the signal is accurately captured! For this reason, a better indication of the uncertainty associated with the present measurements is probably provided by the data scatter between different experimental runs.
References


### Table 1: Properties of the cylindrical PVC pellets used as granular material in the experiments

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Test method</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $\rho_s$ [g/cm$^3$]</td>
<td>1.54</td>
<td>Pycnometer</td>
<td>Average of 3 tests</td>
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<tr>
<td>Particle height [mm]</td>
<td>2.8</td>
<td>Calypter</td>
<td></td>
</tr>
<tr>
<td>Particle base diameter [mm]</td>
<td>3.2</td>
<td>Calypter</td>
<td></td>
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<tr>
<td>Equivalent spherical diameter $d$ [mm]</td>
<td>3.7</td>
<td>Diameter of a sphere of identical volume</td>
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</tr>
<tr>
<td>Maximum concentration $e^{(\text{max})}$ [-]</td>
<td>0.69</td>
<td>Compaction</td>
<td></td>
</tr>
<tr>
<td>Static friction angle $[\text{degrees}]$</td>
<td>35</td>
<td>Tilting board</td>
<td>At loose poured concentration $c_s \approx 0.6$</td>
</tr>
<tr>
<td>Critical friction angle $\theta_c$ [degrees]</td>
<td>31</td>
<td>Simple shear test</td>
<td>Ratio of shear to effective normal stress at steady flow state (see Fig. 3)</td>
</tr>
<tr>
<td>Coefficient of permeability $k$ [m/s]</td>
<td>$k = 0.098 \times e^2 / (1 + e) + 0.024$ obtained by fitting Kozeny-Carmen equation (Das 1990; Spence &amp; Guymer 1997) through the data of Fig. 4</td>
<td>Permeameter / fluidisation cell</td>
<td>Data measured using a permeameter for static assemblies at concentration $c_s = 0.58$ and $0.65$, and using a fluidisation cell for the range of concentration $c_s = 0.4-0.5$</td>
</tr>
<tr>
<td>Specific retention $S_r$ [-]</td>
<td>0.08</td>
<td>Comparison of wet and dry weights of bulk sample</td>
<td>Volume fraction of water retained due to surface tension upon gravity drainage of a saturated sample</td>
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<tr>
<td>Height of capillary rise $h_c$ [mm]</td>
<td>in the interval between 5 and 15 mm</td>
<td>Tulip test</td>
<td>Difference in height between the top of a saturated sample and the water level of a connected reservoir at which the sample desaturates</td>
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</tbody>
</table>
Table 2: Parameters of the experimental runs selected for detailed imaging analysis

<table>
<thead>
<tr>
<th>Flow regime</th>
<th>Run no.</th>
<th>Plot Symbols&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Slope $\beta$ [deg]</th>
<th>Total discharge $Q^{\text{bulk}}$ [l/s]</th>
<th>Transport concentration $C_s$</th>
<th>Flow thickness $H$ [cm]</th>
<th>Mean velocity $U$ [m/s]</th>
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<tr>
<td>Immature, sheet-flow</td>
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<td>Mature debris flow</td>
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<td>10.33</td>
<td>0.529</td>
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<td>0.48</td>
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<tr>
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<td>△ ▲</td>
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<td>0.511</td>
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<td>0.27</td>
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<td>106</td>
<td>△ ▲</td>
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<td>Solid bed debris flow</td>
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<td></td>
<td>42</td>
<td>△ ▲</td>
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<td></td>
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<td>△ ▲</td>
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<td>12.43</td>
<td>0.493</td>
<td>2.7</td>
<td>1.65</td>
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</tbody>
</table>

<sup>1</sup> Hollow symbols denote collisional behaviour, and filled symbols denote frictional behaviour, as determined on the basis of Stokes number $St'$ (see the explanations in section 3.4).
Figure captions

Figure 1. Multiple exposure image of a granular-liquid flow over loose bed, showing the rapidly flowing layer and underlying static deposit (run 85 from the present experiments).

Figure 2. Trento recirculating flume developed for the study of steady uniform debris flows: (a) side view; (b) downstream view; (c) plane view (photos courtesy of L. Guarino).

Figure 3. Results of the simple shear tests. Solid lines: measured stress paths for four different tests, each characterised by distinct void ratios $e$ at the beginning and end of the test; dashed line: steady state locus.

Figure 4. Variation of permeability as a function of void ratio $e$. Crosses (+): data from the permeameter and fluidisation cell tests; solid line: Kozeny-Carmen fit.

Figure 5. Pattern-based particle tracking algorithm (Capart et al. 2002): (a) image detail with positioned particle centres; (b) Voronoï diagram constructed on the sets of particle centres identified on one frame (thin lines) and the next (thick lines); (c) displacement vectors obtained by pairing nearby particles having similar Voronoï cells.

Figure 6. Velocimetry results: (a) sample image frame (from sheet-flow run 85); (b) displacement vectors obtained by tracking grains on two successive frames (grain positions for which no matches were found are circled on the plot); (c) granular trajectories derived from 100 successive image frames; (d) convected trajectories obtained by subtracting the mean flow component from particle tracks extracted from 50 successive frames.

Figure 7. Averaging of raw velocity data (from sheet-flow run 85): (a) cloud of circa 170,000 velocity data points obtained by tracking particles over 500 frames (black symbols: longitudinal velocity; gray symbols: vertical velocity; lines: bin limits); (b) bin-averaged velocity profiles (solid lines: mean velocities; dashed lines: root-mean-squared fluctuation velocity plotted on both sides of the corresponding means).
Figure 8. Relationship between the channel inclination $\beta$ and the Froude number $Fr$ for uniform granular-liquid flows: (+) solid bed, transport concentration $C_s^{(\text{bulk})} = 0.51 - 0.53$; (+) solid bed, transport concentration $C_s^{(\text{bulk})} = 0.53 - 0.55$; (×) loose bed. Solid bed data are derived from direct depth and discharge measurements, and loose bed data from depth-integrated imaging estimates.

Figure 9. Relationship between channel inclination $\beta$ and transport concentration $C_s$ for uniform loose bed flows. Crosses (×): transport concentration data (bulk measurements $C_s^{(\text{bulk})}$ obtained at the channel outlet) for a set of 40 experimental runs; Solid line: equation (23) based on macroscopic force balance.

Figure 10. Typology of the examined flows: (a) loose bed, immature flow (sheet-flow); (b) loose bed, mature flow; (c) loose bed, plug flow; (d) solid bed flow.

Figure 11. Kinetic energy time history used as a check of the steady character of the observed flows: (a) solid bed runs 41 and 46. (b) loose bed runs 85, 90, and 105.

Figure 12. Particularly strong roll waves observed in solid bed run 46: phase trajectory on a plot of potential versus kinetic energy.

Figure 13. Velocity distribution over the channel cross-section: (a) surface velocity across the channel width (run 41); (b) velocity depth profile at the sidewall (run 41); (c) hypothetical 3D velocity distribution for a typical loose bed run.

Figure 14. Depth-integrated estimates based on sidewall imaging measurements compared with bulk measurements at the outlet: (a) total discharge $Q^{(\text{imag})}$ vs. $Q^{(\text{bulk})}$; (b) transport granular concentration $C_s^{(\text{imag})}$ vs. $C_s^{(\text{bulk})}$.

Figure 15. Depth profiles for the sheet-flow regime: (a) mean velocity; (b) solid concentration (gray band indicates range in which concentration measurements are highly uncertain); (c) shear rate; (d) fluctuation velocity. The various quantities are plotted in dimensionless form as described in the text. See Table 2 for symbol legends.
Figure 16. Depth profiles for the mature flow regime: (a) mean velocity; (b) solid concentration; (c) shear rate; (d) fluctuation velocity.

Figure 17. Depth profiles for the plug flow regime: (a) mean velocity; (b) solid concentration; (c) shear rate; (d) fluctuation velocity.

Figure 18. Depth profiles for the solid bed flows: (a) mean velocity; (b) solid concentration; (c) shear rate; (d) fluctuation velocity.

Figure 19. Variation of the normalised shear stress \( \tau /[\mu_w^2 \lambda / (\rho_s d^2)] \) with the local Bagnold number \( Ba = (\rho_s \lambda^{1/2} d^2 / \mu_w) \dot{\gamma} \). See Table 2 for symbol legends. The relationships proposed by Bagnold (1954) for the macro-viscous and grain-inertia regimes are shown as dashed and solid lines, respectively.

Figure 20. Variation of the normalised effective normal stress \( \sigma'/[\mu_w^2 \lambda / (\rho_s d^2)] \) with the local Bagnold number \( Ba = (\rho_s \lambda^{1/2} d^2 / \mu_w) \dot{\gamma} \). The relationships proposed by Bagnold (1954) for the macro-viscous and grain-inertia regimes are shown as dashed and solid lines, respectively.

Figure 21. Variation of the stress ratio \( \tau / \sigma' \) with the local Bagnold number \( Ba = (\rho_s \lambda^{1/2} d^2 / \mu_w) \dot{\gamma} \). The constant values proposed by Bagnold (1954) for the macro-viscous and grain-inertia regimes are shown as dashed lines, while the solid line represents the stress ratio measured with the simple shear apparatus (Table 1).

Figure 22. Relationship between shear rate \( \dot{\gamma} \) and fluctuation velocity \( T^{1/2} \), made dimensionless using Stokes numbers \( St = \frac{1}{18} (\rho_s / \rho_w) d^2 \dot{\gamma} / \nu_w \) and \( St' = \frac{1}{18} (\rho_s / \rho_w) dT^{1/2} / \nu_w \).

Figure 23. Macroscopic behaviour of uniform debris flows over loose beds: dependence of the normalised maximum shear rate \( St_{\text{max}} = \frac{1}{18} (\rho_s / \rho_w) d^2 \dot{\gamma}_{\text{max}} / \nu_w \) on flow inclination \( \beta \).

Figure 24. Comparison of the present collisional data (\( St > St^* \)) with Bagnold’s normal stress function. Gray band indicates range in which concentration measurements are highly
uncertain.

Figure 25. Comparison of the present collisional data \((St > St^*)\) with Bagnold’s shear stress function.

Figure 26. Comparison of the present collisional data \((St > St^*)\) with the normal stress function of kinetic theory. Solid line: basic theory (Jenkins & Hanes 1998); dashed line: revised theory with added mass correction.

Figure 27. Comparison of the present collisional data \((St > St^*)\) with the shear stress function of kinetic theory. Solid line: basic theory (Jenkins & Hanes 1998); dashed line: revised theory with added mass correction.

Figure 28. Comparison of measured inverse correlation time with the collision frequency predicted by kinetic theory.

Figure 29. Monte-Carlo simulations of PTV measurements: (a) fluctuation velocity histories (solid line: ‘true’ signal \(u'(t)\), dots: PTV data \(\hat{u}'\)); (b) velocity autocovariance functions (dashed line: true acf, solid line: attenuated acf, dots: measured acf; (c) measured fluctuation velocity against true value for various signal-to-noise ratios (black dots: direct estimate, white squares: corrected estimate, diagonal: line of perfect agreement).

Figure 30. Estimation of fluctuation velocities based on the Lagrangian autocovariance function (acf): (a) acf measured at mid-depth for run 85 (dots: measured acf; dashed line: exponential fit); (b) comparison of direct (solid line/black dots) and corrected estimates (dashed line/white squares) of the fluctuation velocity profile for run 85. Error bars indicate ±2 standard deviations estimated by coarse-graining.
Figure 3

Figure 4
Direction of increasing slope $\beta$ and concentration $C_s$

Direction of increasing Froude number $F_r$

Figure 10

![Figure 10](image)

Figure 11

![Figure 11](image)

Figure 12

![Figure 12](image)
Figure 13

Figure 14
Figure 15

\[ S_t = \frac{1}{18} \frac{\rho_s}{\rho_w} \frac{d^2 \gamma}{v_w} \]

\[ S'_t = \frac{1}{18} \frac{\rho_s}{\rho_w} \frac{dT^{1/2}}{v_w} \]
Figure 16

\[ St = \frac{1}{18} \frac{\rho \cdot d^2 \gamma}{\rho_v \cdot v_w} \]

\[ St' = \frac{1}{18} \frac{\rho \cdot dT^{1/2}}{\rho_v \cdot v_w} \]
Figure 17

\( St = \frac{1}{18} \frac{\rho_s}{\rho_w} \frac{d^2 \dot{\gamma}}{v_w} \)

\( St' = \frac{1}{18} \frac{\rho_s}{\rho_w} \frac{dT^{1/2}}{v_w} \)
Figure 18
Figure 19

\[
Ba = \frac{\rho \lambda^{1/2} d^2}{\mu_w} \gamma
\]

Figure 20

\[
\frac{\mu^2 \lambda}{(\rho, d^2)}
\]

\[
\frac{\mu^2 \lambda}{(\rho, d^2)}
\]
\[ \tau / \sigma' \]

Figure 21

\[ B_a = \frac{\rho_s \lambda^{1/2} d^2}{\mu_w} \gamma \]

Figure 22

\[ S_{t, \max} = \frac{1}{18 \rho_w v_w} \frac{\rho_s d^2 \gamma}{\beta [\text{deg}]} \]

Figure 23
Figure 26

Figure 27
Figure 28