Boundary effects on the creeping-flow and thermophoretic motions of an aerosol particle in a spherical cavity

Huan J. Keh* and Jih H. Chang

Department of Chemical Engineering, National Taiwan University, Taipei 106-17, Taiwan, R.O.C.

Abstract—An analytical study is presented for the quasisteady translation, rotation, and thermophoresis of an aerosol sphere located at the center of a spherical cavity. The Knudsen number is assumed to be small so that the fluid flow is described by a continuum model with a temperature jump, a thermal creep, and a frictional slip at the solid surfaces. The particle and the cavity surroundings may differ in thermal conductivity and in surface properties. In the limit of small Reynolds and Peclet numbers, the appropriate energy and momentum equations are solved for each system and the particle velocities for an applied force, torque, and temperature gradient are derived individually in closed forms. In general, the boundary effects of the cavity wall on the isothermal creeping-flow and thermophoretic motions of an aerosol particle can be quite significant in appropriate situations. In practical aerosol systems, the boundary effect on thermophoresis is much weaker than that on the motion driven by a gravitation field. © 1998 Elsevier Science Ltd. All rights reserved.

Keywords: Colloidal phenomena; interfacial phenomena; hydrodynamics; heat conduction; multiphase flow; particle.

INTRODUCTION

When a particle is suspended in a gaseous medium possessing a temperature gradient, it will move in the direction of decreasing temperature. This motion is known as thermophoresis and has been the subject of considerable study in the past (Waldmann and Schmitt, 1966; Derjaguin *et al.*, 1976; Bakanov, 1991). The physical explanation of this phenomenon is based on the kinetic theory of gases (Kennard, 1938; Loyalka, 1992). The higher-energy molecules in the hot region of the gas impinge on the particle with greater momenta than molecules coming from the cold region, thus resulting in the migration of the particle in the direction opposite to the temperature gradient. Being a mechanism for the capture of aerosol particles on cold surfaces, thermophoresis is important in many practical applications, such as sampling of aerosol particles (Friedlander, 1977), cleaning of air (Sasse *et al.*, 1994), scale formation on surfaces of heat exchangers (Montassier *et al.*, 1991), modified chemical vapor deposition (Weinberg, 1982), micro-electronic manufacturing (Ye *et al.*, 1991), and nuclear reactor safety (Williams and Loyalka, 1991).

Based on the assumptions of small Knudsen number (l/a , where a is the radius of the particle and l is the mean free path of the gas molecules), small Peclet number, small Reynolds number, as well as the effects of temperature jump, thermal creep, and frictional slip at the gas-particle surface, Brock (1962) obtained the thermophoretic velocity of an aerosol sphere in a constant temperature gradient ∇T_x as

$$\mathbf{U} = - \left[\frac{2C_s(k + k_1 C_t l/a)}{(1 + 2C_m l/a)(2k + k_1 + 2k_1 C_t l/a)} \right] \times \frac{\eta}{\rho T_0} \nabla T_x. \quad (1)$$

Here, ρ , η , and k are the density, viscosity, and thermal conductivity, respectively, of the gas; k_1 is the thermal conductivity of the particle; T_0 is the bulk-gas absolute temperature at the particle center in the absence of the particle (or the mean gas temperature in the vicinity of the particle); C_s , C_t , and C_m are dimensionless coefficients (which are positive values) accounting for the thermal creep, temperature jump, and frictional slip phenomena, respectively, at the particle surface and must be determined experimentally for each gas-solid system. Very accurate values of these slip and jump coefficients for simple gases and gas mixtures, as based on the solutions of the Boltzmann

* Corresponding author.

equation and arbitrary gas-solid surface conditions, have been reported (Loyalka, 1990). A set of reasonable kinetic-theory values for complete thermal and momentum accommodations appear to be $C_s = 1.17$, $C_t = 2.18$, and $C_m = 1.14$ (Talbot *et al.*, 1980). Brock's (1962) analysis was extended to spheroidal particles by using prolate and oblate spheroidal coordinate systems (Leong, 1984).

On the other hand, aerosol particles can be driven to move by a body force, such as gravity. Basset (1961) derived the following expression for the translating velocity of an aerosol sphere under the action of an applied force $\mathbf{F}_A$ in the creeping-flow situation:

$$\mathbf{U} = \left(\frac{1 + 3C_m l/a}{1 + 2C_m l/a} \right) \frac{1}{6\pi\eta a} \mathbf{F}_A. \quad (2)$$

In the particular case of $C_m l/a = 0$, there is no frictional slip at the particle surface and eq. (2) degenerates to Stokes' law. In the limit $C_m l/a \rightarrow \infty$, there is a perfect frictional slip at the particle surface.

In addition, an aerosol particle can be driven to rotate by a torque. The angular velocity Ω of a spherical particle can be expressed as (Basset, 1961)

$$\Omega = (1 + 3C_m l/a) \frac{1}{8\pi\eta a^3} \mathbf{T}_A \quad (3)$$

where $\mathbf{T}_A$ is the torque applied on the sphere about its center.

In many practical applications, aerosol particles are not isolated and will move in the presence of neighboring particles and/or boundaries. Although the particle interactions in isothermal creeping-flow motion (Davis, 1972; Reed and Morrison, 1974; Ying and Peters, 1989; Keh and Chen, 1997) and in thermophoresis (Reed and Morrison, 1975; Keh and Chen, 1995, 1996) of aerosol spheres were analyzed extensively in the past, very little work about the boundary effects on the creeping-flow and thermophoretic motions of aerosol particles has been reported. Through an exact representation in spherical bipolar coordinates, the isothermal creeping-flow motion and thermophoresis of an aerosol sphere normal to an infinite plane wall were recently semi-analytically investigated (Chen and Keh, 1995a,b). Numerical results of corrections to eqs (1) and (2) for the particle velocities were presented for various cases. The retarding effects of the plane wall on the motions of the particle can be very significant when the surface-to-surface distance gets close to zero.

The purpose of this work is to obtain insights into the boundary effects on the creeping-flow and thermophoretic motions of an aerosol particle inside a small pore. This type of problem is difficult to solve due to the complexity of the actual system geometry. In order to overcome the mathematical complexities that occur in the problem of a sphere in a cylinder (which is a widely used model for particles in pores), we choose to examine the translational, rotational, and thermophoretic motions of a spherical particle situated at the center of a spherical cavity. Although

the geometry of spherical cavity is an idealized abstraction of any real system, the results obtained in this geometry have been shown to be in good agreement with available expressions for boundary effects on the partition coefficient (Giddings *et al.*, 1968; Glandt, 1981), settling velocity (Bungay and Brenner, 1973; Happel and Brenner, 1983), and electrophoretic mobility (Zydney, 1995; Keh and Chiou, 1996) of a spherical particle in a cylindrical pore. The spherical symmetry in this model system allows exact analytical solutions to be obtained, and the results show that the boundary effects on the creeping-flow motion and thermophoresis of an aerosol particle can be significant in general situations.

TRANSLATION OF A SLIP SPHERE IN A SPHERICAL CAVITY

In this section we consider the quasisteady translational motion of a spherical particle of radius a in a concentric spherical cavity (or pore) of radius b filled with an incompressible Newtonian fluid, as illustrated in Fig. 1. The fluid may slip frictionally at both surfaces of the particle and of the inside wall of the cavity, and they are allowed to differ in slip coefficient. The particle velocity equals $U\mathbf{e}_z$, where $\mathbf{e}_z$ is the unit vector in the positive z (axial) direction. The Reynolds number is assumed to be sufficiently small so that the inertial terms in the fluid momentum equation can be neglected, in comparison to the viscous terms. Our purpose here is to determine the correction to the Basset equation (2) for the particle due to the presence of the cavity.

The fluid flow between the particle and the cavity is governed by the Stokes equations,

$$\eta \nabla^2 \mathbf{v} - \nabla p = 0 \quad (4a)$$

$$\nabla \cdot \mathbf{v} = 0 \quad (4b)$$

where $\mathbf{v}$ is the fluid velocity field and p is the dynamic pressure distribution. Since the flow field is axially symmetric, it is convenient to introduce the Stokes stream function $\Psi(r, \theta)$ which satisfies eq. (4b) and is related to the velocity components in the spherical coordinate system (r, θ, ϕ) , with its origin at the particle center, by

$$v_r = -\frac{1}{r^2 \sin \theta} \frac{\partial \Psi}{\partial \theta} \quad (5a)$$

$$v_\theta = \frac{1}{r \sin \theta} \frac{\partial \Psi}{\partial r}. \quad (5b)$$

Taking the curl of eq. (4a) and applying eqs (4b) and (5) gives a fourth-order linear partial differential equation for Ψ ,

$$E^4 \Psi = E^2(E^2 \Psi) = 0 \quad (6)$$

where the axisymmetric Stokes operator E^2 is given by

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right). \quad (7)$$

Fig. 1. Geometrical sketch for the motion of a spherical particle in a concentric spherical cavity.

Since the relative tangential velocity of the fluid at each solid surface is proportional to the local tangential stress (Basset, 1961; Happel and Brenner, 1983) and there is no relative normal flow at the solid surfaces, the boundary conditions for the velocity field are

$$r = a: \quad v_r = U \cos \theta \quad (8a)$$

$$v_\theta = -U \sin \theta + \frac{C_m l}{\eta} \tau_{r\theta} \quad (8b)$$

$$r = b: \quad v_r = 0 \quad (9a)$$

$$v_\theta = -\frac{\hat{C}_m l}{\eta} \tau_{r\theta}. \quad (9b)$$

Here, C_m and $\hat{C}_m$ are the frictional slip coefficients about the surfaces of the particle and the cavity wall, respectively, and $\tau_{r\theta}(r, \theta)$ is the shear stress for the fluid flow,

$$\tau_{r\theta} = \eta \left[\frac{1}{r} \frac{\partial v_r}{\partial \theta} + r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \right]. \quad (10)$$

A solution of eq. (6) suitable for satisfying boundary conditions on the spherical surfaces is (Happel and Brenner, 1983)

$$\Psi = (Cr^{-1} + Dr + Er^2 + Fr^4) U \sin^2 \theta. \quad (11)$$

The constants C , D , E , and F are found from eqs (8) and (9) using eq. (5). The procedure is straightforward, with the result

$$C = \frac{1}{4} a^3 \omega [(1 + 3\hat{C}_m^*) - (1 - 3C_m^*)\lambda^3] \quad (12a)$$

$$D = -\frac{3}{4} a \omega [(1 + 2C_m^*)(1 + 3\hat{C}_m^*) - (1 - 3C_m^*)(1 - 2\hat{C}_m^*)\lambda^5] \quad (12b)$$

$$E = \frac{1}{8} \omega [9(1 + 2C_m^*)(1 + 2\hat{C}_m^*)\lambda - 5\lambda^3 - 4(1 - 3C_m^*)(1 - 3\hat{C}_m^*)\lambda^6] \quad (12c)$$

$$F = -\frac{3}{8} a^{-2} \omega [(1 + 2C_m^*)\lambda^3 - (1 - 2\hat{C}_m^*)\lambda^5] \quad (12d)$$

where

$$C_m^* = C_m l / a \quad (13a)$$

$$\hat{C}_m^* = \hat{C}_m l / b \quad (13b)$$

$$\lambda = a/b \quad (14)$$

$$\begin{aligned} \omega = & [(1 + 3C_m^*)(1 + 3\hat{C}_m^*) \\ & - \frac{9}{4}(1 + 2C_m^*)(1 + 2\hat{C}_m^*)\lambda + \frac{5}{2}\lambda^3 \\ & - \frac{9}{4}(1 - 2C_m^*)(1 - 2\hat{C}_m^*)\lambda^5 \\ & + (1 - 3C_m^*)(1 - 3\hat{C}_m^*)\lambda^6]^{-1}. \end{aligned} \quad (15)$$

With this solution, the components of the fluid velocity in this axisymmetric flow (with $v_\phi = 0$) can be calculated by using eq. (5).

The drag force (in the z direction) exerted by the fluid on the particle is (Happel and Brenner, 1983)

$$F_d = 8\pi\eta U D. \quad (16)$$

The particle velocity can be expressed in the form of eq. (2) with a wall correction,

$$U = -M \left(\frac{1 + 3C_m^*}{1 + 2C_m^*} \right) \frac{1}{6\pi\eta a} F_d \quad (17)$$

where the dimensionless mobility coefficient

$$M = -\frac{3a(1 + 2C_m^*)}{4D(1 + 3C_m^*)} \quad (18)$$

and D has been given by eq. (12b). Note that $0 \leq M \leq 1$. The presence of the cavity wall always enhances the hydrodynamic drag on the particle since the normal flow vanishes at the wall as required by eq. (9a).

When $C_m^* = \hat{C}_m^* = 0$, eq. (16) reduces to

$$\begin{aligned} U = & - \left(1 - \frac{9}{4}\lambda + \frac{5}{2}\lambda^3 - \frac{9}{4}\lambda^5 + \lambda^6 \right) \\ & \times (1 - \lambda^5)^{-1} \frac{1}{6\pi\eta a} F_d. \end{aligned} \quad (19)$$

This is the result for the translation of a no-slip sphere in a no-slip cavity.

In the limit $C_m^* \rightarrow \infty$ and $\hat{C}_m^* = 0$, eq. (16) becomes

$$U = - (1 - \frac{3}{2}\lambda + \frac{3}{2}\lambda^5 - \lambda^6) (1 + \frac{3}{2}\lambda^5)^{-1} \frac{1}{4\pi\eta a} F_d. \quad (20)$$

This formula is identical to that for a gas bubble in a no-slip cavity filled with a liquid.

When $C_m^* = 0$ and $\hat{C}_m^* \rightarrow \infty$, eq. (16) reduces to

$$U = - (1 - \frac{3}{2}\lambda + \frac{3}{2}\lambda^5 - \lambda^6) (1 + \frac{3}{2}\lambda^5)^{-1} \frac{1}{6\pi\eta a} F_d. \quad (21)$$

This is the case for a no-slip sphere in an outer frictionless spherical envelope of fluid.

Fig. 2. Plots of the mobility coefficient M for a translating sphere situated at the center of a spherical cavity as a function of λ : (a) $\hat{C}_m^* = \lambda C_m^*$; (b) $\hat{C}_m^* = 0$.

In the limit $C_m^* \rightarrow \infty$ and $\hat{C}_m^* \rightarrow \infty$, eq. (16) becomes

$$U = -(1 - \lambda) \frac{1}{4\pi\eta a} F_d. \quad (22)$$

This expression is the same as that for a gas bubble in an outer frictionless envelope of liquid. Formulas (19)–(21) for the special cases have also been derived by a number of previous investigations as discussed by Happel and Brenner (1983).

The dimensionless mobility coefficient M of the particle, as calculated from eqs (12b) and (18), is plotted vs the separation parameter λ in Fig. 2 for the cases of $\hat{C}_m^* = \lambda C_m^*$ (or $\hat{C}_m = C_m$) and $\hat{C}_m^* = 0$ with C_m^* as a parameter. It can be seen that the boundary effect of the cavity wall on the motion of the particle decreases (or M increases) with the increase of the slip parameters C_m^* and $\hat{C}_m^*$ for a given value of λ . As expected, M is a monotonic decreasing function of λ ,

$M = 1$ as $\lambda = 0$ (the fluid surrounding the particle is unbounded), and $M = 0$ as $\lambda = 1$ (the cavity is filled with the particle) for all cases. Generally speaking, the cavity wall effect on the particle motion is quite significant.

Compared with the available solutions for the translation of a no-slip sphere or a spherical gas bubble along the axis of a no-slip circular tube with radius b provided by the literature (Haberman and Sayre, 1958; Coutanceau and Thizon, 1981), the formulas (19) and (20) for the corresponding motion in a spherical cavity predict somewhat smaller values for the particle mobility at the same value of λ ; however, the difference in this boundary effect between these two geometrical cases is quite small. The similarity in the hydrodynamic hindrance to the particle in these two geometries reflects the fact that the viscous retardation in the circular tube is greatest at the point of closest approach of the particle to the tube wall, with the geometry of this region being very similar to that of a spherical particle at the center of a spherical cavity (Zydney, 1995).

ROTATION OF A SLIP SPHERE IN A SPHERICAL CAVITY

In this section we consider the steady rotational motion of a spherical particle situated at the center of a spherical cavity. Again the fluid may slip at the solid surfaces. The angular velocity of the particle is $\Omega \mathbf{e}_z$. The objective here is to correct eq. (3) for the particle rotation owing to the presence of the cavity.

The fluid flow between the particle and the cavity is still governed by eq. (4), and it must be solved subject to the following boundary conditions:

$$r = a: \quad v_r = v_\theta = 0 \quad (23a)$$

$$v_\phi = \Omega r \sin \theta + \frac{C_m l}{\eta} \tau_{r\phi} \quad (23b)$$

$$r = b: \quad v_r = v_\theta = 0 \quad (24a)$$

$$v_\phi = -\frac{\hat{C}_m l}{\eta} \tau_{r\phi}. \quad (24b)$$

Here, $\tau_{r\phi}(r, \theta)$ is the shear stress,

$$\tau_{r\phi} = \eta \left[\frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} + r \frac{\partial}{\partial r} \left(\frac{v_\phi}{r} \right) \right]. \quad (25)$$

A solution of eq. (4) suitable for satisfying the boundary conditions (23) and (24) is

$$p = \text{constant} \quad (26a)$$

$$v_r = v_\theta = 0 \quad (26b)$$

$$v_\phi = (Gr^{-2} + Hr)\Omega \sin \theta. \quad (26c)$$

The constants G and H are determined from the boundary conditions, with the result

$$G = a^3 [(1 + 3C_m^*) - (1 - 3\hat{C}_m^*)\lambda^3]^{-1} \quad (27a)$$

$$H = -(1 - 3\hat{C}_m^*)\lambda^3 [(1 + 3C_m^*) - (1 - 3\hat{C}_m^*)\lambda^3]^{-1} \quad (27b)$$

where parameters C_m^* , $\hat{C}_m^*$, and λ have been defined by eqs (13) and (14). Note that, in this creeping flow, only the ϕ component of the fluid velocity is nonzero.

The torque exerted on the particle about its center by the fluid is

$$T_d = -8\pi\eta\Omega G. \quad (28)$$

The angular velocity of the particle can be written as the form of eq. (3) with a wall correction,

$$\Omega = -N(1 + 3C_m^*) \frac{1}{8\pi\eta a^3} T_d \quad (29)$$

where the dimensionless rotational mobility coefficient

$$N = 1 - \frac{1 - 3\hat{C}_m^*}{1 + 3C_m^*} \lambda^3. \quad (30)$$

In the particular situation with $\hat{C}_m^* = C_m^* = 0$ and $\lambda = 1$ (namely, a no-slip cavity is filled with a no-slip particle), one obtains $N = 0$. In all the other situations, one has $N > 0$. Also, as expected, $N = 1$ as $\lambda = 0$.

Interestingly, eq. (30) predicts that $N = 1$ if $\hat{C}_m^* = 1/3$, irrespective of the values of C_m^* and λ . It can be found, in this special case, the solution of the fluid flow given by eq. (26) is the same as that for an identical slip sphere rotating in an unbounded fluid of identical properties. This phenomenon does not occur in the case of particle translation presented in the previous section due to the existence of the r component of the fluid velocity field.

When $C_m^* = \hat{C}_m^* = 0$, eq. (29) reduces to

$$\Omega = -(1 - \lambda^3) \frac{1}{8\pi\eta a^3} T_d. \quad (31)$$

This is the result for the rotation of a no-slip sphere in a no-slip cavity.

The rotational mobility coefficient N of the particle, as calculated from eq. (30), is shown as a function of parameter λ in Fig. 3 for the cases of $\hat{C}_m^* = \lambda C_m^*$ (or $\hat{C}_m^* = C_m^*$) and $\hat{C}_m^* = 0$ with C_m^* as a parameter. It can be found that, in general, N increases with the increase of the slip parameters C_m^* and $\hat{C}_m^*$ for a fixed value of λ . If $\hat{C}_m^* < 1/3$ (the hydrodynamic resistance on the particle produced by the cavity wall is greater than that exerted by an unbounded fluid), then $N \leq 1$ and N is a monotonic decreasing function of λ for fixed values of C_m^* and $\hat{C}_m^*$. If $\hat{C}_m^* > 1/3$ (the hydrodynamic hindrance to the particle in the cavity is smaller than that in the unbounded situation), then $N \geq 1$ and N is a monotonic increasing function of λ for given values of C_m^* and $\hat{C}_m^*$. Note that, the highest value of N for the case of $\hat{C}_m^* = \lambda C_m^*$ is 2, which appears in the situation with $\hat{C}_m^* \rightarrow \infty$ and $\lambda = 1$ (namely, a frictionless envelope of radius b is filled with a rotating gas bubble). In general, the boundary effect of the cavity wall on the particle rotation can be quite significant as the value of λ approaches unity.

Fig. 3. Plots of the mobility coefficient N for a rotating sphere situated at the center of a spherical cavity as a function of λ : (a) $\hat{C}_m^* = \lambda C_m^*$; (b) $\hat{C}_m^* = 0$.

THERMOPHORESIS OF A SPHERE IN A SPHERICAL CAVITY

We now consider the quasisteady thermophoretic motion of a spherical particle located at the center of a spherical cavity. A linear temperature field $T_x(r, \theta)$ with a uniform thermal gradient $-E_x \mathbf{e}_x$ (equal to ∇T_x) is prescribed in the cavity surroundings far away from the particle [as described by eq. (36)]. The fluid is allowed to slip, both thermally and frictionally, and the temperature jump may occur at the particle surface and the cavity wall (Kennard, 1938; Brock, 1962). The Reynolds and Peclet numbers in the fluid phase are assumed to be quite small. The aim in this section is to determine the correction to eq. (1) for the particle due to the presence of the cavity.

The energy equations governing the temperature distribution are

$$\nabla^2 T_1 = 0 \quad (r < a) \quad (32a)$$

for the particle,

$$\nabla^2 T = 0 \quad (a < r < b) \quad (32b)$$

for the fluid, and

$$\nabla^2 T_2 = 0 \quad (r > b) \quad (32c)$$

for the cavity surroundings. The boundary conditions at the solid surfaces require that the normal heat fluxes be continuous and a temperature jump which is proportional to the normal temperature gradient (Kennard, 1938) occur. Also, the temperature in the cavity surroundings must approach the linear prescribed field far from the particle and the temperature inside the particle is finite everywhere. Thus, one has

$$r < a: \quad T_1 \text{ is finite} \quad (33)$$

$$r = a: \quad k \frac{\partial T}{\partial r} = k_1 \frac{\partial T_1}{\partial r} \quad (34a)$$

$$T - T_1 = C_t l \frac{\partial T}{\partial r} \quad (34b)$$

$$r = b: \quad k \frac{\partial T}{\partial r} = k_2 \frac{\partial T_2}{\partial r} \quad (35a)$$

$$T - T_2 = -\hat{C}_t l \frac{\partial T}{\partial r} \quad (35b)$$

$$r \rightarrow \infty: \quad T_2 \rightarrow T_\infty = T_0 - E_\infty r \cos \theta. \quad (36)$$

Here, k , k_1 , and k_2 are the thermal conductivities of the fluid, the particle, and the cavity surroundings, respectively, which are assumed to be independent of temperature; C_t and $\hat{C}_t$ are the temperature jump coefficients about the surfaces of the particle and the cavity wall, respectively. The undisturbed temperature at the plane $z = 0$ has been set equal to T_0 .

The solution of eqs (32)–(36) is

$$T_1 = T_0 - 3AE_\infty r \cos \theta \quad (r < a) \quad (37a)$$

$$T = T_0 - A \left[P + Q \left(\frac{a}{r} \right)^3 \right] E_\infty r \cos \theta \quad (a < r < b) \quad (37b)$$

$$T_2 = T_0 - \left[1 - AB \left(\frac{b}{r} \right)^3 \right] E_\infty r \cos \theta \quad (r > b) \quad (37c)$$

where

$$A = 3k_2^* [(1 + 2k_2^* + 2k_2^* \hat{C}_t^*)P - 2(1 - k_2^* + 2k_2^* \hat{C}_t^*)Q\lambda^3]^{-1} \quad (38a)$$

$$B = \frac{1}{3k_2^*} [(1 - k_2^* - k_2^* \hat{C}_t^*)P - (2 + k_2^* - 2k_2^* \hat{C}_t^*)Q\lambda^3] \quad (38b)$$

$$P = 2 + k_1^* + 2k_1^* C_t^* \quad (39a)$$

$$Q = 1 - k_1^* + k_1^* C_t^* \quad (39b)$$

$$k_1^* = k_1/k \quad (40a)$$

$$k_2^* = k_2/k \quad (40b)$$

$$C_t^* = C_t l/a \quad (41a)$$

$$\hat{C}_t^* = \hat{C}_t l/b. \quad (41b)$$

In eq. (38), the definition of parameter λ is given by eq. (14). For the special case of $k_2^* = 1$ and $\hat{C}_t^* = 0$ (which implies $A = 1/P$ and $B = -Q\lambda^3$), the temperature field given by eq. (37) is the same as that for a spherical particle in an unbounded medium.

The fluid motion caused by the thermophoresis of the particle is also governed by eq. (4). Owing to the thermal creep and frictional slip velocities along the solid–fluid interfaces, the boundary conditions for the fluid velocity are

$$r = a: \quad v_r = U \cos \theta \quad (42a)$$

$$v_\theta = -U \sin \theta + \frac{C_m l}{\eta} \tau_{r\theta} + \frac{C_s \eta}{\rho T_0} \frac{\partial T}{r \partial \theta} \quad (42b)$$

$$r = b: \quad v_r = 0 \quad (43a)$$

$$v_\theta = -\frac{\hat{C}_m l}{\eta} \tau_{r\theta} + \frac{\hat{C}_s \eta}{\rho T_0} \frac{\partial T}{r \partial \theta}. \quad (43b)$$

Here, C_s and $\hat{C}_s$ are the thermal slip coefficients of the particle surface and the cavity wall, respectively, U is the instantaneous thermophoretic velocity of the particle to be determined, and $\partial T/\partial \theta$ can be evaluated from eq. (37b). The validity of the expressions for the thermal creep velocities in eqs (42b) and (43b) is based on the assumption that the fluid is only slightly nonuniform in the undisturbed temperature on the length scale of the particle and cavity radii.

Since the governing equation and the boundary conditions are linear, the total flow can be decomposed into two parts. First, we consider the fluid motion about a spherical particle translating with velocity $U\mathbf{e}_z$ at the center of the spherical cavity with frictional slip but with no thermal creep velocities at the solid surfaces. The stream function for this flow has been obtained in eqs (11) and (12), and the force on the particle is given in eq. (16). We shall use superscript (1) to the variables Ψ , F_d , and D to represent this flow.

Next, we consider the flow caused by the thermal creep velocities at the frictionally slip surfaces of the stationary spherical particle and cavity:

$$r = a: \quad v_r^{(2)} = 0 \quad (44a)$$

$$v_\theta^{(2)} = \frac{C_m l}{\eta} \tau_{r\theta} + \frac{C_s \eta}{\rho T_0} \frac{\partial T}{r \partial \theta} \quad (44b)$$

$$r = b: \quad v_r^{(2)} = 0 \quad (45a)$$

$$v_\theta^{(2)} = \frac{\hat{C}_m l}{\eta} \tau_{r\theta} + \frac{\hat{C}_s \eta}{\rho T_0} \frac{\partial T}{r \partial \theta}. \quad (45b)$$

Superimposing the velocity field $\mathbf{v}^{(2)}$ with that derived from $\Psi^{(1)}$ yields the total velocity field produced by the thermophoretic motion of a sphere at the center of a spherical cavity. By obtaining the force exerted on

the stationary sphere, adding this to the force $F_d^{(1)}$, and equating the result to zero, the thermophoretic velocity of the particle will result.

The stream function $\Psi^{(2)}$ for the flow caused by the thermal creep velocities at the surfaces of the stationary spherical particle and cavity can also be expressed by eq. (11), and the coefficients C , D , E , and F should be determined by the boundary conditions (44) and (45) using eq. (5). The result is

$$C = -a^3 K \omega \{ C_s (P + Q) [2(1 + 3\hat{C}_m^*) - 3(1 + 2\hat{C}_m^*)\lambda + \lambda^3] + \hat{C}_s [P - 3P(1 - 2\hat{C}_m^*)\lambda^2 + (2P(1 - 3\hat{C}_m^*) + Q)\lambda^3 - 3Q(1 - 2\hat{C}_m^*)\lambda^5 + 2Q(1 - 3\hat{C}_m^*)\lambda^6] \} \quad (46a)$$

$$D = a K \omega \{ C_s (P + Q) [2(1 + 3\hat{C}_m^*) - 5\lambda^3 + 3(1 - 2\hat{C}_m^*)\lambda^5] + \hat{C}_s [3P(1 + 2\hat{C}_m^*) - 5P\lambda^2 + 3Q(1 + 2\hat{C}_m^*)\lambda^3 + (2P(1 - 3\hat{C}_m^*) - 5Q)\lambda^5 + 2Q(1 - 3\hat{C}_m^*)\lambda^8] \} \quad (46b)$$

$$E = -K \omega \{ C_s (P + Q) [3(1 + 2\hat{C}_m^*)\lambda - 5\lambda^3 + 2(1 - 3\hat{C}_m^*)\lambda^6] + \hat{C}_s [2P(1 + 3\hat{C}_m^*) - (5P - 2Q(1 + 3\hat{C}_m^*))\lambda^3 + 3P(1 - 2\hat{C}_m^*)\lambda^5 - 5Q\lambda^6 + 3Q(1 - 2\hat{C}_m^*)\lambda^8] \} \quad (46c)$$

$$F = a^{-2} K \omega \{ C_s (P + Q) [\lambda^3 - 3(1 - 2\hat{C}_m^*)\lambda^5 + 2(1 - 3\hat{C}_m^*)\lambda^6] + \hat{C}_s [2P(1 + 3\hat{C}_m^*)\lambda^2 - 3P(1 + 2\hat{C}_m^*)\lambda^3 + (P + 2Q(1 + 3\hat{C}_m^*))\lambda^5 - 3Q(1 + 2\hat{C}_m^*)\lambda^6 + Q\lambda^8] \} \quad (46d)$$

where $K = A\eta E_x / 4\rho T_0 U$; the frictional slip parameters C_m^* and $\hat{C}_m^*$ are defined by eq. (13). The force exerted on the stationary sphere by the fluid due to the thermal creep motion, $F_d^{(2)}$, can be obtained by the substitution of eq. (46b) into eq. (16).

Since the net force acting on the thermophoretic particle must vanish, we have

$$F_d^{(1)} + F_d^{(2)} = 8\pi\eta U (D^{(1)} + D^{(2)}) = 0 \quad (47)$$

where $D^{(1)}$ and $D^{(2)}$ are given by eqs (12b) and (46b), respectively. Solution of eq. (47) yields the thermophoretic velocity of the particle, which can be expressed as

$$U = U_0 + M_t \left[\frac{2C_s(1 + k_T^* C_r^*)}{P(1 + 2C_m^*)} \right] \frac{\eta E_x}{\rho T_0} \quad (48)$$

Here,

$$U_0 = \frac{1}{3} \hat{C}_s A \{ 3P(1 + 2\hat{C}_m^*) - 5P\lambda^2 + 3Q(1 + 2\hat{C}_m^*)\lambda^3 + [2P(1 - 3\hat{C}_m^*) - 5Q]\lambda^5 + 2Q(1 - 3\hat{C}_m^*)\lambda^8 \} [(1 + 2\hat{C}_m^*)(1 + 3\hat{C}_m^*) - (1 - 3\hat{C}_m^*)(1 - 2\hat{C}_m^*)\lambda^5]^{-1} \frac{\eta E_x}{\rho T_0} \quad (49)$$

representing the velocity of a sphere with $C_s = 0$ in the cavity, which is due to the thermosmotic flow that arises from the interaction of the prescribed temperature gradient with the slip cavity wall. The second term on the right-hand side of eq. (48) is expressed as the form of eq. (1), with the dimensionless wall correction factor (or the thermophoretic mobility coefficient)

$$M_t = \frac{1}{2} AP(1 + 2C_m^*) [2(1 + 3\hat{C}_m^*) - 5\lambda^3 + 3(1 - 2\hat{C}_m^*)\lambda^5] [(1 + 2\hat{C}_m^*)(1 + 3\hat{C}_m^*) - (1 - 3\hat{C}_m^*)(1 - 2\hat{C}_m^*)\lambda^5]^{-1} \quad (50)$$

Note that M_t is not a function of the thermal slip coefficients C_s and $\hat{C}_s$.

If we consider the case that the uniform temperature gradient $-E_x e_x$ is prescribed along the inside wall of the cavity, then the heat conduction in the cavity surroundings should not be taken into account. In this case, the boundary conditions given by eqs (35) and (36) are replaced by

$$r = b: \quad T = T_0 - E_x r \cos \theta - \hat{C}_t l \frac{\partial T}{\partial r} \quad (51)$$

The solution of the governing equations (32a) and (32b) subject to the boundary conditions (33), (34), and (51) is also given by the form of eqs (37a) and (37b), with coefficient A defined as

$$A = [(1 + \hat{C}_t^*)P + (1 - 2\hat{C}_t^*)Q\lambda^3]^{-1} \quad (52)$$

Thus, the thermophoretic mobility coefficient of a sphere in this concentric cavity can still be expressed as eq. (50) with A given by eq. (52).

A sketch of the heat conduction lines (with a constant value of the heat conduction function V for each) and the isotherms around a spherical particle situated at the center of a spherical cavity with $\lambda = 0.3$ is exhibited in Fig. 4 for the temperature field prescribed by either eq. (36) or eq. (51) (with arbitrary values of k_T^* and $\hat{C}_t^*$). In this axisymmetric case, the heat conduction function is orthogonally related to the temperature distribution by the formulas (Li and Keh, 1997)

$$\frac{\partial V}{\partial R} = R \frac{\partial T}{\partial z} \quad (53a)$$

$$\frac{\partial V}{\partial z} = -R \frac{\partial T}{\partial R} \quad (53b)$$

where $R = r \sin \theta$, which is the radial cylindrical coordinate. These relations can be integrated after the substitution of the temperature profile provided by eq. (37) to yield

$$V_1 = -\frac{3}{2} A E_x r^2 \sin^2 \theta \quad (r < a) \quad (54a)$$

$$V = -A \left[\frac{1}{2} P - Q \left(\frac{a}{r} \right)^3 \right] E_x r^2 \sin^2 \theta \quad (a < r < b) \quad (54b)$$

$$V_2 = - \left[\frac{1}{2} + AB \left(\frac{b}{r} \right)^3 \right] E_x r^2 \sin^2 \theta \quad (r > b) \quad (54c)$$

(a)

(b)

Fig. 4. Heat conduction lines (solid curves) and isotherms (dashed curves) around a spherical particle in a concentric spherical cavity with $\lambda = 0.3$ and $C^* = 0.2$: (a) $k_1^* = 100$; (b) $k_1^* = 0.1$. The dimensionless conduction function $\Gamma = -V/AE_x a^2$ and the dimensionless temperature $\Phi = -(T - T_0)/AE_x a$.

where the value of V along the z axis (with $\theta = 0$ and $\theta = \pi$) is set equal to zero for convenience. In Fig. 4, only the contours in a half of a meridian plane (with $0 \leq \theta \leq \pi/2$) are depicted due to the axial and fore/aft symmetry. It can be found (Li and Keh, 1997) that the heat conduction lines are attracted by the particle if $k_1^* - K_1^* C_1^* > 1$, and they are repelled by the particle if $k_1^* - K_1^* C_1^* < 1$. Note that the isotherms are not continuous at the particle surface, as specified by eq. (34b).

The thermophoretic mobility coefficient M_t of the particle, as calculated from eq. (50), for the temperature field prescribed by eqs (33)–(36) with $k_2^* = 1$ and $\hat{C}_1^* = 0$ is plotted vs parameter λ in Fig. 5. It can be found from eq. (50) that M_t is independent of the parameter k_1^* under this situation. For the case of $\hat{C}_m^* = \lambda C_m^*$ (or $\hat{C}_m = C_m$), as shown in Fig. 5(a), M_t increases with the increase of the slip parameters

(a)

(b)

Fig. 5. Plots of the mobility coefficient M_t for a thermophoretic sphere situated at the center of a spherical cavity as a function of λ for the temperature field prescribed by eq. (36) with $k_2^* = 1$, $\hat{C}_1^* = 0$, and $C_1^* = 2C_m^*$: (a) $\hat{C}_m^* = \lambda C_m^*$; (b) $\hat{C}_m^* = 0$.

C_m^* and $\hat{C}_m^*$ for a fixed value of λ . This trend is the same as that for a translating sphere presented in Fig. 2(a). However, for the case of $\hat{C}_m^* = 0$, M_t decreases with the increase of C_m^* for a given value of λ , as illustrated in Fig. 5(b). A comparison between Figs 5(a) and (b) shows that M_t increases with the increase of $\hat{C}_m^*$ for constant values of C_m^* and λ . Similar to the mobility coefficient M for a translating sphere, M_t is a monotonic decreasing function of λ , $M_t = 1$ as $\lambda = 0$, and $M_t = 0$ as $\lambda = 1$ for both cases. Note that M_t is not necessarily equal to unity as $\lambda = 0$ for general cases with $k_2^* \neq 1$ [$M_t = 3k_2^*/(1 + 2k_2^*)$].

The calculated values of M_t for a sphere in a cavity with the temperature field prescribed by eqs (33), (34), and (51) are shown as a function of parameter λ in Fig. 6 for the case of $\hat{C}_1^* = \lambda C_1^*$ (or $\hat{C}_1 = C_1$) and $C_1^* = 2C_m^*$. Under this situation, M_t again increases

Fig. 6. Plots of the mobility coefficient M_t for a thermophoretic sphere situated at the center of a spherical cavity as a function of λ for the temperature field prescribed by eq. (51) with $\hat{C}_t^* = \lambda C_m^*$, $C_t^* = 2C_m^*$, and $k_2^* = 100$: (a) $\hat{C}_m^* = \lambda C_m^*$; (b) $\hat{C}_m^* = 0$.

with the increase of $\hat{C}_m^*$ for given values of C_m^* and λ , but now this mobility coefficient is a weak increasing function of the parameter k_2^* . It can be seen that M_t in general decreases as C_m^* or λ is increased. However, as indicated in Fig. 6(a), the thermophoretic mobility coefficient is not necessarily a monotonic function of either C_m^* (as λ approaches unity) or λ (as C_m^* is large). These complex results are generated from the combined effects of particle-cavity interactions on the temperature gradient and fluid velocity fields. A comparison between the coefficient M_t illustrated in Figs 5 and 6 and the coefficient M illustrated in Fig. 2 shows that, for real aerosol systems (with $C_m^* \leq 0.1$), the boundary effect on thermophoresis is much weaker than that on the motion in a gravitation field.

The effects of the thermoosmotic flow along the cavity wall on the movement of a particle with no thermal creep at the surface ($C_s = 0$) can be evaluated using eq. (49). The fluid streamlines in a meridian plane associated with this thermoosmotic flow are drawn in Fig. 7(a) for a cavity with $\lambda = 0.3$. The fluid at $\theta = \pi/2$ moves in the direction parallel to that of the prescribed temperature gradient near the cavity wall, but as the fluid approaches the z axis the streamlines become opposite to the direction of the prescribed gradient. This recirculation flow pattern in the cavity exerts a drag force on the particle and results in a migration velocity U_0 in the opposite direction to the prescribed temperature gradient. The next contribution of the cavity wall to the particle velocity is of $O(\lambda^2)$, as indicated in eq. (49), with this behavior arising from the deformation of the streamlines as the recirculated fluid passes around the particle. For a comparison, Fig. 7(b) displays streamlines for a thermophoretic sphere ($C_s \neq 0$) in a corresponding cavity with no thermal creep at the wall ($\hat{C}_s = 0$) [the streamline pattern for an isolated thermophoretic sphere in an unbounded fluid was exhibited by Keh and Yu (1995)]. It can be seen that the motion in the fluid around the equator of the particle ($\theta = \pi/2$) for this case is quite different from that shown in Fig. 7(a). The thermal creep velocity along the surface of the thermophoretic sphere drives the fluid motion which is aligned with the prescribed temperature gradient and, therefore, against the direction of particle movement. As illustrated in eq. (50), the next contribution of the cavity wall to the thermophoretic velocity is of $O(\lambda^3)$, instead of $O(\lambda^2)$ for the effect on U_0 . Note that, the streamline patterns displayed in Figs 6 and 7 are valid for the temperature field prescribed by either eq. (36) or eq. (51) and are independent of the values of k_2^* and $\hat{C}_t^*$.

The dependence of the normalized particle velocity $U_0 \rho T_0 / \hat{C}_s \eta E_x$ [calculated from eq. (49)] on the parameter λ is shown in Fig. 8 for the temperature field prescribed by eqs (33)–(36) with $k_2^* = 1$ and $\hat{C}_t^* = 0$. For the case of $\hat{C}_m^* = 0$, this normalized velocity increases with the increase of the slip parameter C_m^* for a given value of λ , as illustrated in Fig. 8(b). This trend is the same as that for the translation of a sphere presented in Fig. 2(b). However, as indicated in Fig. 8(a) for the case of $\hat{C}_m^* = \lambda C_m^*$ (or $\hat{C}_m = C_m$), this normalized velocity may not be a monotonic function of C_m^* for a fixed value of λ . A comparison between Figs 8(a) and (b) shows that $U_0 \rho T_0 / \hat{C}_s \eta E_x$ decreases (while M_t increases) with the increase of $\hat{C}_m^*$ for constant values of C_m^* and λ . Although $U_0 \rho T_0 / \hat{C}_s \eta E_x$ is a monotonic decreasing function of λ which equals unity as $\lambda = 0$ and vanishes as $\lambda = 1$ for both cases shown in Fig. 8, this normalized velocity is not necessarily equal to one [but equals $3k_2^*/(1 - 2k_2^*)$] as $\lambda = 0$ for general cases with $k_2^* \neq 1$.

The normalized particle velocity $U_0 \rho T_0 / \hat{C}_s \eta E_x$ of the particle, as calculated from eq. (49), for the temperature field prescribed by eqs (33), (34), and (51) is plotted vs parameter λ in Fig. 9 for the situation of

Fig. 7. Fluid streamlines for the recirculation flow around a sphere situated at the center of a spherical cavity with $\lambda = 0.3$, $\hat{C}_m^* = \lambda C_m^*$, $C_t^* = 2C_m^* = 0.2$, and $k_1^* = 100$: (a) $C_s = 0$; (b) $\hat{C}_s = 0$.

$\hat{C}_t^* = \lambda C_t^*$ (or $\hat{C}_t = C_t$) and $C_t^* = 2C_m^*$. For both cases of $\hat{C}_m^* = \lambda C_m^*$ and $\hat{C}_m^* = 0$ under this situation, this normalized velocity decreases monotonically with the increase of the slip parameter C_m^* for a fixed value of λ , and is a monotonic decreasing function of λ which equals unity as $\lambda = 0$ and disappears as $\lambda = 1$ for a given value of C_m^* . Similar to the situation presented in Fig. 8, Fig. 9 also shows that $U_0 \rho T_0 / \hat{C}_s \eta E_\infty$ decreases with increase of $\hat{C}_m^*$ for constant values of C_m^* and λ . It can be found from eq. (49) that $U_0 \rho T_0 / \hat{C}_s \eta E_\infty$, in general, is a weak decreasing function of the parameter k_1^* .

To the author's knowledge, the thermophoretic motion of an aerosol particle inside a small pore of other geometries has not yet been studied. However, the electrophoretic mobility of a charged spherical particle in a concentric spherical cavity (Zydney, 1995) agrees quite well with that in a coaxial circular cylinder (Keh and Chiou, 1996). This agreement is due to

Fig. 8. Plots of the normalized velocity $U_0 \rho T_0 / \hat{C}_s \eta E_\infty$ for a sphere with $C_s = 0$ situated at the center of a spherical cavity as a function of λ for a temperature field prescribed by eq. (36) with $k_2^* = 1$, $\hat{C}_t^* = 0$, $C_t^* = 2C_m^*$, and $k_1^* = 100$. (a) $\hat{C}_m^* = \lambda C_m^*$; (b) $\hat{C}_m^* = 0$.

the fact that the deformation of the electric potential field and the viscous retardation in the cylindrical pore geometry are greatest at the point of closest approach of the sphere to the pore wall, with the geometry of this region being very similar to that found in the spherical cavity. Since the mechanism of thermophoresis/thermoosmosis is analogous to that of electrophoresis/electroosmosis due to the analogy between heat and electric conduction, it is reasonable to expect that the thermophoretic mobility of an aerosol sphere in concentric spherical cavity with $\hat{C}_s = 0$ obtained here agrees well with that in a circular cylinder or in other more realistic models for a small pore. On the other hand, the effect of the thermoosmotic flow on the particle motion in the spherical cavity should be markedly different from that in an open cylindrical pore. The reason for this difference is that the spherical cavity is closed so that there is no net thermoosmotic flow through the pore, while there is

(a)

(b)

Fig. 9. Plots of the normalized velocity $U_0 \rho T_0 / \hat{C}_s \eta E_\infty$ for a sphere with $C_s = 0$ situated at the center of a spherical cavity as a function of λ for a temperature field prescribed by eq. (51) with $\hat{C}_t^* = \lambda C_t^*$, $C_t^* = 2C_m^*$, and $k_t^* = 100$. (a) $\hat{C}_m^* = \lambda C_m^*$; (b) $\hat{C}_m^* = 0$.

a particle thermoosmotic flow in the open cylindrical system.

SUMMARY

The translation, rotation, and thermophoresis of a spherical particle in a concentric spherical cavity filled with an incompressible Newtonian fluid have been analyzed in this study. The particle and the cavity surroundings are allowed to differ in surface properties (including frictional slip coefficient, thermal creep coefficient and temperature jump coefficient) and in thermal conductivity. Based on the assumption of very small Peclet and Reynolds numbers, the temperature and flow fields for these axisymmetric motions are analytically solved and the particle velocities are obtained in the closed-form expressions (17), (29), and (48). The results, which provide important insights into the actual phenomena governing the particle motions under different conditions, show that

the boundary effects of the cavity wall on the creeping-flow and thermophoretic motions of an aerosol particle can be significant in appropriate situations. In practical aerosol systems, the boundary effect on thermophoresis of a particle is much weaker than that on the particle motion driven by gravity. More detailed analyses of the fluid flow and temperature fields for an aerosol particle in open and closed cylindrical pores will clearly be required to quantify the actual behavior in this system and to determine the overall applicability of the results obtained in this paper for the spherical cavity to more realistic pore geometries.

Acknowledgement

This research was supported by the National Science Council of the Republic of China under Grant NSC85-2214-E-002-025.

NOTATION

a	particle radius, m
A	coefficient defined by eq. (38a) or eq. (52)
b	radius of the spherical cavity, m
B	coefficient defined by eq. (38b)
C, D, E, F	coefficients defined by eq. (11), m^3 , m, dimensionless numbers, m^{-2}
C_m, C_s, C_t	coefficients accounting for the frictional slip, thermal creep, and temperature jump, respectively, at the particle surface
$\hat{C}_m, \hat{C}_s, \hat{C}_t$	coefficients accounting for the frictional slip, thermal creep, and temperature jump, respectively, at the cavity wall
C_m^*, C_t^*	equal to $C_m l/a$ and $C_t l/a$, respectively
$\hat{C}_m^*, \hat{C}_t^*$	equal to $\hat{C}_m l/b$ and $\hat{C}_t l/b$, respectively
e_z	unit vector in z direction
E^2	Stokes operator defined by eq. (7), m^{-2}
E_∞	prescribed temperature gradient in the absence of the particle, $K m^{-1}$
F_A	applied force on the particle, N
F_d	drag force exerted on the particle, N
G, H	coefficients defined by eq. (26c), m^3 , dimensionless numbers
k	thermal conductivity of the fluid, $W m^{-1} K^{-1}$
k_1	thermal conductivity of the particle, $W m^{-1} K^{-1}$
k_2	thermal conductivity of the cavity surroundings, $W m^{-1} K^{-1}$
k_1^*, k_2^*	equal to k_1/k and k_2/k , respectively
l	mean free path of the gas molecules, m
M	mobility coefficient defined by eq. (17)
M_t	thermophoretic mobility coefficient defined by eq. (48)
N	rotational mobility coefficient defined by eq. (29)
P	dynamic pressure distribution, $N m^{-2}$
P, Q	variables defined by eq. (39)
r	radial spherical coordinate, m
R	radial cylindrical coordinate, m
T	temperature distribution in the fluid phase, K

T_1	temperature distribution inside the particle, K
T_2	temperature distribution in the cavity surroundings, K
T_0	absolute temperature at the position of particle center in the absence of the particle, K
T_∞	prescribed temperature distribution in the absence of the particle, K
T_A	applied torque on the particle, N m
T_d	torque exerted by the fluid on the particle, N m
U, U	translational velocity of the particle, m s^{-1}
U_0	particle velocity defined by eq. (49), m s^{-1}
$\mathbf{v}$	fluid velocity field, m s^{-1}
v_r, v_θ, v_ϕ	components of fluid velocity in spherical coordinates, m s^{-1}
V	heat conduction function in the fluid phase, m K
V_1	heat conduction function inside the particle, m K
V_2	heat conduction function in the cavity surroundings, m K
z	axial coordinate shown in Fig. 1, m

Greek letters

η	viscosity of the fluid, $\text{kg m}^{-1} \text{s}^{-1}$
θ, ϕ	angular spherical coordinates
λ	equal to a/b
ρ	density of the fluid, kg m^{-3}
$\tau_{r\theta}, \tau_{r\phi}$	shear stresses in the fluid phase, N m^{-2}
Ψ	Stokes stream function of the fluid, $\text{m}^3 \text{s}^{-1}$
ω	variable defined by eq. (15)
Ω, Ω	angular velocity of the particle, s^{-1}

REFERENCES

- Bakanov, S. P. (1991) Thermophoresis in gases at small Knudsen numbers. *Aerosol Sci. Technol.* **15**, 77–92.
- Basset, A. B. (1961) *A Treatise on Hydrodynamics*, Vol. 2, pp. 270–272. Dover, New York, U.S.A.
- Brock, J. R. (1962) On the theory of thermal forces acting on aerosol particles. *J. Colloid Sci.* **17**, 768–780.
- Bungay, P. M. and Brenner, H. (1973) The motion of a closely-fitting sphere in a fluid-filled tube. *Int. J. Multiphase Flow* **1**, 25–56.
- Chen, S. H. and Keh, H. J. (1995a) Axisymmetric motion of two spherical particles with slip surfaces. *J. Colloid Interface Sci.* **171**, 63–72.
- Chen, S. H. and Keh, H. J. (1995b) Axisymmetric thermophoretic motion of two spheres. *J. Aerosol Sci.* **26**, 429–444.
- Coutanceau, M. and Thizon, P. (1981) Wall effect on the bubble behaviour in highly viscous liquids. *J. Fluid Mech.* **107**, 339–373.
- Davis, M. H. (1972) Collisions of small cloud droplets: Gas kinetic effects. *J. Atmos. Sci.* **29**, 911–915.
- Derjaguin, B. V., Rabinovich, Ya. I., Storozhilova, A. I. and Shcherbina, G. I. (1976) Measurement of the coefficient of thermal slip of gases and the thermophoresis velocity of large-size aerosol particles. *J. Colloid Interface Sci.* **57**, 451–461.
- Friedlander, S. K. (1977) *Smoke, Dust and Haze*, pp. 42–44. Wiley, New York, U.S.A.
- Giddings, J. C., Kucera, E., Russell, C. P. and Myers, M. N. (1968) Statistical theory for the equilibrium distribution of rigid molecules in inert porous networks. Exclusion chromatography. *J. Phys. Chem.* **72**, 4397–4408.
- Glandt, E. D. (1981) Distribution Equilibrium between a bulk phase and small pores. *A.I.Ch.E. J.* **27**, 51–59.
- Haberman, W. L. and Sayre, R. M. (1958) Motion of rigid and fluid spheres in stationary and moving liquids inside cylindrical tubes. David Taylor Model Basin Report No. 1143, U.S. Navy Dept., Washington, D.C., U.S.A.
- Happel, J. and Brenner, H. (1983) *Low Reynolds Number Hydrodynamics*. Nijhoff, Dordrecht, The Netherlands.
- Keh, H. J. and Chen, S. H. (1995) Particle interactions in thermophoresis. *Chem. Engng Sci.* **50**, 3395–3407.
- Keh, H. J. and Chen, S. H. (1996) Thermophoresis of an arbitrary three-dimensional array of N interacting arbitrary spheres. *J. Aerosol Sci.* **27**, 1035–1061.
- Keh, H. J. and Chen, S. H. (1997) Low-Reynolds-number hydrodynamic interactions in a suspension of spherical particles with slip surfaces. *Chem. Engng Sci.* **52**, 1789–1805.
- Keh, H. J. and Chiou, J. Y. (1996) Electrophoresis of a colloidal sphere in a circular cylindrical pore. *A.I.Ch.E. J.* **42**, 1397–1406.
- Keh, H. J. and Yu, J. L. (1995) Migration of aerosol spheres under the combined action of thermophoretic and gravitational effects. *Aerosol Sci. Technol.* **22**, 250–260.
- Kennard, E. H. (1938) *Kinetic Theory of Gases*, pp. 291–337. McGraw-Hill, New York, U.S.A.
- Leong, K. H. (1984) Thermophoresis and diffusio-phoresis of large aerosol particles of different shapes. *J. Aerosol Sci.* **15**, 511–517.
- Li, W. J. and Keh, H. J. (1997) Boundary effects on the bipolar behavior of a spherical particle in an electrolytic cell. *J. Electrochem. Soc.* **144**, 3536–3544.
- Loyalka, S. K. (1990) Slip and jump coefficients for rarefied gas flow: variational results for Lennard-Jones and $n(r)$ -6 potentials. *Physica A* **163**, 813–821.
- Loyalka, S. K. (1992) Thermophoretic force on a single particle—I. Numerical solution of the linearized Boltzmann equation. *J. Aerosol Sci.* **23**, 291–300.
- Montassier, N., Boulaud, D. and Renoux, A. (1991) Experimental study of thermophoretic particle deposition in laminar tube flow. *J. Aerosol Sci.* **22**, 677–687.
- Reed, L. D. and Morrison, F. A. (1974) Particle interactions in viscous flow at small values of Knudsen number. *J. Aerosol Sci.* **5**, 175–189.
- Reed, L. D. and Morrison, F. A. (1975) Particle interactions in low Knudsen number thermophoresis. *J. Aerosol Sci.* **6**, 349–365.
- Sasse, A. G. B. M., Nazarov, W. W. and Gadgil, A. J. (1994) Particle filter based on thermophoretic deposition from natural convection flow. *Aerosol Sci. Technol.* **20**, 227–238.

- Talbot, L., Cheng, R. K., Schefer, R. W. and Willis, D. R. (1980) Thermophoresis of particles in a heated boundary layer. *J. Fluid Mech.* **101**, 737–758.
- Waldmann, L. and Schmitt, K. H. (1966) Thermophoresis and diffusiophoresis of aerosols, in *Aerosol Science*, ed. C. N. Davies, pp. 137–162. Academic Press, New York, U.S.A.
- Weinberg, M. C. (1982) Thermophoretic efficiency in modified chemical vapor deposition process. *J. Amer. Ceram. Soc.* **65**, 81–87.
- Williams, M. M. R. and Loyalka, S. K. (1991) *Aerosol Science: Theory and Practice, with Special Applications to the Nuclear Industry*. Pergamon Press, Oxford, U.K.
- Ye, Y., Pui, D. Y. H., Liu, B. Y. H., Opiolka, S., Blumhorst, S. and Fissan, H. (1991) Thermophoretic effect of particle deposition on a free standing semiconductor wafer in a clear room. *J. Aerosol Sci.* **22**, 63–72.
- Ying, R. and Peters, M. H. (1989) Hydrodynamic interaction of two unequal-sized spheres in a slightly rarefied gas: resistance and mobility functions. *J. Fluid Mech.* **207**, 353–378.
- Zydney, A. L. (1995) Boundary effects on the electrophoretic motion of a charged particle in a spherical cavity. *J. Colloid Interface Sci.* **169**, 476–485.