A System Dynamics Model of Fire-Tube Shell Boiler

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A system dynamics model of fire-tube shell boiler was developed. The derivation of the dynamics model started with a nonlinear time-variant dynamic modeling based on the transport phenomena in the fire-tube boiler. A linear time-invariant perturbed model around steady-state operating points was then derived. The identifiable parameters τ_{mw} , τ_{wa} , K , β , and τ_d were identified by using field test data and least-squares estimation method; the coefficients C 's were, meanwhile, directly predicted by using the small-perturbation relations. Empirical correlations of the identifiable parameters were further derived to account for the variation of parameters with operating conditions. The present perturbed model is thus semi-empirical and can describe the dynamic behaviour of fire-tube boilers over a wide range of operating conditions. The predictions of dynamic responses using the present model were shown to agree very well with the test results.

I Introduction

Fig. 1 Schematic of fire-tube shell boilers

Fire-tube shell boiler or package boiler (Fig. 1) has been widely used in industries and residential areas for process heating and hot water supply. The design of fire-tube shell boilers consists of a bundle of fire tubes contained in a shell; severe boiling and evaporating processes take place outside the fire tubes and steam is generated. The heat transfer process from the combustion gas to the boiling water via the tube surface is extremely complicated as it involves combustion, radiation, convection, and boiling processes.

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The thermal performance of boiler is then highly nonlinear and can be significantly affected by the operating conditions such as steam pressure and flowrate, ambient conditions, excess-air quantity, degree of fuel and combustion air mixing etc. The performance can be further disturbed by the variation of steam or combustion air flow during operation. Time-variant phenomena will also appear inevitably due to the integral effect produced by the variation of holdup water or water level in the shell side. The dynamic model of fire-tube boilers should apparently be in the following nonlinear time-variant and multi-input-multi-output (MIMO) form:

$$\dot{\mathbf{X}}(t) = f[\mathbf{X}(t), \mathbf{U}(t)] \quad (1)$$

$$\mathbf{Y}(t) = g[\mathbf{X}(t), \mathbf{U}(t)]. \quad (2)$$

The system dynamics of water-tube power boilers for the design of high performance control system of power plants has been extensively studied in the past (for examples, Chien et al., 1958; Nicholson, 1964, 1965, 1967; Anderson, 1969; Kwan, 1978; McDonald and Kwatny, 1973; Ray and Bowman, 1976; Aleksandrov and Rassokhin, 1985). These studies, however, have been mainly theoretical due to their complexity. The studies of dynamic models of fire-tube shell boilers have been sparse. Only a few simple lumped single-input-single-output (SISO) models for domestic fire-tube hot water heaters have appeared in literature (Lebrun et al., 1985; Malmstrom et al., 1985; Claus and Stephan, 1985). An attempt has then been made here for developing a MIMO system dynamic model of fire-tube shell boilers.

The system dynamics of fire-tube boilers can be identified

by using the estimation methods such as generalized least-squares, instrumental-variables and maximum likelihood algorithms etc. (Hsia, 1979). The system parameters identified in this way may, however, be in large errors at operating conditions different from the one used in the identification, unless the plant is linear and time-invariant which is obviously not the case for fire-tube boilers. The application of this approach is thus limited.

A model-based approach has been used here in the derivation and identification of system dynamics of fire-tube boilers. A nonlinear time-variant dynamic model was first derived based on the transport phenomena in the fire-tube boiler. A linearization using small perturbation around a steady-state operating point was applied and resulted in a linear time-invariant perturbed model with some well-defined and identifiable system parameters and some predictable coefficients. The identifiable parameters clearly possess a physical meaning and their values at various operating conditions can be identified by using test data and least-squares estimation method; the predictable coefficients are, meanwhile, a function of steady-state operating conditions and can be directly evaluated. Empirical correlations of the parameters in terms of steady-state operating conditions

were further derived to account for the variation of parameters with operating conditions. The present perturbed model, written in the form of Eqs. (3) and (4), is thus semi-empirical and can accurately describe the dynamic behaviour of fire-tube boilers over a wide range of operating conditions.

$$\dot{\tilde{\mathbf{X}}}(t) = A(\bar{\mathbf{X}}, \bar{\mathbf{U}})\tilde{\mathbf{X}}(t) + B(\bar{\mathbf{X}}, \bar{\mathbf{U}})\tilde{\mathbf{U}}(t) + E(\bar{\mathbf{X}}, \bar{\mathbf{U}})\tilde{\mathbf{F}}(t) \quad (3)$$

$$\tilde{\mathbf{Y}}(t) = C(\bar{\mathbf{X}}, \bar{\mathbf{U}})\tilde{\mathbf{X}}(t) + D(\bar{\mathbf{X}}, \bar{\mathbf{U}})\tilde{\mathbf{U}}(t) \quad (4)$$

II Physical Modeling and Governing Equations

A three-node lumped model is derived here for the sake of simplicity. It was assumed that the boiler can be divided into three different phases: namely, the holdup water in the shell side as the liquid phase, the metal of the heating surface (fire tubes) as the solid phase, and the combustion gaseous product in the combustion chamber and fire tubes as the gaseous phase. Each phase was assumed to have a uniform temperature. The heat transfer between the three phases is shown in Figs. 1 and 2. The dynamic model can be obtained here by applying an energy and mass balance to each phase.

Gaseous Phase. An energy balance to the gaseous phase will lead to

Nomenclature

A_o = theoretical air flowrate per unit fuel flowrate for stoichiometric combustion, Eq. (20), $\text{Nm}^3/\text{kg oil}$	M_g = mass of combustion gas in the boiler, kg	T_f = temperature of preheated fuel oil, $^{\circ}\text{C}$
A_l = heating surface area of fire tubes, m^2	M_w = mass of holdup water in the shell side, kg	T_{fw} = temperature of feedwater, $^{\circ}\text{C}$
A_{mw} = heat transfer surface area of fire tubes, m^2	m_a = mass flowrate of combustion air, kg/hr	T_{go} = flue gas temperature, $^{\circ}\text{C}$
A_{wa} = area used in evaluating the heat transfer from the boiling water to the ambient, m^2	m_f = mass flowrate of fuel oil, kg/hr	T_m = metallic fire-tube (solid phase) temperature, $^{\circ}\text{C}$
C_f = heat capacity of fuel oil, $\text{kJ/kg}^{\circ}\text{C}$	m_s = mass flowrate of steam, kg/hr	T_o = temperature of datum state, $^{\circ}\text{C}$
C_g = heat capacity of flue gas, $\text{kJ/kg}^{\circ}\text{C}$	m_w = mass flowrate of feedwater, kg/hr	γ_a = absolute humidity of combustion air, kg vapor/kg dry air
C_m = heat capacity of fire tube, $\text{kJ/kg}^{\circ}\text{C}$	N_2 = volumetric concentration of gaseous N_2 in flue gas, %	ϕ_a = ratio of actual to theoretical combustion air flowrates, dimensionless
C_s = heat capacity of steam, $\text{kJ/kg}^{\circ}\text{C}$	O_2 = volumetric concentration of gaseous O_2 in flue gas, %	ρ_a = density of combustion air, kg/m^3
C_w = heat capacity of water, $\text{kJ/kg}^{\circ}\text{C}$	Q_E = net rate of heat carried out by steam from the boiler, Watt	τ_{mw} = time constant for the response of fire-tube temperature due to heat transfer from solid phase to boiling water, s
CO = volumetric concentration of gaseous CO in flue gas, %	Q_{g1} = radiative heat transfer rate from the combustion gas to fire tube surface (solid phase), Watt	τ_{wa} = time constant for the response of water temperature due to heat transfer from boiling water to ambient, s
CO_2 = volumetric concentration of gaseous CO_2 in flue gas, %	Q_{go} = rate of energy loss from the flue gas, Watt	
H_f = heating value of fuel oil, KJ/kg	Q_{in} = rate of energy input resulting from fuel combustion, Watt	Superscripts
h_{mw} = heat transfer coefficient from fire tube to water, $\text{W/m}^2\text{C}$	Q_L = rate of heat loss from the liquid phase to the ambient, Watt	$\sim$ = small perturbation
h_{wa} = heat transfer coefficient from boiling water to ambient, $\text{W/m}^2\text{C}$	T = temperature of boiling water in shell side or exit steam temperature, $^{\circ}\text{C}$	$\bar{}$ = steady-state operating condition
i = enthalpy, KJ/kg	T_a = ambient temperature, $^{\circ}\text{C}$	$\hat{}$ = estimated variable
i_{fw} = enthalpy of feedwater, KJ/kg

Fig. 2 Schematic of 3-node model of fire-tube shell boilers

$$Q_{in} - Q_{go} - Q_{g1} = \frac{d}{dt}(M_g u_g), \quad (5)$$

where Q_{in} can be evaluated by the relation (Cheng et al., 1988; McLean and Murdock, 1972)

$$Q_{in} = Q_{in}(m_f) = m_f[H_f + C_f(T_f - T_o)]; \quad (6)$$

Q_{go} can be evaluated by the relation

$$\begin{aligned} Q_{go} &= Q_{go}(m_f, m_a, T) \\ &= m_f[V_d C_{go}(T_{go} - T_o) + G_{wf} \Delta H + G_{wa} C_s(T_{go} - T_o) \\ &\quad + 30.5 V_d(CO)] \end{aligned} \quad (7)$$

where G_{wf} = the water formation weight of the combustion = .09h + .01w, where h and w are the weight compositions of hydrogen and water in the fuel oil; G_{wa} = water vapor content in the combustion air = 1.293 $\gamma_a \phi_a A_o$, in kg vapor/kg oil, where γ_a is the absolute humidity. ΔH = latent heat of water vapor = 597.5 + $C_s(T_{go} - T_o)$, kcal/kg oil; $V_d = \phi_a A_o$. The semi-empirical relation developed by Huang et al. (1988) can be applied for Q_{g1} , the heat transfer to the tube walls.

$$\begin{aligned} Q_{g1} &= Q_{g1}(m_f, m_a, T, T_{go}) \\ &= f_r \sigma A_1 (T_G^4 - T_1^4) + h_c A_1 (T_G - T_1), \end{aligned} \quad (8)$$

where

$$T_G \equiv T_{go} + \phi \frac{Q_{g1} T_{af} - T_o}{Q_{in} T_{af}}, \quad (9)$$

$$T_{af} \equiv \frac{Q_{in}}{m_a C_g} = \frac{m_f[H_f + C_f(T_f - T_o)]}{m_a C_g}, \quad (10)$$

$$T_1 \equiv r_w \frac{Q_{g1}}{A_1} + C \left(\frac{Q_{g1}}{A_1} \right)^n + T, \quad (11)$$

where f_r , h_c , ϕ , C , n , and r_w are the empirical constants; σ is the Stefan-Boltzman constant, $5.669 \times 10^{-8} \text{ W/m}^2\text{K}$.

The combustion gas contained in the combustion chamber of the boiler has a very small thermal capacity in practice as compared to the boiling water to which the radiative and convective heat Q_{g1} will be transferred. The energy storage rate due to the thermal capacity effect of the combustion gas can then be relatively very small too as compared to Q_{in} and Q_{g1} . This is due to a rapid chemical combustion process with heat generation rate Q_{in} and a fast heat transfer process Q_{g1} . For the sake of simplification, the thermal capacity effect of the combustion gas is neglected here. This may cause a larger error in predicting the flue gas temperature response since the flue gas loss rate Q_{go} may be closely related to the energy storage rate of the combustion gas.

This, however, may not be too serious since Q_{go} is relatively small as compared to Q_{in} and Q_{g1} and the efficiency control of a boiler is of the major concern in many applications. A quasi-steady approximation can therefore hold and the energy equation for the gaseous phase becomes

$$Q_{go} = Q_{in} - Q_{g1}. \quad (12)$$

Solid Phase. Applying an energy balance to the solid phase will lead to

$$M_m C_m \frac{dT_m}{dt} = Q_{g1} - h_{mw} A_{mw} (T_m - T) \quad (13)$$

where the last term $h_{mw} A_{mw} (T_m - T)$ represents the heat convection from tube surface to boiling water.

Liquid Phase. Applying an energy balance to the liquid phase for zero blowdown ($Q_{BD} = 0$) will lead to

$$\frac{d}{dt}[M_w C_w (T - T_o)] = h_{mw} A_{mw} (T_m - T) - Q_E - Q_L \quad (14)$$

where Q_E is the net rate of heat carried out by steam from the boiler; Q_L represents the rate of heat loss from the liquid phase to the ambient, i.e.,

$$Q_E = m_s(i_s - i_o) - m_w(i_{fw} - i_o) \quad (15)$$

$$Q_L = h_{wa} A_{wa} (T - T_a) \quad (16)$$

where i_o is the enthalpy of water at datum state.

Stoichiometric Relation of Gaseous Phase. In addition to the energy and mass balance relations, a stoichiometric relation for the chemical combustion process in the boiler can be derived for the flowrate of combustion air (Cheng et al., 1983; McLean and Murdock, 1972),

$$m_a = \rho_a m_f A_a, \quad (17)$$

where

$$A_a = \phi_a (1 + 1.61 \gamma_a) A_o, \quad \text{in Nm}^3/\text{kg oil} \quad (18)$$

$$\phi_a = \frac{21}{21 - 79 \left(\frac{O_2 - 0.5 CO}{N_2} \right)}, \quad (19)$$

$$A_o = .0889c + .267 \left(h - \frac{o}{8} \right) + 0.333s, \quad \text{in Nm}^3/\text{kg oil}. \quad (20)$$

A_o is the theoretical combustion air flowrate per unit fuel flowrate for stoichiometric combustion, where c , o , and s are, respectively, the weight concentrations of carbon, oxygen, and sulfur in the fuel oil.

The above modeling results in the governing equations of fire-tube boilers, i.e., Eqs. (12), (13), (14), and (17).

III Small Perturbations

Using the small perturbations around a steady state: $Q_{in}(t) = \bar{Q}_{in} + \tilde{Q}_{in}(t)$; $Q_{go}(t) = \bar{Q}_{go} + \tilde{Q}_{go}(t)$; $Q_E(t) = \bar{Q}_E + \tilde{Q}_E(t)$; $T(t) = \bar{T} + \tilde{T}(t)$; $T_m(t) = \bar{T}_m + \tilde{T}_m(t)$; $m_w(t) = \bar{m}_w + \tilde{m}_w(t)$; $m_s(t) = \bar{m}_s + \tilde{m}_s(t)$; $Q_{g1}(t) = \bar{Q}_{g1} + \tilde{Q}_{g1}(t)$, a perturbed model is obtained here from Eqs. (12) to (14) and the mass

balance relation $d\tilde{M}_w/dt = \tilde{m}_w - \tilde{m}_s$ and neglecting the higher-order effects,

$$\tilde{Q}_{go} = \tilde{Q}_{in} - \tilde{Q}_{g1} \quad (21)$$

$$\tau_{mw} \frac{d\tilde{T}_m}{dt} + \tilde{T}_m = K_m \tilde{Q}_{g1} + \tilde{T} \quad (22)$$

$$\tau_{wa} \frac{d\tilde{T}}{dt} + (\beta + 1)\tilde{T} = \beta \tilde{T}_m - \beta K_m \tilde{\zeta} \quad (23)$$

where

$$\tilde{\zeta} \equiv \tilde{Q}_E + C_w(\tilde{m}_w - \tilde{m}_s)(\bar{T} - T_o); \quad (24)$$

$$\tau_{mw} \equiv \frac{M_m C_m}{h_{mw} A_{mw}}, \quad \tau_{wa} \equiv \frac{\bar{M}_w C_w}{h_{wa} A_{wa}},$$

$$K_m \equiv \frac{1}{h_{mw} A_{mw}}, \quad \beta \equiv \frac{h_{mw} A_{mw}}{h_{wa} A_{wa}}.$$

The higher-order terms essentially represent the integral effect due to the variation of water holdup in the shell side and the unbalance of steam and feedwater flowrates. This makes the boiler to behave like a nonlinear time-variant system. However, for safety reason, the water level in the fire-tube boiler is allowed to vary only within a finite range, usually within 2 cm. The variation of holdup water volume is therefore small as compared to the total holdup volume. The integral or time-variant effect can then be neglected by using small-perturbation approximation. In other words, the holdup water volume in the shell side can be assumed to be approximately constant.

Equations (21) to (23) are the small-perturbation equations in terms of the perturbed heat flows $\tilde{Q}$'s which are not the primary system inputs. From the point of view of system dynamics, the direct or primary inputs of the boiler are the fuel oil flowrate $\tilde{m}_f$, the combustion air flowrate $\tilde{m}_a$ and the feedwater flowrate $\tilde{m}_w$. The measurable outputs are the water or steam temperature in the shell side $\tilde{T}$, the flue gas temperature $\tilde{T}_{go}$ and the oxygen concentration $\tilde{O}_2$ of the flue gas, the steam load $\tilde{m}_s$, meanwhile, acts as a disturbance input to the boiler. The model obtained so far is apparently nonlinear in terms of these primary inputs since the heat flows $\tilde{Q}_{go}$, $\tilde{Q}_{g1}$, and $\tilde{Q}_E$ appearing in the above equations are nonlinear functions of these input variables. Linearizations using small perturbation around a steady state are further required.

Applying small perturbation to Eq. (6) and using a first-order approximation will lead to

$$\tilde{Q}_{in} = C_{if} \tilde{m}_f \quad (25)$$

where

$$C_{if} \equiv \left(\frac{\partial \tilde{Q}_{in}}{\partial \tilde{m}_f} \right)_o = H_f + C_f(T_f - T_o).$$

Taking small perturbation of Eq. (15) will lead to

$$\tilde{Q}_E = C_{ew} \tilde{m}_w + C_{es} \tilde{m}_s + C_{eT} \tilde{T}, \quad (26)$$

where

$$C_{ew} \equiv \left(\frac{\partial \tilde{Q}_E}{\partial \tilde{m}_w} \right)_o = -(i_{fw} - i_o)$$

$$C_{es} \equiv \left(\frac{\partial \tilde{Q}_E}{\partial \tilde{m}_s} \right)_o = \bar{i} - i_o$$

$$C_{eT} \equiv \left(\frac{\partial \tilde{Q}_E}{\partial \tilde{T}} \right)_o = \bar{m}_s \left(\frac{\partial \bar{i}}{\partial \bar{T}} \right)_o.$$

The subscripts "o" denote the derivative in respect to a steady-state point. If the datum state is chosen as the feedwater state at T_{fw} , then $T_o = T_{fw}$; $i_{fw} = i_o$; and $C_{ew} = 0$ and we obtain

$$\tilde{Q}_E = C_{es} \tilde{m}_s + C_{eT} \tilde{T}. \quad (27)$$

Similarly achieved here for the flue gas heat loss $\tilde{Q}_{go}$ is

$$\tilde{Q}_{go} = C_{of} \tilde{m}_f + C_{oa} \tilde{m}_a + C_{og} \tilde{T}_{go}, \quad (28)$$

where

$$C_{of} \equiv \left(\frac{\partial \tilde{Q}_{go}}{\partial \tilde{m}_f} \right)_o = V_d C_{go} (\bar{T}_{go} - T_o) + G_{wf} \Delta H$$

$$+ G_{wa} C_s (\bar{T}_{go} - T_o) + 30.5 V_d \bar{CO},$$

$$C_{oa} \equiv \left(\frac{\partial \tilde{Q}_{go}}{\partial \tilde{m}_a} \right)_o = \frac{C_{go}}{\rho_a} (\bar{T}_{go} - T_o) + 1.293 \gamma_a \frac{\bar{m}_f}{\rho_a} C_s (\bar{T}_{go} - T_o)$$

$$+ \frac{30.5}{\rho_a} \bar{CO},$$

$$C_{og} \equiv \left(\frac{\partial \tilde{Q}_{go}}{\partial \tilde{T}_{go}} \right)_o = \bar{m}_f [V_d C_{go} + G_{wf} C_s + G_{wa} C_s].$$

For the radiative heat transfer $\tilde{Q}_{g1}$, small perturbation on Eq. (8) and using quasi-steady approximation yields

$$\tilde{Q}_{g1} = C_{rf} \tilde{m}_f + C_{ra} \tilde{m}_a + C_{rT} \tilde{T} + C_{rg} \tilde{T}_{go}, \quad (29)$$

where

$$C_{rf} \equiv \left(\frac{\partial \tilde{Q}_{g1}}{\partial \tilde{m}_f} \right)_o = \frac{A_1 (4f_r \sigma \bar{T}_G^3 + h_c) [F_{fi} \bar{e} C_{if} / A_1 - (\phi \bar{e} / \bar{m}_f) (1 - F_{af})]}{4f_r \sigma (\bar{T}_G^3 F_{fi} - \bar{T}_1^3 F_{rw}) + h_c (F_{fi} - F_{rw}) - 1}$$

$$C_{ra} \equiv \left(\frac{\partial \tilde{Q}_{g1}}{\partial \tilde{m}_a} \right)_o = \frac{A_1 (4f_r \sigma \bar{T}_G^3 + h_c) (\phi \bar{e} / \bar{m}_a) (1 - F_{af})}{4f_r \sigma (\bar{T}_G^3 F_{fi} - \bar{T}_1^3 F_{rw}) + h_c (F_{fi} - F_{rw}) - 1}$$

$$C_{rT} \equiv \left(\frac{\partial \tilde{Q}_{g1}}{\partial \tilde{T}} \right)_o = \frac{A_1 (4f_r \sigma \bar{T}_G^3 + h_c)}{4f_r \sigma (\bar{T}_G^3 F_{fi} - \bar{T}_1^3 F_{rw}) + h_c (F_{fi} - F_{rw}) - 1}$$

$$C_{rg} \equiv \left(\frac{\partial \tilde{Q}_{g1}}{\partial \tilde{T}_{go}} \right)_o = \frac{-A_1 (4f_r \sigma \bar{T}_G^3 + h_c)}{4f_r \sigma (\bar{T}_G^3 F_{fi} - \bar{T}_1^3 F_{rw}) + h_c (F_{fi} - F_{rw}) - 1}$$

$$F_{af} \equiv \frac{\bar{T}_{af} - T_o}{\bar{T}_{af}}; \quad F_{fi} \equiv \phi \frac{A_1 F_{af}}{\bar{Q}_{in}}; \quad F_{rw} \equiv r_w$$

$$+ n C \left(\frac{\bar{Q}_{g1}}{A_1} \right)^{n-1}$$

$$\bar{T}_G \equiv \bar{T}_{go} + \phi \frac{\bar{Q}_{g1}}{\bar{Q}_{in}} \frac{\bar{T}_{af} - T_o}{\bar{T}_{af}}; \quad \bar{e} \equiv \frac{\bar{Q}_{g1}}{\bar{Q}_{in}}$$

$$\bar{T}_{af} \equiv \frac{\bar{Q}_{in}}{\bar{m}_a C_g} = \frac{\bar{m}_f [H_f + C_f (T_f - T_o)]}{\bar{m}_a C_g} = \frac{\bar{m}_f C_{if}}{\bar{m}_a C_g}$$

$$\bar{T}_1 = r_w \frac{\bar{Q}_{g1}}{A_1} + C \left(\frac{\bar{Q}_{g1}}{A_1} \right)^n + \bar{T}.$$

For the combustion air, small perturbation to Eq. (17) yields, neglecting the effects of CO concentration since it is usually very low in commercial boilers:

$$\tilde{m}_a = C_{mf}\tilde{m}_f + C_{mo}\tilde{O}_2, \quad (30)$$

where

$$C_{mf} \equiv \left(\frac{\partial m_a}{\partial m_f} \right)_o = \rho_a [V_o + (\bar{\Phi}_a - 1)A_o]$$

$$C_{mo} \equiv \left(\frac{\partial m_a}{\partial O_2} \right)_o = \frac{79\rho_a \bar{m}_f A_o \bar{\Phi}_a}{21\bar{N}_2 - 79(\bar{O}_2 - 0.5\bar{CO})}$$

All the coefficients C 's are a function of the steady-state operating conditions and can be directly evaluated.

IV System Dynamic Model

The state-space model is derived here first. The states are defined as

$$x_1 \equiv \tilde{T}$$

$$x_2 \equiv \tilde{T}_m. \quad (31)$$

The above small-perturbation relations are then substituted into Eqs. (21)–(23) to lead to

$$\dot{x}_1 = -\frac{\beta + 1 + KC_{eT}}{\tau_{wa}}x_1 + \frac{\beta}{\tau_{wa}}x_2 - \frac{K[C_{es} - C_w(\bar{T} - T_o)]}{\tau_{wa}}\tilde{m}_s - \frac{KC_w(\bar{T} - T_o)}{\tau_{wa}}\tilde{m}_w \quad (32)$$

$$\dot{x}_2 = \frac{[\beta + K(C_{rT} - R_T C_{rg})]}{\beta\tau_{mw}}x_1 - \frac{1}{\tau_{mw}}x_2 + \frac{K(C_{rf} + R_f C_{rg})}{\beta\tau_{mw}}\tilde{m}_f + \frac{K(C_{ra} - R_a C_{rg})}{\beta\tau_{mw}}\tilde{m}_a \quad (33)$$

$$\tilde{T}_{go} = -R_T x_1 + R_f \tilde{m}_f - R_a \tilde{m}_a \quad (34)$$

$$\tilde{O}_2 = -\frac{C_{mf}}{C_{mo}}\tilde{m}_f + \frac{1}{C_{mo}}\tilde{m}_a. \quad (35)$$

where

$$K = \beta K_m = \frac{1}{h_{wa}A_{wa}}; \quad R_f = \frac{C_{if} - C_{of} - C_{rf}}{C_{og} + C_{rg}};$$

$$R_a = \frac{C_{ra} + C_{oa}}{C_{og} + C_{rg}}; \quad R_T = \frac{C_{rT}}{C_{og} + C_{rg}}.$$

The state vector $\tilde{\mathbf{X}}$, the manipulating input vector $\tilde{\mathbf{U}}$, the output vector $\tilde{\mathbf{Y}}$, and the disturbance input vector $\tilde{\mathbf{I}}$ are defined as

$$\tilde{\mathbf{X}} \equiv \begin{pmatrix} \tilde{T} \\ \tilde{T}_m \end{pmatrix} \quad (36)$$

$$\tilde{\mathbf{U}} \equiv \begin{pmatrix} \tilde{m}_f \\ \tilde{m}_a \\ \tilde{m}_w \end{pmatrix} \quad (37)$$

$$\tilde{\mathbf{Y}} \equiv \begin{pmatrix} \tilde{T} \\ \tilde{T}_{go} \\ \tilde{O}_2 \end{pmatrix} \quad (38)$$

$$\tilde{\mathbf{I}} \equiv (\tilde{m}_s), \quad (39)$$

the state-space model is then obtained here in the form of Eqs. (3) and (4) with

$$\mathbf{A}(\bar{\mathbf{X}}, \bar{\mathbf{U}}) = \begin{pmatrix} -\frac{\beta + 1 + KC_{eT}}{\tau_{wa}} & \frac{\beta}{\tau_{wa}} \\ \frac{\beta + K(C_{rT} - R_T C_{rg})}{\beta\tau_{mw}} & -\frac{1}{\tau_{mw}} \end{pmatrix} \quad (40)$$

$$\mathbf{B}(\bar{\mathbf{X}}, \bar{\mathbf{U}}) = \begin{pmatrix} 0 & 0 & -\frac{KC_w(\bar{T} - T_{fw})}{\tau_{wa}} \\ \frac{K(C_{rf} + R_f C_{rg})}{\beta\tau_{mw}} & \frac{K(C_{ra} + R_a C_{rg})}{\beta\tau_{mw}} & 0 \end{pmatrix} \quad (41)$$

$$\mathbf{C}(\bar{\mathbf{X}}, \bar{\mathbf{U}}) = \begin{pmatrix} 1 & 0 \\ -R_T & 0 \\ 0 & 0 \end{pmatrix} \quad (42)$$

$$\mathbf{D}(\bar{\mathbf{X}}, \bar{\mathbf{U}}) = \begin{pmatrix} 0 & 0 & 0 \\ R_f & -R_a & 0 \\ -C_{mf}/C_{mo} & 1/C_{mo} & 0 \end{pmatrix} \quad (43)$$

$$\mathbf{E}(\bar{\mathbf{X}}, \bar{\mathbf{U}}) = \begin{pmatrix} -K[C_{es} - C_w(\bar{T} - T_{fw})] \\ \tau_{wa} \\ 0 \end{pmatrix} \quad (44)$$

Taking Laplace transform of Eqs. (3) and (4) will lead to

$$\tilde{\mathbf{Y}}(s) = \mathbf{G}(s)\tilde{\mathbf{U}}(s) + \mathbf{W}(s)\tilde{\mathbf{I}}(s), \quad (45)$$

The MIMO transfer-function plant model $\mathbf{G}(s)$ is then

$$\mathbf{G}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} = \quad (46)$$

$$\begin{pmatrix} \frac{Kf_o}{\alpha_o s^2 + \gamma_1 s + \gamma_2} & \frac{Ka_o}{\alpha_o s^2 + \gamma_1 s + \gamma_2} & \frac{K(w_o s + w_1)}{\alpha_o s^2 + \gamma_1 s + \gamma_2} \\ \frac{-R_T Kf_o}{\alpha_o s^2 + \gamma_1 s + \gamma_2} + R_f & \frac{-R_T Ka_o}{\alpha_o s^2 + \gamma_1 s + \gamma_2} - R_a & \frac{-R_T K(w_o s + w_1)}{\alpha_o s^2 + \gamma_1 s + \gamma_2} \\ \frac{-C_{mf}}{C_{mo}} & \frac{1}{C_{mo}} & 0 \end{pmatrix} \quad (47)$$

where

$$\gamma_1 = \alpha_1 + C_{eT}K\tau_{mw}$$

$$\gamma_2 = 1 + (C_{eT} - C_{rT} + C_{rg}R_T)K$$

$$\alpha_o = \tau_{mw}\tau_{wa}$$

$$\alpha_1 = (1 + \beta)\tau_{mw} + \tau_{wa}$$

$$f_o = C_{rf} + C_{rg}R_f$$

$$a_o = C_{ra} - C_{rg}R_a$$

$$\theta_o = \tau_{mw}[C_w(\bar{T} - T_{fw}) - C_{es}]$$

$$\theta_1 = C_w(\bar{T} - T_{fw}) - C_{es}$$

$$w_o = -\tau_{mw}[C_w(\bar{T} - T_{fw})]$$

$$w_1 = -C_w(\bar{T} - T_{fw}).$$

The plant disturbance model $\mathbf{W}(s)$ can be written as

$$\mathbf{W}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{E} \quad (48)$$

$$= \begin{pmatrix} \frac{K(\theta_o s + \theta_1)}{\alpha_o s^2 + \gamma_1 s + \gamma_2} \\ \frac{R_T K(\theta_o s + \theta_1)}{\alpha_o s^2 + \gamma_1 s + \gamma_2} \\ 0 \end{pmatrix}$$

The resultant MIMO plant model for a fire-tube shell boiler can be seen to be 3×3 . The interactive or cross effects between the inputs and outputs are present since the plant transfer function $G(s)$ is nondiagonal. The variation of plant dynamics with the steady-state operating points can clearly be seen from the present model.

The system dynamic model of fire-tube boilers developed above is based on the physical modeling with several simplifications. Some unmodeled dynamics must then exist which may reflect to the time delay or high-order effects. The time delay effect may not be negligible for the temperature response of boiling water in the shell side $\tilde{T}(t)$. A modification is thus required.

Assuming that the time delays for the boiling water temperature response $\tilde{T}(t)$ with respect to the manipulating inputs $\tilde{U}(t)$ and disturbance input $\tilde{\Gamma}(t)$ are the same, i.e., τ_d , then the first row of $G(s)$ and $W(s)$ have to be modified by multiplying a delay term $e^{-s\tau_d}$. This assumption can approximately hold since the dynamic responses $\tilde{Y}$ (steam and flue gas temperatures, $\tilde{T}$ and $\tilde{T}_{go}$ and flue gas concentration $\tilde{O}_2$) due to the disturbance input $\tilde{\Gamma}$ (steam load $\tilde{m}_s$) and the manipulated input $\tilde{U}$ (mass flowrates of fuel, air and feedwater, $\tilde{m}_f$, $\tilde{m}_a$, $\tilde{m}_w$) are produced by a heat transfer through the holdup water in the shell with a large thermal capacity and are therefore approximately in the same order of magnitude. With this consideration, the system dynamic model of fire-tube boilers finally becomes

$G(s) =$

$$\begin{pmatrix} \frac{K_f e^{-s\tau_d}}{\alpha_o s^2 + \gamma_1 s + \gamma_2} & \frac{K_a e^{-s\tau_d}}{\alpha_o s^2 + \gamma_1 s + \gamma_2} & \frac{K e^{-s\tau_d}(w_o s + w_1)}{\alpha_o s^2 + \gamma_1 s + \gamma_2} \\ \frac{-R_T K_f}{\alpha_o s^2 + \gamma_1 s + \gamma_2} + R_f & \frac{-R_T K_a}{\alpha_o s^2 + \gamma_1 s + \gamma_2} - R_a & \frac{-R_T K(w_o s + w_1)}{\alpha_o s^2 + \gamma_1 s + \gamma_2} \\ \frac{-C_{mf}}{C_{mo}} & \frac{1}{C_{mo}} & 0 \end{pmatrix} \quad (49)$$

$$W(s) = \begin{pmatrix} \frac{K(\theta_o s + \theta_1)e^{-s\tau_d}}{\alpha_o s^2 + \gamma_1 s + \gamma_2} \\ \frac{R_T K(\theta_o s + \theta_1)}{\alpha_o s^2 + \gamma_1 s + \gamma_2} \\ 0 \end{pmatrix} \quad (50)$$

The schematic of system dynamic model in state-space and transfer-function forms is presented in Fig. 3.

Equations (49) and (50) represent the system dynamic model of fire-tube boilers in which all the coefficients involving C 's can be directly predicted by the small-perturbation relations described in Section III; τ_{mw} , τ_{wa} , K , β and

Fig. 3 System block representation of fire-tube shell boilers. (a) Transfer-function model; (b) state-space model

τ_d are the system parameters which can be identified using field test results and thus are referred to as "identifiable system parameters."

V Parameter Estimation

1 Experimental Setup. A half-ton fire-tube shell boiler installed in the laboratory was used for the experiment in the present study. Refer to the recent paper of Huang et al., (1988) for the specification of the test boiler. The empirical constants defined in the radiative heat flux relation Q_{g1} , Eq. (8), for this test boiler are $f_r = 0.38567$; $h_c = 13.99 \text{ W/m}^2\text{C}$; $\phi = 1123.09^\circ\text{C}$; $C = 865.11$; $n = -0.0019912$; $r_w = 3.078 \times 10^{-4} \text{ m}^2\text{C/W}$ (Huang et al., 1988).

Three T-type thermocouples were installed in the shell side to measure the average boiling water or steam temperature in the shell side; among them, two were immersed 5 and 10 cm under the water level to detect the water temperature in shell side; and the other one was 10 cm above the liquid level to measure the water vapor temperature. The average value of these three temperatures was taken as the boiling water (liquid phase) temperature. Other four T-type thermocouples were installed in the chimney, feedwater storage tank, fuel line of the burner, and ambient to respectively measure the temperatures of flue gas, feedwater, fuel oil, and ambient. The precision error is estimated to be $\pm 0.1^\circ\text{C}$.

The pressure in the shell side was measured by a Endevco 8510-200 pressure transducer installed at the steam exit of the boiler. The flowrates of the feedwater and fuel oil were respectively measured by a MK-508 turbine flowmeter and a OVAL EC-250 gear-type flowmeter, both with pulse output signals. OVAL EC-250 has been calibrated to give a sensitivity of .01714 kg/pulse with an uncertainty of ± 1.6 percent. The sensitivity for MK-508 is .00151 kg/pulse with an uncertainty of ± 2.9 percent. The flue gas compositions were measured by a gas analyzer of Energy Efficiency System Co. which gave an uncertainty of ± 0.3 percent for O_2 , ± 2 percent for CO gas and ± 10 percent for CO_2 gas. The steam flowrate was measured by an orifice meter which was composed of an orifice plate and two pressure transducers (Yamatake Honeywell piezoelectric transducers).

The dynamic data were taken using a data acquisition system AD700S. All the analog signals of the transducers and sensors were first picked up by a HP 3456A digital

voltmeter with GPIB output connected to IBM PC; the pulse output signals of the turbine flowmeters were, meanwhile, counted by a counter card (AD50120) within AD700S. Software compensation was used with a cold junction compensation card (AD50010T) for temperature measurements. Two 10-channel multiplexers (AD50010) were used for multiple channel measurements. The maximum reading rate of the data acquisition system was 330 reading/sec. The sampling time interval for the present experiment was 3 seconds.

The feedwater flow was regulated by a water level relay control system. Since the laboratory boiler was equipped with an automatic burner, for safety reason, the primary system inputs (flowrates of combustion air and fuel oil) could not be arbitrarily adjusted in the present experiment. Only the nozzle of the burner could be changed here to vary the oil flowrate. For a given nozzle and a fixed setting of excess air, the burner is automatically operated. It is due to this restriction that the primary inputs (m_f, m_s, m_w, m_a) cannot be chosen as the input signals for system identification in the present study. Instead, the secondary input signal $\tilde{Q}_{g1}$ and $\tilde{\zeta}$ were used.

2 Input Signals. Combining Eqs. (22) and (23), while taking into account the time delay, will lead to

$$\tilde{T}(s) = \frac{Ke^{-s\tau_d}}{\tau_{mw}\tau_{wa}(s+p_1)(s+p_2)} \tilde{Q}_{g1} \quad (51)$$

$$- \frac{Ke^{-s\tau_d}(s+z_e)}{\tau_{wa}(s+p_1)(s+p_2)} \tilde{\zeta} \\ \equiv G_T(s)\tilde{Q}_{g1}(s) + W_T(s)\tilde{\zeta}(s), \quad (52)$$

$$G_T(s) = \frac{Ke^{-s\tau_d}}{\tau_{mw}\tau_{wa}(s+p_1)(s+p_2)}; \\ W_T(s) = \frac{-Ke^{-s\tau_d}(s+z_e)}{\tau_{mw}\tau_{wa}(s+p_1)(s+p_2)}, \quad (53)$$

where $z_e = 1/\tau_{mw}$ and

$$p_1 + p_2 = \frac{(1+\beta)\tau_{mw} + \tau_{wa}}{\tau_{mw}\tau_{wa}}; \quad p_1 p_2 = \frac{1}{\tau_{mw}\tau_{wa}}; \\ \beta = \frac{\tau_{mw}\tau_{wa}(p_1 + p_2) - \tau_{mw} - \tau_{wa}}{\tau_{mw}}. \quad (54)$$

The secondary input signal $\tilde{Q}_{g1}$ can be evaluated by measuring $\tilde{Q}_{in}$ and $\tilde{Q}_{go}$ and by using Eq. (21); $\tilde{\zeta}$ can be evaluated by Eq. (24) and the relation $\tilde{Q}_E = \tilde{Q}_E - \tilde{Q}_E$. Both the input signals $\tilde{Q}_{g1}$ and $\tilde{\zeta}$ were mainly produced by a cyclical operation of a feedwater pump due to the relay on/off during the water level control. It was found in the present experiments that they were approximately persistently exciting over the frequency range of 0 ~ 0.3 Hz. From the above derivations, the parameters to be determined are K, τ_d, z_e, p_1 and p_2 which corresponds to the system parameters $K, \tau_d, \tau_{mw}, \tau_{wa}$, and β .

Equation (52) can be discretized for digital signal processing by z -transform to yield

$$\tilde{T}(z) = z^{-n_d} \left[\frac{B(z^{-1})}{A(z^{-1})} \tilde{Q}_{g1}(z) + \frac{C(z^{-1})}{A(z^{-1})} \tilde{\zeta}(z) \right] \quad (55)$$

where

Fig. 4 System block diagram for parameter identification

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2}; \quad B(z^{-1}) = b_1 z^{-1}; \\ C(z^{-1}) = c_1 + c_2 z^{-1}; \quad (56)$$

with the corresponding coefficients

$$a_1 = -(r_1 + r_2) \\ a_2 = r_1 r_2 \\ b_1 = \frac{K p_1 p_2 (r_2 - r_1)}{p_1 - p_2} \\ c_1 = -K \tau_{mw} p_1 p_2 \\ c_2 = -K c_1 \frac{p_1 r_2 - p_2 r_1 + z_e (r_1 - r_2)}{p_1 - p_2} \quad (57)$$

where r_1 and r_2 are the z -poles of the plant; p_1 and p_2 are s -poles.

3 Signal Pretreatment and Parameter Estimation. Pretreatment or prefiltering before system identification is required since the testing signals could be corrupted by internal and external disturbances or noises. The internal disturbance was mainly generated by the severe agitation effect of the boiling water and evaporating process in the shell side (i.e., an unmodeled dynamics) and imposed on the response of steam or boiling water temperature. The external noises were essentially the measurement noises. Both kinds of disturbances and noises were in high-frequency range, as compared to the system response. A low-pass filtering for the testing signals was therefore applied. The following third-order smoothing filter $F(z^{-1})$ was used in the present study,

$$y_k = \frac{1}{3} \sum_{i=-1}^1 u_{k+i} \quad (58)$$

where u_k and y_k are, respectively, the input and output signals of the filter. The discrete-time model in terms of the filtered signals then becomes, from Eq. (55),

$$\hat{A}(q^{-1})\tilde{T}^*(k) = \hat{B}(q^{-1})\tilde{Q}_{g1}^*(k - n_d) \\ + \hat{C}(q^{-1})\tilde{\zeta}^*(k - n_d) + e(k) \quad (59)$$

where $e(k)$ is the residue; $\hat{A}(q^{-1})$, $\hat{B}(q^{-1})$ and $\hat{C}(q^{-1})$ are the estimates of the system parameters in terms of backward shift operator q^{-1} ; $\tilde{T}^*(k)$, $\tilde{Q}_{g1}^*(k - n_d)$ and $\tilde{\zeta}^*(k - n_d)$ are the filtered signals by the filter $F(z^{-1})$. That is,

$$\tilde{T}^*(k) = F(q^{-1})\tilde{T}(k); \quad \tilde{Q}_{g1}^*(k - n_d) = F(q^{-1})\tilde{Q}_{g1}(k - n_d); \\ \tilde{\zeta}^*(k - n_d) = F(q^{-1})\tilde{\zeta}(k - n_d) \quad (60)$$

The system block diagram for parameter identification is shown in Fig. 4.

Fig. 5 Model fitting results. (a) $\bar{m}_f = 14.6$ kg/hr, $\bar{P}_s = 3.8$ kg/cm²; (b) $\bar{m}_f = 26.0$ kg/hr, $\bar{P}_s = 5.1$ kg/cm²; (c) $\bar{m}_f = 37.9$ kg/hr, $\bar{P}_s = 7.3$ kg/cm²

The least-squares method was used in the present study for estimating the system parameters using Eq. (59). The dynamic data was seen to closely follow the predicted values as shown in Fig. 5 for some of the results.

The parameter estimation is well known to be unbiased if the residue $e(k)$ is a statistically independent random process (Hsia, 1979). The whiteness of the residue can be examined by using the normalized autocorrelation function of $e(k)$, $\rho(n)$,

$$\rho(n) = \frac{R_{ee}(n)}{R_{ee}(0)}, \quad (61)$$

where $R_{ee}(n)$ is the autocorrelation function of the residue.

The residues $e(k)$ for various operating conditions being quite close to a white signal can be seen from Fig. 6.

The present tests were run at six different operating conditions with the fuel flowrate ranging from 14.6 to 37.9 kg/hr; the water flowrate ranging from 181 to 465 kg/hr; the steam pressure ranging from 3.8 to 7.3 kg/cm² gage. The identified system parameters in terms of a_1 , a_2 , b_1 , c_1 , c_2 are presented in Table 1. The steady-state gains K were determined by the relation derived from Eq. (14),

$$K = \frac{1}{h_{wa}A_{wa}} = \frac{\bar{T} - T_a}{Q_{g1} - Q_E}. \quad (62)$$

From the above results, the parameters p_1 , p_2 , n_d , $z_e (= 1/\tau_{mw})$, τ_{wa} were determined and from which β can be evaluated by Eq. (54) using z_e , τ_{wa} , p_1 , and p_2 . The final results are presented in Table 2. All the identifiable system parameters, τ_{mw} , τ_{wa} , β , τ_d , and K , appearing in the state-space equations, Eqs. (40) to (44) and Equations (49) and (50), thus can be determined from these results.

4 Empirical Correlations of Parameters. The parameters p_1 , p_2 , n_d , $z_e (= 1/\tau_{mw})$, τ_{wa} can be seen to vary with the steady-state operating conditions which can be essentially

Fig. 6 Whiteness of residue. (a) $\bar{m}_f = 14.6$ kg/hr, $\bar{P}_s = 3.8$ kg/cm²; (b) $\bar{m}_f = 26.0$ kg/hr, $\bar{P}_s = 5.1$ kg/cm²; (c) $\bar{m}_f = 33.6$ kg/hr, $\bar{P}_s = 6.2$ kg/cm²

represented by the fuel flowrate $\bar{m}_f$ and the steam pressure $\bar{P}_s$. The approximate empirical relation was derived as:

$$\psi = C' \bar{m}_f^m \bar{P}_s^n, \quad (63)$$

where ψ represents the parameters p_1 , p_2 , n_d , z_e , τ_{mw} . The identified results as shown in Table 3 and Fig. 7 have been obtained by using least-squares fitting.

5 Model Verification. The dynamic model previously developed can be verified by substituting the identified parameters τ_{mw} , τ_{wa} , K , β and τ_d and the coefficients C 's into the MIMO model, Eqs. (32) to (35) or Eqs. (3) and (4), for system simulation and comparing with the test results. The parameters τ_{mw} , τ_{wa} , K , β and τ_d were evaluated using the empirical relation, Equation (63) and Table 3; the coefficients C 's were, meanwhile, calculated by the small-perturbation relations derived in Section III.

The test data can be seen from Fig. 8 to coincide very well with the simulations for the response of steam (or boiling water) temperature $\bar{T}(t)$. The simulation results for the flue gas temperature are in a larger error (Fig. 9). This is due to the fact that the present modeling used the quasi-steady approximation and ignored the thermal capacity effect in the gaseous phase. However, the error in predicting the flue gas temperature response is tolerable.

Table 1 Z-plane parameter estimation

$\bar{m}_f$ kg/hr	$\bar{P}_s$ kg/cm ² gage	$\hat{a}_1$	$\hat{a}_2$	$\hat{b}_1$	$\hat{c}_1$	$\hat{c}_2$
14.6	3.8	-1.7063	.7088	-9.303×10^{-9}	-2.222×10^{-9}	-3.632×10^{-11}
21.9	5.2	-1.8572	.8592	1.152×10^{-8}	-9.205×10^{-10}	2.924×10^{-10}
26.0	5.1	-1.6922	.6940	-2.514×10^{-8}	-3.856×10^{-9}	2.211×10^{-9}
33.6	6.2	-1.8818	.8826	-5.639×10^{-10}	-2.191×10^{-9}	1.778×10^{-9}
33.6	4.0	-1.4437	.4447	-1.884×10^{-8}	-1.059×10^{-8}	8.233×10^{-9}
37.9	7.3	-1.8674	.8683	6.656×10^{-10}	-1.838×10^{-9}	1.571×10^{-9}

Table 2 System parameter estimation

$\bar{m}_f$ kg/hr	$\bar{P}_s$ kg/cm ² gage	$\bar{T}_s$ °C	p_1 s ⁻¹	p_2 s ⁻¹	z_e s ⁻¹	K °C/W	n_d Δ	τ_{wa} s
14.6	3.8	344.7	.002868	.1119	.4001	.01644	5	1247.3
21.9	5.2	345.6	.005366	.0452	.2449	.00865	1	1009.0
26.0	5.1	345.3	.001951	.1198	.1694	.01048	3	724.7
33.6	6.2	349.9	.002553	.0391	.0667	.00703	3	668.6
33.6	4.0	344.2	.000575	.2696	.1080	.00836	6	697.3
37.9	7.3	356.2	.002329	.0448	.0519	.00413	4	498.0

Table 3 Semi-empirical correlations of system parameters.

$\psi = C' \bar{m}_f^m \bar{P}_s^n$			
ψ	C'	m	n
p_1, s^{-1}	.02878	-2.4041	3.2525
p_2, s^{-1}	.36795	1.3888	-3.7238
z_e, s	147.90	-1.7757	-.7171
$K, °C/W$	.37991	-.7084	-.9099
n_d, Δ	1.6988	1.0706	-1.7663
τ_{wa}, s	.00014	-.7600	-.2470
$\bar{m}_f$	kg/hr;	$\bar{P}_s$	kg/cm ² gage

Fig. 7 Fitting of empirical relations for system parameters

VI Discussions

The time constant for the response of solid phase temperature can be seen from the estimated parameters presented

Fig. 8 Model verification for steam temperature response. (a) $\bar{m}_f = 21.9$ kg/hr, $\bar{P}_s = 5.2$ kg/cm²; (b) $\bar{m}_f = 33.6$ kg/hr, $\bar{P}_s = 6.2$ kg/cm²

Fig. 9 Model verification for flue gas temperature response. (a) $\bar{m}_f = 33.6$ kg/hr, $\bar{P}_s = 6.2$ kg/cm²; (b) $\bar{m}_f = 37.9$ kg/hr, $\bar{P}_s = 7.3$ kg/cm²

in Table 2 to be due to a heat transfer from the metallic tube (solid phase) to the holdup water in the shell side. The time constant $\tau_{mw}(=1/z_e)$ decreases with an increasing fuel firing rate $\bar{m}_f$ and operating pressure $\bar{P}_s$ and is in the order

of 2 to 20 s. This is due to the fact that the boiling phenomenon in the fire-tube boiler is possibly film boiling. The metal surface temperature tends to rise as the heating rate (i.e. fuel firing rate) increases. This causes the convective heat transfer coefficient h_{mw} to decrease. This makes the time constant $\tau_{mw} = M_m C_m / h_{mw} A_{mw}$ increase.

The time constant τ_{wa} for the response of liquid phase due to the heat loss from the holdup water in the shell side to the ambient increases with increasing fuel firing rate m_f and operating pressure $\bar{P}_s$ and are in the order of 8 to 20 minutes. This is mainly due to the fact that a higher fuel firing rate and operating pressure result in a higher agitation effect of the boiling water (liquid phase). This in turn, increases the overall heat transfer coefficient $h_{wa} A_{wa}$ and thus reduces the time constant τ_{wa} .

The system dynamics of the fire-tube shell boilers is of second order as seen from the present study. Two s-poles, p_1 and p_2 , and one s-zero, z_e exist. The results presented in Table 2 further show that the magnitudes of s-pole p_2 and s-zero z_e are in the same order; the s-pole p_1 is, meanwhile, smaller than p_2 by an order. A order reduction to a first-order model is then possible (Franklin et al., 1986).

Three sources of errors exist in the system identification: the time-variant or integral effect due to the variation of holdup water level in the shell side, the internal disturbance induced by the agitation of boiling water, and the assumption of lumping in the modeling. However, the variation of holdup water was relatively small (<2 percent) as compared to the total holdup water in shell side. The internal disturbance was filtered out by a low-pass smoothing filter $F(z^{-1})$. The assumption of lumping can be corrected by adding a time delay term in the model. The resultant error in the parameter estimation can then be negligible. This was proven by the comparison of the test results with the simulation by using the present model.

The cross or interactive effects between system inputs and outputs are noticeably taken into account in the present MIMO dynamics model. The effects of parameter variations on the plant dynamics is one of the sources causing plant uncertainty as in regard to the issue of robust control using linear controller design procedures. The parameter variations may result from the variation of operating conditions. The dynamics model identified in the present study can be used to accurately estimate this type of plant uncertainty and thus it is applicable to the design of various robust control systems.

VII Conclusion

A system dynamics model of fire-tube shell boilers has been developed here by using a model-based approach. The derivation of the dynamic model started with a nonlinear

time-variant dynamic modeling based on the transport phenomena in the fire-tube boiler. A linear time-invariant perturbed model around steady-state operating points was then derived. The identifiable system parameters τ_{mw} , τ_{wa} , K , β , and τ_d were then identified by using field test data and least-squares estimation method; the coefficients C 's were, meanwhile, predicted directly using the small-perturbation relations. Empirical correlations of the identifiable parameters were further derived to account for the variation of parameters with operating conditions. The present perturbed model is thus semi-empirical and can describe the dynamics behaviour of fire-tube boilers over a wide range of operating conditions. The predictions of dynamic responses using the present model were shown to agree very well with the test results.

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