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Study of Unsteady Flow in a Heat Exchanger by the Method of Characteristics

The one-dimensional, unsteady flow in an air-to-air heat exchanger is studied. The governing equations are derived and the method of characteristics with the uniform interval scheme is used in the analysis. The effect of the fin improvement factor on the air temperature in the heat exchanger and the heat transfer rate of the heat exchanger, and air properties in the heat exchanger are analyzed. The numerical results are compared and show good agreement with the available data.

Introduction

Heat exchangers are used in HVAC systems and many applications. Improper design of a heat exchanger may result in the damage or degradation of the components and the performance of a system. Kraus (1988) dealt with the mathematical representation of individual fins. Rindt et al. (1991) presented a numerical model for an unsteady entrance flow in a 90-deg curved tube. Bulck (1991) determined the optimal distribution of the transfer surface area for the maximum heat exchanger effectiveness and the constant total surface area.

The effects of the unsteady fluid properties are important because the heat transfer rate and the performance of a heat exchanger depend on these properties. The unsteady flow in an air-to-air heat exchanger is complicated. The air flow in the heat exchanger is assumed one-dimensional and unsteady in this study. The method of characteristics (Abbott, 1970) is used to simplify the governing equations into nonhomogeneous first-order partial differential equations. The uniform interval scheme (Soo, 1969) is applied to analyze the effect of the fin improvement factor on the air mean temperature in the heat exchanger and the mean heat transfer rate of the heat exchanger, and air properties in the heat exchanger. The numerical results for specific operating conditions are compared with the available data.

Method of Approach

The air-to-air heat exchanger of interest is assumed to be a straight tube with the constant circular cross-sectional area surrounded by fins. The air flow in the heat exchanger is assumed unsteady and one-dimensional. The continuity equation is

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \quad (1)$$

It is assumed that air is an ideal gas. Thus,

$$p = \rho R T \quad (2)$$

Neglecting the body force for the low-density air in a horizontal heat exchanger, the momentum equation is

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + RT \frac{\partial \rho}{\partial x} + \rho R \frac{\partial T}{\partial x} + \frac{f}{2D} \rho u^2 = 0 \quad (3)$$

For Reynolds number, Re , based on the inside diameter of the tube up to 10^5 , an empirical formula of the friction factor, f , in a smooth tube suggested by Blasius is (Kreith, 1965)

$$f = 0.316 Re^{-0.25} \quad (4)$$

In the temperature range of interest, the specific heat at constant volume, C_v , is nearly constant and the viscosity given by Kays and London (1962) is

$$\mu = 113.2 \times 10^{-8} T^{0.5} \quad (5)$$

Since the Biot number is smaller than 0.1 for the present case and the wall of the heat exchanger is thin, the heat absorbed by the wall is negligible (Kreith, 1965). However, the temperature drop across the wall must be considered. For a fully developed flow in a smooth tube with Reynolds number in the range of 10^4 to 1.2×10^5 and Prandtl number near to one, Dittus and Boelter (Holman, 1972) recommended

$$h_a = 0.04 \frac{K_a}{D} Re^{0.8} \quad (6)$$

The ambient air with the constant temperature and velocity is blown continuously over the heat exchanger by a fan. The average convective heat transfer coefficient of air with Re in the range between 4×10^3 and 4×10^4 is

$$h_{ea} = 0.306 \frac{K_{ea}}{D_o} Re^{0.618} \quad (7)$$

Hence, the heat transfer rate per unit area is

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$$q = - \frac{T - T_{ea}}{D \left\{ \frac{1}{0.04 K_a \text{Re}^{0.8}} + \frac{\ln\left(\frac{D_o}{D}\right)}{2K_h} + \frac{1}{0.306 K_{ea} \text{Re}_{ea}^{0.618}} \right\}} \quad (8)$$

Neglecting the change of the air potential energy, the energy equation is

$$C_v \frac{\partial T}{\partial t} + C_v u \frac{\partial T}{\partial x} + \frac{RT}{j} \frac{\partial u}{\partial x} - \frac{5.15 \times 10^{-3}}{Dj} \left(\frac{u\rho D}{T^{0.5}} \right)^{-0.25} u^3 + \frac{4(T - T_{ea})}{\rho D^2} \times \left\{ \frac{1}{2.29 \times 10^3 K_a \left(\frac{u\rho D}{T^{0.5}} \right)^{0.8}} + \frac{\ln\left(\frac{D_o}{D}\right)}{2K_h} + \frac{1}{0.306 K_{ea} \left(\frac{u_{ea} \rho_{ea} D}{\mu_{ea}} \right)^{0.618}} \right\}^{-1} = 0 \quad (9)$$

The rate of heat transfer is

$$Q = \int q \pi D dx \quad (10)$$

Nondimensional variables are defined as

$$\left. \begin{aligned} \rho^* &= \frac{\rho}{\rho_a} \\ T^* &= \frac{(T - T_a)}{(T_{io} - T_a)} \\ t^* &= \frac{ta_a}{D} \\ u^* &= \frac{u}{a_a} \\ x^* &= \frac{x}{D} \end{aligned} \right\} \quad (11)$$

The speed of sound, a_a , is evaluated at atmospheric conditions. Using Eq. (11) and the relations of

$$a_a^2 = \gamma R T_a \quad (12)$$

and

$$C_v = \frac{R}{(\gamma - 1)j} \quad (13)$$

the continuity equation, the momentum equation and the energy equation in nondimensional form are

$$\frac{\partial \rho^*}{\partial t^*} + u^* \frac{\partial \rho^*}{\partial x^*} + \rho^* \frac{\partial u^*}{\partial x^*} = 0 \quad (14)$$

$$\frac{1}{\gamma} \left\{ T^* \frac{(T_{io} - T_a)}{T_a} + 1 \right\} \frac{\partial \rho^*}{\partial x^*} + \rho^* \frac{\partial u^*}{\partial t^*} + \rho^* u^* \frac{\partial u^*}{\partial x^*} + \frac{(T_{io} - T_a) \rho^*}{\gamma T_a} \frac{\partial T^*}{\partial x^*} + \frac{5.15 \times 10^{-3} \rho^{*0.75} u^{*1.75} \left\{ T^* \frac{(T_{io} - T_a)}{T_a} + 1 \right\}^{0.125}}{(\gamma R)^{0.5} \rho_a D^{0.25}} = 0 \quad (15)$$

and

$$\frac{(T_{io} - T_a)}{T_a} \frac{\partial T^*}{\partial t^*} + \frac{(T_{io} - T_a)}{T_a} u^* \frac{\partial T^*}{\partial x^*} + (\gamma - 1) \left\{ T^* \frac{(T_{io} - T_a)}{T_a} + 1 \right\} \frac{\partial u^*}{\partial x^*} - \frac{5.15 \times 10^{-3} \gamma (\gamma - 1) \left\{ T^* \frac{(T_{io} - T_a)}{T_a} + 1 \right\}^{0.125} u^{*2.75}}{(\gamma R)^{0.5} \rho_a D^{0.25} \rho^{*0.25}} + 4(\gamma - 1)j \times \frac{\{ T^* (T_{io} - T_a) + T_a - T_{ea} \}}{\rho_a (\gamma R)^{0.5} T_a^{1.5} R D \rho^*} \times \left\{ \frac{\left\{ T^* \frac{(T_{io} - T_a)}{T_a} + 1 \right\}^{0.4} \ln\left(\frac{D_o}{D}\right)}{2.29 \times 10^3 K_a \{ (\gamma R)^{0.5} \rho_a D \}^{0.8} u^{*0.8} \rho^{*0.8}} + \frac{1}{2K_h} + \frac{1}{0.306 K_{ea} \left(\frac{u_{ea} \rho_{ea} D}{\mu_{ea}} \right)^{0.618}} \right\}^{-1} = 0 \quad (16)$$

The method of characteristics is used with nondimensional characteristics as

$$\lambda^* = \frac{dx^*}{dt^*} \quad (17)$$

The nondimensional characteristic directions determined from the determinant of Eqs. (14), (15), and (16) are

$$\lambda^* = u^* + a^* \quad (18)$$

$$\lambda^* = u^* - a^* \quad (19)$$

Nomenclature

a = sonic speed, m/s
 C_v = specific heat at constant volume, J/kg-K
 D = inside diameter of heat exchanger, m
 D_o = outside diameter of heat exchanger, m
 f = friction coefficient
 j = heat and work conversion factor, N-m/J
 h = heat transfer coefficient, J/s-m²-K
 K = thermal conductivity

p = pressure, N/m²
 Q = heat transfer rate, J/s
 q = heat transfer rate per unit area, J/s-m²
 R = gas constant, N-m/kg-K
 Re = Reynolds no.
 T = temperature, K
 t = time, s
 u = velocity, m/s
 x = distance from heat exchanger inlet, m
 γ = ratio of specific heats
 λ = characteristic direction

μ = viscosity, kg/m-s
 ρ = density, kg/m³

Subscripts

a = ambient condition or air
 ea = condition surrounding heat exchanger
 h = condition of heat exchanger
 io = initial condition at heat exchanger inlet

Superscripts

* = dimensionless variable

and

$$\lambda^* = u^* \quad (20)$$

The compatibility equations along the characteristic directions, Eqs. (18), (19), and (20), respectively, are

$$d\rho^* + \frac{\gamma\rho^*}{a^*} du^* + \frac{(T_{io} - T_a)}{T_a} \frac{\rho^*}{a^{*2}} dT^* + \left\{ 5.15 \times 10^{-3} \gamma \rho^{*0.75} u^{*1.75} \right. \\ \times \frac{\left\{ T^* \frac{(T_{io} - T_a)}{T_a} + 1 \right\}^{0.125}}{a^* \{ (\gamma R)^{0.5} \rho_a D \}^{0.25}} + \frac{\rho^*}{a^{*2}} \left. \left\{ \frac{-5.15 \times 10^{-3} \gamma (\gamma - 1) u^{*2.75}}{\rho^{*0.25}} \right. \right. \\ \left. \left. + \frac{1}{0.306 K_{ea} \left(\frac{u_{ea} \rho_{ea} D}{\mu_{ea}} \right)^{0.618}} \right\}^{-1} \right\} dt^* = 0 \quad (21)$$

$$d\rho^* - \frac{\gamma\rho^*}{a^*} du^* + \frac{(T_{io} - T_a)}{T_a} \frac{\rho^*}{a^{*2}} dT^* + \left\{ -5.15 \times 10^{-3} \gamma \rho^{*0.75} u^{*1.75} \right. \\ \times \frac{\left\{ T^* \frac{(T_{io} - T_a)}{T_a} + 1 \right\}^{0.125}}{a^* \{ (\gamma R)^{0.5} \rho_a D \}^{0.25}} + \frac{\rho^*}{a^{*2}} \left. \left\{ \frac{-5.15 \times 10^{-3} \gamma (\gamma - 1) u^{*2.75}}{\rho^{*0.25}} \right. \right. \\ \left. \left. + \frac{1}{0.306 K_{ea} \left(\frac{u_{ea} \rho_{ea} D}{\mu_{ea}} \right)^{0.618}} \right\}^{-1} \right\} dt^* = 0 \quad (22)$$

and

Fig. 1 The uniform interval scheme

$$d\rho^* \frac{T_{io} - T_a}{T_a (\gamma - 1) a^{*2}} dT^* - \frac{\rho^*}{(\gamma - 1) a^{*2}} \left\{ \frac{-5.15 \times 10^{-3} \gamma (\gamma - 1) u^{*2.75}}{\rho^{*0.25}} \right. \\ \times \frac{\left\{ T^* \frac{(T_{io} - T_a)}{T_a} + 1 \right\}^{0.125}}{\{ (\gamma R)^{0.5} \rho_a D \}^{0.25}} + \frac{4(\gamma - 1) j \{ T^* (T_{io} - T_a) + T_a - T_{ea} \}}{\rho_a (\gamma R)^{0.5} T_a^{1.5} R D \rho^*} \\ \times \left\{ \frac{\left\{ T^* \frac{(T_{io} - T_a)}{T_a} + 1 \right\}^{0.4}}{2.29 \times 10^3 K_a \{ (\gamma R)^{0.5} \rho_a D \}^{0.8} u^{*0.8} \rho^{*0.8}} + \frac{\ln \left(\frac{D_o}{D} \right)}{2 K_h} \right. \\ \left. \left. + \frac{1}{0.306 K_{ea} \left(\frac{u_{ea} \rho_{ea} D}{\mu_{ea}} \right)^{0.618}} \right\}^{-1} \right\} dt^* = 0 \quad (23)$$

These equations which are nonlinear ordinary differential equations can be solved by the finite difference method. Only the terms of the first-order in Taylor series expansion are used. The uniform interval method is used by dividing the plane with coordinates x^* and t^* into several uniform interval areas as shown in Fig. 1. Each net point stands for a specified location of x^* and time of t^* .

Results

The thermal conductivities of air and the heat exchanger used for analysis are 0.0099 J/s-m-K and 117.49 J/s-m-K, respectively. The inside diameter, the wall thickness and the length of the heat exchanger are 19.1 mm, 1.55 mm and 7.62 m, respectively. The available existing periodic data of air properties at the heat exchanger inlet are used as the boundary condition, and the fluctuating amplitudes of the velocity, pressure, temperature and density of the air used are 129, 10, 3 and 7 percent of the respective mean values of 49 m/s, 273 KPa, 383 K and 2.55 Kg/m³, respectively. The period is equal to 0.003 s which is divided into five sections: 0.0000, 0.0006, 0.0012, 0.0018 and 0.0024 s. These five sections with uniform intervals are used to describe air properties along the heat exchanger.

An assumed initial temperature distribution along the heat exchanger is required. For comparison with the existing data, it is assumed that the maximum temperature at the heat exchanger inlet is linearly decreased to the ambient temperature

at the one-third of heat exchanger length and then remained constant at the ambient temperature for the remainder of the heat exchanger. The final results for air properties in the heat exchanger are independent of the assumed initial temperature

Fig. 2 The effect of the fin improvement factor on the air mean temperature along the heat exchanger

Fig. 3 The effect of the fin improvement factor on the mean heat transfer rate of the heat exchanger

Fig. 4 The mean heat transfer rate out of the heat exchanger versus the length of the heat exchanger

distribution. However, a good assumption can reduce computer time substantially.

The fin improvement factor, n , is used to multiply the conductivity of air surrounding the heat exchanger for the Hilpert average heat transfer coefficient. The effect of the fin improvement factor from 5 to 20 on the air mean temperature along the heat exchanger up to 7.62 m long is shown in Fig. 2. The effect of the fin improvement factor on the mean heat transfer rate out of the heat exchanger is shown in Fig. 3. The mean heat transfer rate out of the heat exchanger with the fin improvement factor equal to 10 is integrated along its length as shown in Fig. 4. The results of the velocity, pressure, temperature and density of the air along the heat exchanger at 0.0000, 0.0006, 0.0012, 0.0018, and 0.0024 s with the fin improvement factor equal to 10 are shown in Figs. 5 through 8.

Discussion

A good choice of certain combinations of the variables can make the derivation of equations easily (Lin, 1950). The sonic speed and the heat transfer rate are functions of temperature. Therefore, temperature is chosen as one variable. Density which proves convenient during the derivation of the governing equations is chosen as another variable. The chosen time interval

Fig. 5 The air velocity along the heat exchanger

Fig. 6 The air pressure along the heat exchanger

Fig. 7 The air temperature along the heat exchanger

Fig. 8 The air density along the heat exchanger

must be small enough to give accurate results and yet it should be large enough to minimize computer time.

The available air temperature at the heat exchanger inlet is about 378 K to 388 K. The larger the fin improvement factor, the faster the temperature decreased along the heat exchanger. A value of the fin improvement factor larger than 10 does not provide much better performance. Therefore, the fin improvement factor of 10 is used. The calculated air temperature at the heat exchanger outlet is 307 K, which is comparable with 304.4 K to 305.5 K measured by Edwards and McDonald (1972).

Comparing the mean heat transfer rates obtained by integrating the heat transfer out of the heat exchanger, 2630 J/s, and by the difference of the air mean enthalpy between the inlet and the outlet of the heat exchanger, 2615 J/s, shows that the analytical tool provides good result. The choice of 50 intervals of x for 7.62 m long and 5 intervals of t for a period of 0.003 s provides satisfactory results. The analytical tool can also be used for analyzing other operating conditions.

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