Study of system dynamics model and control of a high-power LED lighting luminaire
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Abstract
The purpose of the present study is to design a current control system which is robust to the system dynamics uncertainty and the disturbance of ambient temperature to assure a stable optical output property of LED. The system dynamics model of the LED lighting system was first derived. A 96 W high-power LED luminaire was designed and built in the present study. The linearly perturbed system dynamics model for the LED luminaire is derived experimentally. The dynamics model of LED lighting system is of a multiple-input–multiple-output (MIMO) system with two inputs (applied voltage and ambient temperature) and two outputs (forward current and heat conducting body temperature). A step response test method was employed to the 96 W LED luminaire to identify the system dynamics model. It is found that the current model is just a constant gain (resistance) and the disturbance model is of first order, both changing with operating conditions (voltage and ambient temperature). A feedback control system using PI algorithm was designed using the results of the system dynamics model. The control system was implemented on a PIC microprocessor. Experimental results show that the control system can stably and accurately control the LED current to a constant value at the variation of ambient temperature up to 40 °C. The control system is shown to have a robust property with respect to the plant uncertainty and the ambient temperature disturbance.
Keywords: LED lighting; LED lighting control; High-power LED
1. Introduction
The luminous efficiency of high-power light-emitting diode (LED) increases dramatically in recent years. This improves the competitiveness of high-power LED relatively to the traditional lighting devices [1]. According to the estimation of Optoelectronics Industry Development Association (OIDA), the cumulative energy savings for utilization of LED lighting in the US alone could amount to 16.6 Quads (760 GW) of electrical energy by 2020 [2]. A high-power LED lighting system consists of a LED lighting module assembled from plural single LED lamps associated with a proper optical design and a heat dissipation device. The key technology of high-power LED lighting in system application side includes heat dissipation and optical control technique.

Due to low optical energy efficiency of LED, a high-power LED lighting system needs to dissipate heat to the ambient in quantity which is several times of the conventional lighting device. Heat dissipation is thus an important issue in high-power LED lighting technology [3]. Since the optical properties of a high-power LED lighting system such as brightness and chromaticity are affected by the forward current of LED and the temperature of the LED chip, the design of the lighting luminaire including the heat dissipation device thus indirectly affects the final optical property [4]. In order to obtain a good quality in lighting control and performance reliability of LED, the understanding of system dynamics of a LED lighting system is very important. However, the research of this topic is very rare. The present study intends to derive the system dynamics model of high-power LED lighting system. From that, the development of a control system using the results for providing a robust constant-current driving to LED will be carried out.
2. Derivation of system dynamics model of LED lighting system

2.1. System dynamics model of a LED lighting luminaire

The LED lighting system (sometimes called “luminaire”) consists of three major components: LED lighting module, heat conducting body, and heat sink. The lighting module includes the light sources (i.e., LED lamps) and secondary optics component. LED lamps are attached to a board for electrical wire connection. The secondary optics component sometimes is added to LED lamps to yield the final optics to the illuminating target. The heat conducting body acts as a thermal connector to the heat sink which is used to dissipate the heat to ambient. The schematic diagram of a LED lighting luminaire is shown in Fig. 1.

For a high-power luminaire, the heat conducting body and the heat sink are usually heavy (2–10 kg) compared to the LED module. Hence, the thermal response of the whole luminaire is dominated by the connecting body and the heat sink.

Since LED lamp is made from semiconductor, its electrical phenomena is similar to a resistor but with a nonlinear voltage–current relation as shown in Fig. 2. Thermal phenomena also exist in a LED luminaire due to the effect of energy conversion and heat dissipation process which must obey the law of energy conservation. The system dynamics of a LED luminaire thus can be treated as a multiple-input–multiple-output (MIMO) system with two inputs (applied voltage \( V \) and ambient temperature \( T_a \)) and two outputs (forward current \( I \) and body temperature \( T_b \)) as shown in Fig. 3, in perturbed variables defined with respect to the steady value [7]. \( G(s) \) is the 2 \( \times \) 2 system transfer function defined as

\[
G(s) = \begin{bmatrix}
G_{ah}(s) & G_{ab}(s) \\
G_{ai}(s) & G_{ai}(s)
\end{bmatrix}
\]

from which the following linear-perturbed system relation in Laplace transform holds:

\[
\begin{bmatrix}
\tilde{T}_b(s) \\
\tilde{I}(s)
\end{bmatrix} = \begin{bmatrix}
G_{ah}(s) & G_{ab}(s) \\
G_{ai}(s) & G_{ai}(s)
\end{bmatrix} \begin{bmatrix}
\tilde{V}(s) \\
\tilde{T}_a(s)
\end{bmatrix},
\]

where the perturbed variables in time domain are defined as follows:

\[
\tilde{V}(t) = V(t) - \bar{V},
\]

\[
\tilde{T}_b(t) = T_b(t) - \bar{T}_b
\]

**Nomenclature**

- \( G_{ah}(s) \) system dynamics model for ambient temperature to current
- \( G_{ai}(s) \) system dynamics model for voltage to current
- \( H_k(s) \) closed-loop transfer function, Eq. (26)
- \( I \) LED forward current, A
- \( I_{set} \) current setpoint, A
- \( k \) gain of \( G_{ai}(s) \)
- \( K_p \) proportional constant of PI controller
- \( K_i \) integral parameter of PI controller
- \( P \) pole of \( G_{ai}(s) \)
- \( S_k(s) \) sensitivity function, Eq. (28)
- \( T_a \) ambient temperature, °C
- \( T_b \) heat conducting body temperature, °C
- \( T_j \) LED junction temperature, °C
- \( V \) voltage, V

**Greek letter**

- \( \bar{\cdot} \) perturbation
- \( \bar{\cdot} \) average

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**Fig. 1.** Configuration of a high-power LED luminaire.
Eq. (2) indicates that both $T_a$ and $T_b$ are affected by ambient temperature $T_a$ and applied voltage $V$ which can be written as, in transfer-function form:

$$\tilde{T}_a(t) = T_a(t) - T_a,$$  \hspace{1cm} (4)

$$\tilde{T}_b(t) = T_b(t) - T_b,$$  \hspace{1cm} (5)

$$\tilde{I}(t) = I(t) - I.$$  \hspace{1cm} (6)

In identifying $G_{ab}(s)$ and $G_{ab}(s)$, the input $T_a$ (ambient temperature) is kept at constant with $\tilde{T}_a(t) = 0$ so that

$$\tilde{T}_b(s) = G_{ab}(s)\tilde{V}(s) \text{ at } \tilde{T}_a(t) = 0,$$  \hspace{1cm} (9)

$$\tilde{I}(s) = G_{ab}(s)\tilde{V}(s) \text{ at } \tilde{T}_a(t) = 0.$$  \hspace{1cm} (10)

In identifying $G_{ab}(s)$ and $G_{ab}(s)$, the input $V$ (voltage) is kept at constant with $\tilde{V}(t) = 0$ so that

$$\tilde{T}_a(s) = G_{ab}(s)\tilde{T}_a(s) \text{ at } \tilde{V}(t) = 0,$$  \hspace{1cm} (11)

$$\tilde{I}(s) = G_{ab}(s)\tilde{T}_a(s) \text{ at } \tilde{V}(t) = 0.$$  \hspace{1cm} (12)

The models $G_{ab}(s)$ and $G_{ab}(s)$ are related to the response of heat conducting body temperature $T_a$ due to the applied voltage $V$ and the ambient temperature ($T_a$) change.

The ambient temperature $T_a$ will indirectly affect the LED junction temperature as well as the optical properties of output light of the luminaire. The effect of $T_a$ variation in the LED forward current control should be kept minimal. Since the control of the LED light engine body temperature $T_a$ is not very important, the identification of $G_{ab}(s)$ and $G_{ab}(s)$ are ignored and only $G_{ab}(s)$ and $G_{ab}(s)$ are identified in the present study. The final purpose of the present study is to develop a forward current control system for a high-power LED luminaire with robust feedback property against the disturbance of $T_a$. Therefore, the simplified dynamics model as shown in Fig. 6 can be used to describe separately the current model $G_{ab}(s)$ and the ambient temperature disturbance model $G_{ab}(s)$.

![Fig. 2. Typical I-V curve of a LED.](image)

![Fig. 3. Input–output block diagram of LED luminaire.](image)

![Fig. 4. MIMO block diagram of a LED luminaire.](image)
2.3. System identification of 96 W LED lighting luminaire

The LED forward current is a response of the applied voltage, i.e., the model \( G_{vi}(s) = I(s)/V(s) \) represents the current response to the input voltage at constant \( T_a \).

For LED, the current response \( I \) caused by the voltage input \( V \) is much faster than 1 ms and can be approximated as an instantaneous process as compared to the thermal response. Hence, the voltage to current model \( G_{vi}(s) \) can be treated as a quasi-steady system with a constant gain. From the current-to-voltage relation of LED as shown in Fig. 2, \( G_{vi}(s) \) can be derived from a steady-state test to the LED luminaire.

To determine the model \( G_{vi}(s) \), the luminaire is driven by a constant-voltage power supply at a constant ambient temperature. The voltage and current are recorded after a steady state is reached. The results are shown in Fig. 7. Eq. (13) is a current–voltage relation determined from
experiment at $T_b = 30 \pm 2 ^\circ C$.

$$I = 0.4274 V^2 - 9.082 V + 48.29.$$  \hspace{1cm} (13)

Using quasi-steady assumption for current-voltage response of LED, Eq. (13) can be used to derive a system dynamics model. Substituting the perturbed relations (3) and (6), $V(t) = \tilde{V} + \tilde{V}(t)$ and $I(t) = I + \tilde{I}(t)$, into Eq. (13) and using the steady-state relation $\bar{I} = 0.4274 \bar{V}^2 - 9.082 \bar{V} + 48.29$ and neglecting the second-order terms, we obtain a linear perturbation equation:

$$\tilde{I}(t) = 0.8548 \bar{V} \tilde{V}(t) - 9.082 \bar{V}(t).$$  \hspace{1cm} (14)

Taking Laplace transfer to Eq. (14), we obtain the dynamics model:

$$G_{vi}(s) = \frac{\tilde{I}(s)}{\tilde{V}(s)} = 0.8548 \bar{V} - 9.082.$$  \hspace{1cm} (15)

It is seen that $G_{vi}(s)$ varies with operating condition which is represented by $\bar{V}$. For the 96 W LED luminaire with $4 \times 4$ lamp arrangement, the operating voltage will vary between 11 and 14.5 V. Hence, $G_{vi}(s)$ falls between 0.3206 and 3.312. This means that the system gain of the linear perturbation model could vary about 10 times at different operating conditions. Hence, the robustness of a control system with respect to the large plant variation as shown in the shaded area of Fig. 8 is very important. The average value 2.244 which corresponds to $\bar{V} = 13.26$ V can be selected as an average gain for the model $G_{vi}(s)$. Fig. 8 shows that the frequency response of the model $G_{vi}(s)$ at various operating voltage.

To identify the model $G_{ai}(s)$, the step-response test was performed to the 96 W LED luminaire. $T$-type thermocouples are used to measure the temperatures of LED heat conducting body and the air inside the chamber. A 0.005 \Omega 1 W resistor is used as the current detector. All the dynamic data are recorded using a YOKOGAWA MV100 recorder with sampling time 2 s. The test facility is shown in Fig. 9. The ambient temperature is simulated using an enclosed chamber with temperature controller. An electric heater is used to heat the chamber using a temperature controller to maintain the chamber in a steady temperature above the room temperature.

![Fig. 7. I–V relation of the 96 W LED luminaire.](attachment:image1.png)

![Fig. 8. Frequency response of $G_{ai}(s)$ at different $\bar{V}$.](attachment:image2.png)
By suddenly changing the ambient temperature (by increasing the heating rate to the test chamber) from a steady-state operation and measuring the LED current response, we can obtain the dynamics model from the data analysis.

For a step change of $\hat{T}_a(t) = Ku(t)$, $u(t)$ is the unit step function and $\hat{T}_a(s) = K/s$. Taking the time derivative of $\hat{T}_a(t)$, we have

$$\hat{T}_a'(t) \equiv \frac{d\hat{T}_a(t)}{dt} = K \delta(t).$$

(16)

Taking the Fourier transform of the time derivative of $\hat{T}_a(t)$, we then obtain

$$\hat{T}_a'(j\omega) \equiv F\left[\frac{d\hat{T}_a(t)}{dt}\right] = j\omega \hat{T}_a(j\omega) = F[K \delta(t)] = K.$$  

(17)

In the meantime, if we take the Fourier transform of the time derivative of the current response $\hat{I}(t)$, we obtain

$$\hat{I}'(j\omega) \equiv F\left[\frac{d\hat{I}(t)}{dt}\right] = j\omega \hat{I}(j\omega) = F[\hat{I}(t)].$$

(18)

Combining Eqs. (17) and (18) we obtain

$$\frac{\hat{I}'(j\omega)}{\hat{T}_a'(j\omega)} = \frac{\hat{I}(j\omega)}{\hat{T}_a(j\omega)} = G_{ai}(j\omega).$$

(19)

From Eq. (17), we further obtain

$$G_{ai}(j\omega) = \frac{\hat{I}(j\omega)}{K}.$$  

(20)

Eq. (20) depicts that the Fourier transform of the time derivative of the step response function dividing by $K$ is the frequency response of the model $G_{ai}(s)$. Fig. 10 is a typical frequency response of the model $G_{ai}(s)$ at $V = 13.85V$ and $T_a$ is imposed a step change from 31.4 to 36.2°C. The results shown in Fig. 10 indicates that the phase of $G_{ai}(j\omega)$ varies from 0° at low frequency to approaching $-90\degree$ at high frequency and the gain of $G_{ai}(j\omega)$ has a slope $-20$ db/decade. This indicates that the model $G_{ai}(s)$ is probably the first order, Eq. (21).

$$G_{ai}(s) = \frac{\hat{I}(s)}{\hat{T}_a(s)} = \frac{k}{s - p}.$$  

(21)

Fig. 11 is the step response comparison of the measurement with the model prediction using Eq. (21) with $k = 0.000684$ and $p = -0.0617$. Fig. 11 also indicates that the current changes 0.1A for 5°C step change of ambient temperature. This means that the current response due to the ambient temperature variation is not very sensitive for the 96W LED luminaire which is relatively heavy compared to a single LED lamp. Table 2 presents the gain $k$ and pole $p$ identified from the step-response test at four operating conditions. The step sizes are all 5°C. It is seen from Table 2 that the gain $k$ varies from 0.000601 to 0.000709 and the pole $p$ varies from $-0.0447$ to $-0.0617$. Both vary in a narrow range. This implies that the system dynamics model $G_{ai}(s)$ is approximately linear over the operating range. The average value $k = 0.000653$ and $p = -0.0484$ can be used to represent an average model. Fig. 12 is the frequency response of $G_{ai}(s)$ at various operating conditions. It is seen that the phases of the four models with parameters shown in Table 2 almost coincide each other. The gain of the various models deviates from the average model by less than 3 db.

The above discussions conclude that the system dynamics model of the 96W LED luminaire can be approximated by the average model, Eqs. (22) and (23), with uncertainty to some extent.

Current model :  

$$\hat{G}_{ai}(s) = 2.244,$$

(22)
Ambient temperature disturbance model:

\[ G_{ai}(s) = \frac{0.000653}{s + 0.0484}. \]  

(23)

3. Application of system dynamics model in current control

Steadily controlling the driving input of the LED using a smart control technology to keep the LED in good and steady optical output is very important. A constant-current driver can keep the LED input current at a constant value to assure a relatively steady light output from LED. However, LED is a kind of semiconductor with nonlinear resistance as shown in Fig. 2. This makes the current control not simple. A constant-current power supplier available in the market may not be suitable for all kinds of LED luminaire since the dynamic response of every luminaire may not be all the same.

From the ambient temperature disturbance model \( G_{ai}(s) \), Eq. (21), the variation of ambient temperature could cause the current of LED to change even the applied voltage is kept constant. As mentioned previously (Figs. 10 and 11), the response of LED current is not very sensitive to the ambient temperature variation for the 96 W LED luminaire. The control system will have a good property in ambient temperature disturbance rejection.

From the current model \( G_{vi}(s) \), Eq. (15), the current variation due to voltage change falls between 0.3206 and 3.312. This means that the system gain could vary about 10 times at different operating conditions. The feedback system for the current control of LED luminaire as shown in Fig. 13 should be robust with respect to large plant variation.

The average model, Eqs. (22) and (23), can be used as the nominal model for the baseline design of the control system.
3.1. Feedback control design analysis

The system dynamics model identified in the present study can be used in the design of the control system. The design of the feedback control system is as shown in Fig. 13. The controller design analysis can be carried out using the average dynamics model of the 96 W LED luminaire, Eqs. (22) and (23).

The response of current of the 96 W LED luminaire with respect to the input voltage change is fast (in zeroth order
The derivative control is not effective since it is used to suppress the oscillation or overshoot in system response. Proportional-integral (PI) control instead of proportional-integral-derivative (PID) is thus used since the overshoot is not a problem in LED luminaire. Besides, PI control algorithm can have robust property with respect to external disturbance if it was carefully tuned [9, 10]. Eq. (24) is the transfer-function model of the PI controller with proportional constant \( K_p \) and integral parameter \( K_i \).

Eq. (25) is the time function of the input voltage to LED luminaire where \( e(t) \) is the output (current) error.

\[
G_c(s) = \frac{K_p + K_i}{s}, \quad (24)
\]

\[
V(t) = e(t)K_p + K_i \int_0^t e(\tau) \, d\tau. \quad (25)
\]

PI controller will add a zero and a pole (at origin) to the plant, \( G_c(s) \). Properly placing the zero could improve the rise time of the feedback system. PI controller also acts as a low-pass filter which will reject the high-frequency disturbance or noises.

The close-loop transfer function for the set-point (command) response of the feedback system is

\[
H_k(s) = \frac{I(s)}{I_{set}(s)} = \frac{G_c \tilde{G}_{ci}}{1 + G_c \tilde{G}_{ci}} = \frac{2.244 K_p s + 2.244 K_i}{(1 + 2.244 K_p s) s + 2.244 K_i}. \quad (26)
\]

From Routh’s stability criterion, the PI controller will give a stable closed-loop system if \( K_p > -0.45 \) and \( K_i > 0 \). According to the requirement of the feedback system, the PI controller can be further tuned. Since \( G_c(s) \) is a constant gain (\( \tilde{G}_{ci} = 2.244 \)) in the baseline design, the closed-loop is first order without overshoot in step response and the steady-state error will be zero. We define the control system requirements of the 96 W LED luminaire as follows:

1. rise time (time for step response from 10% to 90%) < 1 s;
2. settling time (time to reach 98% of steady value) < 6 s.

We used SIMULINK in MATLAB to tune the parameters of the PI controller \((K_p, K_i)\). From the time-domain simulation, the optimal tuning may be carried out by finding the minimum of the integral of the absolute value of the error \((IAE)\) defined in Eq. (27) which also satisfies the above system requirements.

\[
IAE = \int_0^\infty |e(t)| \, dt. \quad (27)
\]

The simulation in use with anti-windup for integral control at saturation shows that the effect of \( K_p \) on control quality is not significant as \( K_p > 5 \). For the average model, Eqs. (22) and (23), the resultant feedback property of the control system is listed in Table 3. The above analysis uses the average model, Eqs. (22) and (23). We repeat the simulation of the control system using different values of the plant model \( G_c(s) \), varying from 0.32 to 3.3. \( K_p = 3 \) and \( K_i = 1.1 \) are chosen as the acceptable PI controller which satisfies the system requirements and the controller stability margin.

![Fig. 13. Feedback control system of LED luminaire.](image-url)
4. Experimental verification of current control for 96 W LED luminaire

The result of the feedback control system design analysis mentioned above is implemented in hardware, as shown in Fig. 15. The control system utilizes the microprocessor PIC 16F877 as the central processing unit of the digital control system for commanding, error detection and controller calculation. The digital output is sent to a digital resistor (AD8402) which will regulate the voltage to the LED luminaire in a switching power voltage regulating device. The current through LED luminaire is measured using a 5 mΩ resistor with an amplifier (IC MAX4373).

Many experiments of the control system were performed at different current setpoints with varying ambient temperature and stagnant air. The results are satisfactory for the current control with robust property. Fig. 16 shows that, for the setpoint 6 A, the current is kept within $6 \pm 0.01$ A all the time, regardless of the ambient temperature variation. Theoretically, the ambient temperature variation could cause LED to change its $I-V$ relation and the system dynamics behavior as well. The control system is shown to have a robust property regarding to the plant uncertainty $G_v(s)$ and the temperature disturbance $G_d(s)$.

Fig. 17 shows the variation of LED junction temperature which is converted from the measured LED base temperature $T_b$. It is seen that the junction temperature does not exceed 105°C at ambient temperature 40°C which is acceptable in application.
5. Discussion and conclusion

The present study intends to derive a system dynamics model of high-power LED lighting system for the development of a robust control system to improve the performance reliability under various operating and environmental conditions. A 96 W high-power LED luminaire was designed and built for the present study. The linearly perturbed system dynamics model for the LED luminaire is first derived experimentally. The dynamics model of LED lighting system is of a MIMO system with two inputs (applied voltage and ambient temperature) and two outputs (forward current and body temperature). The dynamic behavior can be described separately by the current model and the ambient temperature disturbance model. A step response test method was employed to the 96 W LED luminaire for the identification of system models. It is found that the current model is just a constant gain (resistance) and the disturbance model is of first order, both changing with operating conditions (voltage and ambient temperature). This means that the dynamics model is subject to uncertainty resulting from operating conditions. The feedback control system thus requires robustness in the presence of disturbance and plant uncertainty.

The system dynamics model $G_{d}(s)$ identified in the present study indicates that the LED current will change 0.675 A for a 5 °C step change of ambient temperature, i.e., steady-state gain is 0.0135 A/°C. This means that the current response due to the ambient temperature variation is not so sensitive for the 96 W LED luminaire which is relatively heavy compared to a single LED lamp.
A feedback control system for providing constant-current driving to LED using PI controller was designed and analyzed. An optimal controller was determined using the closed-loop sensitivity function to assure system robustness. The control system was implemented on a PIC microprocessor. Experimental results show that the control system can stably and accurately control the current to a constant value at the variation of ambient temperature up to 40 °C and the LED chip junction can be maintained at a temperature lower than 105 °C at stagnant air condition. It is also shown that the design of luminaire will not affect the control system performance.

The step-response test at four operating conditions indicates that the gain $k$ varies from 0.000601 to 0.000709 and the pole $p$ varies from $-0.0447$ to $-0.0617$. Both vary in a narrow range. This implies that the system dynamics model $G_{ai}(s)$ is approximately linear over the operating range. The average value $k = 0.000653$ and $p = -0.0484$ can be used to represent an average model. Since the pole $p$ is inversely proportional to the mass of the LED luminaire. The above average values are for the 96 W LED luminaire made in the present study. However, we can expect that the steady-state gain of the system dynamics model $G_{ai}(s)$ will increase with decreasing mass of the LED luminaire. If the LED luminaries weight 0.487 kg (about one tenth of the present LED luminaire), the pole will shift to $-0.00484$ and the steady-state gain will increase to 1.35 A/°C (about 22.6% setpoint). This implies that the sensitivity of the LED current to the ambient temperature change will be very high if the LED luminaire becomes light in weight. The controller design needs to be very careful in treating the disturbance rejection. This situation will probably occur in the indoor LED lighting design with light weight.

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**References**