The physical mechanism of symmetric vortex merger: A new viewpoint

Mei-Jiau Huang

Department of Mechanical Engineering, National Taiwan University, Taipei, Taiwan 10617, Taiwan

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The physical mechanism of symmetric vortex merger is investigated in use of a resurrected core-spreading vortex method. By taking advantage of the Lagrangian characteristics of the vortex method, both Eulerian and Lagrangian flow structures are obtained and used to explore the cause of merger. The simulation results suggest that a complete merging process is as follows: the vortices deform first due to the mutually induced straining; the deformation results in elliptical vortices and an angle between the major axis of each elliptical vortex and the line joining the two vortices, which in turn cause an attraction of fluid particles from one vortex to the other; sheetlike vortex structures are thus formed; and finally the velocity field induced by these sheetlike structures readily pushes two vortex cores together. This study suggests that the competition between the self-induced rotation and mutual attraction of vortices governs the formation of the sheetlike structures, and consequently the merger. When the flow is viscous, the separation between vortices reduces and the mutual attraction increases with time by diffusion. As the mutual attraction dominates over the self-induced rotation, sheetlike structures are formed gradually and merger eventually occurs. The onset time of merger is thus found to depend not only on the initial separation but also on the Reynolds number. The former determines when the mutual attraction will become dominant and the latter controls the speed at which sheetlike structures grow. © 2005 American Institute of Physics.

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I. INTRODUCTION

Merger has been the subject of intense research for the last four decades for it plays a major role in two-dimensional flows and quasigeostrophic turbulence.^{1–4} The most important discovery is that an inviscid critical distance between the vorticity centroids exists. When the distance between the vorticity centroids is larger than the critical distance, two patches of vorticity rotate around each other indefinitely. On the other hand, when the distance is below the critical distance, two vortices are rapidly deformed, growing arms of vorticity and merging into a single vortex. A body of work has addressed the question of a critical distance. In a theoretical study, Saffman and Szeto⁵ showed that the inviscid critical distance (b_{cr}) is $3.4a$ ($a/b_{cr}=0.29$), where a is the characteristic radius of the vorticity patches. From numerical simulations, it was found that $a/b_{cr}=0.26–0.32$ (see Overman and Zabusky,⁶ Zabusky, Hughes, and Roberts,⁷ Rossow,⁸ and Dritschel⁹). The first confirmation of a critical distance in an experiment is that of Griffiths and Hopfinger,¹⁰ who found $a/b_{cr}=0.30$. Mitchell and Driscoll¹¹ used pure electron plasma columns to model vortices evolving according to the Euler equation. They found the electron vortices merge immediately for initial separations $b<3.1a$ ($a/b_{cr}=0.32$). Meunier and Leweke¹² impulsively rotated two flat plates with sharpened edges in a water tank to generate vortices. Their data suggested $a/b_{cr}=0.29$. Cerretelli and Williamson also obtained this critical ratio experimentally recently.¹³

When the viscous effect is taken into consideration, the vortex size grows in time and merger always occurs. Meunier and Leweke¹² recognized three stages of merger. The

first stage is a viscous metastable state. Two vortices rotate around each other and the separation distance remains approximately constant. During the second convective stage, the vortex centers get closer and rapidly merge into a single core. In the third stage, the spiral vorticity arms are spread out and smoothed by diffusion. Axisymmetrization of the final vortex through filamentation was observed.¹⁴ Cerretelli and Williamson¹³ further divided the last stage into two—the second diffusive stage and the merged diffusive stage. In the second diffusive stage, the vortex centers stop moving closer but the locations of peak vorticity continue getting closer by diffusion. The ultimate stage is one where the merged vortex core grows by diffusion.

Although merger has been observed numerically and experimentally, its physical mechanism is not clear so far. Generally, it is believed that when vorticity is advected out of the vortex cores and into the filaments, by conservation of angular momentum, the cores correspondingly must move toward each other. The attracting metastable state before merging is a rapid adaptation process of each vortex to the external strain field generated by the other vortex. The strain field tends to elliptically deform the core of the vortex and the vortex ellipticity oscillates during the adaptation process (see Le Dizès and Verga¹⁵). The oscillations are attributed to a linear Kelvin mode.^{15,16} A merging criterion was established based on the stability of the metastable state.¹⁷ Melander, Zabusky, and McWilliams¹⁸ deduced equations for the centroid positions, for the aspect ratio of the two vortices, and for their orientations from a “moment model.” Employing a co-rotating reference frame, they emphasized the existence of an exchange band bounded by homoclinic and heteroclinic manifolds passing the hyperbolic fixed points. Within

the exchange band, two vortices are able to exchange vorticity. Melander *et al.*¹⁸ then proposed that if the vorticity distribution extends to regions outside the exchange band, the vorticity configuration ejects filaments and undergoes merger. A sufficient and necessary condition for merger was so obtained. Cerretelli and Williamson¹³ divided the vorticity field into symmetric and antisymmetric parts. They argued that the antisymmetric vorticity field comprises two counter-rotating vortex pairs, whose induced velocity field readily pushes the two centroids together.

Unlike previous investigations that mainly explored the Eulerian flow structures, Velasco Fuentes¹⁹ dealt with the advection of fluid particles in merger. The flow geometry was used to quantify the efficiency of merger, which is defined as the ratio of the circulation of the resultant vortex to the total circulation of the original vortices. Using the vortex-in-cell method and taking advantage of the finite-time unstable and stable manifolds associated with transient fixed points,²⁰ Velasco Fuentes found that filamentation is not produced by the penetration of a stagnation point of the Eulerian field into the vortex, but by the penetration of a stable manifold of a Lagrangian hyperbolic trajectory. Through observations, Velasco Fuentes further concluded that merger is possible even if the vorticity distribution does not extend to regions outside the exchange band defined by Melander *et al.*,¹⁸ and that the filamentation is not the cause of merger but one of its effects. A use of the Lagrangian flow geometry in studying merger is seemingly advantageous.

In this work, the attempt is to explore the physical mechanism of merger by investigating both the Lagrangian and the Eulerian flow geometries. Here the Lagrangian flow structure means the trace of fluid/vorticity particles as well as the trace of the vorticity contours of each of the two vortices. The numerical method employed is a core-spreading vortex method, resurrected by a vortex splitting technique²¹ and a vortex combining technique.²² From the present simulations, it is observed that the vortices deform first due to the mutually induced straining field, the deformation causes an attracting velocity of the fluid particles from one vortex to the other (a phenomenon very similar to the vortex stripping), a sheetlike vortex structure is then formed beside each of the eroded vortex core structures, and finally the velocity field induced by these sheetlike structures readily pushes two vortex core structures together. Besides, it is also observed that the filamentation is not the cause of merger but one of its effects.

This paper is arranged as follows. The numerical method used is introduced in Sec. II. The so-obtained Lagrangian and Eulerian flow geometries are presented and discussed in Sec. III. A physical mechanism for merger is proposed and examined there as well. Conclusions are given in the last section.

II. NUMERICAL METHOD

The numerical method employed in the present study is the discrete core-spreading vortex method, in which the vorticity field is composed of many Gaussian vorticity (computational) elements as follows:

$$\omega(\vec{x}, t) = \sum_{j=1}^N \frac{\Gamma_j}{\pi \sigma_j^2} \exp\left(-\frac{|\vec{x} - \vec{x}_j|^2}{\sigma_j^2}\right). \quad (1)$$

The location of the center ($\vec{x}_j$), the core width (σ_j), and the strength (Γ_j) of the j th computational element are time-marched according to

$$\frac{d\Gamma_j}{dt} = 0, \quad (2)$$

$$\frac{d\vec{x}_j}{dt} = \vec{u}(\vec{x}_j, t), \quad (3)$$

$$\frac{d\sigma_j^2}{dt} = 4\nu, \quad (4)$$

where ν is the fluid viscosity and the velocity $\vec{u}$ is computed through the Biot–Savart law.²³ The core width, however, cannot grow without limit due to the numerical error caused by the ignorance of the shape deformation of computational elements.²⁴ In the present study, a vortex splitting method slightly different from that proposed by Rossi²¹ is employed. Let σ_{cr} be the maximum allowable core width. A computational element is split into $1+M$ elements of core width $\alpha\sigma_0$, $0 < \alpha < 1$, when its core width (σ_0) has grown greater than σ_{cr} . One of these $1+M$ elements will replace the original element, and the others will be uniformly distributed around the former at a distance r away. The strengths of the $1+M$ elements and the value of r are then determined by preserving the second and the fourth moments of vorticity.²⁵ One may notice that the splitting parameter α is free. The larger it is, the smaller the error is induced in a single splitting event but the fatter elements resulted and consequently more splitting events are required during a fixed time period. A value of $\alpha=0.8-0.9$ is recommended by experience.

As time increases, the total number of the computational elements increases rapidly due to the splitting. To control the computational amount, similar and close-by elements should be properly combined into one.²² The combining method employed herein is almost the same as that developed in a previous work of the author.²⁵ The only difference lies in the method of selecting similar and close-by computational elements. It is designed in a more efficient way herein as described below. First, the region occupied by the whole computational elements is divided into rectangular cells of size $\Delta x \times \Delta y$. Elements within a same cell are then collected and form a group. Next, elements in each group (i.e., in each cell) are examined whether all or a portion of them can be combined into one.

The first criterion for a combination is

$$\Gamma_0 / \Gamma_{ref} < \pi \varepsilon \sigma_{min}^2, \quad (5)$$

where Γ_0 is the total circulation of the computational elements to be combined, σ_{min} is the smallest core width among those elements (that is expected to be equal to or greater than $\alpha\sigma_{cr}$), and Γ_{ref} is a reference circulation. If the criterion Eq. (5) is satisfied, the error induced by such a combination is known to be less than $\varepsilon \Gamma_{ref}$.^{22,25} The circulation Γ_0 , the width σ_0 , and the location $\vec{x}_0$ of the resulting element are

then computed by preserving the total circulation, the first and the second moments of vorticity. If the resulting core width σ_0 is smaller than σ_{cr} , the combination is judged acceptable. In summary, the complete combining algorithm is designed as follows.

- (1) Choose Γ_{ref} and specify the error tolerance ε .
- (2) Find the region occupied by all the computational elements. Divide the region into rectangular cells and recognize elements within each cell.
- (3) Renumber the elements successively from one cell to another.
- (4) For each cell in which there exists more than one element, do the following: Let $r+1, r+2, \dots, r+m$ be the computational elements in the cell. Let $i=r+1$.
- (5) Do while $i < r+m$ and so far no combination has been accepted.
- (6) Set C consisting of $i, i+1, \dots, j$, where $j \leq r+m$ is the largest integer that keeps the total circulation in C less than $\pi\varepsilon(\alpha\sigma_{cr})^2\Gamma_{ref}$.
- (7) Do while $j > i$ and so far no combination has been accepted.
- (8) Compute $\vec{x}_0$ and σ_0 from elements in C . If $\sigma_0 \leq \sigma_{cr}$, replace $\{\Gamma_i, \sigma_i, \vec{x}_i\}$ by $\{\Gamma_0, \sigma_0, \vec{x}_0\}$ and remove all the other vortices in C from the simulation. Otherwise, get rid of the j th element from C and renew j by $j-1$.
- (9) End do while j .
- (10) Renew i by $i+1$.
- (11) End do while i .

The above algorithm searches for a combination to a certain extent and allows at most one combination in each cell. The index-dependence²² is not completely eliminated but is weakened in this newly proposed algorithm.

The advantage of the core-spreading vortex method is that it is purely Lagrangian. Both Eulerian and Lagrangian flow structures can thus be obtained simultaneously. Furthermore, in order to capture the evolution of each vortex, only one of the two vortices in merger is simulated and the effect of the other vortex is counted by the image method. That is, all the computational elements tracked and stored in the computer memory, say $\{\Gamma_j, \sigma_j, \vec{x}_j\}$ for $j=1, 2, \dots, N$, are constituents of just one of the two vortices. Elements, $\{\Gamma_j, \sigma_j, \vec{x}_j\}$ and $\{\Gamma_j, \sigma_j, -\vec{x}_j\}$, are both existent in the flow, however, and make contributions to the velocity field. The simulation flows are thus made symmetric with respect to the origin at all times.

In all the simulations discussed below, they are performed by choosing $\alpha=0.85$, $M=4$, and $\sigma_{cr}=0.25$. Experiences show that a choice of $\Delta x=\Delta y=0.5\sigma_{cr}$ works well and is used herein. The minimum ratio of the core width to the inter-element distance is thus about $\alpha\sigma_{cr}/\sqrt{2}\Delta x \approx 1.2$. The reference circulation Γ_{ref} is taken to be the circulation of one vortex (Γ) and ε is taken to be 0.5%. The initial condition is two Burgers vortices, separated by a distance b_0 and each having a radius of $\sigma=1$. Initially 793 computational elements are placed uniformly within a disk of radius 3σ to constitute the “computed” vortex. The initial ratio of the core width to the inter-element distance is set to be 1.2. The strengths (Γ_j)

FIG. 1. The evolution of the vorticity contours during the merging process. The minimum contour is $\omega_{min}=1$ and the contour interval is $\Delta\omega=15$. A (dotted) circle of a diameter b_0 is also plotted for reference.

of these elements are so adjusted that the vorticity values at their locations are exact. The image technique takes care of the other Burgers vortex (the “imaged” vortex). Finally, the fluid viscosity is $\nu=1$ in all simulations. Besides the computational elements, the fluid particle initially located at the center of the computed vortex is also tracked. Its location will be defined as the center of the computed vortex, which is proper at least before the end of the convective stage. The distance between two vortex centers is thus twice the distance from this fluid particle to the origin.

III. RESULTS AND DISCUSSIONS

A. Merging process

The evolution of the Eulerian vorticity field is presented in Fig. 1, where the Reynolds number is $Re \equiv \Gamma/2\pi\nu=160$ and $b_0=5$. As observed before, two vortices rotate around each other for a while and merge into one at later times. Two spiral arms are gradually formed in the outer region. The evolution of the separation distance $b(t)$ is shown in Fig. 2. Also shown are the evolutions of the maximum vorticity value of the whole flow field, the vorticity value at the origin, and that at the vortex center. As distinguished by Cerretelli and Williamson,¹³ the whole process can be divided into four stages: a first constant- b stage (about $0 \leq t \leq 0.05$), a second convective (merging) stage ($0.05 \leq t \leq 0.31$), a third transi-

FIG. 2. Time evolutions of the separation distance between vortex centers (b) normalized by its initial value and the vorticity value at the origin (ω_0) divided by 2Re . Symbols are vorticity value at the vortex center (Δ) and the maximum vorticity value ($\circ$), also normalized by 2Re .

tion stage ($0.31 \leq t \leq 0.41$), and a merged diffusion stage ($t \geq 0.41$). During the transition stage, two vortex centers no longer get closer and the location of the maximum vorticity gradually moves from the vortex centers to the origin by diffusion. Thereafter, the vorticity at the origin remains the maximum, and the distance between centers oscillates about $0.28b_0$ with decreasing amplitude. A careful examination shows that the oscillation period is half the period associated with the clockwise rotation of the spiral arms relative to the mixed core structure.

The Lagrangian flow structures are characterized by the instantaneous locations of computational elements and the evolution of the vorticity field of the computed vortex as shown in Figs. 3 and 4. In both figures, the x axis has been rotated to coincide with the line-of-center (the line joining the computed vortex center and the imaged vortex center). By observation, the merging process may be described as follows. First, the vortices deform due to the mutually induced straining and become more or less elliptic. Moreover, the major axis misaligns with the line-of-center. Consequently, the vorticity particles in the upper outer region (Fig. 3) are attracted toward the imaged vortex, besides rotating around the computed vortex, implying the mutual attraction motion is stronger than the self-induced rotation. Particles, together with the circulation carried by them, are then entrained by the imaged vortex. When more and more vorticity has been entrained, a sheetlike structure is formed beside the core structure of the imaged vortex ($t=0.1$), makes a U-turn around the core structure ($t=0.2$), and reattaches the eroded core structure of the computed vortex ($t=0.25$). An empty region is observed ($t=0.25$), enclosing the center of the imaged vortex. This region gradually disappears as the vortex centers move closer and as the diffusion continues.

Curiosity may arise about how much this sheetlike structure contributes to the total vorticity field observed in Fig. 1. A careful examination finds that contours in the neighborhood of each vortex center in Fig. 1 are denser (implying larger vorticity gradients or less circulation) on one side than

FIG. 3. The entrainment of the computational elements of one vortex by the other vortex (only those elements with strengths greater than 0.001 are shown). Two (dotted) circles of diameters b_0 and $4b_0$ are also plotted for reference.

on the other side. A superposition of contours in Fig. 1 and those in Fig. 4 finds that this asymmetry is caused by the sheetlike structure; in other word, the additional circulation on the other side comes from the sheetlike structure. Consequently, there are two sources of the circulation in each spiral arm in Fig. 1. At some early time, each vortex sheds some of

FIG. 4. The evolution of the vorticity contours of the computed vortex. The minimum contour is $\omega_{\min}=2$ and the contour interval is $\Delta\omega=15$. The iso-vorticity line of $\omega=0.5\omega_{\max}$ of the imaged vortex and the (dotted) circle of diameter b_0 are also plotted for clarity.

FIG. 5. The radial velocity component induced by an elliptic Gaussian vortex of radii 1 and 0.9 with a circulation $\Gamma=320\pi$ along the directions of, from the top to the bottom, $\theta=3\pi/8$, $\pi/8$, $\pi/3$, $\pi/6$ and $\pi/4$.

its own circulation behind. Later it entrains circulation from the other vortex and throws part of it behind again. The amount of circulation shed later is even more than that shed earlier as can be observed from Fig. 4.

It seems true that two vortex centers start to move toward each other when the sheetlike structure observed in Fig. 4 has grown so strong to be capable of inducing significant merging velocity, which is defined as $-db/dt$. As time goes further, the center of the computed vortex moves so fast that it catches up with the leading sheetlike structure and both turn to appear in the same (right) half-plane ($t=0.35$). It is about at this time that the distance between two vortex centers reaches its minimum [the first dip appearing in the curve of $b(t)$ in Fig. 2]. Thereafter two spiral arms rotate clockwise relative to the mixed core region.

It should be mentioned at last that the spreading of the computational elements in Fig. 3 is a result of both advection and diffusion. The advection effect, however, dominates. In order to confirm the above observed/proposed merging process, some quantitative measurements are required and made in the next subsection.

B. Merging mechanism

The fact that an elliptic vortex can induce an inward radial velocity component is illustrated in Fig. 5, in which an elliptic Gaussian vortex of radii 1 and 0.9 with a circulation 320π is studied. It shows that as long as the vortex is not circular, it tends to attract all particles in the first quadrant but repel particles in the second quadrant (the latter is not shown in Fig. 5 but can be understood by symmetry). The misalignment of the major axes of the core structures with the line-of-center actually appears at a very early time due to the nonuniform velocity distribution over the vortex. Its appearance enhances the entrainment. Shown in Fig. 6 is another illustration of this misalignment from a flow simulation having $b_0=8$ and $\text{Re}=160$. There the core structure is defined as the region with vorticity magnitudes $\geq 0.5\omega_{\max}$ and the major and minor axes are defined by the principal directions of the second moments of vorticity.

Alternatively, the formation of the sheetlike structures

FIG. 6. The vorticity contours of one vortex of the flow with $b_0=8$ and $\text{Re}=160$ at $t=1.2$. The minimum contour is $\omega_{\min}=2$ and the contour interval is $\Delta\omega=8$. The thick contour represents the one with $\omega=0.5\omega_{\max}$. Dashed dotted lines are the line-of-center and the major axis of the ellipse.

may be understood by analog with the vortex-stripping phenomenon.^{26,27} It is known that when a vortex is submitted to the action of an external straining field, the vorticity at the periphery of the vortex is torn away and forms filamentary structures. Unlike previous investigations,^{28–30} however, the associated straining in the present problem is not uniformly distributed in space and varies in time. Two resulting “filamentary structures” are thus not equally strong. Moreover, the stronger filamentary structure is attracted and trapped by the other vortex, instead of freely extending and gradually diffusing to infinity.

The conjecture that the merging velocity is mainly contributed by the sheetlike structures is confirmed in the following way. First, the computed vortex in Fig. 4 is divided into two parts—the eroded core structure (composed of computational elements on the right half-plane) and the sheetlike structure (composed of computational elements on the left half-plane). Similarly, the imaged vortex is also divided into two parts. The inward radial velocities at the center of the computed vortex induced by the above four structures are then computed individually. Those induced by the core and sheetlike structures of the computed (imaged) vortex are indicated by u_1 and u_2 (u_3 and u_4), respectively. The total sum of them is half the merging velocity. These velocities are listed in Table I for $b_0=5$ and plotted in Fig. 7 for $b_0=8$.

TABLE I. The inward radial velocity components at the center of the computed vortex induced by the core structure and sheetlike structure of the computed vortex (u_1 and u_2) as well as those of the imaged vortex (u_3 and u_4) from the flow simulation with $b_0=5$, $\nu=1$ and $\text{Re}=160$.

t	u_2/u_{1+2}	u_4/u_{3+4}	$u_{2+4}/u_{1+2+3+4}$
0.05	-0.09/-0.33	0.18/0.37	0.09/0.04
0.1	-0.27/-0.70	1.32/1.46	1.05/0.76
0.15	-0.40/-1.36	2.07/2.32	1.67/0.96
0.2	-0.67/-1.65	3.20/3.58	2.53/1.93
0.25	-1.41/-2.66	6.53/6.98	5.12/4.32
0.3	-4.10/-3.68	12.39/12.79	8.29/9.11

FIG. 7. (a) The self-induced merging velocities and (b) the mutually induced merging velocities. Solid symbols represent contributions from the core structures (u_1 and u_3) and open symbols from the sheet structures (u_2 and u_4). The solid curve is the total merging velocity.

Because the core and sheetlike structures are hard to distinguish after the convective stage, only data at early times are measured. The data in Table I show that the center of the computed vortex is pushed toward the origin by u_3 as well as u_4 , and mainly by u_4 . Data in Fig. 7 further confirm this fact and moreover suggest that the Kelvin instability at early times¹⁵ is largely related to the self-induced motion of the core structure.

Due to the splitting technique used in the present vortex method, the spreading of computational elements is a result of both advection and diffusion. To highlight the importance of the advection effect, simulations with zero viscosity are also performed. The instantaneous locations of the computational elements (equivalently the fluid particles) are shown in Fig. 8 ($b_0=5$) and Fig. 9 ($b_0=4$), where only elements that were initially within a distance of 1.5σ from the vortex center are presented. The initial ratio of a_0/b_0 is about 0.224 and 0.28, respectively, where a_0 is defined as the distance from the vortex center to the location with the maximum azimuthal velocity. When $a_0/b_0=0.224$, the mutually induced strain is not strong enough to overcome the self-induced rotation and no merger is observed. Contrarily, as the value of a_0/b_0 becomes 0.28, vortices are stripped and sheetlike structures are formed. Furthermore, the eroded core struc-

FIG. 8. The instantaneous locations of fluid particles of the computed vortex from $t=0$ to 0.25 by 0.05 ($b_0=5$, $Re=160$, $\nu=0$).

tures become so weak that they can draw few entrained fluid particles to make a U-turn. The vorticity distribution in the neighborhood of the core structure becomes thus strongly asymmetric. In a word, it seems true that merger is governed by the competition between the self-induced rotation and the mutual attraction of vortices. For symmetric merger, the inviscid merger criterion is thus independent of the absolute strength of each vortex but depends on the relative distance between two vortices, namely b/a . As far as asymmetric merger is concerned, both the ratio of the strengths and the relative distance are of importance.

Finally, it is known that when the flow is viscous, merger always occurs. This may be explained as follows. If two vortices are far separated initially, the mutually induced attraction is weak and vorticity moves toward each other mainly by diffusion. As the relative separation distance (b/a) between vortices decreases due to the diffusion, deformation and the mutual attraction of vorticity gradually increase, and eventually merger occurs. In this sense, merger occurs earlier if the diffusion is stronger and if the attraction is faster. The onset time (t_1) of symmetric vortex merger should therefore depend on the vortex strength Γ , the viscosity ν , and the initial separation b_0/a_0 . A dimensional analysis easily shows that the dimensionless onset time $t_1\nu/b_0^2$ is a function of both the initial separation b_0/a_0 and the Reynolds number. This Reynolds-number dependence has never been reported before. It is measured and presented in Fig. 10 from simulations with a fixed initial ratio of $a_0/b_0=0.224$. Here the onset time (t_1) of the merging process is defined as the time when $b(t_1)/b_0=0.99$. As seen, $t_1\nu/b_0^2$ decreases with increasing Reynolds number. An inverse proportionality is even observed at larger Reynolds numbers. It implies that the mutual attraction of vorticity starts to dominate very soon probably

FIG. 9. The instantaneous locations of fluid particles of the computed vortex from $t=0.05$ to 0.15 by 0.05 ($b_0=4$, $Re=160$, $\nu=0$).

FIG. 10. The dimensionless onset time ($t_1 \nu / b_0^2$) of the merger versus the Reynolds number (Re) with $b_0=5$ and $\nu=1$ fixed.

because the viscosity employed herein is not small ($\nu=1$). The inverse proportionality, however, cannot extend to very large Reynolds numbers when the time to form the sheetlike structures is comparable to or smaller than the diffusion time to have a dominant mutual attraction. It is worth mentioning that such a Reynolds-number dependence of the onset time was also obtained in previous investigations^{12,13} if one examines the associated experimental data carefully. It was ignored, however, before.

IV. CONCLUSION

The symmetric vortex merger is simulated in the present work in use of a resurrected core-spreading vortex method. By the image technique, the evolution of the vorticity field of just one of the two vortices is obtained. The simulation results suggest that merger is governed by a competition between the self-induced rotation and the mutual attraction of vortices. The latter increases as the separation distance decreases. A critical separation therefore exists to merger. When the initial separation is larger than the critical one, diffusion is capable of reducing the relative separation distance and thus increasing the mutual attraction. Once the enhanced mutual attraction has dominated over the self-induced rotation, sheetlike structures are then generated at a speed that is Reynolds-number dependent. Consequently, the onset time of merger depends not only on the initial relative separation but also on the Reynolds number. This Reynolds-number dependence was ignored nonetheless in the past.

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