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Predictive Adaptive Control System for Unmeasured Disturbances

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The algorithms for the estimation, prediction, and compensation for unmeasured disturbances are considered in a predictive adaptive control (PAC) system. According to an identified dynamic model for the process, an equivalent unmeasured disturbance is first estimated. Then, the estimated disturbances are fitted into a difference equation by means of the linear regression. Owing to the nonstationarity tendency of the unmeasured disturbances, an on-line exponential data window is used to update the regression model. Subsequently, a predictive compensation is made by making use of the resulting regression model and the system dynamic model in a feedforward manner and is accompanied by a feedback control loop. Simulations show that the PAC system is superior to a simple feedback control system which is already tuned optimally. It should be mentioned that the identification of the process dynamic model in the implementation of the PAC can be conducted with closed-loop data and the algorithms in the PAC are good for real-time implementation.

Introduction

The existence of unmeasured disturbances will usually result in some defects in a control system design. Although a feedback control system, conventional or optimal, can usually recover from those unknown inputs, the performance of the control system is degraded. Thus, a better approach of control is always desirable. The earliest study on the control of the unmeasured disturbance is, perhaps, the work of Johnson (1968). It concluded that through a proper choice of the quadratic performance index, an optimal control can be implemented by a proportional plus integral feedback control principle. Later, the designs of the state observers for the systems where unmeasured disturbances exist are investigated by Johnson (1975) and by Meditch and Hostetter (1974). Recently, the unmeasured disturbances together with the unmeasured state were considered in a design of the inferential control system by Joseph and Brosilow (1978a,b,c), Morari and Stephanopoulos (1980), and Morari and Fung (1982). In those studies mentioned, it is assumed that the dynamics between the unmeasured disturbances and the state variables of the system are well defined. However, in the practical situation, very often, the dynamics must be modeled through identification procedures and thus there seems to be no way to understand the dynamics for those unmeasured disturbances. On the other hand, the models used in control designs are usually good for some specific conditions only, owing to the nonlinearity and complexity of the chemical processes. Thus, in an ever-changing circumstance, where the system parameters or operation conditions vary from time to time, the resulting systems based on the given model will be defective. One approach to solve the problem mentioned is to continually identify the process on-line. Although some on-line identification algorithms are available in the literature (for example, Saridis, 1974; Isermann et al., 1974; Huang and Chao, 1982), the process usually has to be interrupted from its normal operational conditions. The interruption is conducted either by sequentially switching the form of the regulator from one to the other (Gustavsson et al., 1974) or by introducing some special inputs. Most of these kinds of interruption are strongly undesirable in chemical plants. The other approach is to keep the nominal model un-

changed and lump all the unknown inputs and the effects of modeling errors into an equivalent unmeasured disturbance. The compensation for this unmeasured disturbance can then be conducted with a feedforward scheme if the unmeasured disturbance is estimated on-line. Authors such as Garcia and Morari (1982), Liou and Hsu (1983a,b), Tong (1982), and Liu (1983) discussed the control of the unmeasured disturbance recently. Among these, Tong (1982) and Liu (1983) considered the estimation of the lumped unmeasured disturbance by use of an identified nominal model. Owing to the time delay in the process, a zero-order or a first-order extrapolation for the unmeasured disturbance used by most of these authors is not satisfactory for the control implementations. Besides, the noises that may accompany the outputs will have serious effects on the system proposed by Liou and Hsu (1983b).

In this paper, a predictive adaptive control system as shown in Figure 1 is proposed. In this proposed system, a dynamic model is identified by the data from a closed-loop control operation. Later, any discrepancy between the response of the model and that of the process is considered a lumped effect of an equivalent unmeasured disturbance. This unmeasured disturbance can be identified by a real-time algorithm. A real-time prediction model for the unmeasured disturbance is then identified and is used to predict the disturbance that is needed in computing a delayed compensation. The compensation is arranged as a feedforward control and is appended to a feedback control loop, either a conventional or optimal state feedback one. In this manner, the system possesses the capacity for compensating for the unmeasured disturbance as well as the model errors that may exist. The results of a series of digital simulations show that the system is superior to the simple feedback one which is tuned optimally.

The Estimation and Prediction of the Unmeasured Disturbances

Consider a single-loop control system in Figure 2, where a number of different unmeasured disturbances exist. Because the disturbances are either unmeasurable or unknown, their transfer functions to the output will not be available. However, the transfer function from the manipulation input, i.e., $G_p(s)$, can be identified during a transient period of a feedback operation by the method proposed by Huang and Chao (1982). Some descriptions of the modeling procedures are summarized in the Ap-

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Figure 1. The prediction adaptive control system.

Figure 2. Block diagram for the control system with unmeasured disturbance.

pendix. According to the model thus obtained, the unmeasured disturbances in Figure 2 can be considered a lumped equivalent disturbance as

$$y(s) = G_p(s)u(s) + \sum_{i=1}^r G_{Li}(s)L_i(s) \\ = G_p(s)[u(s) + L_e(s)] \quad (1)$$

where

$$L_e(s) = \sum_{i=1}^r \frac{G_{Li}(s)}{G_p(s)} L_i(s) \quad (2)$$

According to the modeling procedures (Huang and Chao, 1982), a dynamic equation of G_p of a first order with delay or a second order with delay will result. Thus, a time domain equation equivalent by eq 1 will be

$$\dot{y}(t) + a_1 y(t) = b_0 u(t-D) + l_e(t-D) \quad (3)$$

or

$$\ddot{y}(t) + a_1 \dot{y}(t) + a_2 y(t) = b_0 u(t-D) + l_e(t-D) \quad (4)$$

where, l_e is the equivalent unmeasured disturbance to the system. If the observed process output is denoted as y_0 , where

$$y_0 = y(t) + \epsilon(t) \quad (5)$$

it is usually assumed that $\epsilon(t)$ is a normally and independently distributed random variable.

Equations 3 and 4 can be transformed into their equivalent sampled-data equation

$$y_0(k) = \phi_1 y_0(k-1) + \phi_2 y_0(k-2) + f_1 u(k-d-1) + f_2 u(k-d-2) + g_1 l_e(k-d-1) + g_2 l_e(k-d-2) + \xi(k) \quad (6)$$

where

$$\xi(k) = \epsilon(k) - \phi_1 \epsilon(k-1) - \phi_2 \epsilon(k-2) \quad (7)$$

Computation methods for evaluating the coefficients in eq

6 can be found elsewhere, for example, Huang and Chao (1983) and Reid (1983). The index k denotes the value of each variable at the instant of k th sample interval from the origin.

It is assumed that the value of the unmeasured disturbance being estimated at instant k , denoted as $l_e(k-d)$, results from a constant input which lasts for a period of N sample intervals. Thus, the estimated value of $l_e(k-d)$ can be obtained through an averaging filter as

$$\hat{l}_e(k-d) = \frac{1}{N'} \sum_{i=k-N'+1}^k \delta_i \quad (8)$$

where

$$\delta_i = \frac{1}{g_1 + g_2} \{y_0(i) - \phi_1 y_0(i-1) - \phi_2 y_0(i-2) - f_1 u(i-d-1) - f_2 u(i-d-2)\} \quad (9)$$

$$= \frac{1}{g_1 + g_2} \{y_0(i) - \sum_{j=1}^2 [\phi_j y_0(i-j) + f_j u(i-d-j)]\} \quad (9)$$

and N' equals $N-1$ as eq 3 is used or N' equals $N-2$ as eq 4 is used.

It is expected that through a proper choice of N , the value of $\hat{l}_e(k-d)$ will approach

$$l_e(k-d) = \frac{1}{N'} \sum_{i=k-N'+1}^k \{y(i) - \sum_{j=1}^2 [\phi_j y(i-j) + f_j u(i-d-j)]\} / (g_1 + g_2) \quad (10)$$

which corresponds to that in the case of noise-free at the output.

If a moving rectangular data window of length N is used, a recursive estimate of $\hat{l}_e$ can be formulated as

$$\hat{l}_e(k-d) = \hat{l}_e(k-d-1) + \frac{1}{N'} [\delta_k - \delta_{k-N'}] \quad (11)$$

Thus, at instant k and according to the definition of l_e in eq 3 and 4, only the value of $\hat{l}_e(k-d)$ rather than $\hat{l}_e(k)$ will be obtained. It can be found, later, that a prediction value of the unmeasured disturbance, denoted as $\hat{l}_e(k/k-d)$, using all the available information, such as $\hat{l}_e(k-d)$, $\hat{l}_e(k-d-1)$, ..., is required for computing a feedforward compensation at the k th instant.

By the definition of eq 2, the unmeasured disturbance can be considered a dynamic process. To enhance the prediction of $\hat{l}_e(k/k-d)$, a dynamic model for this unmeasured disturbance is desirable. In the many studies of the dynamic systems with unknown and inaccessible inputs, it is usually assumed that the unknown input can be formulated as a linear dynamic equations with some sparsely populated sequences of completely unknown impulses (Johnson, 1975; Meditch and Hostetter, 1974; Davison, 1972, etc.) or step inputs (Johnson, 1968). Thus, it is reasonable to assume that the increment of the unmeasured disturbance between each sample instant, i.e.

$$v(j) = \hat{l}_e(j) - \hat{l}_e(j-1) \quad (12)$$

can be formulated as a difference equation with a proper order, i.e.

$$v(j+1) = h_1 v(j) + h_2 v(j-1) + \dots + h_s v(j-s+1) \quad (13)$$

Thus the coefficients in eq 13 can be found by using a regression model

$$v(j+1) = \hat{h}_1 v(j) + \hat{h}_2 v(j-1) + \dots + \hat{h}_s v(j-s+1) + e(j) \quad (14) \\ = \hat{\mathbf{h}}^T \mathbf{V}(j) + e(j)$$

where

$$\hat{\mathbf{h}} = [\hat{h}_1, \hat{h}_2, \dots, \hat{h}_s]^T$$

and

$$\mathbf{V}(j) = [v(j), v(j-1), \dots, v(j-s+1)]^T$$

The values of $\hat{\mathbf{h}}$ can be calculated by a linear least-square algorithm which minimizes the sum of m squared errors, i.e., $\sum_{j=1}^m e^2(j)$. As the dynamic model in eq 13 may be time-varying, the value of $\hat{\mathbf{h}}$, must be calculated from time to time in accordance with the changing environment of the system. A recursive algorithm with a forgetting factor α is adopted for the real-time estimation of $\hat{\mathbf{h}}$. This updating algorithm is also given in the Appendix.

From eq 14 the prediction value for $\hat{v}(j+1/j)$ will be

$$\hat{v}(j+1/j) = \hat{h}_1 v(j) + \hat{h}_2 v(j-1) + \dots + \hat{h}_s v(j-s+1) \quad (15)$$

so that, for $m \leq s$

$$\hat{v}(j+m/j) = \hat{h}_1 \hat{v}(j+m-1/j) + \dots + \hat{h}_m v(j) + \hat{h}_{m+1} v(j-1) + \dots + \hat{h}_s v(j+m-s) \quad (16)$$

and, for $m > s$

$$\hat{v}(j+m/j) = \hat{h}_1 \hat{v}(j+m-1/j) + \hat{h}_2 \hat{v}(j+m-2/j) + \dots + \hat{h}_s \hat{v}(j+m-s/j) \quad (17)$$

Because of eq 15, $\hat{v}(j+m/j)$ can be expressed in terms of $v(j)$, $v(j-1)$, ..., and $v(j-s+1)$; for example

$$\begin{aligned} \hat{v}(j+2/j) &= \hat{h}_1 \hat{v}(j+1/j) + \hat{h}_2 v(j) + \hat{h}_3 v(j-1) + \dots + \hat{h}_s v(j-s+2) \\ &= (\hat{h}_1^2 + \hat{h}_2) v(j) + (\hat{h}_1 \hat{h}_2 + \hat{h}_3) v(j-1) + (\hat{h}_1 \hat{h}_3 + \hat{h}_4) v(j-2) + \dots \\ &\quad + (\hat{h}_1 \hat{h}_{s-1} + \hat{h}_s) v(j-s+2) + \hat{h}_1 \hat{h}_s v(j-s+1) \\ &= \eta_1^{(2)} v(j) + \eta_2^{(2)} v(j-1) + \dots + \eta_s^{(2)} v(j-s+1) \end{aligned} \quad (18)$$

The same procedures can be carried out to the following result

$$\hat{v}(j+m/j) = \eta_1^{(m)} v(j) + \eta_2^{(m)} v(j-1) + \dots + \eta_s^{(m)} v(j-s+1) \quad (19)$$

with

$$\eta_i^{(m)} = [A^m \cdot C]_i \quad (20)$$

where, $[]_i$ denotes the i th component of a vector in the parentheses, and

$$A = \begin{bmatrix} \hat{h}_1 & 1 & 0 & 0 \cdots & 0 \\ \hat{h}_2 & 0 & 1 & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{h}_{s-1} & 0 & 0 & 0 \cdots & 1 \\ \hat{h}_s & 0 & 0 & 0 \cdots & 0 \end{bmatrix} \quad (21)$$

and

$$C = [1, 0, \dots, 0]^T \quad (22)$$

Finally

$$\begin{aligned} \hat{l}_e(k/k-d) &= \sum_{i=1}^d \hat{v}(k-d+i/k-d) + \hat{l}_e(k-d) \\ &= \sum_{i=1}^s \eta_i v(k-d-i+1) + \hat{l}_e(k-d) \end{aligned} \quad (23)$$

where

$$\eta_i = \eta_i^{(1)} + \dots + \eta_i^{(d)} \quad (24)$$

In order to test the algorithm given above, a system as shown in Figure 3 is used as the example for digital sim-

Figure 3. Block diagram for the control system with lumped disturbance.

Figure 4. The estimation and prediction of an unmeasured disturbance; $G_p(s) = e^{-2s}/(4.6s + 1)$.

Figure 5. The estimation and prediction of an unmeasured disturbance; $G_p(s) = e^{-2s}/(4.6s + 1)$.

Figure 6. The estimation and prediction of an unmeasured disturbance; $G_p(s) = e^{-1.3s}/(4.15s + 1)(1.98s + 1)$.

ulations. In the simulations, a model for $G_p(s)$, $\hat{G}_p(s)$, is first identified during a PID control operation where a small set-point change is introduced. Later, a disturbance, which is assumed unknown, is introduced. The process transfer functions for $G_p(s)$ used in the simulation studies are of a first order or of a second order with time delay. They are

$$G_p(s) = \frac{e^{-2s}}{4.6s + 1} \quad (25)$$

and

$$G_p(s) = \frac{e^{-1.3s}}{(4.15s + 1)(1.98s + 1)} \quad (26)$$

It is first assumed that the identification results are perfect so that these exact models can be used in the estimation of the unmeasured disturbances, which are shown in Figures 4 to 9. In these figures, the original disturbances are also given for comparison.

The Compensation for the Unmeasured Disturbances

Although a conventional feedback control system will, in general, alleviate the effect of external disturbances, it

Figure 7. The estimation and prediction of an unmeasured disturbance; $G_p(s) = e^{-1.5s}/(4.15s + 1)(1.98s + 1)$.

Figure 8. The flow chart for the implementation of the PAC.

Figure 9. The responses of the PAC and the conventional PID system subjected to an unmeasured disturbance; example 1.

is still desirable to compensate for those measurable disturbances in a feedforward manner so as to reduce errors. The efforts on the feedforward control have been initiated and developed during the past 20 years. Examples of work in this area are provided by Ballinger and Lamb (1962), Cadman et al. (1967), Luyben (1963), and Huang (1969). As described in the previous section, through the estimation and prediction procedures, the unmeasured loads can be considered as if they were measurable. Thus, the feedforward compensation becomes feasible. However, as mentioned in most of the literature about feedforward control, a feedback loop is indispensable. According to the dynamic models in eq 3 and 4, a feedforward compensation of the unmeasured disturbance can be given as

$$b_0 u_p(t) = -\hat{L}_e(t/t - D) \quad (27)$$

or

$$b_0 u_p(t) + b_1 \dot{u}_p(t) = -\hat{L}_e(t/t - D) \quad (28)$$

In the case where b_1 is zero, eq 27 and 28 become

$$u_p(t) = -\frac{1}{b_0} \hat{L}_e(t/t - D) \quad (29)$$

where, $u_p(t)$ denotes the predictive feedforward input for the unmeasured disturbance.

In a sampled-data system, the predictive feedforward control input is given from eq 6 as

$$u_p(k) = -\frac{1}{f_1} \{f_2 u_p(k-1) + g_1 \hat{L}_e(k/k-d) + g_2 \hat{L}_e(k-1/k-d)\} \quad (30)$$

There are many reasons that the predictive feedforward control, u_p , cannot stand alone to compensate for the unmeasured input. Thus, u_p must be appended with a feedback loop. The feedback input may be of a conventional PID control, denoted as u_b , or of an optimal state control, (OSC) denoted as u_o . Thus, the overall control input denoted as u will be

$$u = u_p + u_b \text{ (or } u_p + u_o) \quad (31)$$

The sampled-data algorithm for the u_b in the following studies is given as

$$u_b(k) = u_b(k-1) + k_c \frac{\delta}{T_R} w(k) + k_c [w(k) - w(k-1)] \quad (32)$$

where

$$w(k) = w(k-1) + \frac{T_D}{\alpha T_D + \delta} [e(k) - e(k-1)] + \frac{\delta}{\alpha T_D + \delta} [e(k) - w(k-1)]$$

Equation 32 corresponds to an interactive analog PID control algorithm.

The optimal state control input, u_o , in this study starts with the state equation for eq 6 where $\xi(k)$ is assumed as zero, i.e.

$$\mathbf{x}(k+1) = \mathbf{H}_1 \mathbf{x}(k) + \mathbf{H}_2 u(k-d) \quad (33)$$

and

$$y(k) = \mathbf{c} \mathbf{x}(k)$$

where

$$\mathbf{x}(k) = [x_1(k), x_2(k)]^T$$

$$\mathbf{H}_1 = \begin{bmatrix} \phi_1 & 1 \\ \phi_2 & 0 \end{bmatrix}$$

and

$$\mathbf{H}_2 = [f_1, f_2]^T \quad (34)$$

$$\mathbf{c} = [1, 0]$$

so that $u_o(k)$ is given as

$$u_o(k) = [\mathbf{K} \mathbf{H}_1^d]_{(1)} y(k) + [\mathbf{K} \mathbf{H}_1^d]_{(2)} \phi_2 y(k-1) + \mathbf{K} \mathbf{H}_2 u_o(k-1) + \mathbf{K} \mathbf{H}_1 \mathbf{H}_2 u_o(k-2) + \dots + \mathbf{K} \mathbf{H}_1^{d-1} \mathbf{H}_2 u_o(k-d) + [\mathbf{K} \mathbf{H}_1^d]_{(2)} f_2 u(k-d-1) \quad (35)$$

where $[\]_{(i)}$ denotes the i th column of the matrix in the parentheses. The gain for the state feedback, $\mathbf{K}$, is obtained by minimizing a quadratic performance index (Huang and Chao, 1975; Roberts, 1964) of

$$\text{I.P.} = \sum_{k=0}^{\infty} [Qy(k)^2 + Ru(k)^2] \quad (36)$$

The resulting system that uses eq 31 as the control strategy is considered as a predictive adaptive control system, PAC.

Stability Considerations of the PAC

The stability of the PAC system can be analyzed by examining the characteristic equation of the system. The following derivations are based on a second-order process which is the typical model for the dynamics of chemical processes.

According to eq 8, $\hat{l}_e(k-d)$ can be written as

$$\hat{l}_e(k-d) = \frac{1}{N'(g_1 + g_2)} \sum_{i=1}^{N'} B^{i-1} \{ (1 - \phi_1 B - \phi_2 B^2) y(k) - (f_1 B^{d+1} + f_2 B^{d+2}) u(k) \} \quad (37)$$

where B is a backward shift operator so that $B^k X(i)$ stands for $X(i-k)$. Let

$$P_1(B) = \frac{1}{N'(g_1 + g_2)} \sum_{i=1}^{N'} B^{i-1} (1 - \phi_1 B - \phi_2 B^2) \quad (38)$$

and

$$P_2(B) = \frac{-1}{N'(g_1 + g_2)} \sum_{i=1}^{N'} B^{i-1} (f_1 B^{d+1} + f_2 B^{d+2}) \quad (39)$$

Then

$$\hat{l}_e(k-d) = P_1(B)y(k) + P_2(B)u(k) \quad (40)$$

From the definition of eq 12 one has

$$\begin{aligned} v(k-d) &= (1-B)\hat{l}_e(k-d) \\ &= (1-B)P_1(B)y(k) + P_2(B)u(k) \end{aligned} \quad (41)$$

From eq 23

$$\hat{l}_e(k/k-d) = \sum_{i=1}^s \eta_i v(k-d-i+1) + \hat{l}_e(k-d) \quad (42)$$

and by the definitions of eq 40 and 41, eq 42 can be written as

$$\begin{aligned} \hat{l}_e(k/k-d) &= [P_1(B) + \sum_{i=1}^s \eta_i B^{i-1} (1-B)P_1(B)]y(k) + \\ &\quad [P_2(B) + \sum_{i=1}^s \eta_i B^{i-1} (1-B)P_2(B)]u(k) \\ &= [1 + \sum_{i=1}^s \eta_i B^{i-1} (1-B)][P_1(B)y(k) + P_2(B)u(k)] \end{aligned} \quad (43)$$

Assuming that the feedback regulator is

$$u_b(k) \text{ [or } u_o(k)] = R_1(B)y(k) \quad (44)$$

then

$$u(k) = R_1(B)y(k) + R_2(B)\hat{l}_e(k/k-d) \quad (45)$$

where, $R_1(B)$ is the discrete transfer function corresponding to eq 32 or to eq 35 and $R_2(B)$ is the discrete transfer function corresponding to eq 29 or 30. Thus

$$\begin{aligned} u(k) &= R_1(B)y(k) + \\ &\quad R_2(B)[1 + \sum_{i=1}^s \eta_i B^{i-1} (1-B)][P_1(B)y(k) + P_2(B)u(k)] \end{aligned}$$

so

$$u(k) = [Q_1(B)/Q_2(B)]y(k) \quad (46)$$

where

$$Q_1(B) = R_1(B) + R_2(B)[1 + \sum_{i=1}^s \eta_i B^{i-1} (1-B)]P_1(B) \quad (47)$$

and

$$Q_2(B) = 1 - R_2(B)[1 + \sum_{i=1}^s \eta_i B^{i-1} (1-B)]P_2(B) \quad (48)$$

Substitution of eq 46 into the dynamic equation of the process yields

$$\{(1 - \phi_1 B - \phi_2 B^2)Q_2(B) - (f_1 B^{d+1} + f_2 B^{d+2})Q_1(B)\}y(k) = 0 \quad (49)$$

Thus, the characteristic equation of the system is

$$(1 - \phi_1 B - \phi_2 B^2)Q_2(B) - (f_1 + f_2 B)B^{d+1}Q_1(B) = 0 \quad (50)$$

The stability of the system depends on whether all the zeros of eq 50 lie outside the unit circle of $|B| = 1$. It can be seen that the predictive feedforward control will affect the stability of the system in quite a complicated way. This is obviously different from the conventional feedforward control for the measurable disturbance.

From eq 30 $R_2(B)$ is given as

$$R_2(B) = \frac{-(g_1 + g_2 B)}{(f_1 + f_2 B)} \quad (51)$$

Now, from eq 6 by assuming that observation errors are negligible, one has

$$y(k) = \{[B^d(g_1 B + g_2 B^2)Q_2(B)] / [(1 - \phi_1 B - \phi_2 B^2)Q_2(B) - (f_1 B^{d+1} + f_2 B^{d+2})Q_1(B)]\}l_e(k) \quad (52)$$

or equivalently

$$y(z) = \{[z^{-d}(g_1 z^{-1} + g_2 z^{-2})Q_2(z^{-1})] / [(1 - \phi_1 z^{-1} - \phi_2 z^{-2})Q_2(z^{-1}) - (f_1 + f_2 z^{-1})z^{-d-1}Q_1(z^{-1})]\}l_e(z) \quad (53)$$

Since

$$\lim_{z \rightarrow 1} Q_2(z^{-1}) = \lim_{z \rightarrow 1} [1 - R_2(z^{-1})P_2(z^{-1})] = 0 \quad (54)$$

One can conclude that for any bounded input l_e , the final offset of the system will be

$$y(\infty) = \lim_{z \rightarrow 1} \left[\frac{z-1}{z} \times y(z) \right] = 0 \quad (55)$$

Thus, theoretically speaking, there is no need of an integral action in $R_1(B)$. This fact will be very helpful for the stability of the system. If the values of f_1 and f_2 are significantly larger than those values of g_1 and g_2 (this is the case where process gain is high), then the characteristic equation of eq 50 will approach that of a single feedback control system, i.e.

$$(1 - \phi_1 B - \phi_2 B^2) - (f_1 + f_2 B)B^{d+1}R_1(B) = 0 \quad (56)$$

Thus, if an integral action is excluded from $R_1(B)$, then the closed-loop stability will be improved significantly, especially in the presence of the time delay.

Simulation Results for the PAC

In the following studies, implementation of the PAC as shown in Figure 1 with different dynamic processes and disturbances are simulated via a digital computer. The implementation of the PAC consists of two phases. In the first phase, the system is operated by a conventional feedback control from which sufficient data are obtained to identify the dynamic model by the procedures given in Appendix. In the second phase, the dynamic model is considered invariant and the unmeasured disturbance is estimated and compensated. The flow diagram of the system implementation is shown in Figure 8.

In each of the following examples, the PID controller settings are obtained as follows. First, the dynamic equation of the identified model together with a PID

Table I. Simulation Results for Example 1

control algorithm	PID control parameters			OSC controller gain		performance IAE
	K_c	T_R	T_D	K_1	K_2	
conventional PID feedback	3.54	12.94	10.9			28.05
PID in PAC	3.54	1294.0	10.9			7.67
OSC in PAC				-3.80	-3.74	9.80

Table II. Simulation Results for Example 2

control algorithm	PID controller			OSC controller		performance IAE
	K_c	T_R	T_C	k_1	k_2	
conventional PID feedback	2.32	3.06	2.02			200.43
PID in PAC	2.32	306	2.02			50.07
OSC in PAC				-26.76	-26.41	48.84

Table III. Simulation Results for Example 3

control algorithm	PID controller			OSC controller		performance IAE
	K_c	T_R	T_D	k_1	k_2	
conventional PID feedback	2.40	2.20	1.41			69.94
PID in PAC	2.40	220	1.41			33.68
OSC in PAC				-21.19	-20.78	44.87

controller is simulated in a digital computer. Then, following the procedures of the Simplex method (Himmelblau, 1972), the parameters are revised toward a minimal value of the integral of the absolute errors, IAE. These settings are also used for the PID controller in the PAC except that the integral time in the controller is magnified with an order of 10^2 . The reason for this has been given in the previous section. On the other hand, the optimal state controller is calculated according to the performance index in eq 36 with a sampling interval of 0.1. The value of Q used in the examples is 1.0 and the value of R is given as 0.75 in example 1 and as 0.1 in examples 2, 3, and 4. The model of eq 13 used for the prediction of unmeasured disturbance in the examples is second order and the forgetting factor α in the examples is given as 0.9.

Example 1. Consider a second-order process of

$$G_p(s) = \frac{0.154e^{-1.2s}}{(7.74s + 1)(13.1s + 1)} \quad (57)$$

Through the identification procedures mentioned, a differential equation model is obtained as

$$\ddot{y} + 0.206\dot{y} + 0.0099y = 0.0015u(t - 12) \quad (58)$$

The simulation results for an unmeasured disturbance as shown in Figure 5 are summarized in Table I and Figure 9.

Example 2. If the process in example 1 is replaced by

$$G_p(s) = \frac{e^{-1.3s}}{(4.15s + 1)(1.98s + 1)} \quad (59)$$

and the unmeasured disturbance is as shown in Figure 7, then the similar procedures will lead to the results which are summarized in Table II and Figure 10.

Example 3. Consider a high order process of

$$G_p(s) = \frac{1}{(0.5s + 1)(s + 1)^2(2s + 1)} \quad (60)$$

Then, the implementation of the identification procedures results

$$4.07\ddot{y} + 3.67\dot{y} + y = u(t - 0.8) \quad (61)$$

If the unmeasured disturbance is given as Figure 3, the simulation results are summarized in Table III and Figure 11. The responses to the set-point change of the system with the existence of the same unmeasured disturbance

Figure 10. The responses of the PAC and the conventional PID system subjected to an unmeasured disturbance; example 2.

Figure 11. The responses of the PAC and the conventional PID system subjected to an unmeasured disturbance, example 3.

are also summarized in Figure 12.

Example 4. If the process is given as

$$G_p(s) = \frac{e^{-1.2s}}{(8s + 1)} \quad (62)$$

Instead, a model with biases in the parameters is assigned as

$$G_p(s) = \frac{0.8e^{-1.2s}}{(6.5s + 1)} \quad (63)$$

Figure 12. The responses of the PAC and the conventional PID system subjected to an unmeasured disturbance and a set-point change; example 3.

Figure 13. The responses of the PAC and the conventional PID system subjected to an unmeasured disturbance; example 4.

Table IV. Simulation Results for Example 4

control algorithm	PID controller			performance IAE
	K_c	T_R	T_D	
conventional PID feedback	4.34	7.975	0.54	58.49
PID in PAC	4.34	797.5	0.54	11.18
OSC in PAC				18.15
PID in PAC with exact model	4.59	645.45	0.54	9.51

The implementation of the PAC using the biased model in the presence of some unmeasured disturbance is then summarized in Table IV and Figure 13. The result of the PAC with the exact model is also given in the table for comparison. From this table, it can be seen that the PAC is capable of compensating for the model errors as well as the unmeasured disturbance.

Summary

In the predictive adaptive control system as described in the text, the identification of models, estimation, and control of the unmeasured disturbance are considered an integrity. In practical situations where identification of the dynamic model cannot be repeated quite often, this system will provide the capacity for compensating for the unmeasured disturbances and model errors in an adaptive manner. From the analysis and simulations, the following conclusions are reached.

(1) The estimation, prediction, and control of the unmeasured disturbance in the PAC are all formulated as real-time algorithms so that it is ready for an on-line implementation.

Figure 14. The identification procedures for the process dynamic model.

(2) The prediction of the unmeasured disturbances, as shown in the examples, are superior to those of the zero-order or first-order extrapolation methods.

(3) The PAC system is more effective in compensating for the unmeasured disturbances when compared with the simple feedback control system which is tuned optimally.

(4) It seems feasible to append other dead-time compensating feedback control to the predictive feedforward control. Good performance can be expected from the resulting system.

(5) From theoretical derivations, there seems to be no need for an integral action in the appended feedback loop. In practice, a small integral action, as illustrated in the examples, can be added so as to deal with the imperfection of the estimations.

(6) The system is more favorable to those processes whose process gain is high. In those cases, the system is much easier to be kept stable than a conventional feedback system.

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Appendix

I. The Identification of the Dynamic Model for a Process in a Closed Loop. The dynamic model used in the control system is obtained by means of identification procedures by Huang and Chao (1982). The procedures can be summarized in Figure 14.

The identification index (I.D.) used in Figure 14 is defined as

$$\text{I.D.} = \text{MSE}/\text{Det}(\mathbf{H})|_{p^*} \quad (\text{A.1})$$

$$\mathbf{H} = \mathbf{Y}_p^T(p^*) \cdot \mathbf{Y}_p(p^*) \quad (\text{A.2})$$

and

$$Y_p(p^*) = \begin{bmatrix} \frac{\partial y(t_1)}{\partial p_1} & \frac{\partial y(t_1)}{\partial p_2} & \dots & \frac{\partial y(t_1)}{\partial p_s} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y(t_N)}{\partial p_1} & \frac{\partial y(t_N)}{\partial p_2} & \dots & \frac{\partial y(t_N)}{\partial p_s} \end{bmatrix}_{p^*}$$

where MSE is the mean squared error of the model being considered. y is the output of the system, p^* is the optimal parameter vector for the candidate model, and $t_1, \dots, t_N$ are the N sampling instants.

The parameter estimation is implemented through an iteration algorithm of

$$P - P^0 = [Y_p^T(P^0)Y_p(P^0)]^{-1}Y_p^T(P^0)[Y - Y(P^0)] \quad (A.3)$$

where $Y(P^0)$ is the output vector calculated at $P = P^0$. The starting value of P^0 can be found elsewhere (for example, Liu, 1983).

II. The On-Line Algorithm for Updating the Regression Model in Eq 14. A recursive algorithm with a forgetting factor α is adopted for the real time estimation of $\hat{h}$ in eq 14

$$\hat{h}(j+1) = \hat{h}(j) + [\alpha + x^T(j+1)P(j)x(j+1)]^{-1} \times P(j)x(j+1)[v(j+1) - x^T(j+1)\hat{h}(j)]; \quad (j \geq m) \quad (A.4)$$

and

$$P(j+1) = \frac{1}{\alpha} \{I - P(j)[\alpha + x^T(j+1)P(j)x(j+1)]^{-1} \times x(j+1)x^T(j+1)P(j)\}; \quad (j \geq m) \quad (A.5)$$

where

$$x(j) = [v(j), v(j-1), \dots, v(j-s+1)]^T \quad (A.6)$$

$$V(j) = [v(j), v(j-1), \dots, v(j-m+1)]^T \quad (A.7)$$

The starting values of $\hat{h}(m)$ and $p(m)$ for eq A.4 and A.5 are given by

$$\hat{h}(m) = [X^T(m)X(m)]^{-1}X^T(m)V(m) \quad (A.8)$$

and

$$P(m) = [X^T(m)X(m)]^{-1} \quad (A.9)$$

where

$$X(m) = [V(j-1), V(j-2), \dots, V(j-s)] \quad (A.10)$$

The value of m in eq A.8 and A.9 denotes the number of observations that is used in the least-squares calculation for the initial value of $\hat{h}$.

Nomenclature

a = constant coefficient
 A = an $s \times s$ matrix
 B = an $s \times 1$ vector
 b = constant coefficient
 B = a backward shift operator
 C = an $s \times 1$ column vector
 D = time delay
 d = time delay in terms of sampling interval
 e = error between the set-point and the observed output

f = coefficient in the discrete-time dynamic equation
 g = coefficient in the discrete-time dynamic equation
 G_p = continuous process transfer function
 $\hat{G}_p$ = identified process transfer function
 G_L = continuous transfer function for the disturbances
 h = coefficient of a regression model in eq 14
 h = a coefficient vector
 H_1, H_2 = a state expression for a second-order process
 k = optimal state feedback gain
 k_c = the proportional gain of a PID controller
 L = disturbance input
 L_e = the equivalent disturbance input
 l_e = the equivalent disturbance in time domain
 $P(j)$ = a j th iteration value of a $s \times s$ matrix
 Q_1, Q_2 = polynomials of B defined in eq 47 and 48
 R_1 = the discrete-time transfer function for the feedback controller
 R_2 = the discrete-time transfer function for the predictive feedback controller
 s = variable in Laplace transform
 t = time
 T_R = the integral time of a PID controller
 T_D = the derivative time of a PID controller
 u = the control input
 u_p = the predictive feedforward control input
 u_o = the optimal state feedback control input
 u_b = the optimal PID control input
 v = a difference quantity defined in eq 12
 V = a column vector
 w = a dummy variable in eq 32
 y = the process output
 y_o = the observed process output
 z = the variable in z transform

Greek Symbols

α = a forgetting factor in eq A.4 and A.5
 δ = an error quantity defined in eq 9 or a sampling interval in eq 32
 ϵ = observation error
 ϕ = coefficient in the discrete-time dynamic model
 ξ = noise in the dynamic equation in terms of the observed output

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