A study of forming pressure in the tube-hydroforming process

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Abstract

The forming pressure required to produce a desired part using the tube-hydroforming process was investigated in the present study. The relationship between hydraulic pressure, outer corner radius of the deformed tube, tube thickness and tube yield stress was established based on a proposed theoretical model. In the theoretical model, the material hardening property was taken into consideration. Since the friction in the tube-hydroforming is smaller than that in the conventional stamping process, the die-closing force was also calculated according to the forming pressure predicted by the proposed theoretical model under frictionless condition. The finite element analysis was performed to validate the proposed theoretical model. In order to confirm the accuracy of the finite element simulations, two different finite element codes were employed to conduct the analysis and the simulation results were compared. The predicted values calculated by the proposed theoretical model were found to agree well with those obtained from the finite element simulation results.

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1. Introduction

Tube-hydroforming process has been widely used to produce automotive structural components due to the superior properties of the hydroformed parts in lightweight and structure rigidity. Compared to the traditional manufacturing process for a closed-section member, i.e., stamping and then welding process, tube-hydroforming leads to cost saving due to fewer die sets being required. In addition, most of tube-hydroformed structural members are of a one-piece type, resulting in a superior structural integrity to an assembly of stamping parts.

In a hydroforming process, the cross sections along the tube can be varied so long as the length of periphery of each deformed cross section is larger than that of the original tube within a certain limit. A set of dies with the die cavity identical to the part shape is designed to form the part. Most hydroformed parts have a complex shape and need a pre-bending process to bend the tube to the curvature of the parts. An additional pre-forming process may be also required to crush the tube so that the crushed tube can be fitted into the die cavity. The major deformation in the hydroforming process is expanding the tube itself to conform with the die cavity by increasing the internal hydraulic pressure to a designed value. For a part with larger difference

between the perimeters of cross sections may require an axial feed to provide more tube material to the larger cross sections to fill the die cavity, in addition to the expansion by hydraulic pressure. The bursting failure mode may occur if the hydraulic pressure is too high [1,2]. The part corner with a smaller radius, on the other hand, may not be completely filled with insufficient internal hydraulic pressure. The underfilling problem is related to the formability of tube material [3,4]. The finite element method was also widely employed to simulate the hydroforming processes and determine the suitable process parameters [5,6].

The major concern of dimensional accuracy in a hydroformed part is the formation of an edge corner to a desired radius. It is known that the hydraulic pressure required in a tube-hydroforming process to manufacture a structural part depends on the minimum corner radius in the part shape. Therefore, a formability study of the tube material is essential to the tube-hydroforming design. The formability analysis of tube-hydroforming may include the relationship between hydraulic pressure, outer corner radius of the deformed tube, tube thickness and tube yield stress. The press tonnage required to hold the tube in place while it is internally pressurized, which is called die-closing force, is also very important in the tube-hydroforming design. A comprehensive knowledge of tube-hydroforming can be found in Ref. [7]. In Ref. [7], an empirical formula for predicting the pressure to form a corner radius is proposed. However, the author points out that the predicted pressure is an upper-

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bound value and is generally significantly higher than the actual pressure required.

In the present study, a theoretical model was proposed to construct the above-mentioned relationship and calculate the die-closing force. Two different finite element codes were employed to validate the proposed theoretical model. A comparison is subsequently made between the predicted values and the finite element simulation results.

2. Theoretical model

In order to simplify the analysis, a long, straight hydroformed part was considered, and the part shape can be formed by expansion alone without the need of axial feed. Thus, a state of plane strain deformation can be assumed in the theoretical model. In the analysis, the following assumptions are made:

- (i) The tube is homogeneous and isotropic.
- (ii) The elastic deformation is negligible and the tube is considered as rigid-plastic.

With the required internal hydraulic pressure calculated by the proposed theoretical model, the die-closing force exerted by the press can be predicted by carrying out a simple analysis. Both the theoretical model and the simple analysis are depicted in the following.

2.1. Relationship between hydraulic pressure and corner radius

A rectangular cross section, as shown in Fig. 1, was adopted to define the dimensions of the tube thickness (t), inner corner radius (r_i), and outer corner radius (r_o). For simplicity, a corner of an arbitrary die cross section, as shown in Fig. 2, was used to construct the theoretical model. In order to further simplify the analysis, the configuration was made symmetric with respect to the y -axis so that only one half of the configuration is necessary to be considered for the analysis. The die opening angle, as denoted by 2α in Fig. 2, was chosen as an arbitrary value to make the model sufficiently general. For the deformed tube at any stage of hydroforming after die-closing, the tube apex which is not in contact with the die surface can be assumed as a circular arc with a uniform thickness, t , and a mean radius of curvature, $\bar{r}$, as shown by $A-B$ in Fig. 2. It is also assumed that a further

Fig. 1. Dimension of a rectangular cross section.

Fig. 2. A corner of an arbitrary die cross section.

deformation caused by an increment of internal pressure only occurs over the circular arc, and the remaining portion of the tube that is already in contact with the die surface, such as $A-P$ and $B-Q$ shown in Fig. 2, remains as it is. Therefore, it may be convenient to use polar coordinates (r, θ) for the analysis of the incipient plastic deformation of the tube apex.

Consider a differential element of the deformed tube at angle of θ , as shown in Fig. 2. The condition of force equilibrium in the r -direction results in

$$\frac{d\sigma_r}{dr} - \frac{\sigma_\theta - \sigma_r}{r} = 0, \quad (1)$$

where σ_θ and σ_r are the hoop and radial stresses, respectively. Since $\sigma_\theta > \sigma_r$ and σ_z is an intermediate stress in the plane strain condition, the Tresca yield criterion is given by

$$\sigma_\theta - \sigma_r = \bar{\sigma} \quad (2)$$

where σ_z is the axial stress and $\bar{\sigma}$ is the flow stress of the tube material. It is readily seen that the hydraulic pressure (p) required to cause the incipient yield of the tube apex is obtained by setting $\sigma_r = -p$ at $r = r_i$ and $\sigma_r = 0$ at $r = r_o$, resulting in

$$p = \bar{\sigma} \cdot \ln \frac{r_o}{r_i} \quad (3)$$

Rearranging Eq. (3) and taking exponential, we obtain the relationship between the outer corner radius of the deformed tube and the hydraulic pressure as

$$\frac{r_o}{t} = \frac{1}{1 - e^{-(p/\bar{\sigma})}} \quad (4)$$

It is to be noted from Eq. (4) that an exponentially proportional relationship exists between the hydraulic pressure required to expand the tube and the ratio of outer corner radius to the current thickness of the deformed tube, instead of the outer corner radius alone.

Both Eqs. (3) and (4) are applicable to a tube without work-hardening behavior. For tubes which work-harden, use of the stress-strain relations must be made to calculate the flow stress $\bar{\sigma}$. It is assumed that the tube work-hardens according to the stress-strain relations

$$\bar{\sigma} = K(\epsilon_0 + \bar{\epsilon})^n, \quad (5)$$

Fig. 3. Configuration of a corner of deformed tube.

where K and ε_0 are material constants, $\bar{\varepsilon}$ the effective plastic strain, and n is the work-hardening exponent. It is obvious from Eq. (5) that the effective strain $\bar{\varepsilon}$ must be calculated first so that the flow stress $\bar{\sigma}$ can be determined.

In order to calculate the effective strain, the configuration of the deformed tube at any stage of the hydroforming process, as shown in Fig. 3, is considered. Assuming that the tube apex $A-B$ is deformed into a new configuration denoted by $A-D-F-E-B$, with $D-F-E$ a circular arc of a mean radius $\bar{r} + d\bar{r}$ and of a uniform thickness $t + dt$, as shown in Fig. 3, when a pressure increment dp is applied. It is noticed that both $d\bar{r}$ and dt have negative values. The thickness of the tube portion which is in contact with the die surface, i.e., $A-D$ and $E-B$, can be assumed as being changed from t to $t + dt$ linearly. The preservation of volume in the plastic deformation results in

$$\bar{r} \cdot 2\alpha \cdot t = (\bar{r} + d\bar{r}) \cdot 2\alpha \cdot (t + dt) - d\bar{r} \cdot \tan \alpha \left[\frac{t + (t + dt)}{2} \right] \quad (6)$$

Neglecting the higher order terms, Eq. (6) becomes

$$\bar{r} \cdot \alpha \cdot dt = d\bar{r} \cdot t \cdot (\tan \alpha - \alpha). \quad (7)$$

The radial strain increment, $d\varepsilon_r$, is expressed by dt/t , or

$$d\varepsilon_r = \frac{dt}{t} = \frac{\tan \alpha - \alpha}{\alpha} \cdot \frac{d\bar{r}}{\bar{r}}. \quad (8)$$

The preservation of volume under the plane strain condition ($d\varepsilon_z = 0$) also implies that

$$d\varepsilon_\theta = -d\varepsilon_r, \quad (9)$$

and the effective increment strain $d\bar{\varepsilon}$ associated with the von Mises yield criterion reduces to:

$$\begin{aligned} d\bar{\varepsilon} &= \sqrt{\frac{2}{9} [(d\varepsilon_r - d\varepsilon_\theta)^2 + (d\varepsilon_\theta + d\varepsilon_z)^2 + (d\varepsilon_z - d\varepsilon_r)^2]} \\ &= \frac{2}{\sqrt{3}} |d\varepsilon_r| \end{aligned} \quad (10)$$

The accumulated effective plastic strain accounts for the entire strain history for the tube, which is pressurized from the initial corner radius $\bar{R}$ to any stage of the hydroforming process, is the integrated value of the effective incremental strain $d\bar{\varepsilon}$, and is given by

$$\bar{\varepsilon} = \int d\bar{\varepsilon} = \frac{2}{\sqrt{3}} \int_{\bar{R}}^{\bar{r}} \frac{\tan \alpha - \alpha}{\alpha} \cdot \left| \frac{d\bar{r}}{\bar{r}} \right| = \frac{2}{\sqrt{3}} \frac{\tan \alpha - \alpha}{\alpha} \cdot \ln \frac{\bar{R}}{\bar{r}} \quad (11)$$

It is to be noted that the effective plastic strain obtained from Eq. (11) is the mean value for the tube apex. The substitution of Eq. (11) into Eq. (5) yields

$$\bar{\sigma} = \left(\varepsilon_0 + \frac{2}{\sqrt{3}} \frac{\tan \alpha - \alpha}{\alpha} \cdot \ln \frac{\bar{R}}{\bar{r}} \right)^n. \quad (12)$$

Combining Eqs. (12) and (3) or Eq. (4), the relationship between the hydraulic pressure, corner radius, and thickness of tube which work-hardens during the hydroforming process can be obtained.

2.2. Die-closing force

The die-closing force calculation is essential for determining the press capacity required for hydroforming a tubular structural member. Since the die force depends on the tube thickness and yield stress in a tube-hydroforming process, in addition to the hydraulic pressure applied, a detailed analysis including the geometric factors and the tube yield stress is necessary in predicting the die-closing force required in a realistic manner.

Fig. 4(a) shows the cross section of an arbitrarily shaped die profile with the deformed tube in the die cavity. The arbitrary die shape was adopted to make the analysis sufficiently general. To facilitate the analysis, it is helpful to make use of the free body diagram of the upper half of the deformed tube as shown in Fig. 4(b). In Fig. 4(b), F_x and F_y denote the reaction forces exerted by the top die in the x and y directions, respectively, p represents the uniform hydraulic pressure acting normally on the inner tube surface, and σ_θ is the hoop stress unevenly distributed along the tube thickness at the end sections, which were chosen horizontally oriented so that the hoop force points downward. The frictional forces are not considered in the free body diagram. Since the press capacity is mainly determined by the vertical component of the die force, the force balance in the y direction is considered in the present study, and thus the die force, F , mentioned in the rest of this paper refers to F_y only.

Fig. 4. An arbitrary die profile with deformed tube. (a) Cross section, (b) free body diagram.

Under frictionless condition, the force balance in the y direction is given by

$$\int_{S_p} P_y dA - \int_{S_t} \sigma_\theta dA - F = 0, \quad (13)$$

where P_y denotes the vertical component of the hydraulic pressure, and S_p and S_t represent the surface areas which the hydraulic pressure and the hoop stress act on, respectively. Note that the deformed tube considered in Fig. 4(b) is assumed to have a unit length, so the quantity F is the die force generated by a unit length of the tube.

The first term in Eq. (13) can be easily evaluated by the product of hydraulic pressure and die width regardless of the die shape. With this in mind, we can calculate the first term in Eq. (13) as

$$\int_{S_p} p_y dA = p \cdot (d - 2t) \quad (14)$$

where t is the tube thickness. It is to be noted that the two times tube thickness is subtracted from the die width, as shown in the parentheses in Eq. (14) because in the actual process the hydraulic pressure acts on the inner surface of the deformed tube instead of the die surface.

Since the hoop stress distribution is not known a priori, an approximation, which assumes that the hoop stress is uniformly distributed across the tube thickness with a magnitude equal to the tube yield stress, was proposed for the calculation. Thus, the second integral in Eq. (13) is given by:

$$\int_{S_t} \sigma_\theta dS = 2\bar{\sigma}t \quad (15)$$

Substituting Eqs. (14) and (15) into (13), we finally obtain the die force as:

$$F = p(d - 2t) - 2\bar{\sigma}t \quad (16)$$

It is to be noted that Eq. (16) only gives the die force for a unit length of a deformed tube. The total force required to hydroform a tubular structural member is the integration of forces calculated from Eq. (16) along the whole deformed length of the tube.

3. Finite element model

The finite element simulations were performed to validate the proposed theoretical models. In order to confirm the accuracy of the finite element simulation results, two different finite element codes, ABAQUS/Standard and PAM.STAMP, were adopted for the analysis. The former was used to perform 2-D plane strain analysis and the later was applied to 3-D simulations. The configuration, as shown in Fig. 5, with various corner radii was adopted to construct the finite element models for both 2-D and 3-D simulations. In the finite element model, the die surface is considered to be rigid. As for the tube, 4-node plane-strain elements and 4-node shell elements were adopted to establish the mesh systems for 2-D and 3-D simulations, respectively. The hydraulic pressure is imposed normally to the inner surface of the tube incrementally and a coefficient of Coulomb friction of 0.05 is assumed. The material properties of SPCC steel are adopted to perform the simulations and the stress–strain relations obtained from experiments is given by $\bar{\sigma} = 645(0.0093 + \bar{\epsilon})^{0.238}$ (MPa).

The relationship between hydraulic pressure and corner radius obtained from the simulation results using both finite element codes for a 2 mm thick tube with an initial corner radius of 40 mm is shown in Fig. 6. It is clearly seen that the values predicted by both finite element codes agree very well with each other. It thus confirms the validity of the finite element analysis.

Fig. 5. Configurations for (a) 3-D and (b) 2-D finite element models.

Fig. 6. Relations between hydraulic pressure and corner radius obtained from different finite element codes.

4. Results and discussions

The configuration with a 90° die opening angle, as shown in Fig. 5, was used to determine the relationship between the hydraulic pressure and the outer corner radius. Since the deformed tube shape at die-closing is unpredictable, finite element simulations were conducted for tube shapes with different initial radii (R) ranging from 10 to 50 mm in an increment of 10 mm. The initial tube thickness is 2 mm and the simulation results are displayed in Fig. 7. A deformation pattern that the curves of the pressure required to deform the tube to a given corner radius almost converge to the curve of $R = 50$ mm is observed, as shown in Fig. 7. This deformation pattern depends on the material properties of the tube. For materials with less work-hardening effect, this deformation pattern will be more obvious as predicted by Eq. (4). It is also seen in Fig. 7 that the internal hydraulic pressure required to start the deformation for a tube with $R = 10$ mm is about 80 MPa. Below this pressure, the tube is almost rigid.

The trend of the relations between hydraulic pressure and outer corner radius predicted by the theoretical model agrees well with the finite element simulation results for all tubes with different initial radii, as shown in Fig. 7. In order to examine the difference between the predicted values and the finite element simulation results more closely, the results for tube of $R = 40$ mm was re-plotted in Fig. 8, as indicated by curves A and B. As seen

Fig. 7. Relations between hydraulic pressure and outer corner radius.

Fig. 8. Comparison between predicted values and finite element results for $R = 40$ mm.

in Fig. 8, the tube starts to deform at about 12 MPa and continues the deformation afterwards. It is also noticed from curves A and B shown in Fig. 8 that the predicted values agree very well with those obtained from the finite element simulations in the lower range of the hydraulic pressure. However, the difference between the predicted values and the simulation results becomes significant at hydraulic pressure higher than 40 MPa, though still in a reasonable range. The difference gets more significant as the hydraulic pressure increases, as shown in Fig. 8. This discrepancy is attributed to the negligence of the variation of tube thickness during the pressurizing process because the original tube thickness is used in Eq. (4) to calculate the required hydraulic pressure. In order to take the tube thickness variation into account, the instantaneous tube thickness during the hydroforming process obtained from the finite element simulation is substituted into Eq. (4) to calculate the relationship between hydraulic pressure and corner radius and the results is plotted as curve C shown in Fig. 8. The comparison between the three curves shown in Fig. 8 reveals that the discrepancy between the predicted values and the finite element simulation results become insignificant as the variation of tube thickness is taken into account in Eq. (4). The validity of the theoretical model is thus confirmed by the finite element analysis.

Once the hydraulic pressure required to deform the tube to a designed value is determined, the die-closing force is readily calculated from Eq. (16). So the die-closing force is not discussed in this paper.

5. Concluding remarks

The theoretical model that predicts the relationship between the hydraulic pressure and the corner radius of the tube to be deformed was validated by the finite element analysis. The comparison between the theoretical predictions and the finite element simulation results also reveals that the relationship between hydraulic pressure and corner radius predicted by the theoretical model could be much more consistent with the finite element analysis if the variation of tube thickness during the hydroforming process is taken into consideration. However, a theoretical model to derive the variation of tube thickness dur-

ing the hydroforming process is yet to be developed. According to the theoretical model, the hydraulic pressure required to deform a tube into a desired part shape depends on the material properties of the tube, tube thickness, and the minimum corner radius of the part shape. Knowing the minimum outer corner radius, which can be tube-hydroformed, the designer is able to design the cross section of a structural member without experiencing underfilling problem in the pressurizing process.

The die-closing force was also calculated by a simple force equilibrium analysis, which could be used to determine the press tonnage required in the tube-hydroforming process. To study the frictional effect on die force, we need to know both the direction and the magnitude of the frictional force at the die-tube interface. The direction of frictional force can be determined only after the neutral point is located. However, due to the difficulty in locating the neutral points around a deformed tube within an arbitrarily shaped die, the directions of frictional force are not known a priori and are die-shape dependent. Hence, a theoretical model that can predict the frictional effect on the die force is not available at this time. However, since the coefficient of friction is generally very small, the die-closing force calculated by Eq. (16) is still useful for the hydroforming process design.

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References

- [1] L.P. Lei, B.S. Kang, S.J. Kang, Prediction of the forming limit in hydroforming process using the finite element method and a ductile fracture criterion, *J. Mater. Proc. Tech.* 113 (2001) 673–679.
- [2] N. Boudeau, A. Lejeune, J.C. Gelin, Influence of material and process parameters on the development of necking and bursting in flange and tube hydroforming, *J. Mater. Proc. Tech.* 125/126 (2002) 849–855.
- [3] S. Fuchizwa, Influence of strain-hardening exponent on the deformation of thin-walled tube of finite length subjected to hydraulic internal pressure, *Proc. Adv. Tech. Plast.* 1 (1984) 297–302.
- [4] D.M. Woo, Tube-bulging under internal pressure and axial force, *J. Eng. Mater. Tech.* 95 (4) (1973) 219–223.
- [5] K. Trana, Finite element simulation of tube hydroforming process-bending, performing and hydroforming, *J. Mater. Proc. Tech.* 127 (2002) 401–408.
- [6] J.B. Yang, B.H. Jeon, S.I. Oh, The tube bending technology of a hydroforming process for an automotive part, *J. Mater. Proc. Tech.* 111 (2001) 175–181.
- [7] H. Singh, *Fundamentals of Hydroforming*, Society of Manufacturing Engineers, USA, 2003.