On the Modal Testing of Microstructures: Its Theoretical Approach and Experimental Setup

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Small and lightweight structures have very high natural frequencies and small elastic displacement compared to ordinary structures. Due to limited excitation bandwidth and relatively large size of pick up devices, conventional modal testing facilities are not feasible for testing MEMS structures. A modal testing system based on the base excitation principle was developed in this research. In the meantime, the associated mathematical model for frequency response functions was derived as well. Testing structures are mounted on a rigid platform that can move essentially with only one translational degree of freedom. An electric discharge pulse strikes the platform to provide a very wide band excitation. Laser Doppler vibrometers are used to pick up both input and output signals. Since these signals are picked up without contacting structures, the structural dynamic characteristics remain intact without any distortion. With this system, modal parameters of a miniature structure can be extracted, and the receptance functions are synthesized successfully. [DOI: 10.1115/1.1320814]

Introduction

Modal parameters govern the dynamic performance of mechanical systems. These parameters can be identified by modal testing which is an indispensable procedure in the development of mechanical systems. The growing interest to develop devices of micro electromechanical systems (MEMS) has led to the creation of many small and lightweight mechanical structures. Compared to regular mechanical structures, these structures possess very high natural frequencies and small elastic displacement. Unfortunately, conventional modal testing facilities cannot be employed to identify the dynamic characteristics of microstructures due to their limited excitation bandwidth and relatively large size of pickup devices. Therefore, it is desirable to develop a system capable of conducting modal testing for microstructures.

As far as measurement is concerned, the excitation method is vital for modal testing. A qualified device should have the capability to excite all the interested modes significantly. In other words, the excitation bandwidth needs to cover the natural frequencies of these modes. Conventional excitation tools used for modal testing are shakers and hammers. For instance, mechanical, hydraulic, and electromagnetic shakers are widely adopted in vibration testing. Nevertheless, the excitation bandwidth for mechanical shakers can hardly exceed 120 Hz [1] due to the attainability of mechanical parts. Besides, the dynamic characteristics of the electrohydraulic valves restrict the performance of hydraulic shakers in terms of which it cannot satisfy excitation bandwidth requirements beyond 2 kHz [1]. In general, electromagnetic shakers have the widest excitation bandwidth among conventional excitation devices. Although they can provide excitation up to 20 kHz [2], it is still far beyond the need to perform modal testing for microstructures. As for impact hammers, they are easy to use but their size and excitation bandwidth cannot meet the requirements for testing microstructures either. Therefore, tools and methods other than conventional ones have been developed to find the dynamic characteristics of miniature structures. In 1987, Patton and Trethway [3] applied electromagnetic force to excite the suspension of a hard disk drive without any direct contact. They

measured the spectrum of coil current of the electromagnet instead of obtaining the excitation force directly. However, the nonlinear relationship between the air gap and the electromagnetic force makes the estimation of excitation force an uneasy task. Another disadvantage is that the dynamic characteristics of the suspension can be distorted with the attachment of a ferromagnetic target. Wilson and Bogy [4–6] installed a load cell beneath the electromagnet to measure the excitation force directly, but the impact of having a ferromagnetic target attached to the suspension cannot be eliminated either. Besides, the excitation bandwidth is limited to 12.8 kHz. Miu, Frees, and Gompertz [7,8] used an air hammer as a high frequency excitation device, but the excitation force cannot be measured. Zhang et al. [9] employed a high voltage electric pulser in getting an excitation with a frequency bandwidth higher than 200 kHz. Inability to measure the excitation force means that no receptance function can be synthesized to predict the force response quantitatively. Instead of using concentrated force excitation, the driven-base technique can be used as another alternative to perform modal testing for fixed-base structures. With this technique, the dynamic behavior can be characterized quantitatively by measuring driven-base motion and the measurement of excitation force is not needed. Lee and Chou [10] gave formulation to describe the driven-base excitation for continuous structures with both input and output being accelerations. Chen, Su, and Chou [11] derived a similar formulation by having displacement as output instead. In these two articles, structures were excited with the assistance of electromagnetic shakers; however, the bandwidth cannot satisfy the testing requirements for microstructures.

A quantitative modal testing system along with its theoretical basis is developed in this work. This proposed system can identify modal parameters and synthesize receptance functions for MEMS structures. The modal testing setup is designed based on the driven-base excitation principle. Testing structures are mounted on an elastically supported rigid platform which can move essentially with one translational degree of freedom only. An electric discharge pulse is generated to strike the platform and provide a very wide band excitation. Laser Doppler vibrometers are used to measure both excitation and response velocities. Since non-contact pickup devices are put to work, the structural dynamic characteristics will not be distorted. Concurrently, a mathematical model for frequency response functions between input and output velocities is derived in this paper. Frequency response functions

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between measured velocities are obtained with a spectrum analyzer. These frequency response functions are processed to extract modal parameters with which the receptance functions can be synthesized. Experimental results indicate that the proposed system can successfully perform modal testing for microstructures and their receptance functions can be synthesized from the extracted modal parameters.

Base Excitation

The motion $\mathbf{u}(\mathbf{x}, t)$ of a linear elastic body occupying a domain V can be modeled as [12]

$$\mathbf{L}\mathbf{u}(\mathbf{x}, t) = \rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x}, t) \quad \text{in } V, \quad (1)$$

where $\rho(\mathbf{x})$ is the mass density and the definition of linear operator $\mathbf{L}$ is

$$\mathbf{L}\mathbf{u}(\mathbf{x}, t) \equiv \partial_j(c_{ijkl}\partial_k)u_l(\mathbf{x}, t)\mathbf{e}_i. \quad (2)$$

The elastic moduli c_{ijkl} in Eq. (2) are functions of spatial coordinates $\mathbf{x}$, and $\mathbf{e}_i$ are unit vectors. If the elastic body is mounted on a rigid platform and excited with a unidirectional base translation $\mathbf{d}_u(t)$, the boundary conditions can be expressed as

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{d}_u(t) \quad \text{on } S_u, \quad (3)$$

$$\mathbf{T}\mathbf{u}(\mathbf{x}, t) = 0 \quad \text{on } S_\sigma, \quad (4)$$

where S_u is the part of the boundary over which the displacement is prescribed and the traction is applied over the remaining part of the boundary S_σ . The traction operator $\mathbf{T}$ is defined as

$$\mathbf{T}\mathbf{u}(\mathbf{x}, t) \equiv n_j(c_{ijkl}\partial_k)u_l(\mathbf{x}, t)\mathbf{e}_i, \quad (5)$$

where n_j is the normal of the surface. The eigenvalue problem associated with Eq. (1) is

$$\mathbf{L}\Phi(\mathbf{x}) + \rho\omega^2\Phi(\mathbf{x}) = 0 \quad \text{in } V, \quad (6)$$

where the eigenfunctions $\Phi(\mathbf{x})$ satisfy the homogeneous boundary conditions

$$\Phi(\mathbf{x}) = 0 \quad \text{on } S_u, \quad (7)$$

$$\mathbf{T}\Phi(\mathbf{x}) = 0 \quad \text{on } S_\sigma. \quad (8)$$

The eigenfunctions of a hyperelastic solid are orthogonal [12]

$$\int_V \rho \Phi_r \cdot \Phi_s d\mathbf{x} = \delta_{rs} M_r, \quad (9)$$

$$\int_V \Phi_r \cdot \mathbf{L}\Phi_s d\mathbf{x} = -\delta_{rs} M_r \omega_r^2, \quad (10)$$

where M_r are modal masses, ω_r^2 are eigenvalues, and δ_{rs} is the Kronecker delta.

In the case when the elastic body is subject to the excitation as described by Eqs. (3) and (4), the solution can be expressed as

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{U}(\mathbf{x}, t) + \mathbf{d}_u(t). \quad (11)$$

where $\mathbf{U}(\mathbf{x}, t)$ satisfies homogeneous boundary conditions

$$\mathbf{U}(\mathbf{x}, t) = 0 \quad \text{on } S_u. \quad (12)$$

$$\mathbf{T}\mathbf{U}(\mathbf{x}, t) = 0 \quad \text{on } S_\sigma. \quad (13)$$

Substituting Eq. (11) into Eq. (1) results in

$$\mathbf{L}\mathbf{U}(\mathbf{x}, t) = \rho(\mathbf{x})\ddot{\mathbf{U}}(\mathbf{x}, t) + \rho(\mathbf{x})\ddot{\mathbf{d}}_u(t). \quad (14)$$

Introducing base excitation velocity

$$\mathbf{v}_u(t) = \dot{\mathbf{d}}_u(t), \quad (15)$$

Eq. (14) becomes

$$\mathbf{L}\mathbf{U}(\mathbf{x}, t) = \rho(\mathbf{x})\ddot{\mathbf{U}}(\mathbf{x}, t) + \rho(\mathbf{x})\dot{\mathbf{v}}_u(t). \quad (16)$$

Since eigenfunctions $\Phi_r(\mathbf{x})$ are complete in V , $\mathbf{U}(\mathbf{x}, t)$ can be expanded into a series

$$\mathbf{U}(\mathbf{x}, t) = \sum_r q_r(t) \Phi_r(\mathbf{x}) \quad (17)$$

and Eq. (16) is decoupled into

$$\omega_r^2 q_r(t) + \ddot{q}_r(t) = -\dot{Q}_r(t), \quad (18)$$

where

$$Q_r(t) = \frac{1}{M_r} \int_V \rho \Phi_r(\xi) \cdot \mathbf{v}_u(t) d\xi. \quad (19)$$

The solution of Eq. (18) in frequency domain is

$$\tilde{q}_r(\omega) = \frac{-i\omega}{\omega_r^2 - \omega^2} \tilde{Q}_r(\omega), \quad (20)$$

where “ $\sim$ ” denotes Fourier transformed functions. Inevitably, there exists damping in the real structures. As to the light damped structures, modal damping of rate type is introduced without affecting the generality, and Eq. (20) can be modified into

$$\tilde{q}_r(\omega) = \frac{-i\omega}{\omega_r^2 + i2\zeta_r\omega_r\omega - \omega^2} \tilde{Q}_r(\omega), \quad (21)$$

where ζ_r is the damping ratio of the r th mode. The displacement field in frequency domain is obtained from Eqs. (11), (17), and (21) as

$$\tilde{\mathbf{u}}(\mathbf{x}, \omega) = \sum_r \left[\frac{-i\omega \tilde{Q}_r(\omega)}{\omega_r^2 + i2\zeta_r\omega_r\omega - \omega^2} \Phi_r(\mathbf{x}) \right] + \tilde{\mathbf{d}}_u(\omega). \quad (22)$$

Since the relationships between the Fourier transform of displacements and velocities are

$$\tilde{\mathbf{v}}(\mathbf{x}, \omega) = i\omega \tilde{\mathbf{u}}(\mathbf{x}, \omega) \quad \text{and} \quad \tilde{\mathbf{v}}_u(\omega) = i\omega \tilde{\mathbf{d}}_u, \quad (23)$$

the velocity field induced by base velocity excitation is found to be

$$\tilde{\mathbf{v}}(\mathbf{x}, \omega) = \sum_r \left[\frac{\omega^2 \tilde{Q}_r(\omega)}{\omega_r^2 + i2\zeta_r\omega_r\omega - \omega^2} \Phi_r(\mathbf{x}) \right] + \tilde{\mathbf{v}}_u(\omega). \quad (24)$$

Assigning x -axis parallel to $\mathbf{d}_u(t)$, the base excitation velocity can be expressed as

$$\tilde{\mathbf{v}}_u(\omega) = [\tilde{v}_u(\omega) \ 0 \ 0]^T. \quad (25)$$

The frequency response functions between velocities are derived from Eq. (24) and the following relationships can be obtained

$$\frac{1}{\omega^2} \begin{bmatrix} \frac{\tilde{v}_i^x(\omega)}{\tilde{v}_u(\omega)} - 1 \\ \frac{\tilde{v}_i^y(\omega)}{\tilde{v}_u(\omega)} \\ \frac{\tilde{v}_i^z(\omega)}{\tilde{v}_u(\omega)} \end{bmatrix} = \sum_r \left[\frac{P_r}{M_r(\omega_r^2 + i2\zeta_r\omega_r\omega - \omega^2)} \right] \begin{bmatrix} \phi_{ri}^x \\ \phi_{ri}^y \\ \phi_{ri}^z \end{bmatrix}, \quad (26)$$

where the subscript i denotes the response at location $\mathbf{x}_i$, r represents the r th mode, and the superscripts x , y , and z indicate the corresponding components. In the numerator,

$$P_r = \int_V \rho \phi_r^x(\xi) d\xi \quad (27)$$

is the linear modal momentum factor of the r th mode caused by base excitation. Expanding Eq. (26) into fractional form, the following expression is determined

$$\frac{1}{\omega^2} \begin{bmatrix} \frac{\tilde{v}_i^x(\omega)}{\tilde{v}_u(\omega)} - 1 \\ \frac{\tilde{v}_i^y(\omega)}{\tilde{v}_u(\omega)} \\ \frac{\tilde{v}_i^z(\omega)}{\tilde{v}_u(\omega)} \end{bmatrix} = \sum_r \left[\frac{\begin{bmatrix} A_{ri}^x \\ A_{ri}^y \\ A_{ri}^z \end{bmatrix}}{i(\omega - \omega_{dr}) - \sigma_r} + \frac{\begin{bmatrix} A_{ri}^x \\ A_{ri}^y \\ A_{ri}^z \end{bmatrix}^*}{i(\omega + \omega_{dr}) - \sigma_r} \right], \quad (28)$$

where σ_r is the damping factor, ω_{dr} represents damped natural frequency, “*” denotes complex conjugate. And the residues are expressed as

$$A_{ri}^\eta = \frac{P_r}{i2\omega_{dr}M_r} \phi_{ri}^\eta, \quad \eta = x, y, z. \quad (29)$$

In Eq. (28), the left-hand side depicts the formulas to modify frequency response functions between velocities whereas the formulation on the right-hand side represents their associated mathematical models. After modifying the measured frequency response functions and with the aid of curve-fitting, modal parameters ω_{dr} , σ_r , and A_{ri}^η can be extracted. Using the numerical method to calculate

$$R_r^x = i \int_V \rho A_{ri}^x d\mathbf{x}_i, \quad (30)$$

the linear modal momentum factors are found to be

$$P_r = (2\omega_{dr}M_r R_r^x)^{1/2}. \quad (31)$$

In the case of unit modal mass, $M_r = 1$, the value of P_r can be determined, and the corresponding normalized mode shapes are calculated as

$$\phi_{ri}^\eta = \frac{i2\omega_{dr}}{P_r} A_{ri}^\eta, \quad \eta = x, y, z. \quad (32)$$

This approach can obtain the modal parameters ω_{dr} , σ_r , and ϕ_{ri}^η without knowing the excitation force, and it is suitable for the modal testing of microstructures which is very difficult to measure the input force.

Receptance functions provide information of frequency response between input forces and output displacements. The receptance function between β -direction at location $\mathbf{x}_i$, and α -direction at location $\mathbf{x}_j$ is

$$H_{ij}^{\beta\alpha}(\omega) = \sum_r \left[\frac{\gamma_r \phi_{rj}^\alpha \phi_{ri}^\beta}{i(\omega - \omega_{dr}) - \sigma_r} + \frac{\gamma_r^* \phi_{rj}^\alpha \phi_{ri}^\beta}{i(\omega + \omega_{dr}) - \sigma_r} \right], \quad (33)$$

where

$$\phi_{ri}^\beta = \Phi_r(\mathbf{x}_i) \cdot \boldsymbol{\beta} \quad (34)$$

is the mode shape component being normalized in respect to unit modal mass, and

$$\gamma_r = \frac{1}{i2\omega_{dr}}. \quad (35)$$

Since all the modal parameters are acquired from driven-base modal testing, the receptance functions in Eq. (33) can be synthesized. Therefore, the responses of microstructures with respect to force excitation can be predicted.

System Setup

Figure 1 shows the system setup for modal testing of microstructures. The system is composed with an elastically supported rigid platform, an electric discharge device, and two laser Doppler vibrometers. The platform is made of an aluminum cube of size 4 mm × 4 mm × 4 mm which has a very high natural frequency in first mode. Therefore, the cube behaves rigidly when it moves in

Fig. 1 The modal testing system designed for investigating microstructures

the frequency range much lower than its first mode natural frequency. Two flexible L-shaped stainless steel sheets support the platform such that it can move essentially with one translational degree of freedom only. Testing structures are mounted on this rigid platform such that their bases are subject to only translational excitation in one specific direction. This ensures a single input during modal testing.

The electric discharge device consists of a condenser and a pair of electrodes. The charged condenser supplies power for electric discharge. A micrometer is modified into a feeder with a needle electrode mounted on its front end. A flat counterelectrode is adhered to one side of the rigid platform. As soon as the needle electrode is moved close enough to the flat counterelectrode, electric discharge generates a spark to strike the rigid platform. Then, the moving platform shakes the testing structure mounted on its top surface to provide a very wide band excitation. After discharging electricity, the voltage of the condenser drops and no further spark can be induced at the same gap between two electrodes, thus ensuring a single strike. For a gap of 2 mm, the duration of discharge in the air is estimated to be on the order of 1 μ s [13] which can provide an excitation bandwidth up to 1 MHz acting on the platform.

Unit: mm

Fig. 2 The L-shaped plate used for testing

The base excitation applied to the testing structure is the same as the moving velocity of the rigid platform. This velocity is picked up with one laser Doppler vibrometer. The other laser Doppler vibrometer measures the response velocities at interested locations. Since noncontact pickup devices are employed, dynamic characteristics of testing structures will not be distorted. In addition, a spectrum analyzer is used to obtain frequency response functions between excitation and response velocities. These frequency response functions are processed to extract modal parameters.

The modal parameters can be used to synthesize the receptance functions and provides the capabilities to predict structure responses with respect to force excitation.

Experiment

An *L*-shaped AISI-304 stainless steel cantilever plate of 190 μm in thickness and 3.9 mm $\times$ 2.7 mm $\times$ 3.0 mm in size as shown in Fig. 2 was used for testing. The plate was adhered to the top

Fig. 3 The modified function $\tilde{\nu}_A^x(\omega)/(\omega^2 \tilde{\nu}_u(\omega))$ and its corresponding coherence function obtained from measured velocities

Fig. 4 The modified function $(\tilde{\nu}_B^x(\omega)/\tilde{\nu}_u(\omega) - 1)/\omega^2$ and the coherence function for $\tilde{\nu}_B^x(\omega)/\tilde{\nu}_u(\omega)$ obtained from measured velocities

Fig. 5 The fitted mode shapes and natural frequencies

surface of the platform with epoxy. The intersections of dashed lines were locations where the response velocities of the plate were measured. Sparks were generated with a condenser charged with a voltage source of 4 kV. Then, the excitation and response signals are fed into a spectrum analyzer to find frequency response

Fig. 6 Mode shapes and natural frequencies obtained from finite element analysis

Fig. 7 The synthesized receptance function $H_{AB}^{yx}(\omega)$

functions. Figure 3 shows a typical modified function $\bar{v}_A^x(\omega)/(\omega^2 \bar{v}_u(\omega))$ obtained from measured velocities and its corresponding coherence function. The coherence function has acceptable values around natural frequencies. The modified function $(\bar{v}_B^x(\omega)/\bar{v}_u(\omega) - 1)/\omega^2$ and the coherence function for $\bar{v}_B^x(\omega)/\bar{v}_u(\omega)$ obtained from measurement are shown in Fig. 4. The frequency range being covered in this test, identification of four modes can be accomplished by comparing the frequency response functions obtained at different locations. The mode shapes and natural frequencies extracted from all measured frequency response functions are given in Fig. 5. Figure 6 shows the simulated results obtained with finite element analysis. Based on the model of finite element, the epoxy-adhered L-shaped plate was treated with fixed end constraints. It is believed that the differences between Fig. 5 and Fig. 6 are mainly originated from the discrepancy between boundary conditions. After the extraction of modal parameters, receptance functions can be synthesized. Figure 7 is the synthesized receptance function between the y-direction of location A and the x-direction of location B as indicated in Fig. 2. The receptance functions would enable the quantitative predictions of the dynamic responses by means of force excitation.

Conclusions

Conducting modal testing for microstructures requires high frequency excitations and noncontact measuring devices. Employing electric discharge and driven-base excitation principle can set up a modal testing system with which it provides broad band excitation. Due to the fact that Laser Doppler vibrometers can pick up both excitation and response velocities without contact the dynamic characteristics of the testing structures are not distorted. In order to describe the frequency response functions between input and output velocities, a mathematical model has been derived and adopted to fit the measured data to extract modal parameters. The experiment showed that the proposed modal testing system is capable of extracting modal parameters for base-fixed microstructures. Since the mode shapes can be normalized with respect to the assigned modal masses, e.g., $M_r = 1$ in previous derivation, synthesis of receptance functions is enabled. Therefore, prediction of structural dynamic responses caused by the applied forces is actualized.

In summary, the method developed in this paper can obtain modal parameters without measuring excitation force. As a result, it is expected that this proposed system is suitable for conducting modal testing pertaining to microstructures.

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Nomenclature

$\mathbf{C}, \mathbf{L}, \mathbf{T}$	= linear operators
c_{ijkl}	= elastic moduli
$\mathbf{d}_u(t), \tilde{\mathbf{d}}_u(\omega)$	= displacement of driven-base and its Fourier transform
$\mathbf{e}_i$	= unit vector
$H_{ij}^{\beta\alpha}(\omega)$	= receptance function
M_r	= modal mass
P_r	= linear modal momentum factor
$Q_r(t), \tilde{Q}_r(\omega)$	= modal force and its Fourier transform
$q_r(t), \tilde{q}_r(\omega)$	= modal displacement and its Fourier transform
R_r^x	= constant
S	= boundary of an elastic body
S_u	= displacement-prescribed boundary
S_σ	= traction-prescribed boundary
$\mathbf{U}(\mathbf{x}, t), \tilde{\mathbf{U}}(\mathbf{x}, \omega)$	= displacement field caused by elastic deformation and its Fourier transform
$\mathbf{u}(\mathbf{x}, t), \tilde{\mathbf{u}}(\mathbf{x}, \omega)$	= total displacement field and its Fourier transform
u_l	= $\mathbf{e}_l$ component of displacement field
V	= domain of an elastic body
$\mathbf{x}, \mathbf{x}_i, \boldsymbol{\xi}$	= position vectors
$\boldsymbol{\alpha}, \boldsymbol{\beta}$	= unit vectors
γ_r	= scaling factor
δ_{rs}	= Kronecker delta
ζ_r	= damping ratio
ρ	= density
σ_r	= damping factor
$\Phi_r(\mathbf{x})$	= r th mode shape
ϕ_{ri}^x	= x -component of the r th mode shape at location $\mathbf{x}_i$

ω_{dr} = damped natural frequency
 ω_r = natural frequency

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