Adaptive nonlinear control of repulsive maglev suspension systems

Chao-Ming Huang, Jia-Yush Yen*, Min-Shin Chen

Department of Mechanical Engineering, National Taiwan University, No. 1, Roosevelt Road, Section 4, Taipei, Taiwan 10617, ROC

Received 12 April 1999; accepted 23 March 2000

Abstract

Magnetic levitation systems have recently become the focus of many research interests not only because they are most suitable for high-precision engineering applications but also because of the fact that they represent a difficult challenge to control engineers. As a result, most previous studies have focused on the control stabilization problem. In this paper, the issue of performance with respect to uncertainty in order to achieve a desired stiffness is addressed. The proposed controller is an adaptive backstepping controller. The adaptive backstepping controller provides system stability under model uncertainty, and achieves the desired servo performance. The experiments show that the proposed control achieves a superior behavior than other control. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Nonlinear control; Maglev suspension system; Adaptive backstepping control; Parametric uncertainty

1. Introduction

With the development of the industrial technology, numerous motion control devices and advanced fabrication equipment have been designed to meet the high-precision positioning demands. The performance of the guiding system, such as stiffness and resolution, plays an important role in these systems. According to the analyses made by Wang (1993) and Huang (1999), the suspension system based on maglev technology is superior to other guideways, like ball-screw, air-lubricated levitation guideways, and electrostatic levitation guideways, due to the frictionless and active stiffness characteristics.

A maglev system can be classified, based on the source of levitation forces, as an attractive system or a repulsive system. Most of the maglev systems discussed in literatures are the attractive systems, where attractive forces are applied between the moving components and fixed components (Atherton, 1980; Cho, Kato & Spilman, 1993; Tsukamoto, Yasuda & Chen, 1988). And the other maglev system, repulsive maglev systems, uses repulsive forces to push the moving components above the fixed components (Williams, Matey, Arie & Rathee, 1989; Wang, 1993; Wu & Fu, 1996; Huang, 1999). In an attractive system, the moving component (carriage) is always suspended under

the fixed component (guide track). The carried objects can only be accessed from under the guide track and it is not convenient for some applications. Generally, it is desirable that the carriage travel on the top of the guide track for convenient access to the carried objects.

The repulsive maglev suspension system in this paper (Fig. 1) utilizes two groups of permanent magnets with the same magnetic poles facing each other to provide the required repulsive levitation force. One group is attached to the bottom of the carriage, and the other group is fixed on the guide tracks. But this design also causes the carriage to be laterally unstable. Accordingly, electro magnets are placed beside the carriage to help stabilize the carriage dynamics. So, a closed-loop control of maglev suspension system is necessary.

In this paper, a multi-input-multi-output adaptive backstepping controller is derived and implemented for decoupling nonlinearities and eliminating uncertainties. The experimental results show that the adaptive backstepping controller for the maglev system has better abilities in decoupling nonlinearities than the adaptive control without backstepping used in Wu and Fu (1996) and provides high rigidity.

2. System modeling

To achieve levitation in the maglev suspension system (Fig. 2), repulsive forces generated by the interaction

* Corresponding author. Tel.: + 886-2-2366-0734; fax: + 886-2-2366-0734.

E-mail address: jyen@ccms.ntu.edu.tw (J. Y. Yen).

Fig. 1. Photograph of the physical system.

Fig. 2. A 3-D view of the maglev suspension system.

between two groups of permanent magnets are used. One group named carriage magnets are fixed on the carriage, and the other group named track magnets are laid out on the four levitation tracks. By applying the same magnetic poles between the two group magnets (Fig. 3), the permanent magnets in carriage can experience repulsive levitation forces and make the carriage levitate. But this repulsive levitation force also cause the carriage to be laterally unstable, other devices called stabilizers are designed to stabilize the unstable dynamics of the carriage. The stabilizers apply electromagnetic forces to the carriage magnets used for levitation. The carriage magnets, track magnets and stabilizers constitute the levitation tracks and there are four levitation tracks in the system discussed here. In Fig. 2, there are six-degrees-of-freedom in the carriage dynamics, X , Y , Z , θ , ϕ and ψ , but only the unstable degrees of freedom, θ and X , are considered in this paper.

Basically, the maglev suspension system presented in this paper is an electrically and mechanically integrated

Fig. 3. The cross-section of the levitation track.

system (Fig. 4). The mechanical part is the mechanism of the maglev system, and the electrical part includes a PC-based controller, input-output interface devices of the PC, like the analog-to-digital converter (ADC) and digital-to-analog converter (DAC), and linear power amplifiers. First, the control commands calculated by the controller are transmitted to the linear power amplifiers by means of DAC (see Fig. 5). After receiving the voltage commands, the linear power amplifiers generate current flows proportional to the commands. The currents then flow into the coils of the stabilizer electromagnets in the levitation tracks and produce forces against the unstable forces caused by track magnets. Accordingly, it is necessary to decompose the system model into the mechanical dynamics and the electrical dynamics.

2.1. Mechanical dynamics

The mechanical dynamics presented here was originally developed by Wang (1993). And a more complete and accurate nonlinear version was derived by Huang (1999). Here, some results useful to this paper are formulated.

The mechanical dynamics (Huang, 1999) derived by using Newton–Euler Equation of Motion for a body moving in space can be represented in a control-affine form:

$$\begin{aligned}\ddot{x}_1 &= f_1(x_1, x_2) + g_{11}(x_1, x_2)I_1 + g_{12}(x_1, x_2)I_2, \\ \ddot{x}_2 &= f_2(x_1, x_2) + g_{21}(x_1, x_2)I_1 + g_{22}(x_1, x_2)I_2,\end{aligned}\quad (1)$$

where x_1 and x_2 are the system dynamic variables θ and X . I_1 and I_2 are the current flows in the coil of the electromagnets (stabilizers). $f_1, f_2, g_{11}, g_{12}, g_{21}, g_{22}$ are

ordinary differential equations

$$\dot{I}_1 = -\frac{R_1}{L_1}I_1 + \frac{K_{A1}}{L_1}u_1 = \tau_{e1}I_1 + K_{e1}u_1, \quad (8)$$

$$\dot{I}_2 = -\frac{R_2}{L_2}I_2 + \frac{K_{A2}}{L_2}u_2 = \tau_{e2}I_2 + K_{e2}u_2, \quad (9)$$

where R_1, R_2 are the resistors of the electromagnet coils, L_1, L_2 are the inductors of the electromagnet coils, and K_{A1}, K_{A2} are gains of the linear power amplifiers, u_1, u_2 are the control voltages to the amplifiers. By combining both the mechanical dynamics and the electrical dynamics, the model of maglev suspension system can be derived.

3. Analysis of model uncertainties

In the previous section, the mathematical model of the maglev suspension system is obtained. In this section, the parameters in the maglev system which are to be treated as uncertainties are discussed. Mass and inertia of carriage, M and J , are parameters difficult to obtain due to the complex structures and non-uniformity of materials. One can only calculate them approximately by using some analysis software, like Pro Engineering. Other parameters like m_z and I_M , which are unmeasurable magnetic properties of the system, will be the major source of system uncertainties.

Collecting the uncertain parameters in Eqs. (1)–(7), the system equation can be re-grouped as a parameterized linear uncertainty model

$$\ddot{x}_1 = \alpha_1 f_{p1}(x_1, x_2) + \beta_{11}g_{p11}(x_1, x_2)I_1 + \beta_{12}g_{p12}(x_1, x_2)I_2, \quad (10)$$

$$\ddot{x}_2 = \alpha_2 f_{p2}(x_1, x_2) + \beta_{21}g_{p21}(x_1, x_2)I_1 + \beta_{22}g_{p22}(x_1, x_2)I_2, \quad (11)$$

where $\alpha_1, \alpha_2, \beta_{11}, \beta_{12}, \beta_{21}, \beta_{22}$ are treated as unknown system parameters or the so-called linear parametric parameters. $f_{p1}, f_{p2}, g_{p11}, g_{p12}, g_{p21}$ and g_{p22} are known nonlinear functions of the system variables. Linking the electrical dynamics in Eqs. (8) and (9) and the parameterized system in Eqs. (10) and (11), the following overall system dynamics could be obtained:

$$\dot{x}_1 = x_3,$$

$$\dot{x}_2 = x_4,$$

$$\dot{x}_3 = \alpha_1 f_{p1}(x_1, x_2) + \beta_{11}g_{p11}(x_1, x_2)I_1 + \beta_{12}g_{p12}(x_1, x_2)I_2,$$

$$\dot{x}_4 = \alpha_2 f_{p2}(x_1, x_2) + \beta_{21}g_{p21}(x_1, x_2)I_1 + \beta_{22}g_{p22}(x_1, x_2)I_2,$$

$$\dot{I}_1 = \tau_{e1}I_1 + K_{e1}u_1,$$

$$\dot{I}_2 = \tau_{e2}I_2 + K_{e2}u_2. \quad (12)$$

4. Controller design

4.1. Backstepping control

Consider the system model derived in Section 2, it is easy to notice that the model is a cascade connection of a nonlinear system with a linear time invariant (LTI) subsystem. The control objective is to drive the nonlinear system outputs x_1 and x_2 to zero asymptotically. For this kind of cascade nonlinear system, the backstepping control (Krstic, Kanellakopoulos and Kokotović, 1995) is more suitable than input-output feedback linearization (Isidori, 1985). If input-output feedback linearization method is applied, an inverse of a 6×6 decoupling matrix will appear in the control algorithm. This will become a burden to the controller if a high sampling rate is desired. By using the backstepping method, this burden can be reduced to one 4×4 plus one 2×2 matrix inverse, and the latter can be calculated beforehand because it is an LTI system. Recall the system model in Eq. (12) and let the controller design objective be the regulation of x_1 and x_2 , that is, $x_1(t) \rightarrow 0$ and $x_2(t) \rightarrow 0$ as $t \rightarrow \infty$. By investigating the first two equations in Eq. (12), x_3 and x_4 can be treated as the virtual controls to stabilize x_1 and x_2 . If $x_3 = -k_1x_1$ and $x_4 = -k_2x_2$, where k_1 and k_2 are both positive, x_1 and x_2 will become zeros asymptotically. Of course, x_3 and x_4 are just state variables and not the controls. Nevertheless, as their “desired values” are prescribed as

$$x_{3,\text{desired}} = -k_1x_1 = v_1(x_1), \quad (13)$$

$$x_{4,\text{desired}} = -k_2x_2 = v_2(x_2), \quad (14)$$

where $v_1(x_1)$ and $v_2(x_2)$ are the stabilizing functions. Let z_1 and z_2 be the deviations of x_3 and x_4 from their desired values, then

$$z_1 = x_3 - v_1(x_1), \quad (15)$$

$$z_2 = x_4 - v_2(x_2). \quad (16)$$

z_1 and z_2 can also be treated as the corresponding error variables. Now, the first two equations of Eq. (12) can be rewritten as

$$\dot{x}_1 = v_1(x_1) + z_1,$$

$$\dot{x}_2 = v_2(x_2) + z_2. \quad (17)$$

In the new coordinates x_1, x_2, z_1 and z_2 , the system can be expressed as

$$\dot{x}_1 = v_1(x_1) + z_1,$$

$$\dot{x}_2 = v_2(x_2) + z_2. \quad (18)$$

$$\begin{aligned}\dot{z}_1 &= \dot{x}_3 - \frac{\partial v_1}{\partial x_1} \dot{x}_1 \\ &= \alpha_1 f_{p1}(x_1, x_2) + \beta_{11} g_{p11}(x_1, x_2) I_1 \\ &\quad + \beta_{12} g_{p12}(x_1, x_2) I_2 - \frac{\partial v_1}{\partial x_1} x_3,\end{aligned}$$

$$\begin{aligned}\dot{z}_2 &= \dot{x}_4 - \frac{\partial v_2}{\partial x_2} \dot{x}_2 \\ &= \alpha_2 f_{p2}(x_1, x_2) + \beta_{21} g_{p21}(x_1, x_2) I_1 \\ &\quad + \beta_{22} g_{p22}(x_1, x_2) I_2 - \frac{\partial v_2}{\partial x_2} x_4,\end{aligned}$$

$$\begin{aligned}\dot{I}_1 &= \tau_{e1} I_1 + K_{e1} u_1, \\ \dot{I}_2 &= \tau_{e2} I_2 + K_{e2} u_2.\end{aligned}$$

From Eq. (13) to Eq. (17), it is obvious that only when $z_1(t) = 0, z_2(t) = 0$, then $x_3 = -k_1 x_1, x_4 = -k_2 x_2$ and $x_1(t) \rightarrow 0, x_2(t) \rightarrow 0$ as $t \rightarrow \infty$. So, the next step in the controller design is to stabilize z_1 and z_2 . Similarly, the stabilizing functions must be found first. Let stabilizing functions $v_3(x_1, x_2, z_1, z_2), v_4(x_1, x_2, z_1, z_2)$ be

$$\begin{aligned}\begin{bmatrix} v_3 \\ v_4 \end{bmatrix} &= \begin{bmatrix} I_{1,\text{desired}} \\ I_{2,\text{desired}} \end{bmatrix} \\ &= \begin{bmatrix} \beta_{11} g_{p11} & \beta_{12} g_{p12} \\ \beta_{21} g_{p21} & \beta_{22} g_{p22} \end{bmatrix}^{-1} \\ &\quad \times \begin{bmatrix} -\alpha_1 f_{p1} + \frac{\partial v_1(x_1)}{\partial x_1} (z_1 + v_1) - k_3 z_1 - x_1 \\ -\alpha_2 f_{p2} + \frac{\partial v_2(x_2)}{\partial x_2} (z_2 + v_2) - k_4 z_2 - x_2 \end{bmatrix}\end{aligned}\quad (19)$$

and define the corresponding error variables z_3, z_4

$$z_3 = I_1 - v_3(x_1, x_2, z_1, z_2), \quad (20)$$

$$z_4 = I_2 - v_4(x_1, x_2, z_1, z_2). \quad (21)$$

Then Eq. (18) can be rewritten based on the original coordinates, x_1, x_2 , and the new coordinates z_1, z_2, z_3, z_4 :

$$\begin{aligned}\dot{x}_1 &= -k_1 x_1 + z_1, \\ \dot{x}_2 &= -k_2 x_2 + z_2,\end{aligned}\quad (22)$$

$$\begin{aligned}\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} &= \begin{bmatrix} -k_3 z_1 - x_1 \\ -k_4 z_2 - x_2 \end{bmatrix} \\ &\quad + \begin{bmatrix} \beta_{11} g_{p11} & \beta_{12} g_{p12} \\ \beta_{21} g_{p21} & \beta_{22} g_{p22} \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix},\end{aligned}$$

$$\begin{bmatrix} \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} \tau_{e1}(z_3 + v_3) \\ \tau_{e2}(z_4 + v_4) \end{bmatrix} + \begin{bmatrix} K_{e1} u_1 \\ K_{e2} u_2 \end{bmatrix}$$

$$\begin{aligned}&- \begin{bmatrix} \frac{\partial v_3}{\partial x_1} & \frac{\partial v_3}{\partial x_2} \\ \frac{\partial v_4}{\partial x_1} & \frac{\partial v_4}{\partial x_2} \end{bmatrix} \begin{bmatrix} z_1 + v_1 \\ z_2 + v_2 \end{bmatrix} \\ &- \begin{bmatrix} \frac{\partial v_3}{\partial z_1} & \frac{\partial v_3}{\partial z_2} \\ \frac{\partial v_4}{\partial z_1} & \frac{\partial v_4}{\partial z_2} \end{bmatrix} \left\{ \begin{bmatrix} -k_3 z_1 - x_1 \\ -k_4 z_2 - x_2 \end{bmatrix} \right. \\ &\quad \left. + \begin{bmatrix} \beta_{11} g_{p11} & \beta_{12} g_{p12} \\ \beta_{21} g_{p21} & \beta_{22} g_{p22} \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} \right\}.\end{aligned}$$

The final step is to design the real controls, u_1, u_2 , to stabilize z_3, z_4 . Then, other states, x_1, x_2, z_1, z_2 , will be stabilized step-by-step and backwardly. By letting the controls, u_1, u_2 , be

$$\begin{aligned}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} &= \begin{bmatrix} K_{e1} & 0 \\ 0 & K_{e2} \end{bmatrix}^{-1} \left\{ - \begin{bmatrix} \tau_{e1}(z_3 + v_3) \\ \tau_{e2}(z_4 + v_4) \end{bmatrix} \right. \\ &\quad - \begin{bmatrix} k_5 z_3 \\ k_6 z_4 \end{bmatrix} - \begin{bmatrix} \beta_{11} g_{p11} & \beta_{12} g_{p12} \\ \beta_{21} g_{p21} & \beta_{22} g_{p22} \end{bmatrix}^T \\ &\quad \left. \times \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \right\},\end{aligned}\quad (23)$$

where

$$\begin{aligned}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} &= - \begin{bmatrix} \frac{\partial v_3}{\partial x_1} & \frac{\partial v_3}{\partial x_2} \\ \frac{\partial v_4}{\partial x_1} & \frac{\partial v_4}{\partial x_2} \end{bmatrix} \begin{bmatrix} z_1 + v_1 \\ z_2 + v_2 \end{bmatrix} \\ &- \begin{bmatrix} \frac{\partial v_3}{\partial z_1} & \frac{\partial v_3}{\partial z_2} \\ \frac{\partial v_4}{\partial z_1} & \frac{\partial v_4}{\partial z_2} \end{bmatrix} \left\{ \begin{bmatrix} -k_3 z_1 - x_1 \\ -k_4 z_2 - x_2 \end{bmatrix} \right. \\ &\quad \left. + \begin{bmatrix} \beta_{11} g_{p11} & \beta_{12} g_{p12} \\ \beta_{21} g_{p21} & \beta_{22} g_{p22} \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} \right\}.\end{aligned}\quad (24)$$

and substituting Eqs. (23) and (24) into Eq. (22), the dynamical equation can be rearranged as

$$\begin{aligned}\dot{x}_1 &= -k_1 x_1 + z_1 \\ \dot{x}_2 &= -k_2 x_2 + z_2, \\ \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} &= \begin{bmatrix} -k_3 z_1 - x_1 \\ -k_4 z_2 - x_2 \end{bmatrix} \\ &\quad + \begin{bmatrix} \beta_{11} g_{p11} & \beta_{12} g_{p12} \\ \beta_{21} g_{p21} & \beta_{22} g_{p22} \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix},\end{aligned}\quad (25a)$$

$$\begin{aligned}\begin{bmatrix} \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} &= \begin{bmatrix} -k_5 z_3 \\ -k_6 z_4 \end{bmatrix} \\ &- \begin{bmatrix} \beta_{11} g_{p11} & \beta_{12} g_{p12} \\ \beta_{21} g_{p21} & \beta_{22} g_{p22} \end{bmatrix}^T \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}.\end{aligned}\quad (25b)$$

Theorem 1. The proposed control, Eqs. (23) and (24), stabilizes the repulsive maglev suspension system in Eq. (12) asymptotically if all the control gains k_i , $i = 1, 2, \dots, 6$ are positive.

Proof. Define a Lyapunov function V :

$$V = \frac{1}{2} \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} z_1 & z_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} z_3 & z_4 \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} \geq 0. \quad (26)$$

By substituting Eqs. (23)–(25a) and (25b), the time derivative of V becomes

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 z_1^2 - k_4 z_2^2 - k_5 z_3^2 - k_6 z_4^2 \leq 0. \quad (27)$$

If $k_i > 0, i = 1, 2, \dots, 6$. Finally, From LaSalle's Theorem (Krstic et al., 1995), one can obtain

$$\lim_{t \rightarrow \infty} x_1(t) = 0, \quad (28)$$

$$\lim_{t \rightarrow \infty} x_2(t) = 0. \quad (29)$$

The asymptotic stability of the maglev suspension system is thus achieved. $\square$

4.2. Adaptive backstepping control

As discussed in Section 3, there exists some uncertainty parameters in the maglev system, like $\alpha_1, \alpha_2, \beta_{11}, \beta_{12}, \beta_{21}$, and β_{22} . The real values of these parameters cannot be obtained exactly. Accordingly, $\alpha_1, \alpha_2, \beta_{11}, \beta_{12}, \beta_{21}$, and β_{22} are assumed to be unknown, and Eq. (19) must be changed to

$$\begin{bmatrix} v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} \hat{\beta}_{11} g_{p11} & \hat{\beta}_{12} g_{p12} \\ \hat{\beta}_{21} g_{p21} & \hat{\beta}_{22} g_{p22} \end{bmatrix}^{-1} \times \begin{bmatrix} -\hat{\alpha}_1 f_{p1} + \frac{\partial v_1(x_1)}{\partial x_1} (z_1 + v_1) - k_3 z_1 - x_1 \\ -\hat{\alpha}_2 f_{p2} + \frac{\partial v_2(x_2)}{\partial x_2} (z_2 + v_2) - k_4 z_2 - x_2 \end{bmatrix} \quad (30)$$

by replacing $\alpha_1, \alpha_2, \beta_{11}, \beta_{12}, \beta_{21}$, and β_{22} with their estimates $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}_{11}, \hat{\beta}_{12}, \hat{\beta}_{21}$, and $\hat{\beta}_{22}$, respectively. Thus, the dynamics for z_1 and z_2 becomes

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -k_3 z_1 - x_1 \\ -k_4 z_2 - x_2 \end{bmatrix} + \begin{bmatrix} \hat{\beta}_{11} g_{p11} & \hat{\beta}_{12} g_{p12} \\ \hat{\beta}_{21} g_{p21} & \hat{\beta}_{22} g_{p22} \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} \tilde{\gamma}_1^T w_1 \\ \tilde{\gamma}_2^T w_2 \end{bmatrix}, \quad (31)$$

where

$$w_1^T = [f_{p1} \quad g_{p11}(z_3 + v_3) \quad g_{p12}(z_4 + v_4)], \quad (32)$$

$$w_2^T = [f_{p2} \quad g_{p21}(z_3 + v_3) \quad g_{p22}(z_4 + v_4)]. \quad (33)$$

and

$$\tilde{\gamma}_1 = \gamma_1 - \hat{\gamma}_1 = [\alpha_1 \quad \beta_{11} \quad \beta_{12}]^T - [\hat{\alpha}_1 \quad \hat{\beta}_{11} \quad \hat{\beta}_{12}]^T, \quad (34)$$

$$\tilde{\gamma}_2 = \gamma_2 - \hat{\gamma}_2 = [\alpha_2 \quad \beta_{21} \quad \beta_{22}]^T - [\hat{\alpha}_2 \quad \hat{\beta}_{21} \quad \hat{\beta}_{22}]^T, \quad (35)$$

are the estimating error vectors for $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}_{11}, \hat{\beta}_{12}, \hat{\beta}_{21}$, and $\hat{\beta}_{22}$, respectively. Based on Eq. (30), the dynamics for z_3 and z_4 of the last two Eq. (22) becomes:

$$\begin{bmatrix} \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} \tau_{e1}(z_3 + v_3) \\ \tau_{e2}(z_4 + v_4) \end{bmatrix} + \begin{bmatrix} K_{e1} u_1 \\ K_{e2} u_2 \end{bmatrix} - \begin{bmatrix} \frac{\partial v_3}{\partial x_1} & \frac{\partial v_3}{\partial x_2} \\ \frac{\partial v_4}{\partial x_1} & \frac{\partial v_4}{\partial x_2} \end{bmatrix} \begin{bmatrix} z_1 + v_1 \\ z_2 + v_2 \end{bmatrix} - \begin{bmatrix} \frac{\partial v_3}{\partial z_1} & \frac{\partial v_3}{\partial z_2} \\ \frac{\partial v_4}{\partial z_1} & \frac{\partial v_4}{\partial z_2} \end{bmatrix} \left\{ \begin{bmatrix} -k_3 z_1 - x_1 \\ -k_4 z_2 - x_2 \end{bmatrix} + \begin{bmatrix} \hat{\beta}_{11} g_{p11} & \hat{\beta}_{12} g_{p12} \\ \hat{\beta}_{21} g_{p21} & \hat{\beta}_{22} g_{p22} \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} \right\} - \begin{bmatrix} \frac{\partial v_3}{\partial \gamma_1} & \frac{\partial v_3}{\partial \gamma_2} \\ \frac{\partial v_4}{\partial \gamma_1} & \frac{\partial v_4}{\partial \gamma_2} \end{bmatrix} \begin{bmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{bmatrix} - \begin{bmatrix} \tilde{\gamma}_1^T \frac{\partial v_3}{\partial z_1} w_1 + \tilde{\gamma}_2^T \frac{\partial v_3}{\partial z_2} w_2 \\ \tilde{\gamma}_1^T \frac{\partial v_4}{\partial z_1} w_1 + \tilde{\gamma}_2^T \frac{\partial v_4}{\partial z_2} w_2 \end{bmatrix}. \quad (36)$$

The control law Eq. (23) also needs to be modified as

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} K_{e1} & 0 \\ 0 & K_{e2} \end{bmatrix}^{-1} \left\{ - \begin{bmatrix} \tau_{e1}(z_3 + v_3) \\ \tau_{e2}(z_4 + v_4) \end{bmatrix} - \begin{bmatrix} k_5 z_3 \\ k_6 z_4 \end{bmatrix} - \begin{bmatrix} \hat{\beta}_{11} g_{p11} & \hat{\beta}_{12} g_{p12} \\ \hat{\beta}_{21} g_{p21} & \hat{\beta}_{22} g_{p22} \end{bmatrix}^T \times \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} A'_1 \\ A'_2 \end{bmatrix} \right\}, \quad (37)$$

where

$$\begin{bmatrix} A'_1 \\ A'_2 \end{bmatrix} = - \begin{bmatrix} \frac{\partial v_3}{\partial x_1} & \frac{\partial v_3}{\partial x_2} \\ \frac{\partial v_4}{\partial x_1} & \frac{\partial v_4}{\partial x_2} \end{bmatrix} \begin{bmatrix} z_1 + v_1 \\ z_2 + v_2 \end{bmatrix} - \begin{bmatrix} \frac{\partial v_3}{\partial \gamma_1} & \frac{\partial v_3}{\partial \gamma_2} \\ \frac{\partial v_4}{\partial \gamma_1} & \frac{\partial v_4}{\partial \gamma_2} \end{bmatrix} \begin{bmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{bmatrix} - \begin{bmatrix} \frac{\partial v_3}{\partial z_1} & \frac{\partial v_3}{\partial z_2} \\ \frac{\partial v_4}{\partial z_1} & \frac{\partial v_4}{\partial z_2} \end{bmatrix} \left\{ \begin{bmatrix} -k_3 z_1 - x_1 \\ -k_4 z_2 - x_2 \end{bmatrix} + \begin{bmatrix} \hat{\beta}_{11} g_{p11} & \hat{\beta}_{12} g_{p12} \\ \hat{\beta}_{21} g_{p21} & \hat{\beta}_{22} g_{p22} \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} \right\} - \begin{bmatrix} \hat{\beta}_{11} g_{p11} & \hat{\beta}_{12} g_{p12} \\ \hat{\beta}_{21} g_{p21} & \hat{\beta}_{22} g_{p22} \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix}. \quad (38)$$

Thus, Eq. (36) becomes

$$\begin{bmatrix} \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = - \begin{Bmatrix} -k_5 z_3 \\ -k_6 z_4 \end{Bmatrix} + \begin{bmatrix} \hat{\beta}_{11} g_{p11} & \hat{\beta}_{12} g_{p12} \\ \hat{\beta}_{21} g_{p21} & \hat{\beta}_{22} g_{p22} \end{bmatrix}^T \times \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} \tilde{\gamma}_1^T \frac{\partial v_3}{\partial z_1} w_1 + \tilde{\gamma}_2^T \frac{\partial v_3}{\partial z_2} w_2 \\ \tilde{\gamma}_1^T \frac{\partial v_4}{\partial z_1} w_1 + \tilde{\gamma}_2^T \frac{\partial v_4}{\partial z_2} w_2 \end{bmatrix}. \quad (39)$$

The final differential equations, Eqs. (25a), (25b), (31) and (39), still involve the estimating error vectors $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$. These two estimates both have effects on the convergence of system state variables ($x_1, x_2, z_1, z_2, z_3, z_4$). If specifying the adaptive laws of $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$ ($\dot{\gamma}_1$ and $\dot{\gamma}_2$, respectively) as

$$-\dot{\tilde{\gamma}}_1 = \hat{\gamma}_1 \Gamma_1 w_1 (z_1 - \partial v_3 / \partial z_1 z_3 - \partial v_4 / \partial z_1 z_4), \quad (40a)$$

$$-\dot{\tilde{\gamma}}_2 = \hat{\gamma}_2 \Gamma_2 w_2 (z_2 - \partial v_3 / \partial z_2 z_3 - \partial v_4 / \partial z_2 z_4), \quad (40b)$$

where $\Gamma_1 > 0$ and $\Gamma_2 > 0$ are both positive definite and then applying them with the control laws in Eqs. (37) and (38), the system state variables ($x_1, x_2, z_1, z_2, z_3, z_4$) will be driven to approach zeros as t tends to infinity.

Theorem 2. *The proposed control, Eqs. (37) and (38), and adaptive laws, Eqs. (40a) and (40b), stabilizes the repulsive maglev suspension system in Eq. (12) asymptotically if all the control gains $k_i, i = 1, 2, \dots, 6$ are positive and adaptive gains $\Gamma_1 > 0$ and $\Gamma_2 > 0$ are both positive definite.*

Proof. Define a Lyapunov function V_2 :

$$\begin{aligned} V_2 = & \frac{1}{2} [x_1 \quad x_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{1}{2} [z_1 \quad z_2] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \\ & + \frac{1}{2} [z_3 \quad z_4] \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} + \frac{1}{2} \tilde{\gamma}_1^T \Gamma_1^{-1} \tilde{\gamma}_1 + \frac{1}{2} \tilde{\gamma}_2^T \Gamma_2^{-1} \tilde{\gamma}_2 \geq 0. \end{aligned} \quad (41)$$

By substituting Eqs. (25a), (31), (39), (40a) and (40b), the time derivative of V_2 becomes

$$\begin{aligned} \dot{V}_2 = & -k_1 x_1^2 - k_2 x_2^2 - k_3 z_1^2 - k_4 z_2^2 - k_5 z_3^2 - k_6 z_4^2 \\ & + z_1 \tilde{\gamma}_1^T w_1 + z_2 \tilde{\gamma}_2^T w_2 \\ & - z_3 \left(\tilde{\gamma}_1^T \frac{\partial v_3}{\partial z_1} w_1 + \tilde{\gamma}_2^T \frac{\partial v_3}{\partial z_2} w_2 \right) \\ & - z_4 \left(\tilde{\gamma}_1^T \frac{\partial v_4}{\partial z_1} w_1 + \tilde{\gamma}_2^T \frac{\partial v_4}{\partial z_2} w_2 \right) \\ & + \tilde{\gamma}_1^T \Gamma_1^{-1} \dot{\tilde{\gamma}}_1 + \tilde{\gamma}_2^T \Gamma_2^{-1} \dot{\tilde{\gamma}}_2 \\ = & -k_1 x_1^2 - k_2 x_2^2 - k_3 z_1^2 - k_4 z_2^2 - k_5 z_3^2 - k_6 z_4^2. \end{aligned} \quad (42)$$

If $k_i > 0, i = 1, 2, \dots, 6$. Finally, From LaSalles Theorem (Krstic et al., 1995), one can obtain

$$\lim_{t \rightarrow \infty} x_1(t) = 0, \quad (43)$$

$$\lim_{t \rightarrow \infty} x_2(t) = 0. \quad (44)$$

The asymptotic stability of the maglev suspension system is thus achieved.

5. Experimental results and discussion

5.1. Experimental configuration

The maglev suspension system (Fig. 4) consists of the mechanism, a set of power amplifiers as actuators, a Pentium 233 PC as the controller, inductive gauging sensors as feedback sources, a 12-bit ADC and a 12-bit DAC as system input–output devices. The resolution of sensors is 4 μm and the bandwidth of sensors is 3.3 kHz. The control inputs are limited within ± 9 V to protect the stabilization coils. Due to the computational power of Pentium 233 PC, high-speed sampling is possible when complex control algorithm is implemented and the sampling frequency of the controller is 1 kHz.

5.2. Transient responses

The goals set for the maglev suspension system are a demonstration of stability, the decoupling of the degrees-of-freedom, improving the system's rigidity, and robustness to system uncertainties. The stabilization of system is the major part of this research project. A controller is proposed that will stabilize the system outputs, lateral translation (X) and rotation with respect to Z -axis (θ), by two inputs, a control signal for inner tracks' stabilizers (u_1) and the outer tracks' stabilizers (u_2). Four different initial conditions need to be tested as shown in Fig. 6, and the white arrows are the proposed stabilized forces. These initial conditions are caused by the unstable forces generated from the four levitation tracks. These forces are due to the misalignment between carriage magnets and track magnets and only when the system is aligned perfectly, will these forces become zero.

After setting the controller parameters, $k_1, k_2, k_3, k_4, k_5, k_6$, to 200, 200, 200, 200, 400, 400, and adaptive gains, Γ_1, Γ_2 , to

$$\Gamma_1 = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 10000 & 0 \\ 0 & 0 & 10000 \end{bmatrix}, \quad (45)$$

$$\Gamma_2 = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 10000 & 0 \\ 0 & 0 & 10000 \end{bmatrix}. \quad (46)$$

the transient responses of four initial conditions were experimented with and shown in Figs. 7–10. Fig. 7 is the experimental result of initial condition A, the maximum positive translation in X . Fig. 8 is the experimental result

Fig. 6. Four typical cases of initial condition (IC), Case A is the maximum positive translation, Case B is the maximum negative translation, Case C is the maximum positive rotation, Case D is the maximum negative rotation.

Fig. 7. Experimental result of initial condition Case A, the maximum positive translation in X .

of initial condition B, the maximum negative translation in X . Fig. 9 is the experimental result of initial condition C, the maximum positive rotation in θ . Fig. 10 is the experimental result of initial condition D, the maximum negative rotation in θ . All experimental results were tested under the limitation that the control signals, u_1 and u_2 , could not exceed the saturation voltages of the power amplifiers, i.e. 9 V. There are two reasons for this limitation. Firstly,

Fig. 8. Experimental result of initial condition Case B, the maximum negative translation in X .

Fig. 9. Experimental result of initial condition Case C, the maximum positive rotation in θ .

saturation of the power amplifier will become another un-modeled nonlinearity in the system and this will increase the complexity of controller design. Secondly, the saturation limitation can also provide a suitable comparison index in transient responses when different controllers are implemented. All results clearly show that the adaptive backstepping controller achieves a superior performance, shorter settling time and lower overshoot over the adaptive controller without backstepping (Wu & Fu, 1996). This is because the adaptive backstepping control in this paper is better at decoupling nonlinearities and the settling speeds of system variables could be well defined easily when backstepping is applied.

Fig. 10. Experimental result of initial condition Case D, the maximum negative rotation in θ .

5.3. Bandwidth tests of closed-loop system

Because the application of the maglev system in this paper is on a linear guiding system, the ability to reject disturbances is a very important specification. When a linear axial drive, like voice coil motor, is integrated (Fig. 2), the dynamics of the maglev system will be influenced by the disturbances caused by the axial drive. These disturbances can be treated as $d_1(s)$ as in Fig. 11 and the relationship between system outputs and disturbances is

$$\frac{y(s)}{d_1(s)} = \frac{1}{1 + K(s)G(s)} = S(s), \quad (47)$$

where $K(s)$ and $G(s)$ are the transfer functions of controller and plant, respectively. But actually, the behavior of the disturbance $d_1(s)$ is not clear and it is hard to apply $d_1(s)$ to system for bandwidth test. An alternative method is to apply disturbance $d_2(s)$ to the system, and the relationship between system outputs and disturbances $d_2(s)$ is

$$\frac{y(s)}{d_2(s)} = \frac{K(s)G(s)}{1 + K(s)G(s)} = T(s). \quad (48)$$

In Eqs. (47) and (48), $S(s)$ is the so-called sensitivity function. $T(s)$ is the closed-loop system transfer function. It is obvious that

$$T(s) + S(s) = 1. \quad (49)$$

So, the behavior of $S(s)$ can be obtained from $T(s)$. In this paper, the closed-loop system bandwidth will be experimented based on this concept. Disturbances will be applied to system with respect to X and θ individually and the closed-loop transfer functions $T_{\theta\theta}(s)$, $T_{X\theta}(s)$, $T_{\theta X}(s)$

Fig. 11. The block diagram of the closed-loop system bandwidth test.

and $T_{XX}(s)$ are shown in Fig. 12. Each of them represents the relationship between system output and disturbance:

$$T_{\theta\theta}(s) = \frac{\theta(s)}{d(s)}, \quad \text{when disturbance is applied to } \theta. \quad (50)$$

$$T_{\theta X}(s) = \frac{X(s)}{d(s)}, \quad \text{when disturbance is applied to } \theta. \quad (51)$$

$$T_{X\theta}(s) = \frac{\theta(s)}{d(s)}, \quad \text{when disturbance is applied to } X. \quad (52)$$

$$T_{XX}(s) = \frac{X(s)}{d(s)}, \quad \text{when disturbance is applied to } X. \quad (53)$$

The higher bandwidths of $T_{\theta\theta}(s)$ and $T_{XX}(s)$ mean better disturbance rejection abilities. The lower gains of $T_{X\theta}(s)$ and $T_{\theta X}(s)$ mean a better decoupling ability between X and θ . As shown in Fig. 12, the bandwidths of $T_{\theta\theta}(s)$ and $T_{XX}(s)$ are close to 36 and 40 Hz, respectively. The low-frequency gains of $T_{X\theta}(s)$ and $T_{\theta X}(s)$ are both close to 10^{-2} .

5.4. Discussion

Based on the experimental results in this section, adaptive backstepping control in this paper not only provides a simple way to design the desired bandwidth of closed-loop system for disturbance rejection but also improves the ability to decouple nonlinearities. In the transient response experiments, the adaptive backstepping control also gives the system faster response speeds and lower overshoots.

Fig. 12. The results of closed-loop system bandwidth test.

6. Conclusions

In this paper, the classification of the maglev suspension systems was introduced, and the working principles of the design, which is a repulsive maglev suspension system was clarified. Due to the lack of information about magnetic parameters in the system, the plant model was transformed into a parameterized form. By applying adaptive backstepping control, the objective of asymptotically stabilizing the system was achieved. The experimental results are also presented and compared with adaptive control without backstepping (Wu & Fu, 1996). The results demonstrate that adaptive backstepping control performs better.

Acknowledgements

This work was supported by the National Science Council, Taiwan R. O. C. under project No. NSC87-2213-E-002-091.

Appendix A

In Eqs. (2)–(7), the nonlinear scalar functions ξ_1 , ξ_2 , ξ_3 and ξ_4 are:

$$\xi_1(x_1, x_2) = \frac{\mu_0}{2\pi} \left\{ -\frac{[e_M^2 - (c_M - a_1 x_1 + x_2)^2]}{[(c_M - a_1 x_1 + x_2)^2 + e_M^2]^2} + \frac{[e_M^2 - (-c_M - a_1 x_1 + x_2)^2]}{[(-c_M - a_1 x_1 + x_2)^2 + e_M^2]^2} \right\}, \quad (\text{A.1})$$

$$\xi_2(x_1, x_2) = \frac{\mu_0}{2\pi} \left\{ -\frac{[e_M^2 - (c_M + a_2 x_1 + x_2)^2]}{[(c_M + a_2 x_1 + x_2)^2 + e_M^2]^2} + \frac{[e_M^2 - (-c_M + a_2 x_1 + x_2)^2]}{[(-c_M + a_2 x_1 + x_2)^2 + e_M^2]^2} \right\}, \quad (\text{A.2})$$

$$\xi_3(x_1, x_2) = \frac{\mu_0 N}{2\pi} \left\{ \frac{-(d - a_1 x_1 + x_2)^2}{[(d - a_1 x_1 + x_2)^2]^2} + \frac{-(-d - a_1 x_1 + x_2)^2}{[(-d - a_1 x_1 + x_2)^2]^2} \right\}, \quad (\text{A.3})$$

$$\xi_4(x_1, x_2) = \frac{\mu_0 N}{2\pi} \left\{ \frac{-(d + a_2 x_1 + x_2)^2}{[(d + a_2 x_1 + x_2)^2]^2} + \frac{-(-d + a_2 x_1 + x_2)^2}{[(-d + a_2 x_1 + x_2)^2]^2} \right\}, \quad (\text{A.4})$$

where dimensional parameters, $a_1, a_2, b_1, b_2, c_M, d, e_M$ and N are listed in Table 1. μ_0 , the permeability of free space is constant and equal to $4\pi \times 10^{-7} \text{H/m}$.

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