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The Impact of Sliding Blades in a Rotary Compressor

The purpose of this study was to analyze the impact of blades with slots in a rotary compressor. The equations for contact forces were derived from Newton's law. Updated moving Lagrangian descriptions with the hybrid technique and the Newark time integration method were used to determine the nodal displacement vectors. The finite element method was used to analyze the contact forces and the stresses of blades. The maximum blade contact force of 2.3 KN was larger than the maximum static force of 1 KN. The maximum blade impact Von Mises stress of 12.3 Mpa that occurred at the contact point was greater than the previous calculated static stress of 5.15 Mpa. Hence, the impact effect was significant. Experimental modal analysis was also used to determine the first and second natural frequencies of blades. [DOI: 10.1115/1.1416150]

Introduction

Rotary compressors with sliding blades have high volumetric efficiency and small output pulsation of high pressure air and thus may present significant challenges to reciprocating compressors. Rotary compressors have been developed and used for small air conditioning systems. Fukuta [1] analyzed the behavior of blades during start up and before a rise in pressure acted at the blade bottom. Fukuta et al. [2] generated a mathematical model of compressors. Gorrepati [3] determined modal parameters of blades by applying the finite element method and modal strain energy. Ooi and Wang [4] studied rotary compressors for household refrigerators. Huang and Chiu [5] proposed a compressor with a composite curve for the stator inner contour.

Blades with rotor slots undergo greater stress from high rotational speeds and thus have shorter lives. At high rotational speeds, significant stress may arise from collision of sliding blades with the contour of the stator due to chattering phenomenon. Additional stress may arise from collision between sliding blades and the slots of the rotor due to blade slapping motion. Padovan et al. [6] utilized the moving updated Lagrangian observer for the analysis of moving contact fields. Fukuta [7] utilized pressure acting at the blade bottom to reduce the chattering phenomenon of blades. Amirouche [8] proposed a simulated model and discussed the impact of rolling contact. Sisir [9] studied contact between sliding blades and slots to analyze the reaction force at the contact point. Malone [10] analyzed impact phenomena of plates by using the finite element method and contact mechanics. Choi [11] discussed the contact of the crack element.

A schematic of a rotary compressor with sliding blades and an elliptical stator is shown in Fig. 1. The static contact force and the static contact stress at the contact point were studied [12]. The prototype of this compressor was built and tested. However, there was a high level of noise, and distortion of the blades occurred during the operation. Therefore, the purpose of this study was to analyze the impact between the blades and slots of the rotor to improve the existing analyses.

There are several numerical methods by which the finite element can be used to study the impact of blades. Simo et al. [13] contrasted the perturbed method with the augmented method. In particular, the penalty method and the Lagrange multiplier method have been used most frequently [14,15]. The penalty method introduces a lot of numerical values to simulate the stiffness between node points, but it maintains the original size and the bandwidth of the matrix. However, the difficulty in using this method

lies in selecting the penalty value. The mathematical calculation becomes difficult with a big penalty value. Also, the result may not be accurate with a small penalty value. The Lagrange multiplier method does not have such problems. Nevertheless, it increases the size and the bandwidth of the matrix, and the calculation becomes complicated. In contrast, the hybrid technique does not require a penalty value and it does not alter the size or bandwidth of the matrix [16]. Therefore, the hybrid technique and the Newark time integration method were used to analyze the contact force as well as the stress of blades in this study. The frequency response due to the impact of blades and rotor slots was also studied. By clarifying the impact of blades and rotor slots it is expected that the results of this investigation can be used to reduce the noise, improve the design and prolong the life of compressors.

Method of Approach

When the blades rotate with the rotor, air is drawn in and compressed. The polytropic process is assumed during compression. The air pressure p in the blade segment volume V can be obtained from

$$p = p_i \left(\frac{V_i}{V} \right)^n \quad (1)$$

where p_i is the atmospheric pressure and V_i is the volume at the completion of the inlet process [17]. The types of a sliding blade contacted with a slot of the rotor are shown in Fig. 2. Contact type 1 occurs when the blade out of the rotor increases to a certain length and the moment due to the frictional force at the blade tip increases. The blade contacts with the slot at points A and B, and the blade contacts with the stator at point C. Decreasing the inclined angle of the blade enhances this type of contact. Contact type 1 changes to contact type 2 when the length of blade out of the rotor decreases, the effect of the moment due to the frictional

Fig. 1 Rotary compressor with sliding blades

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Fig. 2 Contact types between blade and rotor slot

force at the blade tip decreases and the moment due to the normal force at the blade tip becomes relatively large. Contact type 2 changes to contact type 3 when the length of the blade out of the rotor is short, the effect of the moment due to the frictional force at the blade tip is very small and the moment due to the normal force at the blade tip becomes relatively large.

The length of the blade out of the rotor, the relative velocity and the relative acceleration of the blade to the slot can be derived, respectively, and are

$$\ell_s = \sqrt{(y_C - y_A)^2 + (x_C - x_A)^2} \quad (2)$$

$$v_r = \frac{d(\ell_s)}{dt} = \frac{d(\ell_s)}{d\theta} \times \frac{d\theta}{dt} = \frac{d(\ell_s)}{d\theta} \times \omega \quad (3)$$

and

Fig. 3 Forces acted on the blade

$$a_r = \frac{d(v_r)}{dt} = \frac{d(v_r)}{d\theta} \times \frac{d\theta}{dt} = \frac{d(v_r)}{d\theta} \times \omega = \frac{d^2(\ell_s)}{d\theta^2} \times \omega^2 \quad (4)$$

where the subscripts C and A are contact points of the blade with the stator and rotor, respectively.

The forces acted on the blade for contact type 1 are shown in Fig. 3. The equations of motion based on Newton's law and Coulomb's law are

$$\left. \begin{aligned} N_{Cx} + f_{Cx} + P_{nx} + N_{Ax} + f_{Ax} + N_{Bx} + f_{Bx} &= m(a_{rx} + a_{\omega x} + a_{cx}) \\ N_{Cy} + f_{Cy} + P_{ny} + N_{Ay} + f_{Ay} + N_{By} + f_{By} &= m(a_{ry} + a_{\omega y} + a_{cy}) \\ -N_C \frac{\ell}{2} \sin \psi + f_C \frac{\ell}{2} \cos \psi + P_n \left(\frac{\ell}{2} - \frac{AC}{2} \right) - N_A \left(\frac{\ell}{2} - \frac{AC}{2} \right) - f_A \frac{e}{2} - N_B \frac{\ell}{2} + f_B \frac{e}{2} &= 0 \end{aligned} \right\} \quad (5)$$

The contact forces can be determined as

$$\left. \begin{aligned} N_A &= \frac{\Delta_A}{\Delta} \\ N_B &= \frac{\Delta_B}{\Delta} \\ N_C &= \frac{\Delta_C}{\Delta} \end{aligned} \right\} \quad (6)$$

where Δ_A , Δ_B , Δ_C and Δ are functions of the configuration, the operation conditions and material properties of the compressor, and the location of the blade.

When a body is subjected to loads while in a state of equilibrium, any point in the body should satisfy [18]

$$\rho \ddot{u} = \sigma_{ji} + \rho b_i \quad (7)$$

Applying the virtual work principle, Eq. (7) is multiplied by the virtual displacement vector δu_i and integrated for the whole domain. Using the Gauss divergence theorem, the shape function with the isoparametric interpolation and separate variables while also considering damping effects, the elements in the equation are assembled and the non-linear equation can be written as

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\} \quad (8)$$

The hybrid technique that combines the displacement method and the force method can be used for solving the matrix.

From the nodal displacement at the gap and the force method, the force acted on the gap can be obtained. Using the Newmark time integration method yields

$$\left. \begin{aligned} \{\ddot{u}_{n+1}\} &= \frac{1}{\alpha \Delta t^2} (\{u_{n+1}\} - \{u_n\}) - \frac{1}{\alpha \Delta t} \{\dot{u}\} - \left(\frac{1}{2\alpha} - 1 \right) \{\ddot{u}_n\} \\ \{\dot{u}_{n+1}\} &= \{\dot{u}_n\} + \Delta t(1 - \delta)\{\ddot{u}_n\} + \delta \Delta t \{\ddot{u}_{n+1}\} \end{aligned} \right\} \quad (9)$$

where $\{u_n\}$ and $\{u_{n+1}\}$ are the nodal displacements at the time t_n and t_{n+1} , respectively, and Δt is the time increment from t_n to t_{n+1} .

Substituting Eq. (9) into Eq. (8) yields

$$\begin{aligned} & (a_0[M] + a_1[C] + [K])\{u_{n+1}\} \\ &= \{F\} + [M](a_0\{u_n\} + a_2\{\dot{u}_n\} + a_3\{\ddot{u}_n\}) \\ &+ [C](a_1\{u_n\} + a_4\{\dot{u}_n\} + a_5\{\ddot{u}_n\}) \end{aligned} \quad (10)$$

where

$$\left. \begin{aligned} a_0 &= \frac{1}{\alpha \Delta t^2}, \quad a_1 = \frac{\delta}{\alpha \Delta t}, \quad a_2 = \frac{1}{\alpha \Delta t}, \\ a_3 &= \frac{1}{2\alpha} - 1, \quad a_4 = \frac{\delta}{\alpha} - 1, \quad a_5 = \frac{\Delta t}{2} \left(\frac{\delta}{\alpha} - 2 \right) \end{aligned} \right\} \quad (11)$$

Fig. 4 Experimental set up for determination of frequency response

Fig. 5 Locations of test points on blade

Modal Analysis

When a blade is subjected to dynamic force, it vibrates and creates noise. The different states and locations of the forces excite various mode shapes of vibration. To analyze the noise and the composition of the noise frequency, it is necessary to determine the natural frequencies of the blade. Experimental set up with a two-frequency Fast Fourier Transform (FFT) analyzer and a blade suspended by a slender wire is shown in Fig. 4. An accelerometer is attached at a point on the blade. A hammer with a force transducer sensor is used to hit the blade. The signals of the hammer and the accelerator are transferred to the FFT analyzer to obtain a frequency response. The six points d_1 , d_2 , d_3 , d_4 , d_5 and d_6 located on the blade are shown in Fig. 5 and the accelerometer is attached at point d_1 . Each point is hit ten times and the average value is used.

The natural frequencies of various modes can be obtained after the frequency response function is obtained from

$$h_{ij} = \frac{u_i}{F_j} \quad (12)$$

where the subscripts i and j are the locations of the accelerator and the point hit by the hammer, respectively.

Noise Measurement

The sound power level (SPL) is defined as

$$SPL = 10 \times \log \left(\frac{W_i}{10^{-12}} \right) dB \quad (13)$$

where W_i is the sound power measured by the sound level meter.

Table 1 Data of Compressor

item	dimension
major axis length	76.20 mm
minor axis length	58.45 mm
stator depth	154.8 mm
rotor radius	58.45 mm
slot thickness	5.02 mm
slot width	43.00 mm
blade thickness	4.95 mm
blade width	40.00 mm
blade depth	154.8 mm
radius of blade tip	2.50 mm
blade number	10

Fig. 6 Blade angular location θ

Fig. 7 Length of blade out of rotor

Fig. 8 Relative velocity of blade to slot

Fig. 9 Relative acceleration of blade to slot

Fig. 10 Pressure difference on both sides of blade

Fig. 11 Force on point C

Fig. 12 Force on point A

Fig. 13 Force on point B

Fig. 14 Finite element model of blade and rotor slot wall

Fig. 15 Contact force of blade versus time

Fig. 16 Von Mises stress of blade versus time

Fig. 17 Contour plot of Von Mises stresses in blade

Results

The data of the compressor are shown in Table 1. The stator and the rotor are made of carbon steel, and the blades are made of aluminum alloy with a density of 2770 kg/m^3 and a friction coefficient of 0.15. The measured angular location is shown in Fig. 6. A software program was generated for analysis. When the inclined angle of the blade is 16° and the rotational speed of the rotor is 809 rpm, the length of the blade out of the rotor, the relative velocity and the relative acceleration of the blade to the slot are shown in Figs. 7, 8 and 9, respectively. The calculated pressure difference on both sides of the blade is shown in Fig. 10, and the calculated static forces acting on points C, A and B are shown in Figs. 11, 12 and 13, respectively. The rotational speed of the rotor of 2000 rpm and the blade inclined angle of 25° are analyzed further for comparison with the previous data. The finite element model of the blade with the software COSMOS was used for the analysis. The finite element modal of the blade and the rotor slot wall is shown in Fig. 14, and the model consists of 923 triangular and quadrangular elements with 1059 nodes. The calculated contact force versus time is shown in Fig. 15. The Von Mises stresses of the blade at the contact point versus time with the calculated time step of $3 \times 10^{-5} \text{ s}$ are shown in Fig. 16. The contour plot of Von Mises stresses of the blade when the contact maximum Von Mises stress occurs in the blade is shown in Fig. 17. The real parts and the imaginary parts of the frequency response function when

(a) real part of h_{11} (scale)

(b) real part of h_{12} (scale)

(c) real part of h_{13} (scale)

(d) real part of h_{14} (scale)

(e) real part of h_{15} (scale)

(f) real part of h_{16} (scale)

Fig. 18 Real part of frequency response function

the accelerator is located at point d_1 and the blade is hit at various locations from d_1 to d_6 are shown in Figs. 18 and 19, respectively.

The calculated speed of the blade when it hits the slot of the rotor is 182.546 mm/s. This value is used in the experiment to simulate the blade hitting the slot of the rotor. At room temperature of 22°C, pressure at 1 atm and the sound level meter located 2 m from the source of noise, the results of noise are shown in Table 2. Energy propagation losses of 4×10^{-12} W due to viscosity and heat conduction during transmission of sound waves are relatively small and are therefore neglected [19].

Discussion

Maximum displacement of the blade relative to the rotor slot occurs when the blade is located at 90° as shown in Fig. 7, and the blade speed relative to the rotor slot is zero as shown in Fig. 8. When the speed of the blade relative to the rotor slot is greatest, the acceleration of the blade relative to the rotor slot is zero as

shown in Fig. 9. The force difference between two sides of the blade due to air pressure increases during compression and decreases during discharge as shown in Fig. 10.

It is assumed that the material is elastic and the properties and structure of the material remain unchanged during the experiment. Fig. 15 shows the maximum impact force is 2.3 kN occurring at point A, which is larger than the maximum static force of 1 kN as shown in Figs. 11, 12 and 13. Figures 16 and 17 show the impact maximum Von Mises stress at contact point is 12.3 MPa, which is larger than the calculated static contact stress of 5.15 MPa [12]. Therefore, the impact of the blade on the rotor slot is significant. The most substantial blade noise comes from the first and second natural frequencies of 1075 Hz and 2963 Hz, respectively, as shown in Figs. 18 and 19 [20].

Since the impact of blades on the rotor slots is significant, the current blade design should be improved. Based on the data obtained in this study, adjusting the pressure acting at the blade bottoms [7] and using the composite contour of the stator [5] can reduce the impact stresses on the blades and prolong the life of

(a) imaginary part of h_{11} (scale)

(b) imaginary part of h_{12} (scale)

(c) imaginary part of h_{13} (scale)

(d) imaginary part of h_{14} (scale)

(e) imaginary part of h_{15} (scale)

(f) imaginary part of h_{16} (scale)

Fig. 19 Imaginary part of frequency response function

Table 2 Results of Noise

source of sound	Power W_i (W)	SPL (dB)
environmental noise	2.818×10^{-8}	44.5
noise of environment and collide	9.734×10^{-5}	79.9
noise of environment and collide of key board	3.388×10^{-7}	55.3

blades. In addition, some conceptual designs of compressors are recommended. Figure 20(a) shows a design with connecting rods. Figure 20(b) shows a design with a cam. Figure 20(c) shows a design with a spring.

Conclusion

The calculated impact force is larger than the static force and the calculated impact stress is larger than the static stress. Therefore, the impact of the blade on the rotor slot is significant and relevant to the design of rotary compressors with sliding blades. By clarifying the impact of blades and rotor slots, the current

Fig. 20 Conceptual designs of rotary compressor

blade design or the configuration of the compressor should be improved. It is expected that the results of this investigation can be used to reduce the noise, improve the design and prolong the life of compressors.

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Nomenclature

a_r	= relative acceleration of blade to slot
a_ω	= centrifugal acceleration
a_c	= Coriolis acceleration
b_i	= body force vector
$[C]$	= damping matrix
e	= blade thickness
f	= friction force; noise frequency
F	= force
h	= frequency response
$[K]$	= stiffness matrix
l	= blade length
l_s	= length of blade out of rotor
$[M]$	= mass matrix
m	= blade mass
N	= contact force
n	= polytropic process exponent
P_n	= total force on blade due to air pressure
p	= air pressure
t	= time
u	= displacement vector
V	= blade segment volume
v_r	= relative velocity of blade to slot
W_i	= sound power
x	= horizontal coordinate
y	= vertical coordinate
α, δ	= Newark integration parameters
σ_{ij}	= Cauchy stress tensor
θ	= angular location of blade center line
ψ	= angle between contact force and blade centerline
ρ	= density
ω	= rotational speed of rotor
ϕ	= blade inclined angle

Subscripts

A, B, C	= contact points
i	= inlet condition; location of accelerator

j = hit point on blade
 x, y = components in x and y directions

Superscripts

$\cdot$ = first derivative
 $\ddot{}$ = second derivative

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