Data Bank

Performance evaluation of pairing between sites and wind turbines

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Abstract

This paper presents a simple method for determination of pairing between sites and wind generators. It requires six parameters to describe the matching between turbine models and site characteristics, and the energy output performance can thus be easily estimated and used as the index of pairing effectiveness. To describe a Weibull model of wind speed distribution, the shape parameter and the scale parameter are necessarily required. Besides, four other parameters are chosen to specify the characteristics of the power curve of a wind generator: the cut-in speed, the rated speed, the cut-off speed and the nominal power. By combining these six parameters, the average power output of some particular wind turbine at a specific site can be practically and quickly approximated as a reference for turbine siting consideration. An example is also shown to demonstrate the utilization of the proposed method to choose between a group of wind sites and a list of commercial wind turbines. © 2006 Elsevier Ltd. All rights reserved.

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1. Introduction

The best pairing of a wind turbine and a site is to design the cut-in speed and rated speed of the Wind Energy Conversion System (WECS) exactly according to the wind characteristics of the site. However, this case-by-case process is economically unpractical. A more applicable method is to match a given site with a most suitable machine among

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Nomenclature

β	shape parameter of Weibull distribution
θ	scale parameter of Weibull distribution
$f(v)$	probability density function (PDF) of wind speed
$C(v)$	cumulative density function of wind speed
v_c	cut-in speed of wind generator
v_r	rated speed of wind generator
v_f	cut-off speed of wind generator
P_r	rated (nominal) power output of wind generator
P_{avg}	average power output
PP	pairing performance

existing ones. Numerous criteria have been proposed for the pairing procedure since 1979, and all of them basically bear the similar form of the combination of a statistical model of wind speed distribution and a power curve model of a WECS. Although this is the only procedure one can follow, the results are not always reliable due to the lack of consideration of the degree of approximation [1]. Besides, some of the derived criteria are too complicated to be practically applicable, especially when the number of sites or turbines is large, or when performing iterative procedures such as optimization.

The aim of this paper is to suggest a simple method for estimation of the average power output of the pairing between arbitrary sites and wind turbines. Instead of dealing with tedious bar-chart data of wind distribution and power curves, statistical approximation could easily describe them with several parameters, from which enormous time and storage resource can be saved. This is especially useful when cross-matching between a large number of sites and turbines, or when optimizing the configuration of turbine installation sites on wind farms. By constructing a generalized index for power output performance of specified pairing, the most economically suitable turbine for a given site can be easily chosen, and vice versa.

2. Theories

Before the installation of any wind turbine, it is necessary to estimate the expected power output in order to assess the economic viability of the project, usually based on wind statistics measured over a period of at least 1 year [2]. It has been concluded by Garcia et al. that the two-parameter Weibull probability model fits the real wind data better than the lognormal, gamma and Rayleigh models [3,5,6]. In other words, most wind speed distribution characteristics at any site can be described by two parameters: the shape parameter β , and the scale parameter θ . The fraction of time duration that the wind blows at speed v is thus determined by

$$f(v) = \frac{\beta}{\theta} \left(\frac{v}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{v}{\theta}\right)^{\beta}\right]. \quad (1)$$

Moreover, the cumulative density function of the Weibull distribution is defined as

$$C(v) = \int_0^v f(v) dv = 1 - \exp \left[- \left(\frac{v}{\theta} \right)^\beta \right]. \quad (2)$$

As for the power curve model of a wind turbine, it can be modelled by four spec parameters: the cut-in speed v_c , the rated speed v_r , the cut-off speed v_f , and the nominal power P_r . According to Stevens and Smulders [4] and Pallabazzer [6], The power curve of a wind turbine can be well approximated with the parabolic law:

$$P(v) = \begin{cases} P_r \frac{v^2 - v_c^2}{v_r^2 - v_c^2} & \text{if } v_c \leq v < v_r, \\ P_r & \text{if } v_r \leq v < v_f, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

This approximation works well for pitch-controlled turbines, but for stall-controlled turbines it might fail to fit when v is large. However, this could be compensated by adjusting v_f to account for the discrepancy.

Although some may doubt that the above approximations could be rough or even inaccurate due to the limited number of parameters, and detailed information would thus be ignored, it should be noticed that even the raw data from experiments and observations are merely approximations to the real situations. The wind regime varies from time to time, and the performance of different turbines could fluctuate due to manufacturing or operating conditions. The idea is to facilitate evaluations by using the most easily acquired parameters, to avoid unnecessary or misleading site–turbine pairings, and to provide a quick and applicable result for easy reference.

For example, some might instinctly decide that a good pairing means to match a site of steady and high-speed winds with a wind turbine of higher nominal power. However, this does not always work, and the reasons can be illustrated by Fig. 1. Turbine A represents most of the small-scale wind turbines with lower v_c , v_f and P_r , while turbine B stands for the larger turbines with higher v_c , v_f and P_r . Site A is of steady and faster winds, while site B is characterized by averagely slower but erratic winds. One might match site A for turbine B, in hopes of the merit from the combination of strong winds and higher power output. However, since turbine B has a higher v_c , its power curve results in little overlap with the wind profile of site A. Therefore, the small-scale turbine A would make a much better pairing with site A than turbine B. On the other hand, the wind speed distribution of site B seems to favor turbine A, but in fact turbine B might have a chance to capture more energy since it has higher P_r .

Because it is difficult to predict the overlapped range from Weibull parameters without plotting the distribution curve, the most straightforward and simple solution is to calculate the resulting power output directly. Since the probability density function (PDF) $f(v)$ represents the fraction of duration of wind speed v , the average power output of a turbine can be defined as

$$P_{\text{avg}} = \int_{v_c}^{v_f} P(v) f(v) dv, \quad (4)$$

Fig. 1. Example of turbine-site pairing.

which can be further derived as

$$P_{\text{avg}} = \int_{v_c}^{v_r} P_r \frac{v^2 - v_c^2}{v_r^2 - v_c^2} f(v) dv + \int_{v_r}^{v_f} P_r f(v) dv. \quad (5)$$

This is the general form of power estimation, in which the second term of the right-hand side of Eq. (5) can be derived from Eq. (2) as $P_r[C(v_f) - C(v_r)]$. However, the first term requires some additional work. Pallabazzer suggested solving this term by introducing a $v_{m2,1}$ term, so that Eq. (5) can be further derived as

$$P_{\text{avg}} = P_r \frac{v_{m2,1}^2 - v_c^2}{v_r^2 - v_c^2} [C(v_r) - C(v_c)] + P_r [C(v_f) - C(v_r)], \quad (6)$$

where $v_{m2,1}$ represents the root-mean-square speed:

$$v_{m2,1} = \sqrt{\frac{\int_0^{T_1} v^2 dt}{T_1}} = \left[\frac{\int_{v_c}^{v_r} v^2 f(v) dv}{\int_{v_c}^{v_r} f(v) dv} \right]^{1/2} \quad (7)$$

in which T_1 represents the time duration of wind speeds ranging from v_c to v_r within total time T :

$$T_1 = T[C(v_r) - C(v_c)]. \quad (8)$$

Although Eq. (6) gives a neat form for power estimation, the root-mean-square speed $v_{m2,1}$ must be determined numerically for each value of β and each couple of $v_c - v_r$ [5]. This is quite inconvenient especially when there are many sites to be considered. If an explicit

form of approximation of Eq. (5) can be obtained, the evaluation process can be conducted more efficiently.

3. Modification

Beginning from Eq. (4), the power estimation equation can be derived into another form by different perspective. First it can be rewritten by substituting Eq. (2) into the PDF as

$$P_{\text{avg}} = \int_{v_c}^{v_f} P(v)f(v) dv = \int_{v_c}^{v_f} P(v)C'(v) dv. \quad (9)$$

Using integration by parts, it can be further derived as

$$\begin{aligned} P_{\text{avg}} &= [P(v)C(v)]_{v_c}^{v_f} - \int_{v_c}^{v_f} P'(v)C(v) dv \\ &= [P(v)C(v)]_{v_c}^{v_f} - \frac{2P_r}{v_r^2 - v_c^2} \int_{v_c}^{v_f} v \left[1 - \exp \left[-\left(\frac{v}{\theta}\right)^\beta \right] \right] dv \\ &= [P(v)C(v)]_{v_c}^{v_f} - P_r + \frac{2P_r}{v_r^2 - v_c^2} \int_{v_c}^{v_f} v \exp \left[-\left(\frac{v}{\theta}\right)^\beta \right] dv. \end{aligned} \quad (10)$$

Although Eq. (10) can be further derived by introducing the incomplete gamma functions to yield a much simpler form, the incomplete gamma functions are neither supported on most calculators nor embedded in common programming environments. Since the incomplete gamma function curves usually bear similar characteristics with cubic polynomial curves in relevant cases, we try to approximate the last term of Eq. (10) with a cubic polynomial, from which the average power output can be estimated by Simpson's three-eighths rule as

$$\begin{aligned} P_{\text{avg}} &= P_r \left\{ 1 - \exp \left[-\left(\frac{v_f}{\theta}\right)^\beta \right] \right\} - P_r \left\{ 1 - \frac{1}{4(v_c + v_r)} \left(v_c \exp \left[-\left(\frac{v_c}{\theta}\right)^\beta \right] \right. \right. \\ &\quad \left. \left. + (v_c + 2v_r) \exp \left[-\left(\frac{v_c + 2v_r}{3\theta}\right)^\beta \right] + (v_r + 2v_c) \exp \left[-\left(\frac{v_r + 2v_c}{3\theta}\right)^\beta \right] + v_r \exp \left[-\left(\frac{v_r}{\theta}\right)^\beta \right] \right) \right\} \\ &= P_r \left\{ -\exp \left[-\left(\frac{v_f}{\theta}\right)^\beta \right] + \frac{1}{4(v_c + v_r)} \left(v_c \exp \left[-\left(\frac{v_c}{\theta}\right)^\beta \right] \right. \right. \\ &\quad \left. \left. + (v_c + 2v_r) \exp \left[-\left(\frac{v_c + 2v_r}{3\theta}\right)^\beta \right] + (v_r + 2v_c) \exp \left[-\left(\frac{v_r + 2v_c}{3\theta}\right)^\beta \right] + v_r \exp \left[-\left(\frac{v_r}{\theta}\right)^\beta \right] \right) \right\} \end{aligned} \quad (11)$$

or

$$P_{\text{avg}} = P_r \left\{ \frac{1}{4(v_c + v_r)} \left[\zeta(v_c) + 3\zeta\left(\frac{v_c + 2v_r}{3}\right) + 3\zeta\left(\frac{v_r + 2v_c}{3}\right) + \zeta(v_r) \right] - \frac{\zeta(v_f)}{v_f} \right\}, \quad (12)$$

where

$$\zeta(v) = v(1 - C(v)) = v \exp \left[-\left(\frac{v}{\theta}\right)^\beta \right]. \quad (13)$$

As a function of six variables: β , θ , v_c , v_r , v_f and P_r , Eqs. (12) and (13) give the final form of the cubic approximation for the estimated average power output.

For some cases, $\xi(v_f)$ can be ignored when most of the wind speeds range below v_f . That is, when the wind turbine has a relatively higher v_f , or the wind regime at the site has a relatively higher β . For such special cases, Eq. (12) can be simplified as

$$P_{\text{avg}} = \frac{P_r}{4(v_c + v_r)} \left[\xi(v_c) + 3\xi\left(\frac{v_c + 2v_r}{3}\right) + 3\xi\left(\frac{v_r + 2v_c}{3}\right) + \xi(v_r) \right]. \quad (14)$$

Although one usually expects θ to be greater than 5 and β smaller than 2 for most commercial cases, which could easily render $\xi(v_f)$ nontrivial, Eq. (14) might still be applicable for some special conditions such as small or off-grid turbines in the remote area.

4. Pairing performance

The dimensionless capacity factor, or so-called mean power coefficient is defined as the index of turbine–site pairing performance, and is denoted as PP:

$$\text{PP} = \frac{P_{\text{avg}}}{P_r} \quad (15)$$

which can be interpreted as the power output effectiveness with respect to the nominal power. Furthermore, since the cost of the wind turbine is positively correlated to its nominal power, PP can also be an approximate measure of the power output of the unit cost, even though there is no proportionality between WECS cost and nominal power [6].

By substitution of Eq. (12), (15) can be further derived as

$$\text{PP} = \frac{1}{4(v_c + v_r)} \left[\xi(v_c) + 3\xi\left(\frac{v_c + 2v_r}{3}\right) + 3\xi\left(\frac{v_r + 2v_c}{3}\right) + \xi(v_r) \right] - \frac{\xi(v_f)}{v_f} \quad (16)$$

and can be simplified as follows when Eq. (14) is applicable:

$$\text{PP} = \frac{\xi(v_c) + 3\xi((v_c + 2v_r)/3) + 3\xi((v_r + 2v_c)/3) + \xi(v_r)}{4(v_c + v_r)}. \quad (17)$$

It should be noticed that in Eqs. (16) and (17), PP is independent of P_r . Therefore, two wind turbines with the same v_c , v_r and v_f but different nominal power P_r get the same pairing performance at the same site. This is reasonable for performance-oriented cases because the turbine with larger nominal power is equivalent to the combination of several smaller turbines. According to the assumptions, for example, turbine #6 in Table 3 can be substituted by 10 turbines #1, and turbine #7 is equivalent to 4 turbines #4. However, this idealistic substitution holds only when the installation cost of each turbine cannot be definitely determined. Once the real cost of the turbines is decided, Eq. (15) must be corrected by replacing the denominator with the exact cost.

A suggested flow chart of the selection process is shown in Fig. 2. First, wind characteristics of M sites and power output profiles of N turbines are collected. Wind data of the i th site are referred to as f^i , and the power curve of the j th turbine is referred to as P^j . Then the estimated power output of the j th turbine installed at the i th site is designated as P_{avg}^{ij} , and can be calculated according to Eq. (12). If the power output demand per turbine is given as P_{obj} , then the turbine–site pair with P_{avg}^{ij} which is the closest to P_{obj} would make the most suitable choice. If the power demand is yet undecided, then the pairing

Fig. 2. Flow chart of the selection process.

performance PP^{ij} will be calculated by dividing P_{avg}^{ij} with the total installation cost, or with the nominal power P_r^j , depending on the availability of the installation cost. Subsequently the turbine–site pair with the highest PP^{ij} would be recognized as the best choice of pairing. However, if two or more best pairs of the same PP^{ij} are yielded, implying that either one large turbine or several smaller ones can be used to generate equivalent power output, one must also take the facility area into account. A group of smaller turbines has higher flexibility, exchangeability and lower visual impact, but requires more area of installation,

while an equivalent large turbine is just the exact opposite. One may either convert those disadvantages into cost of money and update the installation cost to reveal the refined PP^{ij} for further comparison, or just decide according to actual needs and environmental limitations.

5. Results and discussion

Some real data of wind characteristics and turbine specifications have been collected for the pairing process. Table 1 shows the Weibull parameters of the wind characteristics gathered from non-commercial weather observation reports of 32 sites located at Uganda [7], Italy [8], Finland [9], Hong Kong [10] and Oman [11]. Table 2 shows the specifications of 20 different wind turbines collected by G. Botta et al. [12], R. Pallabazzer [5] and the author. It should be emphasized that all these data are randomly gathered from different

Table 1
Wind characteristics collected from 32 sites

No.	Type	Region	Site	β	θ
1	A	Uganda [7]	Arua	1.54	3.20
2	A		Gulu	1.37	1.90
3	A		Soroti	1.40	2.30
4	A		Masindi	2.20	2.30
5	A		Lira	1.33	2.10
6	A		Kasese	1.10	2.00
7	A		Mbarara	1.66	2.60
8	B		Kabale	1.60	4.10
9	A		Jinja	1.73	3.00
10	B		Tororo	1.80	4.40
11	A	Italy [8]	Entebbe	1.46	2.50
12	C		Somalia	3.30	6.30
13	D		Calabria	1.20	5.80
14	A	Finland [9]	Sodankyla	1.75	3.20
15	D		Pyhatunturi	2.00	11.50
16	D	Hong Kong [10]	Tai Mo Shan	2.00	8.40
17	D		Waglan Island	2.00	7.70
18	D		Tate's Cairn	2.10	7.30
19	D		Green Island	2.00	6.90
20	D		Cheung Chao	2.10	6.00
21	D		Central Plaza in Wan Chai	1.90	5.80
22	B		Ching Pak House in Tsing Yi	2.00	4.50
23	B		Kai Tak	1.70	4.50
24	B		Sha Lo Wan	1.60	4.40
25	B		Lau Fail Shan	2.10	4.30
26	B	Oman [11]	Tai Mei Tuk	1.70	4.30
27	B		Star Ferry in Tsim Sha Tsui	1.90	4.10
28	B		Tap Mun	1.90	3.80
29	C		Mramul	3.44	6.35
30	C		Masirah	3.13	6.18
31	C		Sur	2.73	6.61
32	C		Thumrait	3.34	7.67

Table 2
Specifications of 20 commercial wind turbines (sorted according to P_r)

No.	Manufacturer	Type	P_r (kW)	v_c (m/s)	v_r (m/s)	v_f (m/s)
1	PROVEN	WT600	0.6	2.5	10	30
2	Southwest	Whisper H40	0.9	3.4	12.5	20.3
3	Southwest	Whisper H80	1	3.1	11.6	20.3
4	INCLIN	1500	1.5	3.5	12	14
5	PROVEN	WT2500	2.5	2.5	12	30
6	PROVEN	WT6000	6	2.5	10	30
7	INCLIN	6000	6	3.5	12	14
8	PITCHWIND	PW20/14	20	3	9	30
9	Grumman	M25	20	4	12.5	22
10	NORDTANK	NTK 75F	75	4	18	25
11	ECOTECNIA	20/150	140	4	14.5	25
12	Riva Calzoni	M 30	200	3.5	11	20
13	VERONET	OEV26/220	220	5	14	25
14	Mitsubishi	MWT 250	275	5	13	24
15	WEG	MS 3	300	6.5	11.5	24
16	WEST	MEDIT I	320	5.5	13.3	20
17	VESTAS	WD 34	400	5	13	25
18	BONUS	450	450	5	15	28
19	BONUS	1.3 MW	1300	3	15	25
20	WEST	Gamma 60	1500	5	13.3	27

Fig. 3. Weibull distributions of all 32 sites.

Fig. 4. Representative types of wind distributions.

sources and have no relative correlations. The purpose is merely to adopt realistic values for parameters, not to endorse or compare between any of these commercial products.

First we take a look at the wind data. Fig. 3 shows the Weibull distributions of all 32 sites. Because they all bear similar forms, to distinguish each one of them would be difficult and unnecessary. Basically they can be categorized into four types, as shown in Fig. 4. Type A represents the sites with the lowest wind speed: site #1, #2, #3, #4, #5, #6, #7, #9, #11 and #14. Type B represents sites with mid-wind speed and moderate range of distribution: site #8, #10, #22, #23, #24, #25, #26, #27 and #28. Type C is of those with higher wind speed and moderate range of distribution: site #12, #29, #30, #31 and #32. Type D is of those with higher wind speed and widest range of distribution: site #13, #15, #16, #17, #18, #19, #20 and #21. It is easy to perceive from Fig. 4 that Type D stands for the better candidates for wind energy utilization, while Type A for the worst choices, but it is not so easy to directly tell the exact suitable turbine for each site.

So we calculate the PP index for each turbine–site pair, and the results are listed in Table 3. The result of the best turbine for each site is shown in boldface, while that of the best site for each turbine is in italic. One might be surprised that there seems to be a perfect site for all turbines, and also a perfect turbine for most of the sites. Turbine #8 surpasses all other turbines at 29 out of 32 sites because it has the second widest operation range from 3 to 30 m/s. Although turbine #1, #5 and #6 have the widest range from 2.5 to 30 m/s, higher v_r limits their chances to go to full capacity. Nevertheless, turbine #1 and #6 might still possess higher PP indices than turbine #8 at some special sites. This suggests that a perfect turbine does not really exist. There will always be some unexpected environments while some other wind turbines may be more suitable.

Table 3
Results of pairing performance (PP) in percent (%)

Site#	Type	Turbine #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
		P_r	0.6	0.9	1	1.5	2.5	6	6	20	20	75	140	200	220	275	300	320	400	450	1300	1500
		V_c	2.5	3.4	3.1	3.5	2.5	2.5	3.5	3	4	4	4	3.5	5	5	6.5	5.5	5	5	3	5
		V_r	10	13	12	12	12	10	12	9	13	18	15	11	14	13	12	13	13	15	15	13
	β	θ and V_f	30	20	20	14	30	30	14	30	22	25	25	20	25	24	24	20	25	28	25	27
1	A	1.54 3.2	7.8	3.6	4.7	3.8	5.2	7.8	3.8	8.5	2.9	1.4	2.1	4.7	1.5	1.7	1.1	1.3	1.7	1.3	2.8	1.7
2	A	1.37 1.9	1.9	0.7	1	0.7	1.4	1.9	0.7	1.8	0.5	0.3	0.4	0.9	0.2	0.2	0.1	0.1	0.2	0.2	0.7	0.2
3	A	1.4 2.3	3.4	1.4	1.9	1.5	2.3	3.4	1.5	3.5	1	0.6	0.8	1.8	0.4	0.5	0.2	0.3	0.5	0.4	1.2	0.5
4	A	2.2 2.3	1.6	0.5	0.8	0.5	1.3	1.6	0.5	1.2	0.2	0.2	0.2	0.5	0	0	0	0	0	0	0.7	0
5	A	1.33 2.1	2.9	1.2	1.6	1.2	2	2.9	1.2	2.9	0.9	0.5	0.7	1.5	0.4	0.4	0.2	0.3	0.4	0.3	1	0.4
6	A	1.1 2	4	1.9	2.9	2	2.8	4	2	4.3	1.5	0.7	1.1	2.4	0.8	1	0.7	0.7	1	0.7	1.5	0.9
7	A	1.66 2.6	3.6	1.5	2	1.5	2.5	3.6	1.5	3.6	1	0.6	0.8	1.8	0.4	0.4	0.1	0.3	0.4	0.4	1.3	0.4
8	B	1.6 4.1	14	7.5	9.4	7.9	9.8	14	7.9	16	6.4	2.9	4.6	9.6	3.7	4.4	3.6	3.6	4.4	3.2	5.4	4.2
9	A	1.73 3	5.4	2.3	3.6	2.4	3.6	5.4	2.4	5.7	1.7	0.9	1.3	2.9	0.7	0.8	0.3	0.5	0.8	0.6	1.9	0.8
10	B	1.8 4.4	16	7.9	10	8.4	10	16	8.4	18	6.8	3	4.8	10	3.8	4.5	3.5	3.6	4.5	3.2	5.7	4.3
11	A	1.46 2.5	4.1	1.7	2.3	1.8	2.7	4.1	1.8	4.2	1.3	0.7	1	2.1	0.6	0.6	0.3	0.4	0.6	0.5	1.5	0.6
12	C	3.3 6.3	31	16	20	17	21	31	17	37	14	5.3	9.5	22	7.7	9.3	6.8	7.3	9.3	6.5	11	8.8
13	D	1.2 5.8	31	21	24	17	25	31	17	33	20	11	16	24	16	18	18	15	18	14	17	17
14	A	1.75 3.2	6.5	2.8	3.8	3	4.3	6.5	3	7	2.1	1.1	1.6	3.6	1	1.1	0.5	0.8	1.1	0.9	2.3	1.1
15	D	2 11.5	68	51	56	35	59	68	35	72	52	34	46	57	46	49	51	43	50	42	45	49
16	D	2 8.4	51	36	41	32	40	51	32	55	35	18	27	43	26	30	31	27	30	23	27	29
17	D	2 7.7	45	31	35	29	35	45	29	50	29	14	22	37	21	25	25	22	25	19	23	24
18	D	2.1 7.3	42	27	32	27	32	42	27	47	26	12	19	34	18	21	21	19	21	16	20	20
19	D	2 6.9	38	24	29	24	29	38	24	43	23	10	17	30	16	18	18	16	18	14	18	18
20	D	2.1 6	30	17	21	18	21	30	18	34	15	6.6	11	22	9.7	11	11	9.7	11	8.2	12	11
21	D	1.9 5.8	28	16	20	17	20	28	17	32	15	6.6	11	21	9.7	11	11	9.7	11	8.3	12	11
22	B	2 4.5	15	7.5	9.6	8	10	15	8	17	6.3	2.8	4.5	9.8	3.4	4	2.8	3.1	4	2.9	5.4	3.8
23	B	1.7 4.5	17	9.1	11	9.6	12	17	9.6	19	7.9	3.5	5.7	12	4.7	5.5	4.6	4.5	5.5	4	6.5	5.3
24	B	1.6 4.4	17	9.2	11	9.6	12	17	9.6	19	8.1	3.6	5.8	12	4.9	5.8	5	4.7	5.8	4.2	6.6	5.5
25	B	2.1 4.3	13	6.1	7.9	6.5	8.5	13	6.5	15	5	2.3	3.5	8	2.5	2.9	1.7	2.2	2.9	2.2	4.4	2.8
26	B	1.7 4.3	15	7.9	10	8.4	10	15	8.4	17	6.8	3	4.9	10	3.9	4.6	3.7	3.7	4.6	3.3	5.7	4.4
27	B	1.9 4.1	12	5.9	7.6	6.3	8.2	12	6.3	14	4.8	2.2	3.5	7.7	2.5	2.9	1.9	2.2	2.9	2.2	4.3	2.8
28	B	1.9 3.8	10	4.5	6	4.8	6.5	10	4.8	11	3.6	1.8	2.6	5.9	1.8	2.1	1.1	1.5	2.1	1.5	3.4	2
29	C	3.44 6.35	32	16	21	18	21	32	18	37	14	5.3	9.6	22	7.7	9.4	6.7	7.3	9.4	6.5	11	8.9
30	C	3.13 6.18	30	15	20	17	20	30	17	35	13	5.2	9.1	21	7.4	8.9	6.5	7	8.9	6.2	11	8.4
31	C	2.73 6.61	35	20	24	21	24	35	21	41	18	7.2	12	26	11	13	12	11	13	9.1	14	12
32	C	3.34 7.67	48	28	35	31	34	48	31	55	26	11	18	37	17	20	19	17	20	14	20	19

On the other hand, site #15 seems to be more adequate for every single turbine than any other sites. This is because site #15, apart from other typical Type D sites, has an excessively wide range of wind speed distribution (Fig. 5). It can be observed in Table 2 that most turbines have a v_f larger than 20 m/s, and site #15 is the only site that exhibits wind speeds over 20 m/s.

Aside from these, Type A sites have very bad PP indices, because they do not have enough wind speed to reach the threshold of cut-in speeds. Besides, turbines with nominal power larger than 200 kW (#13~#20) also exhibit poor performances, because large-scale wind turbines usually have higher cut-in speeds, and thus require stronger winds for operation. Moreover, turbine #2, #4, #5, #7 and #9 also get lower PP indices even if they have low cut-in speeds. This is because their rated speeds are relatively higher, which prevent them from operating at nominal power.

One might wonder about the accuracy of the cubic polynomial approximation in Eq. (11). Therefore, we choose several turbine–site pairs to perform direct numerical integration of Eq. (10), and then compare with those results calculated from Eq. (12) to

Fig. 5. Wind speed distribution of Type D sites.

confirm the accuracy. Every representative site of each type of sites is selected to be paired with five turbines with different v_c , v_r and v_f . The results of the comparison between the numerical integration and the cubic polynomial approximation are shown in Table 4. It should be noticed that although site #2 shows most inaccuracy while its errors range around 10%, this is because Type A sites are of extremely low wind speeds, and thus cause relatively lower power output and higher errors. Basically it can be concluded that the cubic polynomial approximation of Eq. (12) is sufficiently accurate, for most of the errors are lower than 5%.

6. Conclusions

This paper presents a very effective yet simple method for obtaining average power output of any given turbine–site pair. Using only six parameters of the wind characteristics and turbine specifications, one can accurately determine the power output without performing direct numerical integrations, which can be very tedious and time consuming. This method is especially suitable for iterative procedures, since repeated numerical integrations consume huge calculating resources even for modern computers. Although its accuracy would inevitably be compromised for some cases, the evaluations can provide quick and preliminary trends to help eliminate obviously impracticable pairings, and to concentrate the calculating resources on more promising ones.

Moreover, a pairing performance index is also established as a comparing reference between different turbine–site pairings. Although this index is independent of the nominal

Table 4
Comparison between numerical integration and cubic polynomial approximation for average power output

Site #	Type	Turbine #	2				6				7				8				9			
			P_r	V_c'	V_c	V_f'	num.	appr.	err. (%)	num.	appr.	err. (%)	num.	appr.	err. (%)	num.	appr.	err. (%)	num.	appr.	err. (%)	
2	A	1.37	1.9	6.04	6.62	9.6	117	116	-1.1	40.9	44.2	8.0	361	361	-0.1	85.8	95.8	12	20 kW			
	B	1.7	4.5	84.1	81.7	-2.9	1039	1026	-1.3	591	577	-2.4	3864	3843	-0.6	1620	1584	-2.2	4 m/s			
	C	3.44	6.35	155	147	-5.3	1901	1917	0.9	1105	1058	-4.2	7425	7483	0.8	3006	2830	-6	12.5 m/s			
	D	2	8.4	322	322	0.2	3018	3045	0.9	1911	1913	0.1	###	###	0.7	6853	6901	0.7	22 m/s			

power when the installation cost is unknown, one can first use this index to eliminate obviously infeasible choices, and it will thus be easier to select from few remained combinations while bringing other constraints, like the overall power demand and facility area, into consideration.

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