Stability of the frequency adaptive control technique and its application to compact disk drives

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Abstract

This paper presents the stability and implementation of the frequency adaptive control technique (FACT) for periodic disturbance cancellation in compact disk drives. The compact disk drive operates at various rotating speeds and modes, which causes periodic disturbances with different fundamental frequencies. To solve this, FACT provides a suitable framework for periodic disturbance rejection in the cases where the fundamental frequencies of the disturbance are alterable. This paper shows that the convergence property of parameters in the proposed adaptive algorithm is exponentially stable. The FACT controller is also shown to be equivalent to a linear time-invariant controller, and the upper bound of adaptation gains is determined according to system dynamics for the attenuation of periodic disturbances. These results are verified by experiments using the track-following system of a high speed CD-ROM drive. It is found that the stable limits of the adaptation gains match well between analytic and experimental results. The results significantly enhance the ability to design and analyze the FACT controller for a variety of applications where periodic disturbances with various fundamental frequencies are encountered.

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1. Introduction

Adaptive algorithms are widely used for feedforward control in order to minimize the magnitude of the error signals. In many applications where there are periodic disturbances, the adaptive feedforward control is more suitable than methods based on the internal model principal (IMP) (Francis & Wonham, 1976). For example, DC motors exhibit unavoidable vibration at their frequency of rotation and their integer harmonics when these motors are used to drive the disk in the hard disk drives (HDD) or compact disk drives (CDD). The adaptive feedforward cancellation (AFC) (Bodson, Sacks, & Khosla, 1994; Sacks, Bodson, & Messner, 1995) schemes provide useful guidelines for the HDD. Sacks, Bodson, and Khosla (1996) successfully applied the AFC for a high performance magnetic disk drive to

eliminate several harmonics originating from a rotating DC motor. As shown by Kempf, Messner, Tomizuka, and Horowitz (1993), the necessary requirement for such controllers was that the sampling time had to be adjusted so that the duration of an integer number of samples matched the period of the fundamental frequency. In this regard, the implementations of AFC are suitable for cases where the fundamental frequencies are constant.

The method discussed in this paper is the frequency adaptive control technique (FACT) proposed by Liu and Yang (2002, 2004). The FACT is capable of reducing harmonic disturbances, of which the fundamental frequency is changeable. The runout effects are simply cancelled by adding the feedforward terms along with original control input to the dynamic plant. Since the magnitude and phase of the sinusoidal disturbance are generally not known, they must be estimated on-line. A hybrid adaptive algorithm, in which the parameters are adjusted at rates significantly slower than those at which the systems operate, can be used for that purpose. The advantages of this hybrid adaptive scheme over the

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continuous counterparts include less computational effort and more robust properties in the presence of modeling errors (Xie & Wu, 2002; Narendra, Khalifa, & Annaswamy, 1985; Shin, 1992).

In this paper, the analytical results are extended to show that the convergence property of FACT can be asserted through the Lyapunov function approach, even though the infrequent adjustments of parameters occur at fewer discrete instants. Additionally, the results exactly verified that the disturbance rejection by the adaptive FACT algorithm was equivalent to a linear time-invariant (LTI) controller. In fact, it was found possible to predict the upper limit of adaptation gains at every harmonic based on frequency response measurements. The bounds on the adaptation gain obtained by experiment and calculation were verified and compared, showing that the gain limits matched very well. An additional advantage of the FACT, shown experimentally, is that the transient responses are free of oscillation and the adaptive system is less sensitive with smaller adaptation gain. These benefits permit the design of a FACT controller to cancel many harmonics for faster convergence of parameters with smaller adaptation gains.

This paper is organized as follows: Section 2 details the analytic development of FACT on parameters convergence and the limit bound of adaptation gain. In Section 3 the experimental setup and results are described. Discussion and conclusions are presented in Section 4.

2. The frequency adaptive control technique

The target system considered here is a simple continuous-time, dual actuator track-following system

Fig. 1. The plug-in configuration of FACT.

in a CD-ROM drive (Liu & Yang, 2004). The plug-in FACT controller, as shown in Fig. 1, was designed to minimize the tracking error in the track-following system. The transfer functions $G_f(s)$ and $G_c(s)$ represent fine and coarse actuator dynamics, respectively. Two actuators are modeled as a second-order system; and a pair of feedback controllers $C_f(s)$ and $C_c(s)$ are used to stabilize the closed-loop system. The plant $G_f(s)$ effected by the periodic disturbance $d(t)$ is controlled by both the controller $C_f(s)$ and the compensation $v(t)$ through the plug-in FACT. Fig. 2 shows the block diagram of the n th bank FACT for a single frequency at ω_n . Since the bandwidth of the coarse dynamic is much smaller than that of fine dynamic, the transfer function from $v(t)$ to $e(t)$ can be clarified as

$$\frac{e}{v} = \frac{G_f(s)}{1 + G_f(s)C_f(s)} = W(s). \quad (1)$$

Let the periodic error $e(t)$ be given by the M order sinusoidal functions with fundamental frequency ω_1 , i.e.,

$$e(t) = \sum_{n=1}^M (a_n \cos \omega_n t + b_n \sin \omega_n t), \quad (2)$$

where $\omega_n = n\omega_1$, and a_n and b_n are unknown variables. Operated by the frequency sampling filter (FSF), the n th output responding to $e(t)$ is defined by

$$\xi_n(t) = \frac{N}{2}(\alpha_n + j\beta_n), \quad (3)$$

where N is the number of samples in a fundamental period of $e(t)$ and has a minimum value of $2M + 2$. It was shown that the parameters a_n and b_n in (2) can be identified as

$$\begin{aligned} a_n &= \alpha_n \cos \omega_n t + \beta_n \sin \omega_n t, \\ b_n &= \alpha_n \sin \omega_n t - \beta_n \cos \omega_n t. \end{aligned} \quad (4)$$

By defining the output $v(t)$ as

$$v(t) = \sum_{n=1}^M v_n(t) = \sum_{n=1}^M (x_n \cos \omega_n t + y_n \sin \omega_n t) \quad (5)$$

and defining the total energy J of $e(t)$ as

$$J = \sum_{i=1}^M J_i = \sum_{i=1}^M \frac{1}{2}(a_i^2 + b_i^2) = \frac{1}{2}(\mathbf{A}^T \mathbf{A} + \mathbf{B}^T \mathbf{B}) \quad (6)$$

Fig. 2. The block representation of the n th bank FACT.

the adaptation law of x_n and y_n that are used to reduce the energy J can be formulated as

$$\begin{aligned}\dot{x}_n &= -\bar{\mu}_n[W_r(\omega_n)a_n + W_i(\omega_n)b_n], \\ \dot{y}_n &= -\bar{\mu}_n[-W_i(\omega_n)a_n + W_r(\omega_n)b_n],\end{aligned}\quad (7)$$

where $W_r(\omega_n)$ and $W_i(\omega_n)$ are the real and imaginary parts of $W(s)$ at $s = j\omega_n$, respectively. Since the adaptation rate is more than twice as fast as the highest harmonic frequency, i.e.,

$$N\omega_1 \geq (2M + 2)\omega_1 = 2(\omega_M + \omega_1) > 2\omega_M \quad (8)$$

it is therefore sufficient to approximate the adaptation law in (7) with a backward Euler method as

$$\begin{aligned}x_n(i+1) &= x_n(i) - \mu_n[W_r(\omega_n)a_n + W_i(\omega_n)b_n], \\ y_n(i+1) &= y_n(i) - \mu_n[-W_i(\omega_n)a_n + W_r(\omega_n)b_n],\end{aligned}\quad (9)$$

where $\mu_n = \bar{\mu}_n/N\omega_1$.

2.1. Linear representation of the FACT controller

In this section, a general unified formulation of LTI representations is developed for the FACT algorithm. Bayard (1997) showed the necessary and sufficient conditions for LTI representation of a large number of

vector $\theta = [x_n \ y_n]^T$ is tuned in real-time by the adaptation algorithm in (7). The transfer function $\Gamma(s)$ is selected as an integrator $1/s$ especially for the FACT adaptation algorithm.

Letting

$$E = \begin{bmatrix} W_r(\omega_n) & -W_i(\omega_n) \\ W_i(\omega_n) & W_r(\omega_n) \end{bmatrix} \quad (10)$$

and defining

$$\tilde{e}(t) = \begin{bmatrix} \alpha_n(t) \\ \beta_n(t) \end{bmatrix},$$

$$Q(t) = \begin{bmatrix} \alpha_n(t) & \beta_n(t) \\ -\beta_n(t) & \alpha_n(t) \end{bmatrix}$$

and

$$\tilde{\zeta}(t) = E^T \zeta(t)$$

the vector $w(t)$ can be expressed as

$$\begin{aligned}w(t) &= -\bar{\mu}_n Q(t) \tilde{\zeta}(t) \\ &= -\bar{\mu}_n Q(t) E^T \zeta(t) \\ &= -\bar{\mu}_n \tilde{Q}(t) \tilde{e}(t),\end{aligned}\quad (11)$$

where

$$\tilde{Q}(t) = \begin{bmatrix} W_r(\omega_n)\cos\omega_n t + W_i(\omega_n)\sin\omega_n t & -W_i(\omega_n)\cos\omega_n t + W_r(\omega_n)\sin\omega_n t \\ -W_i(\omega_n)\cos\omega_n t + W_r(\omega_n)\sin\omega_n t & -W_r(\omega_n)\cos\omega_n t - W_i(\omega_n)\sin\omega_n t \end{bmatrix}.$$

various adaptive systems with regressors of sinusoidal excitation, especially those used for vibration suppression. The LTI expression permits to precise prediction of the stability boundary of an adaptive system. If the transfer function $W(s)$ is known, then the upper bound of the adaptation gain $\bar{\mu}_n$ can be obtained, where $\bar{\mu}_n$ is regarded as a proportional feedback gain in the LTI representation.

Without loss of generality, the dashed-line box $\tilde{H}_n(s)$ of the LTI structure in Fig. 3 indicates the adaptation process of the FACT scheme in Fig. 2 for a single frequency ω_n . The output $v_n(t)$ of $\tilde{H}_n(s)$ expressed in (5) is constructed as a linear combination of the elements of the regressor vector $\zeta = [\cos\omega_n t \ \sin\omega_n t]^T$, where the

Thus, it is straightforward to show that the right-hand side of (11) is the same as that of (7). Since $\Gamma(s) = 1/s$, it implies that $w(t)$ is equal to $\dot{\theta}(t)$. It follows that (11) is the same as (7) and the dashed-line box in Fig. 3 is equivalent to that in Fig. 2. Finally, by taking the operation $\theta = \Gamma[w]$, we get the relationship from $\tilde{e}$ to $v_n(t)$ as

$$\begin{aligned}v_n(t) &= -\bar{\mu}_n \zeta^T \Gamma[w] \\ &= -\bar{\mu}_n \zeta^T \Gamma[\tilde{Q}\tilde{e}],\end{aligned}\quad (12)$$

where $\Gamma[\cdot]$ denotes $\Gamma(s)$ operating on a time domain vector and is equivalent to $d/dt[\cdot]$ in the current case.

Theorem 1. Let the system from $\tilde{e}(t)$ to $v_n(t)$ in (12) be $v_n = \tilde{H}_n(s)\tilde{e}$.

Its equivalent LTI representation is exactly expressed in the following MISO form

$$\tilde{H}_n(s) = [H_{\alpha_n}(s) \ H_{\beta_n}(s)], \quad (14)$$

where

$$H_{\alpha_n}(s) = -\bar{\mu}_n \frac{W_r(\omega_n)s - W_i(\omega_n)\omega_n}{s^2 + \omega_n^2}$$

$$H_{\beta_n}(s) = \bar{\mu}_n \frac{W_i(\omega_n)s + W_r(\omega_n)\omega_n}{s^2 + \omega_n^2}.$$

Fig. 3. The structure of LTI system $v_n(t) = \tilde{H}_n[\tilde{e}(t)]$.

Proof. see Appendix. $\square$

Theorem 1 shows that an LTI equivalent representation exists for the subsystems from $\alpha_n(t)$ to $v_n(t)$ and from $\beta_n(t)$ to $v_n(t)$. In practice, however, it is more useful to obtain the LTI representation from $e(t)$ to $v_n(t)$ by which the upper limit of $\bar{\mu}_n$ can be decided. According to (4), the value $\alpha_n(t)$ is obtained via a proper coordinate transformation, i.e.,

$$\alpha_n(t) = a_n \cos \omega_n t + b_n \sin \omega_n t. \quad (15)$$

Indeed, the last equation is exactly the same as the n th component of $e(t)$ in (2). In other words, the relationship between $\alpha_n(t)$ to $v_n(t)$ is the same as that between $e(t)$ to $v_n(t)$ at frequency ω_n . Hence, the following results are certified.

Corollary 2. For the FACT scheme at a single frequency ω_n , the equivalent LTI representation is exactly the same as the following closed-form analytic expression

$$H_n(s) = \bar{\mu}_n \frac{W_r(\omega_n)s - W_i(\omega_n)\omega_n}{s^2 + \omega_n^2}. \quad (16)$$

Therefore, the plant $W(s)$ and the equivalent controller $H_n(s)$ function as a standard negative feedback system as depicted in Fig. 4.

For the multi-frequency schemes, it is straightforward to show that the system from $\tilde{e}(t)$ to $v(t)$ is exactly expressed as an LTI system

$$\tilde{H}(s) = [H_A(s) \ H_B(s)], \quad (17)$$

where $H_A(s)$ and $H_B(s)$ are given in closed form as

$$H_A(s) = -\sum_{i=1}^M \bar{\mu}_i \frac{W_r(\omega_i)s - W_i(\omega_i)\omega_i}{s^2 + \omega_i^2},$$

$$H_B(s) = \sum_{i=1}^M \bar{\mu}_i \frac{W_i(\omega_i)s + W_r(\omega_i)\omega_i}{s^2 + \omega_i^2}.$$

Similarly, the equivalent LTI transfer function can be represented as

$$\frac{v(s)}{e(s)} = \sum_{i=1}^M \bar{\mu}_i \frac{W_r(\omega_i)s - W_i(\omega_i)\omega_i}{s^2 + \omega_i^2}. \quad (18)$$

Fig. 4. The equivalent closed-loop system with controller $H_n(s)$ and the plant $W(s)$ in series.

2.2. Upper limit of the adaptation gain

Eq. (18) allows precise prediction of the stability boundary for the adaptive system if the rational transfer function $W(s)$ is known. For the single frequency ω_n , the equivalent open-loop transfer function can be denoted as $H_n(s)W(s)$, where $H_n(s)$ is expressed in (16). Based on the Nyquist criterion (Franklin, Powell, & Emami-Naeini, 1991), a simple root-locus test with $\bar{\mu}_n$ as the proportional feedback gain yields an upper bound on the adaptation gain. In view of the following properties of the frequency response of $H_n(s)$, however, the maximum value of gain $\bar{\mu}_n$ can be calculated directly without the process of root-locus testing.

For example, when the disk is running at constant angular velocity (CAV) 4500 rpm (or 75 Hz), Fig. 5 shows the corresponding dynamic response $W(s)$, in which $W_r(\omega_1) = 1.6239$, and $W_i(\omega_1) = 1.0382$ with $\omega_1 = 75$ Hz. Substituting the values of $W_r(\omega_1)$, $W_i(\omega_1)$ and ω_1 into (16), the frequency response of $H_1(s)$ can be drawn in Fig. 6, where $|H_1(j\omega_1)|$ approaches infinity and the phase approaches -90° as $\omega \gg \omega_1$. Indeed, the magnitude is boundless due to the existence of a pair of poles on the jw -axis at the location corresponding to 75 Hz. Therefore, according to the internal model principle, the linear feedback system in Fig. 4 includes a model of the disturbance generation system, and has perfect disturbance rejection at frequency 75 Hz.

Since the FACT algorithm has an equivalent adaptive rate of $N\omega_1$ and its LTI equivalent $H_1(s)$ can be formulated by (16), the overall system DC gain can be determined by introducing a zero-order holder $Z_0(s)$ into the continuous-time system in Fig. 4. Ideally, the zero-order holder is well known $(1 - e^{-sT})/s$ and can properly be approximated by $e^{-sT/2}$. However, the holder can be approximated by a LTI transfer function which is the following ratio of polynomials in s (Ogata,

Fig. 5. The experimental frequency response results of $W(s)$.

Fig. 6. The frequency response of $H_1(s)$ as the disk running at 4500 rpm.

1987, Chapter 4):

$$\frac{1 - e^{-sT}}{s} = \frac{T}{T/2s + 1}. \quad (19)$$

Substituting the sampling time T as $1/N\omega_1$, the zero-order holder $Z_0(s)$ is chosen to approximate an ideal holder with a DC gain of unity, as follows:

$$Z_0(s) = \frac{2N\omega_1}{s + 2N\omega_1} \quad (20)$$

with its corresponding frequency responses as shown in Fig. 7. Therefore, considering the stability of the compound equivalent system with open-loop transfer function as $H_1(s)Z_0(s)W(s)$, the maximum value of $\bar{\mu}_1$ is determined at the stability margin where $|H_1(s)Z_0(s)W(s)| = 1$ and its phase at -180° . At this gain crossover frequency ω_p , it follows that

$$\arg[H_1(s)]_{s=j\omega_p} + \arg[Z_0(s)W(s)]_{s=j\omega_p} = -180^\circ. \quad (21)$$

Since the argument of $H_1(s)$ approaches to -90° as $\omega_p \gg \omega_1$, in the same way the argument of $Z_0(s)W(s)$ becomes -90° at ω_p . Therefore, the maximum gain $\bar{\mu}_{1,max}$ is obtained by

$$|H_1(j\omega_p)Z_0(j\omega_p)W(j\omega_p)| = 1 \quad (22)$$

and is expressed explicitly as

$$\bar{\mu}_{1,max} = \frac{\omega_p^2 - \omega_1^2}{2\pi|Z_0(j\omega_p)W(j\omega_p)|\sqrt{W_A^2(\omega_1)\omega_p^2 + W_B^2(\omega_1)\omega_1^2}}, \quad (23)$$

where $W_A(\omega_1)$ and $W_B(\omega_1)$ are the real and imaginary parts of $Z_0(j\omega_1)W(j\omega_1)$, respectively. For the implementation of the discrete-time adaptation gain μ_1 , the value $\bar{\mu}_1$ in (23) is modified by its multiplication with the equivalent sampling period $1/N\omega_1$, and hence, the

Fig. 7. The dynamics of approximated zero-order holder $Z_0(s)$ with sampling rate $N\omega_1$.

maximum gain at ω_1 becomes

$$\mu_{1,max} = \frac{\bar{\mu}_{1,max}}{N\omega_1}. \quad (24)$$

By referring to both the condition of $\arg[Z_0(j\omega_p)W(j\omega_p)] = -90^\circ$, and the identified plant $W(s)$ in Fig. 5, the crossover frequency ω_p becomes 835 Hz and $\mu_{1,max}$ is 0.0571, provided N is given as 12.

In general cases, the components in (18) are independent because the FSF is decoupled, so each component has zero contribution to the others. Therefore, the upper limit of μ_i 's is calculated directly by (24), as follows:

$$\mu_{i,max} = \frac{\omega_p^2 - \omega_i^2}{2\pi N\omega_1|Z_0(j\omega_p)W(j\omega_p)|\sqrt{W_A^2(\omega_i)\omega_p^2 + W_B^2(\omega_i)\omega_i^2}} \quad (25)$$

for $i = 1, 2, \dots, M$. It is important to point out that the introduced zero-order holder not only provides a necessary phase lag which matches the sampling time between N samples in a complete disk revolution, but also reduces the magnitude of $Z_0(s)W(s)$ gradually as the frequency increases. Therefore, at the frequency of the highest harmonic, the corresponding value $\mu_{i,max}$ obtained from (25) will be larger than the experimental value, due to the distinct attenuation of the magnitude of $Z_0(s)W(s)$. As shown below, the analytic value of μ_4 is larger than the experimental one for the 4th harmonic of frequency ω_4 .

2.3. Convergence property of FACT

Using Lyapunov theory can help show the convergence of the proposed adaptive algorithm. For simplicity, the following theorem states the convergence

condition for a single harmonic frequency ω_n and is extendable for other harmonics.

Theorem 3. Consider the feedforward adaptive system shown in Figs. 1 and 2 where the adaptive algorithm is designed as (7). The adaptation variables x_n and y_n converge exponentially if and only if the adaptation gain $\bar{\mu}_n$ is less than the value given in (23).

Proof. The proof of necessary condition follows directly from the result that $\bar{\mu}_n$ is the upper bound satisfying the Nyquist criterion. The proof of sufficient condition is complete if we establish that for a $\bar{\mu}_n$ with value less than the one given in (23), the x_n and y_n will converge exponentially. Recall that when vector θ was given by $\theta = [x_n \ y_n]^T$, the in-phase and quadrature coefficients of the n th harmonic of TES can be expressed as

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = E\theta, \quad (26)$$

where E is defined in (10). It is assumed that $\theta^* \in R^2$ is a constant to which the adaptation algorithm drives the vector θ to converge by measuring a_n and b_n . Since the estimation error is defined as

$$\varepsilon = \begin{bmatrix} a_n \\ b_n \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = E(\theta - \theta^*) = E\tilde{\theta}, \quad (27)$$

where $\tilde{\theta} = \theta - \theta^*$ is the parameter error, it is sufficient to show that the trajectory θ in (7) is a candidate so that $\varepsilon \rightarrow 0$ or $\theta \rightarrow \theta^*$ as $t \rightarrow \infty$ by minimizing a certain cost function of ε with respect to θ (Ioannou & Su, 1996). As a matter of fact, the measurements a_n and b_n become zero as the adaptation converges.

As defined in (6), the quadratic cost function can be expressed as

$$\begin{aligned} J &= \frac{1}{2}(a_n^2 + b_n^2) = \frac{1}{2}\varepsilon^T \varepsilon \\ &= \frac{1}{2}\tilde{\theta}^T E^T E \tilde{\theta} = \frac{1}{2}\tilde{\theta}^T \bar{E} \tilde{\theta}, \end{aligned} \quad (28)$$

where

$$\begin{aligned} \bar{E} &= E^T E \\ &= \begin{bmatrix} W_r^2(\omega_n) + W_i^2(\omega_n) & 0 \\ 0 & W_r^2(\omega_n) + W_i^2(\omega_n) \end{bmatrix} \end{aligned} \quad (29)$$

and J can be minimized with respect to $\tilde{\theta}$. Since the quadratic function J is convex over R^2 , its minimum occurs when $\tilde{\theta} = 0$, i.e., $\theta = \theta^*$. In terms of θ and $\tilde{\theta}$, the adaptation law (7) becomes

$$\begin{aligned} \dot{\theta} &= -\bar{\mu}_n E^T E \tilde{\theta} \\ &= -\bar{\mu}_n \bar{E} \tilde{\theta}. \end{aligned} \quad (30)$$

Because $\tilde{\theta} = \theta - \theta^*$, we have

$$\dot{\tilde{\theta}} = \dot{\theta} - \dot{\theta}^* = \dot{\theta} = -\bar{\mu}_n \bar{E} \tilde{\theta}. \quad (31)$$

By using the Lyapunov function approach, the function $V(\tilde{\theta})$ is chosen to be the same as J , a positive definite

function and $V(0) = 0 \ \forall \ t \in R^+$. According to the maximum $\bar{\mu}_n$ for the stability criterion (25), there exists a c_0 such that $\bar{\mu}_n \geq c_0 > 0$. Along any trajectory of compensation gain through its error dynamics (31), we have

$$\begin{aligned} \dot{V} &= \frac{1}{2}(\dot{\tilde{\theta}}^T \bar{E} \tilde{\theta} + \tilde{\theta}^T \bar{E} \dot{\tilde{\theta}}) \\ &= \frac{1}{2}(-\bar{\mu}_n \tilde{\theta}^T \bar{E} \tilde{\theta} - \bar{\mu}_n \tilde{\theta}^T \bar{E} \tilde{\theta}) \\ &= -\bar{\mu}_n \tilde{\theta}^T \bar{E} \tilde{\theta} < -\bar{\mu}_n \tilde{\theta}^T \bar{E} \tilde{\theta} \leq -c_0 \tilde{\theta}^T \bar{E} \tilde{\theta} < 0. \end{aligned} \quad (32)$$

This implies that, as $\tilde{\theta}$ approaches zero, the adaptation algorithm is both convergent and exponentially stable. $\square$

3. Experimental implementation and results

The experimental system used for implementation consists of a commercial high-speed CD-ROM drive and an EPF10K field programmable gate array (FPGA) device (ALTERA, 2002). The drive functioned properly with the original servo processor, including spindle, focus, track-following and track-seeking servos. The experimental system, as shown in Fig. 8, is capable of accessing the track-following error signal (TES) coming from the disk drive through the 8-bit analog-to-digital converter (ADC) at a sampling rate of 176.4 kHz. The tracking output (TRO) is driven by an 8-bits digital-to-analog converter (DAC) that controls the current in the voice coil motor (VCM) of the fine actuator, whereas the feed motor output (FMO) is the output driven by a PWM power module that controls the movement of the sled DC motor. The frequency generator (FG), accounting for the disk position decoder signal from the spindle motor, has six logic outputs in square form for each revolution. In addition to controllers $C_f(s)$ and $C_c(s)$, the proposed FACT algorithms and additional interfacing signals are implemented digitally into the single FPGA chip on the design environment consists of FPGA Express and Quartus II, as provided by Synopsys and Altera Corporation, respectively.

Fig. 8. Experimental setup for tracking-following system in Fig. 1.

Fig. 9. The frequency response measurement setup for plant $W(s)$.

Fig. 9 shows the block diagram indicating the measurement of the frequency response $W(s)$, where the dynamic system analyzer HP-3563A outputs the swept sine signal as input measured on Ch A, and takes the tracking error as output on Ch B for fast Fourier transformation. The resulting frequency response was shown previously in Fig. 5. The coarse actuator with its controller has high gains at low frequencies, so that the smallest sinusoidal frequency sent from the analyzer must be larger than the bandwidth of the coarse dynamics, or else the sled motor will be excited and the track-following system will break down. Thus, to identify the system $W(s)$ correctly, it is necessary to ensure that the coarse actuator is not excited by the external sine signal.

In the following experiments, the FACT controller was able to compensate for any of the first four specific harmonics. In this regard, the least value of spaced points N is 10. However, to make good use of the decoding signal of FG, it is more convenient to adopt N as 12.

Fig. 10 shows a typical TES over three revolutions of the disk when that the disc speed is CAV 4500 rpm. Without FACT, a significant repeatable runout is apparent. Since the track width for the CD-ROM disk is $1.6 \mu\text{m}$, the laser spot deviates about 4% off-track ($0.06 \mu\text{m}$). Fig. 11(a) shows the corresponding spectrum for TES, in which there are peaks at the harmonic frequencies.

3.1. Independence of FACT banks

The previous study (Liu & Yang, 2004) applied FACT to cancel any undesired component in the TES spectrum, showing that only the target components reduced dramatically, while the others remain unchanged. This effect is because the banks of FACT are completely decoupled and the compensation gains converge exponentially. The results in Fig. 11(b), where FACT has been applied to the fundamental frequency (75 Hz), illustrate this property. This character is much different from the AFC method, where the harmonics

Fig. 10. Time domain TES about three revolutions at 4500 rpm (75 Hz) without the FACT.

Fig. 11. (a) Frequency domain TES for Fig. 10 and (b) TES spectrum after the fundamental frequency cancelled by FACT.

other than those being directly cancelled could be attenuated or amplified.

3.2. Convergence verification of compensation gains

The compensation gains $x_n(t)$ and $y_n(t)$ do converge independently, as stated in Theorem 3. For example, Fig. 12 shows time-domain series for the parameters $x_n(t)$ and $y_n(t)$ starting when the adaptation is turned on at time zero. In each sub-figure, part (a) indicates that the n th harmonic is cancelled, whereas in part (b), the four harmonics are cancelled simultaneously. It is clear that each variable approaches the same limit without transitional oscillation, verifying the independence of the FACT algorithm. Constancy in the adaptation gains $x_n(t)$ and $y_n(t)$ results in less convergence time if the tracking ends up at the target track, and their convergent values are saved for the next seeking operation. These adaptation gains can effectively diminish the runout harmonics as soon as the track-following proceeds.

3.3. Determination of adaptation gains

Before implementing (9) to compensate for the undesired harmonic components, it is necessary to determine appropriate values for the adaptation gains. According to (25), the upper limit of adaptation gain

Fig. 12. Time series of $x_n(t)$ and $y_n(t)$ when the FACT start at time zero. In each sub-figure, part (a) indicates that only the n th harmonic is cancelled, and part (b), the four harmonics are cancelled simultaneously.

$\mu_{i,max}$ is obtained for each harmonic component. Table 1 shows a comparison of the analytic and experimental results of $\mu_{i,max}$ for the cases where the disk is running at CAV 4500 rpm (75 Hz) and 6780 rpm (113 Hz). It is found that the experimental limits of $\mu_{i,max}$ decrease as the harmonic order increases, which is due to the decreasing rate of adaptation for higher harmonics. Furthermore, the analytic and experimental gains for the first three frequencies match properly. However, the fourth harmonic exhibits more relative errors due to the introduction of a zero-order holder with magnitudes attenuating at high frequencies. To achieve smaller relative error, a larger value of N must be adopted.

Based on the current value of $N = 12$, the approach provides a method to determine the upper limit and the optimal value of adaptation gain for each harmonic component. For example, Fig. 13 shows the trajectories of the cost J_1 defined as the tracking error energy in (6) of the first harmonic, where the fundamental harmonic

Table 1

The calculated and experimental values of $\mu_{i,max}$

Harmonic i Frequency (Hz)		$\mu_{i,max}$	error $ a - b $	
		Analytic (a)	Experiment (b)	
CAV 4500 rpm				
1	75	0.0571	0.0543	0.0028
2	150	0.0377	0.0366	0.0011
3	225	0.0353	0.0347	0.0006
4	300	0.0398	0.0286	0.0111
CAV 6780 rpm				
1	113	0.0258	0.0270	0.0012
2	226	0.0176	0.0192	0.0016
3	339	0.0190	0.0175	0.0014
4	452	0.0215	0.0127	0.0087

is cancelled with various adaptation gains. The experimental value of 100% $\mu_{i,max}$ is the upper limit where the trajectory J_1 converges to zero exponentially. For

Fig. 13. The trajectories of normalized J_1 in (6) when the fundamental harmonic is reduced at the time zero, where the corresponding adaptation gains μ_i 's are chosen to be the 25%, 50%, 75%, 100%, 110%, and 120% of $\mu_{1,max}$, respectively.

adaptation gains larger than the limit, such as the 110% or 120% of $\mu_{1,max}$, the trajectory J_1 cannot converge to zero. It is seen that when the adaptation gain is less than the limit as those shown in 25%, 50%, and 75% of $\mu_{1,max}$, the transient period behaves a smaller transient oscillation in corresponding to a smaller adaptation gain. As a result, in order to have a better transient performance, it is not proper to use the maximum value $\mu_{i,max}$. Instead, a smaller value of μ_i , for example 1/4 of $\mu_{1,max}$, yields a better transient behavior and a commendable convergence rate. In this way, based on the upper limit provided in (25), the optimal value of adaptation gain can be chosen properly. In applications to CDD, the convergence property and steady-state performance are more important than the convergence time because the transient period takes place only at the first track-following process as the servo system starts up.

3.4. Robust stability of FACT

The function of FACT relies on the values of W_r and W_i , which are identified from the system $W(s)$, as shown in Fig. 9. It is well known that no method can exactly identify a physical system and that the physical parameters can make differences according to the

nominal values depending on the age or environment of the system. The notion of robustness therefore requires that stability is preserved in the face of the modeling errors or system uncertainties. As shown in (16), the FACT controller is equivalent to a LIT controller with a pair of complex poles and a single zero. It can be seen that the location of the complex poles depends only on ω_n so that the convergence property is preserved under the variations associated with W_r and W_i . Such variations, however, move the zero to a different position consequently. In case of both the values of W_r and W_i have positive (or negative) changes with respect to their nominal values, it is straightforward to show that the result is the same as the case that a larger (or smaller) adaptation gain μ_i is adopted. On the other hand, if the values of W_r and W_i deviate individually in opposite direction, the movement of the zero makes a variation in the transient convergence. Depending on the magnitude of variations, the transient behaviors are therefore different. For example, Fig. 14 shows the convergence of trajectory J_1 in the case where W_r and W_i are perturbed in opposite directions with $\pm 20\%$, while Fig. 15 shows the case with perturbation of $\pm 30\%$. It is clear that the values J_1 's are successfully reduced to zero, so the stability is robust with respect to the convergence requirement.

Fig. 14. The trajectories of normalized J_1 converge to zero when the nominal values of W_r and W_i vary from $\pm 20\%$, where (a) indicates the result of $1.2W_r$ and $0.8W_i$ and (b) shows the result of $0.8W_r$ and $1.2W_i$.

Fig. 15. The trajectories of normalized J_1 converge to zero when the nominal values of W_r and W_i vary from $\pm 30\%$, where (a) indicates the result of $1.3W_r$ and $0.7W_i$ and (b) shows the result of $0.7W_r$ and $1.3W_i$.

4. Conclusion

The analytical results presented in this paper provide a useful guideline for the design and implementation of a FACT controller to adaptively compensate the disk runout for the reduction of tracking error. The experiments are conducted on a high speed commercial CD-ROM drive with an FPGA-based development system. First, the frequency response of the optical head with its fine-tune controller is identified to generate a transfer function. Then, the linear time-invariant equivalence of the FACT controller is established and its upper limits of adaptation gains are calculated. The experiments show that the frequency adaptation algorithm can be made exponentially stable and convergent

without oscillation by choosing appropriate adaptation gains less than their upper bounds.

Appendix A

Proof of Theorem 1. Let $r(t)$ be the impulse response of the LTI operator corresponding to $\Gamma(s)$. Eq. (12) can be expressed in terms of convolution integrals,

$$\begin{aligned} v_n(t) = & -\mu_n \cos \omega_n t \int_0^t r(\tau) \{ [W_r(\omega_n) \cos \omega_n(t-\tau) \\ & + W_i(\omega_n) \sin \omega_n(t-\tau)] \alpha_n(t-\tau) \\ & + [-W_i(\omega_n) \cos \omega_n(t-\tau) \\ & + W_r(\omega_n) \sin \omega_n(t-\tau)] \beta_n(t-\tau) \} d\tau \\ & - \mu_n \sin \omega_n t \int_0^t r(\tau) \{ [-W_i(\omega_n) \cos \omega_n(t-\tau) \\ & + W_r(\omega_n) \sin \omega_n(t-\tau)] \alpha_n(t-\tau) \\ & + [-W_r(\omega_n) \cos \omega_n(t-\tau) \\ & - W_i(\omega_n) \sin \omega_n(t-\tau)] \beta_n(t-\tau) \} d\tau. \end{aligned} \quad (\text{A.1})$$

Following the trigonometric identities,

$$\begin{aligned} & \cos \omega_n t [W_r(\omega_n) \cos \omega_n(t-\tau) + W_i(\omega_n) \sin \omega_n(t-\tau)] \\ & + \sin \omega_n t [-W_i(\omega_n) \cos \omega_n(t-\tau) \\ & + W_r(\omega_n) \sin \omega_n(t-\tau)] \\ & = W_r(\omega_n) \cos \omega_n \tau - W_i(\omega_n) \sin \omega_n \tau \end{aligned} \quad (\text{A.2})$$

and

$$\begin{aligned} & \cos \omega_n t [-W_i(\omega_n) \cos \omega_n(t-\tau) + W_r(\omega_n) \sin \omega_n(t-\tau)] \\ & + \sin \omega_n t [-W_r(\omega_n) \cos \omega_n(t-\tau) \\ & - W_i(\omega_n) \sin \omega_n(t-\tau)] \\ & = -W_i(\omega_n) \cos \omega_n \tau - W_r(\omega_n) \sin \omega_n \tau. \end{aligned} \quad (\text{A.3})$$

Eq. (A.1) becomes

$$\begin{aligned} v_n(t) = & -\mu_n \int_0^t r(\tau) [W_r(\omega_n) \cos \omega_n \tau \\ & - W_i(\omega_n) \sin \omega_n \tau] \alpha_n(t-\tau) d\tau \\ & - \mu_n \int_0^t r(\tau) [-W_i(\omega_n) \cos \omega_n \tau \\ & - W_r(\omega_n) \sin \omega_n \tau] \beta_n(t-\tau) d\tau \\ & = -\mu_n r_1(t) * \alpha_n(t) - \mu_n r_2(t) * \beta_n(t), \end{aligned} \quad (\text{A.4})$$

where

$$\begin{aligned} r_1(t) &= r(t) [W_r(\omega_n) \cos \omega_n t - W_i(\omega_n) \sin \omega_n t], \\ r_2(t) &= r(t) [-W_i(\omega_n) \cos \omega_n t - W_r(\omega_n) \sin \omega_n t] \end{aligned} \quad (\text{A.5})$$

and the operator $f_1(t) * f_2(t)$ denotes the convolution of $f_1(t)$ and $f_2(t)$. Since the variable $r_1(t)$ and $r_2(t)$ can be manipulated by standard complex operation, it follows

that

$$\begin{aligned}
 r_1(t) &= W_r(\omega_n)r(t)\cos \omega_n t - W_i(\omega_n)r(t)\sin \omega_n t \\
 &= W_r(\omega_n)r(t)e^{j\omega_n t} - jW_r(\omega_n)r(t)\sin \omega_n t \\
 &\quad + jW_i(\omega_n)r(t)e^{j\omega_n t} - jW_i(\omega_n)r(t)\cos \omega_n t \\
 &= W_r(\omega_n)r(t)e^{j\omega_n t} + W_r(\omega_n)r(t)e^{-j\omega_n t} \\
 &\quad - W_r(\omega_n)r(t)\cos \omega_n t \\
 &\quad + jW_i(\omega_n)r(t)e^{j\omega_n t} - jW_i(\omega_n)r(t)e^{-j\omega_n t} \\
 &\quad + W_i(\omega_n)r(t)\sin \omega_n t \\
 &= W_r(\omega_n)r(t)e^{j\omega_n t} + W_r(\omega_n)r(t)e^{-j\omega_n t} \\
 &\quad + jW_i(\omega_n)r(t)e^{j\omega_n t} - jW_i(\omega_n)r(t)e^{-j\omega_n t} - r_1(t)
 \end{aligned}$$

or

$$\begin{aligned}
 r_1(t) &= \frac{1}{2} W_r(\omega_n)r(t)(e^{j\omega_n t} + e^{-j\omega_n t}) \\
 &\quad + \frac{j}{2} W_i(\omega_n)r(t)(e^{j\omega_n t} - e^{-j\omega_n t}). \quad (A.6)
 \end{aligned}$$

In the similar way, the $r_2(t)$ follows

$$\begin{aligned}
 r_2(t) &= -W_i(\omega_n)r(t)\cos \omega_n t - W_r(\omega_n)r(t)\sin \omega_n t \\
 &= -W_i(\omega_n)r(t)e^{j\omega_n t} + jW_i(\omega_n)r(t)\sin \omega_n t \\
 &\quad + jW_r(\omega_n)r(t)e^{j\omega_n t} - jW_r(\omega_n)r(t)\cos \omega_n t \\
 &= -W_i(\omega_n)r(t)e^{j\omega_n t} - W_i(\omega_n)r(t)e^{-j\omega_n t} \\
 &\quad + W_i(\omega_n)r(t)\cos \omega_n t \\
 &\quad + jW_r(\omega_n)r(t)e^{j\omega_n t} - jW_r(\omega_n)r(t)e^{-j\omega_n t} \\
 &\quad + W_r(\omega_n)r(t)\sin \omega_n t \\
 &= -W_i(\omega_n)r(t)e^{j\omega_n t} - W_i(\omega_n)r(t)e^{-j\omega_n t} \\
 &\quad + jW_r(\omega_n)r(t)e^{j\omega_n t} - jW_r(\omega_n)r(t)e^{-j\omega_n t} - r_2(t)
 \end{aligned}$$

or

$$\begin{aligned}
 r_2(t) &= \frac{j}{2} W_r(\omega_n)r(t)(e^{j\omega_n t} - e^{-j\omega_n t}) \\
 &\quad - \frac{1}{2} W_i(\omega_n)r(t)(e^{j\omega_n t} + e^{-j\omega_n t}). \quad (A.7)
 \end{aligned}$$

Taking the Laplace transform $\mathcal{L}\{\cdot\}$ of (A.6) and (A.7) and using the modulation property,

$$\mathcal{L}\{r(t)e^{j\omega_n t}\} = \zeta(s - j\omega_n t) \quad (A.8)$$

the Laplace transform of (A.4) can be written as

$$\begin{aligned}
 v_n(s) &= -\mu_n \left\{ \frac{W_r(\omega_n)}{2} [\zeta(s - j\omega_n) + \zeta(s + j\omega_n)] \right. \\
 &\quad \left. + \frac{jW_i(\omega_n)}{2} [\zeta(s - j\omega_n) - \zeta(s + j\omega_n)] \right\} \alpha_n(s) \\
 &\quad - \mu_n \left\{ \frac{jW_r(\omega_n)}{2} [\zeta(s - j\omega_n) - \zeta(s + j\omega_n)] \right. \\
 &\quad \left. - \frac{W_i(\omega_n)}{2} [\zeta(s - j\omega_n) + \zeta(s + j\omega_n)] \right\} \beta_n(s). \quad (A.9)
 \end{aligned}$$

For adaptation law used in FACT, the $\zeta(s)$ was chosen as $\zeta(s) = 1/s$, it follows that

$$\begin{aligned}
 v_n(s) &= -\mu_n \frac{W_r(\omega_n)s - W_i(\omega_n)\omega_n}{s^2 + \omega_n^2} \alpha_n(s) \\
 &\quad + \mu_n \frac{W_i(\omega_n)s + W_r(\omega_n)\omega_n}{s^2 + \omega_n^2} \beta_n(s) \quad (A.10)
 \end{aligned}$$

and (14) follows. $\square$

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