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Extended Transfer Matrix Method With Complex Numbers for Branched Torsional Systems

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Abstract: The extended transfer matrix method with complex numbers and the Newton-Raphson technique is used to analyze torsional vibration for branched systems. The relationship between the first station and the last station is established. Equivalent contact disks of the power train with a contact transfer matrix are used to simplify a branched torsional system into a single shaft system. This method reduces calculation of some elements in the complicated procedures of matrix multiplication. A system with three free-end boundaries is analyzed. The calculated results of the eigenvalues and eigenvectors of an undamped branched system are compared and show good agreement with the data obtained by the MATLAB software program. The eigensolutions of a damped branched system are calculated and the results are acceptable. Therefore, the methodology used is feasible for the torsional vibration analysis of damped branched systems.

Key Words: Extended transfer matrix method, branched, damped, torsional system

NOMENCLATURE

c	=	internal damping coefficient
d	=	external damping coefficient
E, G, M, U, V, Y	=	transfer matrices
e, g, m, u, v	=	elements of matrix
I	=	mass moment of inertia
k	=	stiffness
n	=	ratio of tooth numbers of contact gears
T	=	torque matrix
t	=	time
Z	=	state vector
$\bar{Z}$	=	extended state vector
σ	=	negative real part of eigenvalue
λ	=	eigenvalue of system
Θ	=	angular displacement vector
θ	=	angular displacement magnitude
τ	=	torque magnitude

ω = natural circular frequency

Subscripts

a, b, h, q, s, x = locations of disks
 i = i th disk
 ij = i th row and j th column

Superscripts

i = imaginary part
 L = left of disk
 R = right of disk
 r = real part
 T = transposed transfer matrix

1. INTRODUCTION

Many power train systems are branched systems. The input power is split and transmitted to various components for different applications. The torsional vibration analysis at the contact point of two meshed gears is very complicated. Holzer (1922) used the n^2 effect, where n is the ratio of tooth numbers of the driving gear to the driven gear, to analyze an undamped branched system. The mass moments of inertia of gears on the subshaft, the stiffnesses, and the damping coefficients of the subshaft were multiplied by n^2 . Hirschborn (1958) used the node distance to determine natural frequencies based on the Holzer method. Gilbert (1972) analyzed a branched system by the energy method and Lagrange equation. Pestel (1963) introduced the transfer matrix method with the point and field transfer matrices to analyze an undamped branched system. Dawson and Davies (1974) improved the transfer matrix method and used the extended transfer matrix with the Newton-Raphson technique to determine natural frequencies for an undamped system. Sankar (1979) used the extended transfer matrix method and the Newton-Raphson method to determine natural frequencies and mode shapes for an undamped branched system. Doughty (1985) used complex numbers in the transfer matrix method with the Newton-Raphson technique to determine natural frequencies of a damped system. The extended transfer matrix method with complex numbers and the Newton-Raphson technique showed its advantages in analyzing the torsional vibration of the single shaft systems (Huang and Horng, 1999). This study will extend the extended transfer matrix method with the Newton-Raphson technique and derive a contact transfer matrix to analyze a damped branched system with three free-end boundaries.

2. METHOD OF ATTACK

An equivalent mass elastic system of a vibrational system with both ends free is shown in Figure 1. I is the mass moment of inertia of the disk, k is the stiffness of the shaft, and c and d are the internal and external damping coefficients, respectively.

It is assumed that the disks are rigid bodies with infinite stiffnesses and that the shafts are slim with negligible mass moments of inertia. If the damping effect is assumed to be proportional to the relative vibratory angular speed and the damping coefficient is constant, the equation of motion for a disk and a shaft as shown in Figure 2 can be expressed as

Figure 1. Equivalent mass elastic system with both ends free.

$$\begin{cases} \Theta_i^L = \Theta_i^R \\ T_i^R - T_i^L = I_i \frac{d^2 \Theta_i}{dt^2} + d_i \frac{d \Theta_i}{dt} \end{cases} \quad (1)$$

and

$$\begin{cases} T_i^R = T_{i+1}^L \\ T_i^R = k_i (\Theta_{i+1}^L - \Theta_i^R) + c_i \frac{d}{dt} (\Theta_{i+1}^L - \Theta_i^R) \end{cases} \quad (2)$$

or

$$\begin{cases} \theta_i^L = \theta_i^R \\ \tau_i^R - \tau_i^L = I_i (-\sigma + j\omega)^2 \theta_i^L + j d_i (-\sigma + j\omega) \theta_i^L \end{cases} \quad (3)$$

and

$$\begin{cases} \tau_i^R = \tau_{i+1}^L \\ \tau_i^R = k_i (\theta_{i+1}^L - \theta_i^R) + c_i (-\sigma + j\omega) (\theta_{i+1}^L - \theta_i^R) \end{cases}, \quad (4)$$

where $-\sigma$ and ω are the real and imaginary parts of the eigenvalue λ and $\frac{d\Theta}{dt}$ and $\frac{d^2\Theta}{dt^2}$ are the first and the second derivatives of Θ with respect to time, respectively.

The state vector of a station is defined as

$$Z = \begin{Bmatrix} Z^r \\ Z^i \end{Bmatrix} = \begin{Bmatrix} \theta^r \\ \tau^r \\ \theta^i \\ \tau^i \end{Bmatrix}, \quad (5)$$

which is a column matrix of the vibratory angular displacement and the vibratory torque. The superscripts r and i indicate the real and imaginary parts, respectively. The extended state vector can be expressed as

Figure 2. Disk and shaft.

$$\bar{Z} = \left\{ \theta^r, \tau^r, \theta^i, \tau^i, \frac{\partial \theta^r}{\partial \sigma}, \frac{\partial \tau^r}{\partial \sigma}, \frac{\partial \theta^i}{\partial \sigma}, \frac{\partial \tau^i}{\partial \sigma}, \frac{\partial \theta^r}{\partial \omega}, \frac{\partial \tau^r}{\partial \omega}, \frac{\partial \theta^i}{\partial \omega}, \frac{\partial \tau^i}{\partial \omega} \right\}^T. \quad (6)$$

Defining the field transfer matrix, F , and the point transfer matrix, P , the relationship of the n th and the first extended state vectors is

$$\bar{Z}_n^R = P_{n-1} F_{n-1} P_{n-2} F_{n-2} \dots P_1 \bar{Z}_1^L. \quad (7)$$

Differentiating equation (5) with respect to σ and ω , combining with equation (5) and using the extended transfer matrix method, the relationship between the right of the n th extended state vector and the left of the first extended state vector is

$$\begin{pmatrix} \theta^r \\ \tau^r \\ \theta^i \\ \tau^i \\ \frac{\partial \theta^r}{\partial \sigma} \\ \frac{\partial \tau^r}{\partial \sigma} \\ \frac{\partial \theta^i}{\partial \sigma} \\ \frac{\partial \tau^i}{\partial \sigma} \\ \frac{\partial \theta^r}{\partial \omega} \\ \frac{\partial \tau^r}{\partial \omega} \\ \frac{\partial \theta^i}{\partial \omega} \\ \frac{\partial \tau^i}{\partial \omega} \end{pmatrix}_n^R = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{11} & a_{12} & a_{13} & a_{14} & 0 & 0 & 0 & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 & 0 & 0 \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 0 & 0 \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0 & 0 \\ a_{91} & a_{92} & a_{93} & a_{94} & 0 & 0 & 0 & 0 & a_{11} & a_{12} & a_{13} & a_{14} \\ a_{101} & a_{102} & a_{103} & a_{104} & 0 & 0 & 0 & 0 & a_{21} & a_{22} & a_{23} & a_{24} \\ a_{111} & a_{112} & a_{113} & a_{114} & 0 & 0 & 0 & 0 & a_{31} & a_{32} & a_{33} & a_{34} \\ a_{121} & a_{122} & a_{123} & a_{124} & 0 & 0 & 0 & 0 & a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \times \begin{pmatrix} \theta^r \\ \tau^r \\ \theta^i \\ \tau^i \\ \frac{\partial \theta^r}{\partial \sigma} \\ \frac{\partial \tau^r}{\partial \sigma} \\ \frac{\partial \theta^i}{\partial \sigma} \\ \frac{\partial \tau^i}{\partial \sigma} \\ \frac{\partial \theta^r}{\partial \omega} \\ \frac{\partial \tau^r}{\partial \omega} \\ \frac{\partial \theta^i}{\partial \omega} \\ \frac{\partial \tau^i}{\partial \omega} \end{pmatrix}_1^L \quad (8)$$

Figure 3. An equivalent mass elastic damped branched system.

or

$$\bar{Z}_n^R = Y \bar{Z}_1^L. \quad (9)$$

The elements of the matrix are functions of I, k, c, d, σ , and ω .

An equivalent mass elastic damped branched system of a power train is shown in Figure 3. The i th gear and the $(n + 1)$ th gear in contact are replaced by two sets of disks as shown in Figure 4. Two disks in each set are connected by a rigid shaft. The right disk of the upper set and the left disk of the lower set, which mesh with each other, are assumed to be rigid without mass moments of inertia. The left disk of the upper set and the right disk of the lower set have the same dimensions as the original i th and $(n + 1)$ th disks, respectively. The rigid shafts are also assumed without the mass moments of inertia. With these assumptions, the damped branched system shown in Figure 3 can be replaced by the system as shown in Figure 5. It is assumed that the disk without the mass moment of inertia vibrates with the same amplitude as a disk with the mass moment of inertia, which takes into account vibratory torque.

Referring to Figure 5, the relationship of the state vectors between point b and point a is

$$Z_b = E Z_a, \quad (10)$$

where

$$E = \begin{pmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ e_{21} & e_{22} & e_{23} & e_{24} \\ e_{31} & e_{32} & e_{33} & e_{34} \\ e_{41} & e_{42} & e_{43} & e_{44} \end{pmatrix}. \quad (11)$$

Figure 4. Replacement of gears in contact.

Figure 5. Equivalent damped branched system.

Similarly, the relationship of the state vectors between point s and point h is

$$Z_s = MZ_h, \quad (12)$$

where

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix}. \quad (13)$$

The relationship of the state vectors between point x and point q is

$$Z_x = UZ_q, \quad (14)$$

where

$$U = \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix}. \quad (15)$$

The matrices, E , M , and U are the product matrices of point transfer matrices and field transfer matrices. The relationship of the state vectors between points b, h, and q are

$$\begin{cases} \theta_h^r = \theta_b^r \\ \theta_h^i = \theta_b^i \end{cases} \quad (16)$$

$$\begin{cases} \theta_q^r = n\theta_b^r \\ \theta_q^i = n\theta_b^i \end{cases} \quad (17)$$

and

$$\begin{cases} -n\tau_q^r = (\tau_h^r - \tau_b^r) \\ -n\tau_q^i = (\tau_h^i - \tau_b^i) \end{cases}. \quad (18)$$

From equations (14), (16), (17), and (18) with the free boundary condition at point x, the relationship between the state vectors of point h and point b can be obtained as

$$Z_h = VZ_b, \quad (19)$$

where V is the contact transfer matrix and is

$$V = \begin{pmatrix} 1 & 0 & 0 & 0 \\ v_{21} & 1 & v_{23} & 0 \\ 0 & 0 & 1 & 0 \\ v_{41} & 0 & v_{43} & 0 \end{pmatrix}. \quad (20)$$

Table 1. Data of damped branched system.

Disk No.	I (N-m-s ²)	d (N-m-s/rad)	k (KN-m/rad)	c (N-m-s/rad)
1	0.068	7.00	170	0
2	0.068	0.00	290	0
3	1.360	0.00	79	10
4	2.260	0.00	90	0.23
5	0.136	0.00	—	—
6	0.056	0.00	11	0
7	0.045	0.00	11	0.5
8	0.169	0.08	—	—

Therefore, the relationship between the extended state vectors of point s and point a is

$$\bar{Z}_s = MVE\bar{Z}_a = \bar{Z}_a \tag{21}$$

or

$$\begin{pmatrix} \theta^r \\ \tau^r \\ \theta^i \\ \tau^i \\ \frac{\partial \theta^r}{\partial \sigma} \\ \frac{\partial \tau^r}{\partial \sigma} \\ \frac{\partial \theta^i}{\partial \sigma} \\ \frac{\partial \tau^i}{\partial \sigma} \\ \frac{\partial \theta^r}{\partial \omega} \\ \frac{\partial \tau^r}{\partial \omega} \\ \frac{\partial \theta^i}{\partial \omega} \\ \frac{\partial \tau^i}{\partial \omega} \end{pmatrix}_s = \begin{pmatrix} g_{11} & g_{12} & g_{13} & g_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ g_{21} & g_{22} & g_{23} & g_{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ g_{31} & g_{32} & g_{33} & g_{34} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ g_{41} & g_{42} & g_{43} & g_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ g_{51} & g_{52} & g_{53} & g_{54} & g_{11} & g_{12} & g_{13} & g_{14} & 0 & 0 & 0 & 0 \\ g_{61} & g_{62} & g_{63} & g_{64} & g_{21} & g_{22} & g_{23} & g_{24} & 0 & 0 & 0 & 0 \\ g_{71} & g_{72} & g_{73} & g_{74} & g_{31} & g_{32} & g_{33} & g_{34} & 0 & 0 & 0 & 0 \\ g_{81} & g_{82} & g_{83} & g_{84} & g_{41} & g_{42} & g_{43} & g_{44} & 0 & 0 & 0 & 0 \\ g_{91} & g_{92} & g_{93} & g_{94} & 0 & 0 & 0 & 0 & g_{11} & g_{12} & g_{13} & g_{14} \\ g_{101} & g_{102} & g_{103} & g_{104} & 0 & 0 & 0 & 0 & g_{21} & g_{22} & g_{23} & g_{24} \\ g_{111} & g_{112} & g_{113} & g_{114} & 0 & 0 & 0 & 0 & g_{31} & g_{32} & g_{33} & g_{34} \\ g_{121} & g_{122} & g_{123} & g_{124} & 0 & 0 & 0 & 0 & g_{41} & g_{42} & g_{43} & g_{44} \end{pmatrix} \times \begin{pmatrix} \theta^r \\ \tau^r \\ \theta^i \\ \tau^i \\ \frac{\partial \theta^r}{\partial \sigma} \\ \frac{\partial \tau^r}{\partial \sigma} \\ \frac{\partial \theta^i}{\partial \sigma} \\ \frac{\partial \tau^i}{\partial \sigma} \\ \frac{\partial \theta^r}{\partial \omega} \\ \frac{\partial \tau^r}{\partial \omega} \\ \frac{\partial \theta^i}{\partial \omega} \\ \frac{\partial \tau^i}{\partial \omega} \end{pmatrix}_a \tag{22}$$

Since the elements of matrices E, M, U , and V are functions of I, c, d, k, σ , and ω , the elements of matrix G are also functions of these parameters.

Figure 6. An equivalent damped branched system.

If point *a* is free, the torque must be zero. If the vibratory amplitude of point *a* is an arbitrary fixed value and the remaining elements of the extended state vector are equal to zero (Sankar, 1979), the extended state vector of point *a* can be expressed as

$$\bar{Z}_a = \{1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0\}^T. \quad (23)$$

From equation (22), we can obtain

$$\left\{ \begin{array}{l} \tau_s^r = g_{21}\theta_a^r = g_{21} \\ \tau_s^i = g_{41}\theta_a^r = g_{41} \\ \frac{\partial \tau_s^r}{\partial \sigma} = \frac{\partial g_{21}}{\partial \sigma} = g_{61} \\ \frac{\partial \tau_s^i}{\partial \sigma} = \frac{\partial g_{41}}{\partial \sigma} = g_{81} \\ \frac{\partial \tau_s^r}{\partial \omega} = \frac{\partial g_{21}}{\partial \omega} = g_{101} \\ \frac{\partial \tau_s^i}{\partial \omega} = \frac{\partial g_{41}}{\partial \omega} = g_{121} \end{array} \right. \quad (24)$$

If point *s* is free, the torque of point *s* must be zero. Thus, we have

$$\left\{ \begin{array}{l} g_{21}(\sigma, \omega) = 0 \\ g_{41}(\sigma, \omega) = 0 \end{array} \right. \quad (25)$$

With a set of trial values of σ and ω and by utilizing the Newton-Raphson method as

$$\left\{ \begin{array}{l} \sigma \\ \omega \end{array} \right\}^{new} = \left\{ \begin{array}{l} \sigma \\ \omega \end{array} \right\}^{old} + \left\{ \begin{array}{cc} g_{61} & g_{101} \\ g_{81} & g_{121} \end{array} \right\}^{-1} \times \left\{ \begin{array}{l} -g_{21} \\ -g_{41} \end{array} \right\}^{old}, \quad (26)$$

Table 2. Calculated eigensolutions of undamped system.

Disk No.	Mode					
	1st	2nd	3rd	4th	5th	6th
	Eigenvalue ω (rad/s)					
	174	293	723	839	1211	2830
Eigenvector						
1	1.00	1.00	1.00	1.00	1.00	1.00
2	0.99	0.97	0.79	0.72	0.41	-2.21
3	0.98	0.93	0.57	0.44	-0.07	0.06
4	12.60	-0.57	-0.03	-5.90	0.00	-0.00
5	13.20	-0.66	-0.15	92.73	-0.00	0.00
6	0.49	0.46	0.29	0.22	-0.04	0.03
7	-174.6	0.10	-77.3	-0.28	0.01	-0.00
8	-327.9	-0.31	10.41	0.03	-0.00	0.00

Table 3. Eigenvalues and eigenvectors of undamped system obtained by MATLAB.

Disk No.	Mode					
	1st	2nd	3rd	4th	5th	6th
	Eigenvalue ω (rad/s)					
	175	293	724	839	1211	2831
Eigenvector						
1	1.00	1.00	1.00	1.00	1.00	1.00
2	1.00	0.97	0.80	0.72	0.41	-2.21
3	0.98	0.93	0.56	0.44	-0.07	0.06
4	12.89	-0.57	-0.03	-5.87	0.00	-0.00
5	13.62	-0.66	-0.15	92.40	-0.00	0.00
6	0.49	0.46	0.28	0.22	-0.04	0.03
7	-178.7	0.10	-77.3	-0.28	0.01	-0.01
8	-335.6	-0.32	10.96	0.03	-0.00	0.00

only the values of g_{61}, g_{81}, g_{101} , and g_{121} are needed to determine eigenvalues of the system. By trial and error with the assumed initial values of σ and ω , accurate eigenvalues can be obtained when g_{21} and g_{41} in equation (26) converge and are less than 10^{-4} Nm.

3. RESULT

An equivalent damped branched system of a power train as shown in Figure 6 is used for numerical calculation. The ratio of tooth numbers, n , of gear 3 to gear 6 is 0.5. The mass moments of inertia and external damping coefficients of the disks, the stiffnesses, and the internal damping coefficients of shafts in the system used for analysis are shown in Table 1.

The internal damping coefficients and the external damping coefficients in Table 1 are changed to zero for analysis of the undamped branched system. The calculated eigensolutions obtained by this method are shown in Table 2. The counterclockwise direction of the main

Table 4. Calculated eigenvalues and eigenvectors of damped branched system with data in Table 1.

		Mode					
		1st	2nd	3rd	4th	5th	6th
		Eigenvalue $\frac{1}{n}\sigma + j\omega \left(\frac{\text{rad}}{\text{s}}\right)$					
		$-2 + j174$	$-7 + j293$	$-0 + j723$	$-0 + j839$	$-41 + j1211$	$-9 + j2831$
Disk No.		Eigenvector					
1	real	1.00	1.00	1.00	1.00	1.00	1.00
	img	0.00	0.00	0.00	0.00	0.00	0.00
2	real	0.99	0.97	0.79	0.72	0.41	-2.20
	img	0.01	0.00	0.03	0.00	0.00	0.01
3	real	0.97	0.93	0.57	0.44	-0.07	0.06
	img	0.01	0.02	0.04	0.05	0.00	-0.00
4	real	11.78	-0.57	-0.03	-5.88	0.00	-0.00
	img	-3.04	0.00	-0.01	0.18	0.00	-0.00
5	real	12.34	-0.66	-0.15	92.51	-0.00	0.00
	img	3.19	-0.00	-0.03	-3.38	0.00	0.00
6	real	0.49	0.46	0.29	0.22	-0.04	0.03
	img	0.01	0.01	0.02	0.02	0.00	-0.00
7	real	-163.8	0.10	-77.3	-0.28	-0.00	-0.00
	img	40.55	0.01	-3.45	-0.03	-0.00	0.00
8	real	-307.8	-0.30	10.42	0.03	-0.00	0.00
	img	76.05	0.01	0.45	0.00	0.00	-0.00

shaft and the clockwise direction of the subshaft viewed from the left of the system are defined positive.

The eigenvalues and eigenvectors obtained from the MATLAB software program are used for comparison with the calculated results. These eigenvectors of the disks obtained by using the MATLAB software program must be divided by the eigenvectors of disk 1 of the corresponding modes. Furthermore, the eigenvectors of disk 7 and disk 8 must be multiplied by n . The results are shown in Table 3. The calculated eigenvalues and eigenvectors of the damped branched system with the data in Table 1 are shown in Table 4.

4. DISCUSSION

In addition to the elements with the value to be zero, there are 80 elements in the matrix in equation (22). Since only values of the elements g_{61} , g_{81} , g_{101} , and g_{121} are needed, it is not necessary to determine all the elements of the matrix product of equation (22). Some of the complex operations involving the matrix products are reduced. Since the derivatives of the angular displacement and the torque can be determined during the matrix operation and these values are used in the Newton-Raphson technique directly, the computer time should be less than that by using the transfer matrix method.

Divergence and overshooting may occur during the searching for eigenvalues. For example, in the analysis of an undamped branched system, if a trial value of the circular frequency is 150 rad/s, the calculated result is 723 rad/s, which is another eigenvalue of the system. The same situation occurs for the trial value of 160 rad/s in obtaining the circular frequency of 293 rad/s. The converged criterion during the calculation for an undamped branched system requires that the residual torque of τ'_s be less than 0.0001 Nm. To prevent this situation from occurring, the eigenvalues and eigenvectors of the system obtained by neglecting the damping effects are used as the initial trial values for the damped branched system. Since the deviation of frequencies for a damped system from an undamped system is small, the circular frequencies of an undamped system are the best assumed initial values for a damped system.

A contact transfer matrix is derived by using the information of the subshaft. The eigenvalues of the damped branched system with the subshaft might be lost. For example, 174 rad/s and 723 rad/s are lost. Thus, the main shaft is considered as the subshaft to obtain these two lost eigenvalues and eigenvectors.

The calculated results of the undamped branched system in Table 2 are compared and show good agreement with the data in Table 3 obtained by using the MATLAB software program. Based on the comparison of the calculated results of the imaginary parts of the eigenvalues and the real parts of the eigenvectors in Table 4 with the results in Tables 2 and 3, the calculated results of the damped branched system should be acceptable. Therefore, the methodology used in this study is feasible for the torsional vibration analysis of damped systems.

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