On Pressure Distributions of Drum Brakes

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Based on the assumptions that there is perfect contact at the interface between the brake drum and the lining plates, the friction coefficient is constant, the thermal effect is neglected, and the brake drum is a rigid body, the pressure distributions of drum brakes were studied by using the boundary element method. The constant element is used in the two-dimensional model of the drum brake for simplicity and economy. The friction force versus the effective lift at the actuation edge and the location of the maximum pressure are compared and indicate a good correlation with existing data. The effects of the Young's modulus of elasticity of the metal shoe, the arc lengths of the metal shoe and lining plate, the location, the thickness, the friction coefficient, the Young's modulus of elasticity of the lining plate, and the angle of actuation force on the pressure distributions were then studied. By selecting proper values of these parameters, a drum brake can be designed to have a more uniform pressure distribution and a longer life. [DOI: 10.1115/1.1427694]

Introduction

Drum brakes, which are an important device and are installed in vehicles and motor cycles, are used to reduce the speed and to assure the safety of the drivers and passengers on the road. The schematic drawing of a drum brake is shown in Fig. 1. It primarily consists of a brake drum and brake shoes with lining plates. Qualities it should have include: good braking performance; uniform braking on all wheels; good endurance without failure; and easy operation, inspection, adjustment and maintenance. Advantages offered by the drum brake are its capability from the self-energizing effect to reduce the required actuating force and its relatively lesser cost.

Day et al. [1] used the finite element method to analyze a two-dimensional model of the drum brake. Thereafter, the thermal expansion, imperfect contact, the decay of braking capability and all kinds of transient phenomena were analyzed by using the finite element method [2–6]. Watson and Newcomb [7] argued that it is more expensive to use the finite element program in the main frame and that the pressure distribution on the lining plate does not vary significantly along the axis of the brake. The measurement of the torque or the wear can be used to calculate the pressure distribution [8]. Nevertheless, these methods are indirect methods. It is difficult to obtain directly the pressure distribution from an experiment [2,5]. Recently, the applications of the boundary element method (BEM) have been increased and showed that the BEM can reduce the time required to prepare numerical data and can provide good results in the linear problems [9–14].

The purpose of this study is to analyze the effect of the effective lift at the actuation end on the friction force, effects of the Young's modulus of elasticity for metal shoes, the angles of the arc lengths of metal shoes and lining plates, respectively, the leading location of the lining plates, the thickness of the lining plate, the friction coefficient, the Young's modulus of elasticity for lining plates, and the location of the actuating force of the drum brake on the pressure distributions. A two-dimensional model of the drum brake is developed by using the BEM, and independent and less expensive software is generated for the personal computer to reduce the cost for designing drum brakes. By selecting proper values of these parameters, a drum brake can be designed to have a more uniform pressure distribution and a longer life.

Method of Approach

During the derivation of the boundary integration equations, the solution of an infinite domain subjected to a concentrated load at any internal location is required. If a unit force acts on an internal point M , determination of the effect of this force on any point Q must satisfy two requirements. The first one is that all of the stresses must be zero when the distance between the points M and Q becomes infinite. The second one is that the stress becomes infinite, or the point M is a singular point when the distance between the points M and Q becomes zero. Combining the differential equation of the stress, the relationship of the stress and the strain and the relationship of the strain and the displacement yields

$$\left. \begin{aligned} \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \left(\frac{1}{1-2\nu} \right) \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial x \partial y} \right) &= \frac{-f_x}{G_s} \\ \frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \left(\frac{1}{1-2\nu} \right) \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial x \partial y} \right) &= \frac{-f_y}{G_s} \end{aligned} \right\} \quad (1)$$

which is the Navier two-dimensional differential equations and can be solved by using the complementary function and the special integration. If the displacements are expressed in terms of the Galerkin vector,

$$\left. \begin{aligned} u_x &= \frac{\partial^2 \bar{G}_x}{\partial x^2} + \frac{\partial^2 \bar{G}_x}{\partial y^2} - \frac{1}{2(1-\nu)} \left(\frac{\partial^2 \bar{G}_x}{\partial x^2} + \frac{\partial^2 \bar{G}_y}{\partial x \partial y} \right) \\ u_y &= \frac{\partial^2 \bar{G}_y}{\partial x^2} + \frac{\partial^2 \bar{G}_y}{\partial y^2} - \frac{1}{2(1-\nu)} \left(\frac{\partial^2 \bar{G}_y}{\partial y^2} + \frac{\partial^2 \bar{G}_x}{\partial x \partial y} \right) \end{aligned} \right\} \quad (2)$$

the Galerkin vector can be determined as

$$\bar{G}_x = \bar{G}_y = \frac{1}{8\pi G_s} r^2 \ln \left[\frac{1}{r} \right] \quad (3)$$

and the displacement is

$$u_i = U_{ij} e_j \quad (4)$$

Substituting Eq. (4) into the relationship of the stress and the displacement yields the traction force

$$t_i = T_{ij} e_j \quad (5)$$

Contributed by the Design Automation Committee for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received Nov. 1999. Associate Editor: H. Lankarani.

Fig. 1 Schematic drawing of drum brake

To minimize the error of numerical approximation, the basic solution of the displacement as a weighting factor is multiplied to every term and integrated to yield the Somigliana equation

$$u_i^i + \int_{\Gamma} T_{ik} u_k d\Gamma = \int_{\Gamma} U_{lk} t_k d\Gamma + \int_{\Omega} U_{lk} f_k d\Omega \quad (6)$$

The displacement of any internal point can be determined if the displacements of the boundary points and the traction forces are determined. The Somigliana equation can also be applied to the boundary points and yields

$$c_{lk}^i u_k^i + \int_{\Gamma} T_{lk} u_k d\Gamma = \int_{\Gamma} U_{lk} t_k d\Gamma + \int_{\Omega} U_{lk} f_k d\Omega \quad (7)$$

Four variables u_1 , u_2 , t_1 and t_2 exist for each nodal point in a two-dimensional problem; therefore, there will be $4N$ variables for N nodal points. If there is only one solution for an elastic problem, only two values of these four variables are known for each nodal point and $2N$ equations are required to solve the unknown variables.

The geometric configuration, the displacements and the traction forces must be described. If there is a unit force acting on the point i and the effect of body force can be neglected, the two integrated terms related to the nodal points i and j form the influence coefficients, G^{ij} and $\hat{H}^{ij}$. If

$$\left. \begin{aligned} H^{ij} &= \hat{H}^{ij} & i \neq j \\ H^{ij} &= \hat{H}^{ij} + c^i & i = j \end{aligned} \right\} \quad (8)$$

Eq. (7) becomes

$$\sum_{j=1}^N H^{ij} u_j^i = \sum_{j=1}^N G^{ij} t_j^i \quad (9)$$

Applying to all boundary points and using local coordinates as the basis of the boundary value yields

$$HT'U = GT'P \quad (10)$$

where the matrices H and G are $2N \times 2N$ matrices. The values of the displacement matrix U and the traction force matrix P are all relative to global coordinates. The boundary condition should be given after calculation of all coefficients for the matrices H and G in order to obtain the unique solution. Since it is not a symmetric matrix, a direct method such as the Gaussian elimination method can be used.

Model and Numerical Analysis

It is assumed that the variations of the pressure and the deformation in the axial direction are negligible for simplicity, that no deformation occurs during braking, that there is the perfect initial contact between the contact surfaces of the lining plate and the circular profile of brake drum, that the friction coefficient of the lining plate and the brake drum is constant, that the wear is negligible, that the thermal effect is neglected and that the brake drum

is a rigid body. The two-dimensional model of the brake shoe that is used in this analysis can be divided into two parts based on materials, the metal shoe and the lining plate. They are divided into 28 elements, respectively, due to the limited size of the computer memory. The displacement and the traction force at the center point in the constant element are used for the element to simplify the integration. Applying Eq. (10) to the metal shoe and the lining plate, respectively, combining the points on the contact surface of the lining plate and the metal shoe and the independent portion of the lining surface, and combining the points on the contact surface of the metal shoe and the lining plate and the independent portion of the metal shoe with the traction equilibrium condition

$$P_L^B = P_S^C \quad (11)$$

and the displacement compatibility condition

$$U_L^B = -U_S^C \quad (12)$$

yields

$$\begin{bmatrix} H_L^A & H_L^B & -G_L^B & 0 \\ 0 & -H_S^C & -G_S^C & H_S^D \end{bmatrix} \begin{bmatrix} U_L^A \\ U_L^B \\ P_S^C \\ U_S^D \end{bmatrix} = \begin{bmatrix} G_L^A & 0 & 0 & 0 \\ 0 & 0 & 0 & G_S^D \end{bmatrix} \begin{bmatrix} P_L^A \\ 0 \\ 0 \\ P_S^D \end{bmatrix} \quad (13)$$

where the matrices H and G have been multiplied by the transfer matrix, the superscript B stands for the common portion of the lining plate and the metal shoe and the superscript A stands for the independent portion of the lining plate, the superscript C stands for the common part of the metal shoe and the lining plate and the superscript D stands for the independent portion of the metal shoe.

In the local coordinates, four variables for each nodal point are the displacements U_n and U_t in the normal direction and the tangential direction, respectively, and the traction forces P_n and P_t in the normal direction and the tangential direction, respectively. The boundary conditions along the friction surface are

$$\left. \begin{aligned} U_n &= 0 \\ P_t &= -\mu P_n \end{aligned} \right\} \quad (14)$$

Two variables U_t and P_n are unknown. Equation (13) can be written

$$\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ P_n \\ U_t \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ -H_n & 0 & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ U_n \\ 0 \\ \cdot \\ \cdot \end{bmatrix} \quad (15)$$

The boundary conditions for the fixed anchor are

$$\left. \begin{aligned} U_n &= 0 \\ U_t &= 0 \end{aligned} \right\} \quad (16)$$

The supporting portion for the sliding abutment can be rewritten

$$\begin{aligned} & [\cdot \cdot G_n^i H_t^i - G_n^{i+1} H_t^{i+1} \cdot \cdot] [\cdot \cdot P_n^i U_t^i P_n^{i+1} U_t^{i+1} \cdot \cdot]^T \\ & = [\cdot \cdot -H_n^i G_t^i - H_n^{i+1} G_t^{i+1} \cdot \cdot] [\cdot \cdot U_n^i P_t^i U_n^{i+1} P_t^{i+1} \cdot \cdot]^T \end{aligned} \quad (17)$$

where the superscript stands for the number of the nodal point and the subscript stands for the direction. Similarly, the supporting portion for the fixed anchor can be rewritten

$$\begin{aligned} & [..-G_n^i - G_t^i - G_n^{i+1} - G_t^{i+1} ..][..P_n^i P_t^i P_n^{i+1} P_t^{i+1} ..]^T \\ & = [..-H_n^i - H_t^i - H_n^{i+1} - H_t^{i+1} ..][..U_n^i U_t^i U_n^{i+1} U_t^{i+1} ..]^T \end{aligned} \quad (18)$$

After considering the boundary condition, the matrix can be rewritten

$$H'U' = G'P' \quad (19)$$

where the matrices H' and G' are the matrices after arrangement, and the matrix U' is obtained from the unknown displacement and the traction force. The matrix P' is obtained from all the boundary conditions. After obtaining the traction forces and the displacements of nodes, the torque can be obtained from

$$Torque = \sum_{n=1}^m \mu P \Delta A \quad (20)$$

where ΔA is the area of the element.

Result

The data of the first drum brake with the supporting end fixed by a pin as shown in Fig. 1 are listed in Table 1. The angle α is measured from the line of the pin center and the brake drum center to the line of the brake drum center to near edge of the lining plate. The angle ϕ is measured from the line of the actuation force to the normal vector of the surface at the load point. The friction force versus the effective lift at the actuation edge that is normal to the acting surface is shown in Fig. 2. The data obtained from Day and Harding [15] by using the finite element method are also shown for comparison.

The data of the second drum brake are shown in Table 2. If the

Table 1 Data of first drum brake.

	metal shoe	lining plate
equivalent thickness	30 mm	12 mm
angle of arc length	150°	110°
Young's modulus of elasticity	2.07x10 ⁵ N/mm ²	372 N/mm ²
Poisson's ratio	0.3	0.25
friction radius	209.5 mm	
angular location of lining plate α	20°	
lining width	178 mm	
friction coefficient	0.38

Fig. 2 Friction force versus effective lift at actuation end

Table 2 Data of second drum brake.

	metal shoe	lining plate
equivalent thickness	16.89 mm	4 mm
angle of arc length	170°	120°
Young's modulus of elasticity	8.28x10 ⁵ N/mm ²	372 N/mm ²
Poisson's ratio	0.3	0.25
friction radius	70 mm	
angular location of lining plate α	25°	
lining width	28 mm	
friction coefficient	0.4

Fig. 3 Pressure distribution with high Young's modulus of elasticity for rigid metal shoe

Fig. 4 Pressure distribution versus Young's modulus of elasticity of metal shoe

Fig. 5 Pressure distribution versus lining arc for various arc lengths of metal shoe

Fig. 6 Pressure distribution versus lining arc for various arc lengths of lining plate

Fig. 9 Pressure distribution versus lining arc for various friction coefficients of lining plate

Young's modulus of elasticity increases, the material becomes stiffer. At first, the Young's modulus of elasticity of the brake shoe in Table 2 is replaced by a value four times larger, 3.31×10^6 N/mm², and the result of the pressure distribution is compared with the sinusoidal curve of the rigid brake obtained from Offner [16] as shown in Fig. 3. Based on the data shown in Table 1 and the displacement of the surface along the normal direction of the surface to be 0.1 mm, the pressure distributions versus the lining arc for various Young's moduli of elasticity of metal shoes are shown in Fig. 4. The effects of the arc lengths of the metal shoe and the lining plate with the same center location on the pressure

distribution are shown in Figs. 5 and 6, respectively. Shifting the lining plate, the pressure distribution is shown in Fig. 7. The pressure distributions versus the lining arc for the thickness of the lining plate to be 3, 4 and 5 mm are shown in Fig. 8. It is well known that the friction coefficient of material can affect the braking performance. The effect of the friction coefficient of the lining plate on the pressure distribution is shown in Fig. 9. Changing the Young's modulus of elasticity of the lining plate, the pressure distribution is shown in Fig. 10. If the ratio of the Young's modulus of elasticity of the lining plate to the Young's modulus of the metal shoe remains constant and both Young's moduli of elasticity of the lining plate and the metal shoe increase

Fig. 7 Pressure distribution versus lining arc for various leading locations of lining plate

Fig. 10 Pressure distribution versus lining arc for various Young's moduli of elasticity of lining plate

Fig. 8 Pressure distribution versus lining arc for various lining plate thicknesses

Fig. 11 Pressure distribution versus lining arc for various Young's moduli of elasticity of metal shoe and lining plate

Fig. 12 Pressure distribution versus lining arc for various angles of actuation force

20% and decrease 20%, respectively, the pressure distributions are shown in Fig. 11. The effect of the angle measured from the line of the actuating force to the normal vector of the surface at the load point with the traction pressure 50 N/mm² on the pressure distribution is shown in Fig. 12.

Discussion

Watson and Newcomb [7] argued that it is more expensive to use the finite element program in the main frame. This study uses the boundary element method in the personal computer. Nevertheless, the computer running time is not available for comparison. Using the linear elements or the second order elements creates discontinuous phenomena along the irregular boundary. The common nodal point has the different normal vector and boundary conditions. It is necessary to have an extra equation to provide a unique solution for the final linear equation [12]. Therefore, these elements are inappropriate to treat the corner problem. Using the constant element can get rid of this problem. In addition, since there is no shape function and no need of the Jacobin coordinate transform, integrating the equation is easier and writing a computer code is simpler for the constant element than the second order element. Watson and Newcomb [7] also argued that the pressure distribution on the lining plate does not vary significantly along the axis of the brake. Therefore, two-dimensional model is used in this analysis to reduce the computer time.

It is difficult to obtain directly the pressure distribution from an experiment [2,5]. Day [17] studied two distinct brake operating conditions experimentally to investigate the effect of vehicle speed on the pressure distribution with time. However, the dimensions of the brake are not specified completely, and the results are not appropriate for comparison with this study. The friction force is proportional to the effective lift at the actuating edge as shown in Fig. 2. The deviation of the calculated friction force from that obtained by Day and Harding [15] increases when the effective lift increases. It may come from the insufficient number of the elements and the accumulative error from the numerical calculation, and it can be reduced if the number of the elements is increased.

The maximum pressure on the lining plate that occurs at 60° in Fig. 3 is located at 85° from the center of the pin. The conventional angular location of the maximum pressure from the supporting point for the rigid drum brake is at 90° [16]. Figure 4 shows that the angular location of the maximum pressure shifted further from the supporting point when the Young's modulus of elasticity of metal shoe increases. The location of the maximum pressure should be close to 90° from the supporting point if the Young's modulus of elasticity increases to simulate the rigid brake. Therefore, the mathematical model is feasible for the study of the pressure distribution of lining plates. When the Young's modulus of elasticity of the metal shoe decreases, the pressure at

the center of the metal shoe decreases, but the pressure increases toward the supporting point. This result may come from the buckling effect that the pressure at the center of the brake shoe is lowered.

The pressure close to the supporting point decreases and the pressure distribution is more uniform if the arc length of the metal shoe increases as shown in Fig. 5. The similar result of a more uniform pressure can be obtained if the arc length of the lining plate increases as shown in Fig. 6. If the arc length of lining plate decreases, a local high pressure may occur. Figure 7 shows that the maximum pressure decreases if the lining plate is shifted toward the supporting point and the pressure distribution becomes more uniform. If the thickness of the lining plate increases, the maximum pressure decreases and a more uniform pressure distribution can be obtained as shown in Fig. 8. Figure 9 shows that the maximum pressure decreases and the pressure distribution becomes more uniform if the friction coefficient decreases. If the Young's modulus of elasticity of the lining plate decreases, the maximum pressure decreases and a more uniform pressure distribution can be achieved as shown in Fig. 10. Figure 11 shows that the maximum pressure and the pressure distribution decrease if both Young's moduli of elasticity of the metal shoe and the lining plate decrease while their ratio remains the same. The shapes of the pressure distribution remain the same. This result identifies that obtained by Wintle [4]. The maximum pressure decreases and the pressure distribution becomes more uniform if the angle of the actuation force increases as shown in Fig. 12. Reducing the angle of the actuation force decreases the pressure at the actuation edge and may result in the separation of the lining plate at the actuation end from the brake drum.

Frictional work in a brake is primarily converted into heat that increases the temperature of shoes and lining plates of the brake and changes the properties, particularly in the thermal expansion. The circumferential distribution of pressure along the brake lining arc, that plays an important role in determining the torque generated, depends upon the thermal expansion, flexural deflection and contact at the interface. Heat generated at the sliding interface between the friction material and the mating surface of the brake is not uniformly distributed and leads to pronounced variation in high surface temperature and the high interface pressure [2]. Friction interface temperature in the brake is affected by the interface pressure distribution, so that regions of local high interface pressure will also be regions of high surface temperature [6]. The effects in the areas of frictional heat generation and dissipation with thermal consideration on the distribution of pressure, that is very important in minimizing brake thermal problems, is required in order to incorporate the non-linear characteristics of friction materials to obtain the uniform friction interface pressure.

The distribution of pressure is significantly affected by drum flexibility, which relates to the Young's modulus of elasticity. The pressure generated for the flexible drum is reduced because a large actuation displacement is absorbed by drum deflection thereby reducing both effective cam lift and actuation force.

Conclusion

Based on the similarity between the calculated results and the available data, the results suggest that the software developed can be used to predict the pressure distribution of the lining plate on the drum brake. If the deflections of the metal shoe and the lining plate and the thermal effect are neglected, the pressure distribution is a sinusoidal curve. If these deflections are considered in the analysis, the pressure distribution will be changed. In order to design a drum brake with a more uniform pressure acting on the lining plate and to prolong the life of the friction material the arc lengths of the brake shoe and the lining plate should be longer; the

location of the lining plate should be close to the supporting point; the lining plate should be thicker; the friction coefficient should be smaller; the Young's modulus of elasticity for the material used in the brake shoe and the lining plate should be smaller; and the angle of the actuation force should be larger. However, it should be noted that the material with the small Young's modulus of elasticity might wear easily and faster. Although the material with a small friction coefficient can provide more uniform pressure acting on the lining plate, it should not be too small. Otherwise, it will lower the braking effect. The friction coefficient of material about 0.2 is a good selection for application. In addition, a larger actuation force should be applied with the actuation angle of 35° to compensate and increase the braking effort.

Acknowledgment

The authors would like to express their sincere appreciation for the grant no. NSC88-2212-E-002-036 from the National Science Council of the Republic of China for this study and to Grant D. Huang for comments and revisions made on this manuscript.

Nomenclature

A	= area
e	= unit vector in j direction
F	= actuation force
f	= body force in the i direction
$\bar{G}$	= Galerkin vector
G^{ij}, H^{ij}	= influence coefficient matrix regarding points i and j
G_S	= shear modulus
P	= traction force matrix
r	= distance between points M and Q
T	= traction basis solution matrix or transverse matrix
T'	= coordinate transform matrix
t	= traction force basic solution
U	= displacement matrix
u	= displacement basic solution
ν	= Poisson's ratio
x, y	= coordinate
Ω	= integration domain
μ	= friction coefficient
α	= angle measured from line of centers of drum brake and pin to line of center of drum brake and near edge of lining plate
ϕ	= angle measured from the line of actuation force to the normal vector of the surface at the load point
ϕ	= angle of arc length of lining plate

Γ = boundary

Subscripts

ij	= i th row and j th column in matrix
n	= normal component
l	= in l direction
t	= tangential component
L	= lining plate
S	= metal shoe
x, y	= direction

Superscripts

i	= point i
ij	= displacement vector of Q in j direction with a unit force acted on point M in i direction

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