Optimal Tolerance Allocation for a Sliding Vane Compressor

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An optimization model has been built with consideration of the required reliability, the minimum machining cost, and quality loss. The normal and the lognormal distributions of tolerances that depend on the production types of components are used in the reliability model. Cost tolerance data obtained from Bjørke are used to calculate the machining cost. The asymmetric quadratic quality loss model is used to calculate quality loss caused by the deviation and the mean-shift of distributions. Tolerance allocation of a sliding vane rotary compressor is optimized for the required reliability, the minimum cost and quality loss, and optimum tolerances of components are recommended. The results show that high accuracies of the slot length, the vane thickness, and the slot width are required. Hence, their tolerances are smaller than other components. The effects of the correlation coefficient of the bottom cover plate and the top cover plate and the correlation coefficient of the front cover plate and the rear cover plate to total cost are insignificant. Further, the cost of quality loss is reduced when the weighting ratio of the quality loss function weighting coefficient to the machining cost function weighting coefficient is increased. The total cost is increased because tight tolerance allocation increases the machining cost. [DOI: 10.1115/1.2114893]

Introduction

Sliding vane rotary compressors have been developed and used for years because of lightweight, small size, simple mechanism, and easy maintenance. They are commonly used in refrigerators and automotive air conditioning systems. A considerable amount of studies were conducted to improve sliding vane rotary compressors [1].

A sliding vane rotary compressor has been designed and its schematic drawing is shown in Fig. 1 [2]. Some topics regarding this compressor were studied, such as the static contact force, stress and deflection of its vanes, stator contour, impact of vanes on the rotor, configuration, disassembly process, and test of the compressor [2–5]. The results showed that the compression ratio was not high as expected and the noise level was high because of leakage and friction due to improper tolerance.

Tolerance design is an important task in embodiment design. Tolerance allocation of an assembly in the conventional design is based on designers' experience or design handbooks. When tolerance is tight, the cost is high, and vice versa. If accumulated tolerance of components is out of specification and causes improper assembly, tolerances of components have to be revised. The task becomes complicated and very tedious if the number of components is large. Furthermore, the highest reliability, the best quality, and the minimum cost of a product cannot be assured and achieved.

The standard for the exchange of product model data (STEP) proposed by the international standard organization (ISO) and the product data exchange specification (PDES) proposed by the US National Bureau of Standards were very useful for tolerance design. Bjørke [6] and Juster et al. [7] studied the relationship of linear tolerance accumulation and allocation. Feng and Yang [8] generated a model to integrate tolerance information into the product database. Parkinson [9] applied a deterministic method of robust design to determine optimum nominal dimensions of components with dimensional tolerances. Zhou et al. [10] and Jordaan and Ungerer [11] used various numerical methods to approach optimal tolerance allocation. Di Stefano [12] analyzed statistical tolerance by using a mean shift model to evaluate the mean shift

factor. Hong and Chang [13] deal with interactions of the deviation space, deviation volume, and tolerance primitives to determine the relationship between process planning decisions and tolerances. Main and Ferreira [14] addressed the acceptable level of inaccuracy of the machined parts associated with its geometric tolerances. Manarvi and Juster [15] compiled available information and developed an integrated tolerance model and a tolerance allocation process. Hu et al. [16] provided automated methods for the specification of geometric tolerance types. Wu and Rao [17] investigated the modeling and analysis of tolerances and clearances. Mujezinović et al. [18] applied a mathematical model for representing geometric tolerances to polygonal faces and showed its sensitivity to the precedence of datum reference frames.

Dong et al. [19] studied the relationship between tolerance and the machining cost. Shiu et al. [20] discussed misalignment and fabrication error of compliant parts and minimized manufacturing costs associated with tolerances. Xue and Ji [21] investigated a cost-tolerance model and allocated systematically the best tolerance to minimize the total manufacturing cost.

Mahadevan and Ni [22] analyzed tolerance reliability assessment of automotive spot-welded joints. Several researchers implemented the quality loss model [23–26] into tolerance synthesis. Ye and Salustri [27] constructed a nonlinear optimization model with consideration of product design, manufacturing, and quality simultaneously. Li [28] investigated the optimal manufacturing setting to minimize the expected quality loss.

Up to the present, only the combined cost tolerance and reliability model [29,30] and the combined cost tolerance and the quality loss model [23–25,31,32] were utilized for tolerance allocation. The optimal tolerance allocation model is not available based on the simultaneous consideration of the reliability, cost, and quality loss. In addition, semi-tolerances were assumed in existing studies. Therefore, the purpose of this study is to build an optimal tolerance allocation model with simultaneous consideration of the required reliability, the minimum cost, and quality loss. In the meantime, since tolerance allocation of the compressor as shown in Fig. 1 has not been optimized, tolerances between vanes and slots in the rotor, and vanes and the stator will be optimized.

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Fig. 1 Components and schematic drawing of sliding vane rotary compressor

Method of Approach

Basic components of the compressor as shown in Fig. 1 consist of a rotor, a stator, and sliding vanes. A rotor with ten slots for ten sliding vanes is inside and concentric with the stator. The outer surface of the rotor, the inner contour of the stator and the vanes form a volume segment. When the rotor rotates, the vanes slide outward along the slot due to the centrifugal force. The maximum volume segment is obtained at the end of the intake process. When the vanes slide inward, the volume segment is reduced and the air is compressed. The cross section of the compressor is shown in Fig. 2 and 18 independent components are 1—stator, 2—rotor, 3—bottom cover plate, 4—top cover plate, 5—front cover plate, 6—rear cover plate, 7—vane, 8—front bearing, 9—rear bearing, 10—oil ring, 11—bottom bolt, 12—top bolt, 13—front bolt, 14—rear bolt, 15—bottom gasket, 16—top gasket, 17—front gasket, and 18—rear gasket.

The reliability index is the minimum distance from the origin of the coordinate system to the limited surface of the design. The standard parts of an assembly that are manufactured in mass production are considered as the normal distribution. The non-standard parts of an assembly that are produced in batch production are considered as the lognormal distribution with a dimension mean-shift because of the asymmetric distribution [33].

Let $\bar{X}$ be the mean vector of the normal distribution vector or the lognormal distribution X , V be the covariance matrix, and $\text{cov}(x_i, x_j)$ stand for the covariance of the variable dimensions x_i and x_j in an assembly [34] as

$$X = \{x_1, x_2, \dots, x_n\} \quad (1)$$

$$\bar{X} = \{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n\} \quad (2)$$

Fig. 2 Cross section of compressor and part numbers

$$V = \begin{bmatrix} \sigma_1 & \text{cov}(x_1, x_2) & \cdots & \text{cov}(x_1, x_n) \\ \text{cov}(x_2, x_1) & \sigma_2 & \cdots & \text{cov}(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov}(x_n, x_1) & \text{cov}(x_n, x_2) & \cdots & \sigma_n \end{bmatrix} \quad (3)$$

where

$$\text{cov}(x_i, x_j) = E[(x_i - \mu_i)(x_j - \mu_j)] \quad (4)$$

μ_i and μ_j are the mean values of x_i and x_j , respectively, and σ_i is the standard deviation of x_i . The coefficient of variation δ_i of x_i is

$$\delta_i = \frac{\sigma_i}{\mu_i} \quad (5)$$

The variables of the normalized standard distribution for the normal distribution or the lognormal distribution, respectively, are

$$y_i = \frac{x_i - \mu_i}{\sigma_i} \quad (6)$$

and

$$y_i = \frac{\ln x_i - \mu_i}{\sigma_i} \quad (7)$$

The variable vector of the normalized standard distribution is $Y = \{y_1, y_2, \dots, y_n\}$. The correlation coefficient of y_i and y_j is

$$\rho_{ij} = \frac{1}{N} \sum_{i=1}^N y_i y_j \quad (8)$$

where N is the population size. The independent standard normal distribution vector $Z = \{z_1, z_2, z_3, \dots, z_n\}$ of variables z_i can be obtained from

$$y_1 = \alpha_{11} z_1$$

$$y_2 = \alpha_{21} z_1 + \alpha_{22} z_2$$

$$\dots$$

$$y_n = \alpha_{n1} z_1 + \alpha_{n2} z_2 + \dots + \alpha_{nn} z_n \quad (9)$$

where

$$\alpha_{11} = 1.0$$

$$\alpha_{i1} = \rho_{ij} \quad 1 < i$$

$$\alpha_{ik} = \frac{1}{\alpha_{kk}} \left(\rho_{ij} - \sum_{j=1}^{k-1} \alpha_{ij} \alpha_{kj} \right) \quad 1 < k < i$$

$$\alpha_{ii} = \sqrt{1 - \sum_{j=1}^{i-1} \alpha_{ij}^2} \quad (10)$$

Then the variables z_i are

$$z_1 = y_1$$

$$z_2 = \frac{1}{\alpha_{22}} (y_2 - \alpha_{21} z_1)$$

$$\dots$$

$$z_n = \frac{1}{\alpha_{nn}} (y_n - \alpha_{n1} z_1 - \dots - \alpha_{n,n-1} z_{n-1}) \quad (11)$$

Once the independent normal distribution vector Z is available, the reliability index can be obtained by using the Beta algorithm which was developed by Hohenbichler and Rackwitz [35] as shown in Fig. 3, where G is the design function. Details of the Beta algorithm are presented in the Appendix.

Fig. 3 Flowchart of beta algorithm process

In the case of multiple design functions, the minimum reliability index is

$$\beta_{\min} = \min\{\beta_1, \beta_2, \dots, \beta_k, \dots, \beta_p\} \quad 1 \leq k \leq p \quad (12)$$

where p is the number of design functions.

The quality loss model of the-nominal-the-best type is that the best value of dimension is equal to the nominal dimension. If it is used, the expected total loss is [36]

$$E(L_{\text{asm}}) = \sum_{i=1}^n \left(\sum_{\mu_i < m_i} k_{iL} [\sigma_i^2 + (\mu_i - m_i)^2] + \sum_{\mu_i > m_i} k_{iR} [\sigma_i^2 + (\mu_i - m_i)^2] \right) \quad (13)$$

where m_i stands for the nominal value of the i th dimension

$$k_{iL} = \frac{A_h}{\Delta_{iL}^2} \quad (14)$$

$$k_{iR} = \frac{A_h}{\Delta_{iR}^2}$$

are quality loss coefficients, A_h is the cost of the replacement or the repair for the h th component, and Δ_{iL} and Δ_{iR} are specific semi-tolerance, or called customer's semi-tolerance of the i th dimension. The quality loss curve and the probabilistic density function are shown in Fig. 4.

It can be seen that quality loss consists of loss by the deviation, $k_{iL}\sigma_i^2$ and $k_{iR}\sigma_i^2$, and loss by the mean-shift, $k_{iL}(\mu_i - m_i)^2$ and $k_{iR}(\mu_i - m_i)^2$. Quality loss increases when the standard deviation increases or the mean-shift increases.

Since the reliability index stands for the minimum distance from the origin of the coordinates to the limited surface in the standard normal distribution, the reliability corresponding to the reliability index can be obtained from the inverse of the accumulated distribution function based on the specified reliability. The left and right side semi-tolerances provide the required information related to quality loss. Söderberg [23] defined that the customer's lower acceptance limit of a component life is 75% of the nominal value. The reliability index of 75% reliability is

Fig. 4 Asymmetric quality loss curve and asymmetric dimension distribution

$$\beta^* = \Phi^{-1}(0.75) = 0.67449 \quad (15)$$

where Φ is the accumulated functional distribution of the independent normal distribution. Customer's semi-tolerance can be optimized for minimizing the goal of the machining cost through the machining cost-tolerance model $C_m(\Gamma)$ with the constraint

$$\beta^* - \beta_k(\Gamma) \leq 0 \quad 1 \leq k \leq p \quad (16)$$

where

$$\Gamma = \{\sigma_1, \sigma_2, \dots, \sigma_i, \dots, \sigma_n\} \quad 1 \leq i \leq n \quad (17)$$

The process of calculating the quality loss coefficient is shown in Fig. 5.

If the replacement cost A_h is known, a set of Δ_{iL} and Δ_{iR} nearest to the specific customer's quality loss can be obtained from Fig. 4. In the meantime, the quality loss coefficients k_{iL} and k_{iR} of the asymmetric quality loss function can be obtained. Then tolerance can be optimized by a non-linear programming with minimizing the goal of

$$C_t(\Gamma) = W_m C_m(\Gamma) + W_q E(L_{\text{asm}}(\Gamma)) \quad (18)$$

and constraint function

$$\beta^{**} - \beta_k(\Gamma) \leq 0 \quad 1 \leq k \leq p \quad (19)$$

where C_t is the total cost that consists of the machining cost C_m and the quality loss expected value $E(L_{\text{asm}})$. W_m and W_q are the weighting coefficients of the machining cost function and the quality loss function, respectively. β^{**} is the required assembly reliability index, and p is the number of design functions. The reliability index β_k of each design function stands for the assembly yield rate of each component. If β^{**} is larger, the reliability of the assembly will be higher. When the reliability is set at 99.0%, the corresponding reliability index is 2.32635. The processes to obtain semi-tolerance are shown in Fig. 6.

The minimization of machining cost and quality loss is a multi-attribute decision-making problem. Also, the desired reliability in-

Fig. 5 Flowchart of calculating quality loss coefficient

Fig. 6 Flowchart of determining optimal semi-tolerance

dex can be set in a penalty function form as the third objective function. Due to the consideration of computational efficiency, the attribute functions can be added together with different weighting coefficients. Cheng and Maghsoodloo [26] directly summed the cost-tolerance and quality loss for tolerance allocation, but they only considered the normal distribution condition without dimensional correlation. In this study, the replacement cost in the quality loss function has been carefully calculated in order to have an equivalent unit as machining cost in Bjørke's cost-tolerance model. Therefore, machining cost function and quality loss function can be combined into a single objective function with selected weighting coefficients. The weighting ratio W_q/W_m represents the relative importance between quality loss and machining cost.

Oriented Functional Relationship. If an assembly that consists of many components is complicated, it is not worth analyzing all dimension features because of the cost. The assembly feature graph or the functional feature graph can be used to determine key functional dimensions. The representation of the oriented functional relation for dimension tolerance proposed by Zhang and Porchet [37] is used in this study, and its main purpose is to organize features of components according to assembly drawings. Users can find functional components, which really affect the performance of a product, quickly from the data bank and analyze tolerance features.

The oriented functional relationship is shown in Fig. 7. Each node stands for an independent component, and a line with an arrow stands for the mating relation between components. If an arrow points from A to B, it stands for component B is fixed on component A. A line without an arrow stands for a sub-assembly with components as points C and D, and these components are assembled into a sub-assembly before being assembled with other components. A dashed line stands for the functional dimension that needs to be controlled for the dimension and tolerance, as point E to point F.

Six symbols to represent relations between components are shown in Table 1. The connection between planes and the connection between cylinders stand for a tight fitting between two planes and two cylinders, respectively. The connection by planes and planes and the connection by cylinders and cylinders stand for a distance existed and loose running fit for planes and cylinders, respectively. The connection by cylinders and planes stands for simultaneous existence of connection between planes and connec-

Fig. 7 Oriented functional relationship

Table 1 Symbols for feature fit in oriented functional relationship

Relation of connection	Symbol
Connection between planes	p
Connection between cylinders	c
Connection by planes and planes	pp
Connection by cylinders and cylinders	cc
Connection by cylinders and planes	cp
Thread connection	t

tion between cylinders.

The oriented functional relationship graph of the compressor as shown in Figs. 1 and 2 is shown in Fig. 8. The bottom cover plate that is used as the basis is at the top. Only a line contact, but without the assembly relationship, exists between vanes and the stator inner contour. The rotor connects to the inner rings of two bearings. Each bearing is considered as a sub-assembly that consists of the outer ring and the inner ring with the relative motion between them and cannot be counted as one node. Hence, nodes 8 and 9 are divided into 8_1 and 8_2 , and 9_1 and 9_2 , respectively. The rotor and vanes are considered as the third sub-assembly.

It is necessary to find tolerance chains of functional dimensions in order to analyze dimension tolerance. There are four dashed lines. If a functional dimension chain consists of these specific dimensions, all tolerances in the dimension chain should be analyzed. The dashed line from node 1 to node 7 stands for the contact between vanes and the stator inner contour. When the rotor is stationary, vanes may not contact with the stator. When the rotor rotates, vanes slide outward due to the centrifugal force. The tip of the vane slides along the stator inner contour. Although the dashed line represents the special contact situation in the oriented functional relationship graph, there actually is no functional tolerance between vanes and the stator. Therefore, it is not necessary to analyze this dimension chain.

Honda et al. [38] utilized a numerical method to analyze the gap flow between the side cover plate and the rotor and proved the importance of the clearance to the compressor performance. Two dashed lines from node 2 to nodes 5 and 6 stand for functional dimensions of the rotor to the front cover plate and the rear cover plate, respectively. These two clearances are very important to the compressor. When the clearance is decreased, friction between the rotor and two cover plates, power loss and wear of components increase. If the clearance is increased, leakage of fluid occurs and reduces the compression performance. Therefore, all the dimen-

Fig. 8 Oriented functional relationship graph of rotary compressor

Fig. 9 First functional dimension chain

sion chains related to these two clearances need to be controlled.

The first functional dimension chain consists of six nodes as shown in Fig. 9(a). The dimension variables are shown in Fig. 9(b). The length of slot in the rotor, x_2 , plus two specific clearances C_1 and C_2 should equal the stator length x_1 plus the thickness of two gaskets L_1 and L_2 . The constraint is

$$x_1 + L_1 + L_2 = x_2 + C_1 + C_2 \quad (20)$$

There are four nodes in the second functional dimension chain as shown in Fig. 10(a). Nodes 9_1 and 9_2 stand for the outer ring and the inner ring of the bearing. The thickness of the bearing plus the wall thickness of the slot L_3 should equal the thickness of the real cover plate x_4 . The locations of dimension variables are shown in Fig. 10(b). The constraint can be written as

$$x_3 + L_3 + C_1 = x_4 + C_1 \quad (21)$$

The third functional dimension chain as shown in Fig. 11 is similar to the second functional dimension chain, except with one more oil ring, node 10. The bearing thickness L_4 , the oil ring

thickness plus the wall thickness x_5 should be equal to the front cover plate thickness x_6 as shown in Fig. 11(b). The constraint can be written as

$$C_2 + L_4 + L_5 + x_5 = C_2 + x_6 \quad (22)$$

In addition, a dashed line that connects nodes 2 and 7 stands for the clearance between vanes and the rotor. The rotor and the vane form a sub-assembly, and the fourth functional dimension chain is shown in Fig. 12(a). The schematic drawing of the vane and the rotor slot is shown in Fig. 12(b), and the dimension relation is

$$x_7 + C_3 + C_4 = x_8 \quad (23)$$

Synthesis and Optimization of Tolerance. The design functions as shown in Fig. 13 can be rewritten into

$$G_1 = (x_1 + L_1 + L_2) - (x_2 + C_1 + C_2) + 0.1 \quad (24)$$

$$G_2 = x_3 + L_3 - x_4 + 0.1 \quad (25)$$

Fig. 10 Second functional dimension chain

Fig. 11 Third functional dimension chain

Fig. 12 Fourth functional dimension chain

$$G_3 = x_4 - x_3 - L_3 + 0.1 \quad (26)$$

$$G_4 = L_4 + L_5 + x_5 - x_6 + 0.1 \quad (27)$$

$$G_5 = x_6 - L_4 - L_5 - x_5 + 0.1 \quad (28)$$

$$G_6 = x_8 - (x_7 + C_3 + C_4) + 0.02 \quad (29)$$

The design function G_1 stands for the clearance constraint between the front cover plate and the rear cover plate. The sum of clearances is within 0.1 mm. The design functions G_2 and G_3 mean that the thickness variation between the rear cover plate and the bearing is within 0.1 mm. The design functions G_4 and G_5 mean that the thickness variation between the front cover plate, the bearing and the oil ring is within 0.1 mm. The purpose of the first five design functions is apparently to maintain the appropriate clearances C_1 and C_2 between two cover plates and bearings in order to prevent leakage and reduce friction. The design function G_6 stands for the required clearance between the vanes and the slots of the rotor to allow vanes to slide smoothly. The combined nominal clearance of C_3 and C_4 is 0.05 mm and the minimum clearance is 0.02 mm.

The covariance matrix of dimension variables is

$$V' = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{bmatrix} \quad (30)$$

According to Parkinson [9], the dimensions on the same component are correlated. It can be considered that the different dimensions at the same component may be manufactured by the same datum setup during the process. Holes for the bearings and the oil ring in the front cover plate and the rear cover plate are manufactured by the milling process. The parts are set up on the same fixture when machining for the plate thickness and the holes. The dimension variables x_3 and x_4 located at the rear cover plate are dependent, and the dimension variables x_5 and x_6 located at the front cover plate are dependent. Therefore, the values of the correlation coefficients ρ_{34} and ρ_{56} are not zeros. The other dimension variables are assumed independent and their correlation coefficients are zero. Equation (30) can be rewritten as

$$V' = \begin{bmatrix} 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & \rho_{34} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{43} & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & \rho_{56} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{65} & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0 & 0 \end{bmatrix} \quad (31)$$

Optimal tolerance allocation can be achieved by following the process mentioned above.

Results

The combined model is evaluated for optimal tolerance allocation of an assembly with normal distributions of independent variables without the mean-shift and quality loss. The results are compared and show comparable agreement with the existing data [39] to assure the feasibility of the combined methodology.

Table 2 Statistical distribution types of dimensions

Symbol	Name	Distribution type
x_1	Stator length	Lognormal
x_2	Rotor center length	Lognormal
x_3	Rear cover plate sink thickness	Lognormal
x_4	Rear cover plate thickness	Lognormal
x_5	Front cover plate sink thickness	Lognormal
x_6	Front cover plate thickness	Lognormal
x_7	Vane thickness	Lognormal
x_8	Slot width	Lognormal
L_1	Gasket thickness	Normal
L_2	Gasket thickness	Normal
L_3	Real cover plate bearing thickness	Normal
L_4	Front cover plate bearing thickness	Normal
L_5	Oil ring thickness	Normal
C_1	Rear cover plate clearance	Fixed target value
C_2	Front cover plate clearance	Fixed target value
C_3	Clearance between vane and slot	Fixed target value
C_4	Clearance between vane and slot	Fixed target value

The statistical distributions of components depend on manufacturing processes. The oil ring, bearings, and gaskets are standard parts made in mass production, and the dimensions are assumed to be normal distributions. The other components are manufactured in batch production, and the dimensions are assumed to be lognormal distributions as shown in Table 2.

Only the lognormal distributions of dimension variables x_1 to x_8 will be optimized, and these tolerances must be specified in design drawings. Dimension variables L_1 to L_5 are from standard parts made in mass production and will not be optimized. Normal distributions of these dimensions affect the reliability index in the reliability model. Hence, the standard deviations of these dimensions must be controlled to have good quality products.

Since costs of some compressor components are not available, the process of this study is based on the cost tolerance model proposed by Bjørke [6]. The numerical function in the reciprocal power type is used for the curve fitting, and the coefficients of the curve are listed in Table 3. The cost tolerance coefficients of all functional dimensions of components are listed in Table 4. Because L_1 to L_5 are the dimensions of the standard parts and C_1 to C_4 are specified clearances, they are not included in the optimization model. If the required reliability is 75% and the required reliability index β^* is equal to 0.67449, customer's semi-tolerances of all dimension variables are listed in Table 5.

Based on the manufacturing cost, the replacement costs of the stator A_1 , the rotor A_2 , the rear cover plate A_3 , the front cover plate A_4 , and the set of vanes A_5 are 9.0, 5.0, 1.0, 1.5, and 0.3, respectively [6]. Quality loss coefficients of the functional dimensions obtained are listed in Table 6. Because the replacement cost of the rotor is high and the customer's semi-tolerances of x_2 and x_8 are smaller than other dimensions, the quality loss coefficients of x_2 and x_8 are much larger than other dimension variables. It can be expected that the dimension variables with the required tightest tolerances are x_2 and x_8 in tolerance optimization.

Table 3 Coefficients of curve fitting with $C_m = c_0 t^{-c_1}$ for Bjørke's cost-tolerance models

No.	Process type	c_0	c_1
1	External cylindrical surface (rotating)	1.192	0.591
2	Internal cylindrical surface (rotating)	7.602	0.814
3	Face surrounding bore, to a datum (rotating)	18.064	1.115
4	Internal cylindrical surfaces (non-rotating)	12.435	0.566
5	Face surrounding a bore, to a datum (non-rotating)	18.622	0.564
6	Plane surface	13.457	0.764

Table 4 Nominal dimensions and coefficients of cost tolerance function $C_m = c_0 t^{c_1}$

Symbol	Nominal dimension (mm)	Coefficients of cost tolerance function	
		c_0	c_1
x_1	152.0	18.622	0.564
x_2	154.8	18.064	1.115
x_3	5.0	12.435	0.566
x_4	20.0	13.457	0.764
x_5	5.0	12.435	0.566
x_6	30.0	13.457	0.764
x_7	4.95	13.457	0.764
x_8	5.0	13.457	0.764
L_1	1.5		
L_2	1.5		
L_3	15.0		
L_4	15.0		
L_5	10.0		
C_1	0.1		
C_2	0.1		
$C_3 + C_4$	0.05		

Distribution of Vane Thickness. Two sets of 10 vanes, a total of 20 vanes, are measured by a digital micrometer to obtain the mean-shift value of the thickness. The thickness of each vane is measured seven times. The largest and the smallest values are obsolete, and the other five values are used to calculate the mean value to reduce the measured error. The mean values of the measured vane thickness are listed in Table 7.

Table 5 Customer's semi-tolerance of dimension variables

Symbol	Left customer's semi-tolerance Δ_{iL} (mm)	Right customer's semi-tolerance Δ_{iR} (mm)
x_1	0.3341	0.3347
x_2	0.1303	0.1304
x_3	0.2808	0.2945
x_4	0.2103	0.2121
x_5	0.2801	0.2938
x_6	0.2100	0.2112
x_7	0.0509	0.0513
x_8	0.0507	0.0511

Table 6 Quality loss coefficients of functional dimensions

Symbol	Left quality loss coefficient k_{iL}	Right quality loss coefficient k_{iR}
x_1	80.6491	80.3548
x_2	294.4430	294.0312
x_3	12.6799	11.5238
x_4	22.6086	22.2152
x_5	19.1211	17.3823
x_6	34.0203	33.6257
x_7	115.9523	113.9805
x_8	3500.6748	3441.9239

Table 7 Mean values of measured vane thickness

No	Thickness (mm)	No	Thickness (mm)	No	Thickness (mm)	No	Thickness (mm)
1	4.917	6	4.949	11	4.976	16	4.943
2	4.950	7	4.958	12	4.945	17	4.945
3	4.941	8	4.946	13	4.939	18	4.969
4	4.935	9	4.951	14	4.925	19	4.974
5	4.930	10	4.949	15	4.965	20	4.977

Fig. 14 Probability density functions of normal and lognormal distributions for vane thickness

The mean value and the standard deviation of the normal distribution for the vane thickness are 4.948 and 0.0178, respectively. The middle value and the coefficient of variation of the lognormal distribution are 4.947 and 0.0403, respectively. The curves of the probability density function of the normal distribution and the lognormal distribution and the practical distribution histogram are shown in Fig. 14.

The chi-square test is used to verify the fitting goodness of the normal distribution and the lognormal distribution. The results are shown in Table 8. The symbol n_i is the observed frequency for each interval and e_i is the theoretical frequency. From the test results, fitting of the lognormal distribution is better than that of the normal distribution for the vane thickness.

Since only the dimension variables of the lognormal distributions are used for tolerance optimization in this study, three parameters that are needed for optimization are the mean-shift value, the correlation coefficient of the lognormal distribution, and the standard deviation of dimension variables of the normal distribution. First, it is assumed that the vanes, the front cover plate, and the rear cover plate are to be processed by the same machine. Therefore, based on the mean-shift value of the vane thickness x_7 , the mean-shift values of the rear cover plate x_4 and the front cover plate x_6 are assumed to be 0.003 mm. It is assumed that other dimensions do not have the mean-shift. Second, the correlation coefficients of two parts depend on the machining process and the fixture datum setup. In this case, the dimensions of x_3 and x_4 , and x_5 and x_6 are assumed to be independent. Third, the statistical data of the mass production parts are not available. So, it is assumed that standard deviations of these standard parts are 0.01 mm. Tolerances of these standard parts are 0.06 mm obtained from the concept of six-sigma, $\pm 3\sigma$. If all of the correlation coefficients are assumed to be zero, the result of tolerance optimization is listed in Table 9. The dimensions with specified tolerances for practical engineering drawings are listed in Table 10. The upper and lower tolerances of functional dimensions are shown in Fig. 15.

The correlation coefficients ρ_{34} of x_3 and x_4 and ρ_{56} of x_5 and x_6 can be set as variables based on optimal tolerances obtained to perform the parameter analysis. Since the machining processes are similar, these two correlation coefficients are set as the same vari-

Table 8 Normal distribution and lognormal distribution verified by chi-square test

Interval (mm)	Real number n_i	Theoretical number e_i		chi-square value $(n_i - e_i)^2 / e_i$	
		Normal	Lognormal	Normal	Lognormal
<4.920	1	1.16	1.74	0.022	0.315
4.920–4.930	1	1.97	2.19	0.478	0.647
4.930–4.940	3	3.40	3.32	0.047	0.031
4.940–4.950	7	4.36	4.72	1.599	1.101
4.950–4.960	3	4.11	2.87	0.300	0.006
4.960–4.970	2	2.84	2.66	0.248	0.164
>4.970	3	2.16	2.50	0.327	0.100
Sum	20	20.0	20.0	3.021	2.364

able ρ . The relation between the correlation coefficient ρ and the cost is shown in Fig. 16. The second parameter analysis performed is to set the standard deviations of the mass production parts as the variables and analyze the relation between the quality of standard parts and the cost. The result is shown in Fig. 17. Increasing the weighting ratio W_q/W_m increases the relative im-

Table 9 Results of normal distribution with standard deviation of 0.01 mm and zero correlation coefficients

Variable	Optimum standard deviation (mm)	Lower limit of tolerance (mm)	Upper limit of tolerance (mm)	Total tolerance (mm)
x_1	0.03788	0.0921	0.0921	0.1842
x_2	0.01461	0.0355	0.0355	0.0710
x_3	0.03168	0.0766	0.0776	0.1542
x_4	0.02510	0.0610	0.0611	0.1221
x_5	0.03078	0.0744	0.0753	0.1497
x_6	0.02423	0.0589	0.0590	0.1179
x_7	0.00753	0.0183	0.0183	0.0366
x_8	0.00640	0.0155	0.0156	0.0311
Optimum machining cost C_m is 5.1128				
Quality loss expected value $E(L_{asm})$ is 0.3888				
Total cost C_t is 5.5017				
The result of Monte Carlo simulation is 93.71% by 10,000 samples				

Table 10 Functional dimensions with optimal tolerances specified

Variable	Dimension description	Dimension (mm)
x_1	Stator length	152 ± 0.092
x_2	Rotor slot length	154.8 ± 0.036
x_3	Rear cover plate wall thickness with cut off	$5 + 0.077$ -0.078
x_4	Rear cover plate thickness	20 ± 0.061
x_5	Front cover plate thickness with cut off	$5 + 0.074$ -0.075
x_6	Front cover plate thickness	30 ± 0.059
x_7	Vane thickness	4.95 ± 0.018
x_8	Rotor slot width	5 ± 0.016

Fig. 15 Upper and lower tolerances of function dimensions

Fig. 16 Relation between correlation coefficient of cover plate thickness and cost

portance of quality loss in the combined objective function. The effect of weighting ratio on the cost is studied, and the results are shown in Fig. 18.

Discussion

The required reliability, the minimum machining cost and quality loss are considered simultaneously for tolerance allocation design to improve existing models. Consideration of the required reliability, the minimum machining cost and quality loss simultaneously is a multi-attribute decision-making problem. In general, the multi-attribute utility analysis is a good way to obtain the optimum results in many applications. Since the manufacturing costs of compressor components are obtained from machining factories in this study, and the replacement cost A_h is converted into the equivalent cost as the same unit as Bjørke's cost-tolerance model, the gradients of the distribution curves can be obtained. The advantage of single-objective optimization is that the efficiency is higher and calculation time is less than the multi-attribute utility analysis. Therefore, the Beta algorithm process and the non-linear programming are used in this study. Hohenbichler and Rackwitz's Beta algorithm is used to calculate the reliability index [35] because the efficiency of the transformation is higher than those of other transformation methods when the distribution model is non-normal and asymmetric.

Table 5 shows that right customer's semi-tolerance Δ_{iR} is larger than left customer's semi-tolerance Δ_{iL} . Therefore, the left quality loss coefficient k_{iL} is larger than the right quality loss coefficient

Fig. 17 Relation between standard deviations of mass production parts and cost

Fig. 18 Relation between weighting ratio and cost

K_{IR} as shown in Table 6. The quality loss coefficients of x_2 and x_8 are much higher than those of other dimensions because the stator and the rotor are the most expensive components in the rotary compressor.

The chi-square value 2.364 of the lognormal distribution is smaller than the value 3.021 of the normal distribution as shown in Table 8. It demonstrates that the lognormal distribution has better fitting than the normal distribution for the vane thickness when mean-shift and asymmetry exist. The mean value of vanes as shown in Table 7 is 4.947 mm and the nominal dimension as shown in Table 4 is 4.95 mm. The mean-shift value of the lognormal distribution due to machining is 0.003 mm, which is used for tolerance optimization in this study.

Design functions control tolerances of the vane thickness and the slot width, and the slot length affects clearances of the front cover plate and the rear cover plate. The optimization generates tighter tolerances for the slot length x_2 , the vane thickness x_7 and the slot width x_8 , which are critical function dimensions for manufacturing, than other functional dimensions as shown in Fig. 15. The unit value of 0.01 mm is used in Table 10 in order to match the precision of the regular machining process.

When correlation coefficients $\rho_{x_3x_4}$ of x_3 and x_4 and $\rho_{x_5x_6}$ of x_5 and x_6 vary, the cost variation is insignificant as shown in Fig. 16. One reason is that dimension variables x_3 and x_4 , and x_5 and x_6 have larger tolerances than others. The variation of correlation coefficients does not affect the total cost significantly. When the standard deviation of standard parts increases, the total cost increases as shown in Fig. 17. It means that other components of the assembly made by batch production need to have high manufacturing accuracy in order to compensate for poor quality mass production parts.

The quality loss in the total cost function is small compared with the total cost as shown in Figs. 16 and 17. The effect of the weighting ratio W_q/W_m varied from 1 to 8 on the cost is investigated. The results as shown in Fig. 18 indicate that the cost of quality loss is reduced when the weighting ratio is increased. The total cost is increased because tight tolerance allocation increases the machining cost. However, the situation can be changed if the part is made of high-cost material and the replacement cost of a component is much greater than the machining cost.

Conclusion

An optimization model has been established with consideration of the required functional reliability, the minimum machining cost and quality loss in this study. The procedure to obtain optimal customer's semi-tolerance with the combined model is revealed. The reliability index model is capable of handling the normal distribution and the non-normal distribution. Tolerance allocation of the compressor is optimized. The data of the vane thickness is better fitted by the lognormal distribution than the normal distribution, and it is verified by chi-square test.

The oriented relationship functional graph has been used to analyze the dimension connection of the components in the sliding vane rotary compressor. Through the graph analysis, the functional dimension chains are sorted out and then the design functions for optimization are generated. Optimal tolerance allocation of functional dimensions in the rotary compressor is obtained by the non-linear programming with the efficient beta algorithm [35]. According to the optimization results, tolerances of the rotor slot length, the rotor slot width, and the vane thickness are tighter than those of other functional dimensions. Furthermore, the effect of correlation coefficients has been analyzed. The effects of the correlation coefficients of the bottom cover plate and the top cover plate, and the front cover plate and the rear cover plate on the cost are insignificant. When the standard deviation of standard parts increases, the total cost increases because other components of the assembly made by batch production need to have high manufacturing accuracy in order to compensate for poor quality mass production parts. The cost of quality loss is reduced when the weight-

ing ratio is increased. The total cost is increased because tight tolerance allocation increases the machining cost.

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Nomenclature

A	= replacement cost
C	= cost
E	= expected
G	= design function
J	= Jacobian coordinates transform matrix
k	= quality loss coefficient
L	= quality loss
m	= nominal and target value
n	= number of components
p	= number of design functions
V	= covariance matrix
X	= dimension vector
x	= dimension variable
y	= normalized dimension variable
W	= weighting coefficient of objective function
Z	= vector of independent standard distribution variable
z	= independent standard distribution variable
β	= reliability index of design function
μ	= mean value of dimension
σ	= standard deviation
Γ	= set of standard deviation
δ	= coefficient of variation
Δ	= specific semi-tolerance
ρ	= correlation coefficient

Subscripts

0	= initial condition
1, 2, ..., n	= dimension
asm	= assembly
h	= h th component
i	= i th dimension
j	= j th dimension
ij	= relation between i th and j th dimension
k	= k th design function
L	= left side of mean value
m	= machining
min	= minimum
q	= quality loss
R	= right side of mean value
t	= total
x	= dimension variables
z	= normalized dimension variable

Superscripts

*	= required
-	= mean
t	= transport

Appendix

The beta algorithm that was developed by Hohenbichler and Rackwitz [35] is an iteration method to calculate the reliability index. The steps for the normal distribution or the lognormal distribution are listed as follows [40]:

- Select a starting point in the variable space, the vector X_0 .

- b. Using Rosenblatt transformation to transfer X_0 into an independent normalized vector Z_0 .
- c. Determine the Jacobian matrix

$$J = \frac{\partial(z_1, z_2, \dots, z_n)}{\partial(x_1, x_2, \dots, x_n)} = \begin{bmatrix} \frac{\partial z_1}{\partial x_1} & \frac{\partial z_1}{\partial x_2} & \dots & \frac{\partial z_1}{\partial x_n} \\ \frac{\partial z_2}{\partial x_1} & \frac{\partial z_2}{\partial x_2} & \dots & \frac{\partial z_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial z_n}{\partial x_1} & \frac{\partial z_n}{\partial x_2} & \dots & \frac{\partial z_n}{\partial x_n} \end{bmatrix} \quad (A1)$$

- d. Obtain the value of the design function G and its gradient ∇G

$$G_z(Z_0) = G_X(X_0) \quad (A2)$$

$$\nabla G_Z = (J^{-1})^T \nabla G_X \quad (A3)$$

- e. The new point can be obtained from the gradient of the vector

$$Z^* = \frac{1}{\nabla G_Z^T \nabla G_Z} [\nabla G_Z^T Z_0 - G_Z(Z_0)] \nabla G_Z \quad (A4)$$

- f. The new iteration point in X space can be approached by the first-order linear approximation

$$X^* \cong X_0 + J^{-1}(Z^* - Z_0) \quad (A5)$$

- g. Obtain the reliability index for this iteration

$$\beta = (Z^* Z^*)^{1/2} \quad (A6)$$

- h. Input X^* into the iteration process again until the convergence is reached.

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