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On the Nonuniform Deformation of the Cylinder Compression Test

A theoretical model based on Hill's general method is developed in the present study to calculate the flow stress in a cylindrical specimen under axial compression in the presence of friction at the die-specimen interface. Unlike most of the published methods which studied the incipient barreling only, the proposed theoretical model takes the barreled shape of the deforming specimen into account. In order to construct the stress-strain curve, the mean effective strain of the barreled specimen was also calculated on the basis of an assumed velocity field. As the present study shows, the proposed theoretical model provides good results, both in magnitude and in trend, for the prediction of flow stresses in the barreled specimen during the compression test. [S0094-4289(00)00602-2]

1 Introduction

The cylinder compression test has been widely adopted for determining a material's flow stress as a function of the strain during the compression of cylindrical specimens between two flat dies. If the friction is absent at the die-specimen interface, the deformation in the cylinder is uniform and the free surface of the cylinder remains straight during the compression. Thus, the flow stress of the specimen is simply the value obtained by dividing the applied force (P) by the contact area of die-specimen interface (A), and the effective strain is the same as the axial compressive strain given by $\ln(2H_0/2H)$, where $2H_0$ and $2H$ are the initial height and the instantaneous height of the deforming specimen, respectively. In reality, due to the presence of friction at the die-specimen interface, the radial displacement of the material near the interface is restrained while the remaining portion of the specimen bulges out changing the free surface into the form of a barrel. The barreled shape leads to an inhomogeneity of deformation, and therefore, the method used to calculate the flow stress and the effective strain for the uniformly deformed specimen is not applicable.

Many theoretical methods have been proposed to derive the flow stress for a barreled specimen. Siebel [1] assumed no barreling and a low coefficient of friction, and derived a simple well-known formula for the relationship between the mean axial pressure acting at the die-specimen interface and the flow stress of the material under frictionless uniaxial compression. With the use of the upper-bound method, Avitzur [2] has determined a relationship between the applied load and flow stress of the material for a cylinder under uniaxial compression with friction at the die-specimen interface. Later, Lee and Altan [3] also proposed an upper-bound velocity field that considers bulging of the cylinder during the upsetting. They developed computer programs to determine the strain, strain-rate, velocity, and flow-stress distributions. Although the previous works mentioned above have taken the effect of friction into consideration, the efforts were mainly focused on the incipient barreling of the free surface. In other words, the specimen with a straight free surface was used for the analysis, and no attempt was made to consider a barrel-shaped specimen for the analysis.

Since the compression load acting on any cross-sectional area of the barreled specimen has the same value, another way to calculate the flow stress of the specimen may rest on the analysis for the mean cross sectional area. Following this concept, a simple expression for the flow stress (Y) is therefore suggested as

$$Y = \frac{P}{\bar{A}}, \quad (1)$$

where $\bar{A}$ is the mean cross-sectional area obtained by dividing the volume of the specimen (V) by the instantaneous height $2H$ of the specimen. This method is termed the "mean flow stress method" thereafter in this paper.

In 1983, Ettouney and Hardt [4] extended the concept, which Bridgman [5] had developed for calculating the stress distribution at the neck of tensile specimen, to determine the flow stress of the barreled specimen. Considering the barreled shape in the analysis, they derived a formula of the following nature:

$$Y = (\sigma_z)_m \left\{ \left[1 - \frac{2R}{R_2} \right] \cdot \ln \left[1 - \frac{R_2}{2R} \right] \right\}^{-1}, \quad (2)$$

where

$$(\sigma_z)_m = \frac{P}{\pi R_2^2},$$

Y is the flow stress of the material under frictionless uniaxial compression, P is the compression load, R is the radius of the bulge curvature, and R_2 is the maximum radius of the barreled specimen, as shown in Fig. 1.

In the present study, a theoretical model based on Hill's general method was developed to calculate the flow stress of the barrel-shaped specimen, which is compressed under frictional condition. In the theoretical model, the longitudinal nonuniformity was under investigation, and an expression for the flow stress in terms of the compression load and the geometric parameters defining the barreled specimen was obtained. Finite element simulations were also performed in the present study in order to validate the theoretical calculation. The flow stresses calculated according to the proposed theoretical model are compared with the true flow stress and with those predicted by the Ettouney and Hardt's formula (termed the "E-H formula" for short), and the mean flow stress method, based on the compression loads and the geometric parameters obtained from the finite element simulations.

It should be noted that the strain distribution of the barreled specimen is also not uniform and the effective strain cannot therefore be determined explicitly. Nevertheless, the overall axial compressive strain has been commonly used by researchers, without proof, to approximately represent the effective strain. In the present study, an admissible velocity field was proposed to calculate the mean effective strain, and the detailed derivation is presented following the calculation of the flow stress.

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Fig. 1 Configurations of barrel-shaped and equivalent specimens

2 Calculation of the Flow Stress

In 1963, Hill [6] proposed a new method of analysis for metal-forming processes based on a variational principle. Although Hill believed that the proposed method comes close to the ideal, very few investigations on the method have appeared in the published literature. Among them, Lahoti and Kobayashi [7] carried out the analysis of ring compression with barreling, spread in Steckel rolling, and thickness change in tube sinking.

Hill's method begins by selecting a class of velocity field v_i from which the best approximation will eventually be taken. The selected velocity field must satisfy all the kinematic conditions. However, the associated stress field in the deformation zone, which is determined according to the material constitutive law, will not generally satisfy all the static requirements. Hence, the selection criterion for the best approximating stress field σ_{ij} may be considered as

$$\int_V \sigma_{ij} \frac{\partial w_j}{\partial x_i} dV = \int_{S_I} \tau_j w_j dS + \int_{S_C} [(n_i \tau_i) n_j + m k l_j] w_j dS \quad (3)$$

for a sufficiently wide subclass of virtual orthogonalizing motion w_j , where τ_i denotes the surface traction computed from the considered approximating stress field σ_{ij} , n_i , and n_j are the local unit outward normals, l_j is a unit tangent vector opposite in sense to the relative velocity of sliding in the approximating field, and mk represents the constant frictional stress with $0 \leq m \leq 1$, k being the shear yield stress. For metal-forming process, the surface S of the deforming zone usually consists of three distinct parts: S_C is the interface between the die and the workpiece; S_F is the unconstrained surface; S_I is the interface between the deforming zone and the rigid zone. It is to be noted that use of the reverse of the virtual work-rate principle has been made to derive Eq. (3), and although the method is applicable to all types of friction, constant friction is adopted in Eq. (3) for simplicity.

In applying Eq. (3), the orthogonalizing family w_j must be sufficiently wide and extensive to identify a single approximating velocity field in the particular class constructed for satisfying the kinematic conditions. Once the orthogonalizing family is chosen, the calculus of variations technique is applied to Eq. (3), treating w_j as a variation parameter. This furnishes a system of equilibrium equations and boundary conditions, suited to the particular approximating class, allowing us to determine its best member (Lahoti and Kobayashi [7]). The detailed description of the method can be found in (Hill [6]).

To facilitate the analysis, the following assumption are made:

- 1 The deforming material is isotropic and incompressible.
- 2 The elastic deformation is negligible, i.e., the material is considered as rigid-plastic.

Now, consider a barrel-shaped specimen under compression between two flat dies which move toward each other with a unit speed at any stage of compression. The specimen has an instantaneous height of $2H$ and a radius of R_1 at the die-specimen interface, as shown in Fig. 1. Since the nonuniform deformation in the z -direction is our main concern, the selected velocity field must be a function of z . A simple velocity field representing the incipient barreling of the specimen has been shown by Kobayashi et al. [8] to be in the form

$$v_r = \frac{1}{2} r \Phi'(z), \quad v_\theta = 0, \quad \text{and} \quad v_z = -\Phi(z), \quad (4)$$

where v_r, v_θ, v_z are the velocities in the r, θ , and z directions, respectively, the prime denotes differentiation, Φ is a function that is sufficiently differentiable. The velocity field given in Eq. (4) implies the incompressibility of material, while $\Phi(z) = -\Phi(-z)$, $\Phi(H) = -\Phi(-H) = 1$ due to the symmetry with respect to the mid-plane and the velocity boundary conditions, respectively. It is reasonable to suppose that the velocity field given in Eq. (4) continues to hold during a finite compression of the barreled cylinder, and it is therefore adopted in the present study to calculate the flow stress for the barreled cylinder.

One way to choose the virtual orthogonalizing velocities is just to take the similar form as that given in Eq. (4), such as

$$w_r = \frac{1}{2} r \phi'(z), \quad w_\theta = 0, \quad \text{and} \quad w_z = -\phi(z), \quad (5)$$

with $\phi(z) = -\phi(-z)$.

Making use of the following conditions: $S_I = 0$, $\{n_r, n_\theta, n_z\} = \{0, 0, 1\}$, and $\{l_r, l_\theta, l_z\} = \{-1, 0, 0\}$, Eq. (3) can be written as

$$2 \int_0^H \left\{ \int_A \left[\sigma_r \frac{\partial w_r}{\partial r} + \sigma_\theta \frac{w_r}{r} + \sigma_z \frac{\partial w_z}{\partial z} + \tau_{rz} \left(\frac{\partial w_r}{\partial z} + \frac{\partial w_z}{\partial r} \right) \right] dA \right\} dz \\ = 2 \int_A [\sigma_z w_z - m k w_r]_{z=H} dA, \quad (6)$$

noting that n_j and l_i are the unit normal vectors in the z and r directions, respectively.

Substituting Eq. (5) into Eq. (6), yields

$$2 \int_0^H \left\{ \int_A \left[\frac{\sigma_r}{2} \phi' + \frac{\sigma_\theta}{2} \phi' - \sigma_z \phi' + \frac{\tau_{rz} r}{2} \phi'' \right] dA \right\} dz \\ = -2 \int_A \left[\sigma_z \phi + \frac{m k r \phi'}{2} \right]_{z=H} dA. \quad (7)$$

Applying integration by parts and rearranging terms, Eq. (7) becomes

$$\int_0^H \int_A \left[(\sigma_r + \sigma_\theta - 2\sigma_z) \phi' - r \frac{\partial \tau_{rz}}{\partial z} \phi' \right] dA dz \\ + \int_0^H \int_A \frac{\partial (\tau_{rz} r \phi')}{\partial z} dA dz \\ = 2P \phi(H) - m k \phi'(H) \int_{A_H} r dA, \quad (8)$$

where P is the compression load given by $P = -\int_{A_H} \sigma_z dA$, and A_H is the contact area at the die-specimen interface.

Since the area of cross section of the barreled cylinder is itself a function of z , it is convenient to consider Eq. (8) in relation to an equivalent specimen having a uniform cross section with a mean area $\bar{A} = V/(2H)$, where V is the volume of the specimen which remains constant during compression. The configuration of the equivalent specimen is also shown in Fig. 1 by the dashed-line, the equivalent radius being denoted by $\bar{R}$. With the above as-

sumption in mind, the integrations with respect to dA and dz in Eq. (8) can then be interchanged. Consequently, the second integral on the left-hand side of Eq. (8) can be written as

$$\int_0^H \int_{\bar{A}} \frac{\partial(\tau_{rz} r \phi')}{\partial z} dA dz = \int_{\bar{A}} \left[\int_0^H \frac{\partial(\tau_{rz} r \phi')}{\partial z} dz \right] dA = \int_{\bar{A}} \tau_{rz}(H) r \phi'(H) dA, \quad (9)$$

with $\tau_{rz}=0$ at $z=0$.

In the cylinder compression with friction at the die-specimen interface, Hill [6] has shown that the von Mises yield criterion can be taken in the form

$$\sigma_z - \frac{1}{2}(\sigma_r + \sigma_\theta) = -Y, \quad (10)$$

to a close approximation.

Substituting Eq. (9) and Eq. (10) into Eq. (8) and noting that $P\phi(H) = P \int_0^H \phi'(z) dz$, results in

$$\int_0^H \left\{ \int_{\bar{A}} \left(2Y - r \frac{\partial \tau_{rz}}{\partial z} \right) dA - 2P \right\} \phi' dz - \phi'(H) \left\{ mk \int_{A_H} r dA + \int_{\bar{A}} \tau_{rz}(H) \cdot r dA \right\} = 0, \quad (11)$$

where the first and the third integrals are evaluated over the mean area of the barreled cylinder.

Since ϕ' is arbitrary, it follows from Eq. (11) that

$$\int_{\bar{A}} \left(2Y - r \frac{\partial \tau_{rz}}{\partial z} \right) dA - 2P = 0, \quad (12)$$

and

$$mk \int_0^{R_1} 2\pi r^2 dr + \int_0^{\bar{R}} (\tau_{rz})_{z=H} 2\pi r^2 dr = 0, \quad (13)$$

where $\bar{R}$ is the radius corresponding to the mean area $\bar{A}$, and R_1 is the radius of the surface of contact, as shown in Fig. 1.

Integrating out Eq. (13), yields

$$(\tau_{rz})_{z=H} = - \left(\frac{R_1}{\bar{R}} \right)^3 mk. \quad (14)$$

Integrating Eq. (12) with respect to z between the limits 0 to H , and noting the fact that $\tau_{rz}=0$ at $z=0$, gives

$$\int_{\bar{A}} [2YH - r(\tau_{rz})_{z=H}] dA - 2PH = 0. \quad (15)$$

Evaluating Eq. (15) at $z=H$ and substituting from Eq. (14), we obtain the expression for the compression load as

$$P = Y\bar{A} + \frac{\pi R_1^3 mk}{3H}. \quad (16)$$

When the barreling is disregarded, i.e., $R_1 = \bar{R}$, and mk is replaced with μY , Eq. (16) reduces to the well known Siebel formula.

For the von Mises yield criterion, $k = Y/\sqrt{3}$, Eq. (16) becomes

$$P = Y \left(\bar{A} + \frac{\pi R_1^3 m}{3\sqrt{3}H} \right). \quad (17)$$

The expression of the flow stress therefore becomes

$$Y = P \left(\bar{A} + \frac{\pi R_1^3 m}{3\sqrt{3}H} \right)^{-1}. \quad (18)$$

3 Calculation of Effective Strain

Since the barreling of the cylinder leads to a nonuniform deformation, the stress-strain relation may be best represented by adopting the mean flow stress and the associated mean effective strain of the deforming cylinder. In order to determine the mean effective strain, the upper half of the barreled specimen, as shown in Fig. 1, was considered, with a velocity field suggested by Chakrabarty [9] being adopted, which has the form

$$v_r = -\dot{H} \left\{ \frac{r}{2H} + \mu \psi' \left(\frac{z}{H} \right) \right\}, \quad v_\theta = 0,$$

and

$$v_z = \dot{H} \left\{ \frac{z}{H} + \frac{\mu H}{r} \psi \left(\frac{z}{H} \right) \right\}, \quad (19)$$

where $\dot{H}$ is the rate of change of the specimen height, μ is an arbitrary small coefficient representing the frictional condition at the die-specimen interface, and ψ is an approximating function of z/H . Also, $\psi(0) = \psi(1) = \psi(-1) = 0$. The arbitrariness of the function ψ , which represents the barreling effect, makes the velocity field flexible enough to include the whole range of possible shapes of the barreled specimen. The velocity field given in Eq. (19) characterizes the feature of longitudinal nonuniformity in the barreled specimen, and the associated strain rates are

$$\dot{\epsilon}_r = \frac{\partial v_r}{\partial r} = -\frac{\dot{H}}{2H},$$

$$\dot{\epsilon}_\theta = \frac{v_r}{r} = -\frac{\dot{H}}{H} \left\{ \frac{1}{2} + \frac{\mu H}{r} \psi' \left(\frac{z}{H} \right) \right\},$$

$$\dot{\epsilon}_z = \frac{\partial v_z}{\partial z} = \frac{\dot{H}}{H} \left\{ 1 + \frac{\mu H}{r} \psi' \left(\frac{z}{H} \right) \right\},$$

$$\dot{\gamma}_{rz} = \frac{1}{2} \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right) = -\frac{\mu \dot{H}}{2H} \left\{ \psi'' \left(\frac{z}{H} \right) + \frac{H^2}{r^2} \psi \left(\frac{z}{H} \right) \right\}. \quad (20)$$

The corresponding expression for the effective strain rate $\dot{\epsilon}$ is given by

$$\dot{\epsilon}^2 = \frac{2}{3} (\dot{\epsilon}_r^2 + \dot{\epsilon}_\theta^2 + \dot{\epsilon}_z^2 + 2\dot{\gamma}_{rz}^2) = \left\{ 1 + \frac{2\mu H}{r} \psi' \left(\frac{z}{H} \right) \right\} \left(\frac{\dot{H}}{H} \right)^2, \quad (21)$$

the terms of order μ^2 being omitted since they are small compared to unity. This gives the effective strain rate as

$$\dot{\epsilon} = -\frac{\dot{H}}{H} \left[1 + \frac{\mu H}{r} \psi' \left(\frac{z}{H} \right) \right], \quad (22)$$

with the terms of order μ^2 being omitted. The negative sign on the left-hand side of Eq. (22) is used to ensure a positive value for the effective strain rate.

A mean effective strain rate $\dot{\bar{\epsilon}}$ may be defined with sufficient accuracy as

$$\begin{aligned} \dot{\bar{\epsilon}} &= \frac{1}{\pi \bar{R}^2 H} \int_0^H \int_0^{\bar{R}} \dot{\epsilon} 2\pi r dr dz \\ &= -\frac{\dot{H}}{H} \left\{ 1 + \frac{2\mu}{\bar{R}^2} \int_0^{\bar{R}} dr \int_0^H \psi' \left(\frac{z}{H} \right) dz \right\} \\ &= -\frac{\dot{H}}{H}, \end{aligned} \quad (23)$$

since ψ vanishes at both limits of integration, i.e., $\psi(0)=\psi(1)=0$, $\bar{R}$ being the radius of the mean area. The total mean effective strain $\bar{\epsilon}$ at any stage is therefore given by

$$\bar{\epsilon} = \int \dot{\epsilon} dt = \int -\frac{1}{H} \frac{dH}{dt} dt = -\int_{H_0}^H \frac{dH}{H} = \ln\left(\frac{H_0}{H}\right), \quad (24)$$

where $2H_0$ is the initial height of the undeformed specimen. Equation (24) indicates that the mean effective strain for a barreled specimen is the same as in the case of compression without barreling to a first order of approximation. This result provides the theoretical justification for using the axial compressive strain, $\ln(H_0/H)$, or $\ln(2H_0/2H)$, as the effective strain to construct the stress-strain curve for a barreled specimen in the compression test.

4 Finite Element Model

In an actual compression test, it is very difficult to determine the exact frictional condition at the interface between die face and specimen; while the coefficient of friction can be easily incorporated in the finite element method which has been considered as a well-developed technology for the analysis of metal forming processes. It is therefore convenient to perform the finite element simulations, instead of conducting experiments, to validate the proposed theoretical model. The procedure begins with the construction of the finite element model, as shown in Fig. 2. Because of symmetry, only a quarter of the specimen is analyzed. Both the top and bottom dies are considered as rigid and the specimen is modeled by 4-node axisymmetric elements, as shown in Fig. 2. It is seen in this figure that the mesh is very dense so that the profile of the deformed specimen can be determined with sufficient accuracy. The material constitutive relation of the specimen used in the simulations is $Y = K\bar{\epsilon}^n$ with $K = 516$ MPa and $n = 0.23$, where $\bar{\epsilon}$ is the effective plastic strain. Since the barreling of the specimen depends also on the ratio of the initial height ($2H_0$) to the initial diameter (d_0), known as the aspect ratio, in addition to the friction coefficient, specimens of 10 mm in diameter and of two different heights of 15 mm and 10 mm were used in the simulations, the corresponding aspect ratios being 1.5 and 1.0, respectively.

The simulations were performed for both the specimen sizes under the frictional conditions specified by $m = 0$, $m = 0.3$, and $m = 0.5$, respectively. The compression load and the geometric parameters of the deforming specimen, such as the radius of the contact area between die and specimen (R_1), the maximum radius corresponding to the central section of the specimen (R_2), and the instantaneous height of the specimen ($2H$), as shown in Fig. 1,

Fig. 2 Initial mesh for the finite element simulation (unit: mm)

obtained from the finite element simulations are used to calculate the flow stresses Y according to the proposed theoretical model, the E-H formula, and the mean flow stress method. The predicted flow stresses are then compared with each other and with the given material stress-strain relation for the required validation.

The finite element program DEFORM was adopted to perform the simulations, and the program was run on a SGI R4000 workstation.

5 Results and Discussions

In order to validate the finite element simulation itself as an effective method for providing the necessary information for the compression loads and the geometric parameters of the barreled specimen, the simulation was first performed for the specimen that is 10 mm in height ($2H_0/d_0 = 1.0$) under frictionless condition, so that the simulation results can be compared with theoretical predictions. The free surface of the deforming cylinder, obtained from the finite element simulation, remains straight even under a large reduction in height to 4 mm. The flow stresses calculated according to the relation P/A at various effective strains (or axial compressive strains) during the compression are almost the same as those given by the stress-strain curve, as shown in Fig. 3, where P is the compression load, and A is the cross-sectional area of the specimen obtained from the finite element simulation. Although this good agreement between the simulation results and the theoretical predictions only validates the finite element simulations under the frictionless condition, it also implies the feasibility of using this method to simulate the cylinder compression in the presence of friction. It is therefore meaningful to compare the true flow stress given in the finite element method as input data with the flow stresses calculated according to the present theoretical model, the E-H formula, and the mean flow stress method, using the compression loads and the geometric parameters of the deforming specimen furnished by the finite element simulation results.

The compression load at various height reduction ratios ($H_0 - H)/H_0$ for both the specimens under different frictional condi-

Fig. 3 Comparison of the simulated flow stresses (P/A) with the true flow stresses

Fig. 4 Compression load at various height reduction ratios ($2H_0/d_0 = 1.0$)

Fig. 5 Radius of bulge profile at various effective strains ($2H_0/d_0=1.0$)

tions were obtained from the finite element simulations. The results for the specimen of aspect ratio of 1.0 are plotted in Fig. 4. As seen in the figure, the compression load increases as the coefficient of friction increases.

In the E-H formula, the bulge profile is treated as a circular arc and the radius of the bulge curvature R is determined from an empirical formula (Horton et al. [10]) which has the form

$$R = \frac{(2H)^2 + (d_2 - d_1)^2}{4(d_2 - d_1)}, \quad (25)$$

where d_1 , d_2 are the minimum and maximum diameters of the barreled specimen, respectively, and $2H$ is the instantaneous height of the specimen. The finite element simulation results were utilized to calculate the corresponding radius of the bulge curvature for the specimens under different frictional condition according to Eq. (25). Figure 5 shows the calculated radii of the bulge profile versus the mean effective strain of the cylinder with an aspect ratio of 1.0 under constant shear friction given by $m = 0.1, 0.3$, and 0.5 , respectively. As seen in Fig. 5, the radius of the bulge profile decreases almost exponentially with the current height, and the specimen under higher friction coefficients has smaller radii of curvature as expected.

With the compression loads and the geometric parameters generated from the finite element simulation results, the flow stresses calculated according to the proposed theoretical model, the E-H formula, and the mean flow stress method under various frictional conditions were evaluated. To limit the length of this paper, only the results of higher friction coefficients are discussed. The flow stress calculated from the three methods and the true flow stress are plotted against the mean effective strain in Figs. 6 and 7, respectively, for the specimen of aspect ratio 1.5 under the frictional conditions corresponding to $m = 0.3$ and $m = 0.5$. It is seen in Fig. 6 and Fig. 7 that the mean flow stress method shows good results over a range of low compressive strains in which the barreling is not significant. However, when the cylinder is deformed

Fig. 6 Comparison of flow stresses at various effective strains ($2H_0/d_0=1.5$; $m=0.3$)

Fig. 7 Comparison of flow stresses at various effective strains ($2H_0/d_0=1.5$; $m=0.5$)

to large effective strains, say $\bar{\epsilon} \geq 0.8$, the calculated mean flow stress deviates from the true flow stress and the difference becomes larger as the reduction increases, and the overestimated value of the flow stress at higher compressive strains becomes more significant as the coefficient of friction increases. It follows that the calculation based on the mean cross sectional area ($\bar{A}$) is not sufficiently accurate, and the flow stress calculated from this method requires correction at higher reductions in height.

It is also seen in Fig. 6 and Fig. 7 that the flow stresses predicted by the E-H formula agree reasonably well with the true flow stresses at lower compressive strains for both the frictional conditions of $m = 0.3$ and $m = 0.5$, but significant differences are noted at compressive strains higher than 0.4. As noted in both the figures, the calculated flow stress according to the E-H formula increases very rapidly when the compressive strain is greater than 1.0 for $m = 0.3$, and greater than 0.8 for $m = 0.5$. The difference between the predicted flow stresses and the true flow stresses may be attributed to the magnification of error in the approximate formula (Eq. (25)) for the radius of curvature R of the bulge curvature. The rapid increase in the calculated flow stress is due to the fact that R_2 approaches the value of $2R$ as the strain increases, which makes the term $[1 - R_2/(2R)]$ appearing in Eq. (2) tend to zero. When $R_2 > 2R$ the flow stress is not obtainable from Eq. (2). This suggests that the E-H formula is applicable only to cylinders of relatively large aspect ratios.

The flow stresses calculated according to the proposed theoretical model for $m = 0.3$ are also plotted in Fig. 6. It is seen in this figure that the theoretical model underestimates the flow stresses over a range of axial compressive strains lower than 1.2, and overestimates the flow stresses at higher axial compressive strains. However, no marked difference is observed almost throughout the compression process. Although the difference would tend to increase with the increasing compressive strains, the variation is small and the flow stress still agrees reasonably well with the true flow stress. When the friction coefficient is increased to $m = 0.5$, the flow stress calculated by the proposed theoretical model agrees very well with the true flow stress, even for large compressive strains, as indicated in Fig. 7, while the other two theories result in significant deviations from the true flow stress. Figure 7 also suggests that the proposed theoretical model would provide better results for specimens with significant barreling. On the whole, the proposed theoretical model is an improvement over the E-H formula and the mean flow stress method, both of which predict significantly higher flow stresses at higher compressive strains, and therefore increasingly deviate from the true flow stress of the material.

The comparison of the flow stresses calculated by the three methods with the true flow stress is also made in Fig. 8 for the specimen with an aspect ratio of 1.0 and a friction coefficient of 0.5. The lower the aspect ratio, the less is the radius of the barrel curvature, implying that an infinitely long cylinder would not barrel at all, as suggested by Johnson and Mellor [11]. Hence, decreasing the aspect ratio from 1.5 to 1.0 significantly decreases the

Fig. 8 Comparison of flow stresses at various effective strains ($2H_0/d_0=1.0$; $m=0.5$)

barrel radius when subjected to the same height reduction ratio, thereby significantly increasing the inhomogeneity of deformation. It is seen in Fig. 8 that the proposed theoretical model still predicts the flow stress consistently in close agreement with the true flow stress, and is superior to those given by the other two theories. This confirms the validity of the proposed theoretical model for calculating the flow stress for a barreled specimen under axial compression over the whole range of reductions in height.

6 Summary and Concluding Remarks

The purpose of this paper is to calculate the flow stresses for a barreled specimen, resulting from the presence of frictional force at the die-specimen interface during the compression test. The finite element simulations, instead of experimental results, were employed to generate the compression loads and the geometric parameters of the barreled specimen to validate the theoretical model developed in the present study. Unlike most of the published methods, which studied the incipient barreling only, the proposed theoretical model takes the continued barreling of the deforming specimen into account. As the present study shows, the analysis which is based on Hill's general method for metal forming problems has yielded better results, both in magnitude and in trend, for the prediction of flow stresses in the barreled specimen in a compression test. It is also noted from the comparison made between the three methods that the proposed theoretical model can be used more effectively to the cylinder compression test with significant degrees of barreling.

In spite of extensive derivations and complex integrations required in its application, Hill's general method is certainly based on sound mathematical principles, and is flexible enough for in-

roducing various simplifying assumptions. The satisfactory estimate of the flow stress for the barreled specimen under inhomogeneous compression indicates that the assumptions made in the proposed theoretical model are sufficiently realistic.

The velocity field adopted in the present study to calculate the mean effective strain for the barreled specimen also proves to be effective. The fact that the mean effective strain of the barreled specimen is approximately equal to the overall axial compressive strain to the first order simplifies the construction of the stress-strain relation in the compression test.

Furthermore, although the analysis correctly predicts the main trend for the flow stresses in a barreled specimen during the compression test, the proposed theoretical model would tend to underestimate the flow stress of specimen at lower compressive strains. However, the difference between the calculated flow stresses and the true flow stresses is not significant. It is suggested that the inclusion of a shape factor in Eq. (18), which can allow for the change in the barrel geometry during the compression, might improve the accuracy of the proposed theoretical model.

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