Transient deformation of a poroelastic channel bed

P. C. Hsieh^{a,†}, W. P. Shih^b and L. H. Huang^b

^a *Department of Soil and Water Conservation, National Chung-Hsing University, Taichung 40227, Taiwan, ROC*

^b *Department of Civil Engineering, National Taiwan University, Taipei 106, Taiwan, ROC*

SUMMARY

The coupled transient response of a poroelastic bed form due to stream flow and non-linear water waves is investigated numerically. The theory of potential flow is applied to channel flow while Biot's theory of poroelasticity (*J. Appl. Phys.* 1962; **33**(4):1482) is adopted to deal with the deformable porous bed. A boundary-fitted co-ordinate system is used to calculate the variation in the bed form. The result of a simple periodic wave form over a soft poroelastic bed agrees well with the analytical solution of Hsieh *et al.* (*J. Eng. Mech., ASCE* 2000; **126**(10):1064). However, due to the rapidly damping second dilatational wave inside the soft poroelastic bed, the solution for transient bed form near the interface is not easy to compute accurately. In order to overcome this difficulty, a simplified numerical model based on the boundary layer correction concept proposed by Hsieh *et al.* (2000) is established, which neglects Darcy's terms. The transient deformation of an irregular poroelastic bed that includes a trench and a downward step at the channel bed is simulated successfully. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: Transient deformation; soft poroelastic bed; boundary layer correction concept; boundary-fitted co-ordinate transformation

1. INTRODUCTION

The problem of flow past a porous deformable bed is always an important topic to hydraulic and ocean engineers. In addition to the complexity of interrelated mechanics between homogeneous water flow and porous deformable bed, the interaction between the skeleton and the pore-saturated fluid of the porous bed is even more complicated. For these subjects, some researches based on experiments [1–3] or simplified theories, which may apply empirical mass transport formulas [4–7] to avoid the analysis of complicated deformable porous media mechanics, were published to explain the physical behaviour. In addition, Darwin [8] conducted experiments on sand-ripple caused by oscillatory bed motion and concluded that the eddies induced by a series of vortex acting on the sandy bed would probably render unstable ripple into stable state. Bagnold [9] explained that the instability of dune in the desert was due to the impact

[†]Correspondence to: P. C. Hsieh, Department of Soil and Water Conservation, National Chung-Hsing University, Taichung 40227, Taiwan, R.O.C.
E-mail: pchsieh@dragon.nchu.edu.tw

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of sand by wind. Exner [10] established a differential erosion equation for two-dimensional flow, which indicated that the change in bed elevation was due to dilatational variation of bottom velocity. Anderson [11] applied Exner's [10] erosion equation to explain the mechanism of ripple formation. However, the mechanism of dune and antidune formation is somewhat different from that of ripple. Kennedy [12] applied an empirical sediment transport formula to govern the continuity of the porous bed. Also, he used instability analysis of potential theory to obtain his famous discovery about the dune and antidune formations in alluvial channels. Unfortunately, owing to the constraint of instability analysis, Kennedy [12] could only find the dominant wavelength of stable bed forms instead of the whole bed form.

In fact, fluid within porous material interacting with a deforming solid skeleton is a more complicated two-phase flow problem for a realistic analysis. Biot [13] developed a poroelasticity theory to describe elastic waves in a fluid saturated porous solid. Mei and Foda [14] proposed a boundary layer correction to simplify the analysis; however, their approach was based on physical intuition but without systematic analysis. By treating the bed as a poroelastic material, Huang and Song [15] solved the problem of oscillatory linear water waves interacting with a deformable bed by using three decoupled Helmholtz equations derived by Huang and Chwang [16]. In their solution, five non-dimensional parameters were derived. Chen *et al.* [17] also applied Huang and Chwang's [16] approach along with the conventional Stokes expansion of deep water waves based on the perturbation parameter $\varepsilon_1 = k_0 a$ (k_0 and a are the wave number and the amplitude of incoming water wave, respectively) to investigate the dynamic response of permeable bed material to non-linear water waves. They found that the conventional Stokes expansion is only valid for hard poroelastic bed material but invalid for soft materials even though the Ursell parameter is small. (Here 'hard' and 'soft' are defined according to a parameter Π , a Mach number of the second longitudinal wave inside the poroelastic material, in their paper.) Huang and Chiang [18] simulated stable bed forms under a constant current and linear oscillatory water waves by an approach similar to that of Huang and Song [15]. In Huang and Chiang [18], various bed forms such as dune, antidune and flat bed under linear oscillatory water wave accompanied with a constant current were obtained. The ambiguous lagged distance δ , introduced in Kenney [12], which has bothered researchers in river mechanics for many years was clarified. Huang *et al.* [19] and Hsieh *et al.* [20] used a two-parameter ($\varepsilon_1, \varepsilon_2 = k_0/k_2$, k_2 is the wave number of the second longitudinal wave inside the poroelastic material) perturbation expansion to discuss the dynamic response of soft poroelastic bed to linear and non-linear oscillatory water waves by a boundary layer correction approach.

The aforementioned studies are mostly focused on the stable bed form problem, however, the transient response of a constant current flowing over a semi-infinite soft poroelastic deforming bed along with non-linear water waves (Figure 1) remains to be analysed. In the present study, a finite difference numerical model based on the poroelastic theory is proposed to discuss the transient bed form.

2. FORMULATION

This study analyses the problem of a non-linear water wave along with a uniform channel flow over a poroelastic deformable bed, as defined schematically in Figure 1. Region 1 is homogeneous water, considered as potential flow, while region 2 is a semi-infinite porous medium saturated with water governed by Biot's theory of poroelasticity [21]. The co-ordinates

Figure 1. Definition sketch.

of region 1 range from $y = \gamma(x, t)$ to $y = h + \zeta(x, t)$, and region 2 from $y = \gamma(x, t)$ to $y \rightarrow -\infty$. The symbols ζ and γ represent the displacements of waves from the mean free surface ($y = h$) and mean bed interface ($y = 0$), respectively.

2.1. Boundary value problem

Assuming that the homogeneous channel flow is a potential flow, the flow velocity can be written as a given constant current (U) in the x direction (i.e. $U\mathbf{e}_x$) plus a perturbed velocity $\mathbf{u}_1$. Thus the perturbed velocity potential ϕ_1 satisfies

$$\nabla \phi_1 = \mathbf{u}_1 \quad (1)$$

The equations of continuity and momentum in terms of velocity potential ϕ_1 become

$$\nabla^2 \phi_1 = 0 \quad (2)$$

$$\frac{\partial \phi_1}{\partial t} + \frac{1}{2} \left[2U \frac{\partial \phi_1}{\partial x} + \left(\frac{\partial \phi_1}{\partial x} \right)^2 + \left(\frac{\partial \phi_1}{\partial y} \right)^2 \right] + \frac{P_1}{\rho_f} = 0 \quad (3)$$

where P_1 is the perturbed pressure, and ρ_f is the water density.

Referring to Biot's theory of poroelasticity [13], the linear momentum equations of solid skeleton and fluid for the porous bed may be written as

$$\nabla \cdot \underline{\underline{\sigma}} = \rho_{11} \frac{\partial^2 \underline{\underline{d}}}{\partial t^2} + \rho_{12} \frac{\partial^2 \underline{\underline{D}}}{\partial t^2} + b \left(\frac{\partial \underline{\underline{d}}}{\partial t} - \frac{\partial \underline{\underline{D}}}{\partial t} \right) \quad (4)$$

$$\nabla \cdot \underline{\underline{S}} = \rho_{12} \frac{\partial^2 \underline{\underline{d}}}{\partial t^2} + \rho_{22} \frac{\partial^2 \underline{\underline{D}}}{\partial t^2} - b \left(\frac{\partial \underline{\underline{d}}}{\partial t} - \frac{\partial \underline{\underline{D}}}{\partial t} \right) \quad (5)$$

with

$$\underline{\underline{\sigma}} = \underline{\underline{\tau}} - (1 - n_0) P_2 \underline{\underline{I}} \quad (6)$$

$$\underline{\underline{\tau}} = 2G \underline{\underline{e}} + \lambda (\nabla \cdot \underline{\underline{d}}) \underline{\underline{I}} \quad (7)$$

$$\underline{\underline{e}} = \frac{1}{2}[\nabla \underline{\underline{d}} + (\nabla \underline{\underline{d}})^t] \quad (8)$$

$$\underline{\underline{S}} = -n_0 P_2 \underline{\underline{I}} \quad (9)$$

$$\rho_{11} = (1 - n_0)\rho_s + \rho_a \quad (10)$$

$$\rho_{12} = -\rho_a \quad (11)$$

$$\rho_{22} = n_0 \rho_f + \rho_a \quad (12)$$

$$b = \mu n_0^2 / k_p \quad (13)$$

where $\underline{\underline{\sigma}}$ is the solid stress tensor, $\underline{\underline{\tau}}$ the effective stress tensor of the solid, $\underline{\underline{e}}$ the strain tensor of solid, $\underline{\underline{S}}$ the normal stress tensor of the fluid, $\underline{\underline{d}}$ and $\underline{\underline{D}}$ are the solid and fluid displacement vectors, respectively, P_2 the perturbed pressure of the fluid inside the porous medium; ρ_s the solid density; ρ_a the mass coupling effect (neglected in this study), n_0 the porosity, μ the fluid viscosity, k_p the specific permeability, G and λ are the Lamé's constants of elasticity and $\underline{\underline{I}}$ the identity matrix.

Combining the continuity equations of the solid and the fluid with the state equation of the fluid, and after linearization of the porosity (see Reference [22]), we find

$$\frac{\partial P_2}{\partial t} = -\frac{K}{n_0} \left[(1 - n_0) \nabla \cdot \left(\frac{\partial \underline{\underline{d}}}{\partial t} \right) + n_0 \nabla \cdot \left(\frac{\partial \underline{\underline{D}}}{\partial t} \right) \right] \quad (14)$$

for the perturbed pressure. In Equation (14), K is the bulk modulus of compressibility of fluid inside the porous bed.

According to Huang and Chwang [16], the displacement vectors of skeleton and fluid could be separated into dilatational and rotational parts

$$\underline{\underline{d}} = \nabla \varphi_d + \nabla \times \underline{\underline{H}}_d \quad (15)$$

$$\underline{\underline{D}} = \nabla \varphi_D + \nabla \times \underline{\underline{H}}_D \quad (16)$$

The first terms on the right-hand sides of Equations (15) and (16) represent the dilatational components (two dilatational waves) of the skeleton and fluid displacements, respectively; while the second terms represent rotational components of the skeleton and fluid displacements. The former ones are irrotational while the latter ones are solenoidal. Referring to Morse and Feshbach [23], $\underline{\underline{H}}_d$ and $\underline{\underline{H}}_D$ can be expressed as

$$\underline{\underline{H}}_d = \omega_d \underline{\underline{e}}_z \quad \text{and} \quad \underline{\underline{H}}_D = \omega_D \underline{\underline{e}}_z \quad (17)$$

where $\underline{\underline{e}}_z$ is the unit vector normal to the x - y plane.

Substituting Equations (15)–(17) into Equations (4) and (5), we obtain

$$(2G + \lambda) \nabla^2 \varphi_d - (1 - n_0) P_2 = \rho_{11} \frac{\partial^2 \varphi_d}{\partial t^2} + \rho_{12} \frac{\partial^2 \varphi_D}{\partial t^2} + b \left(\frac{\partial \varphi_d}{\partial t} - \frac{\partial \varphi_D}{\partial t} \right) \quad (18)$$

$$-n_0 P_2 = \rho_{12} \frac{\partial^2 \varphi_d}{\partial t^2} + \rho_{22} \frac{\partial^2 \varphi_D}{\partial t^2} - b \left(\frac{\partial \varphi_d}{\partial t} - \frac{\partial \varphi_D}{\partial t} \right) \quad (19)$$

$$G\nabla^2\omega_d = \rho_{11}\frac{\partial^2\omega_d}{\partial t^2} + \rho_{12}\frac{\partial^2\omega_D}{\partial t^2} + b\left(\frac{\partial\omega_d}{\partial t} - \frac{\partial\omega_D}{\partial t}\right) \quad (20)$$

$$0 = \rho_{12}\frac{\partial^2\omega_d}{\partial t^2} + \rho_{22}\frac{\partial^2\omega_D}{\partial t^2} - b\left(\frac{\partial\omega_d}{\partial t} - \frac{\partial\omega_D}{\partial t}\right) \quad (21)$$

Substituting Equations (15)–(17) into Equation (14), we get

$$\frac{\partial P_2}{\partial t} = -\frac{K}{n_0}\frac{\partial}{\partial t}[(1-n_0)\nabla^2\varphi_d + n_0\nabla^2\varphi_D] \quad (22)$$

Equations (18)–(22) are the governing equations.

There are three boundaries that require boundary conditions. They are (i) the free surface [$y = h + \zeta(x, t)$], (ii) the channel bed [$y = \gamma(x, t)$], and (iii) the deep far field of the porous bed [$y \rightarrow -\infty$].

On the free surface, the kinematic boundary condition is

$$\frac{\partial\phi_1}{\partial y} = \frac{\partial\zeta}{\partial t} + \frac{\partial\zeta}{\partial x}\left(U + \frac{\partial\phi_1}{\partial x}\right) \quad (23)$$

and the dynamic boundary condition is

$$\frac{\partial\phi_1}{\partial t} + \frac{1}{2}\left[2U\frac{\partial\phi_1}{\partial x} + \left(\frac{\partial\phi_1}{\partial x}\right)^2 + \left(\frac{\partial\phi_1}{\partial y}\right)^2\right] + g\zeta = 0 \quad (24)$$

On the porous bed, the continuity of pressure implies that

$$P_1 = P_2 \quad (25)$$

while the continuity of fluid flux gives

$$\begin{aligned} \frac{\partial\phi_1}{\partial y} - \frac{\partial\gamma}{\partial x}\frac{\partial\phi_1}{\partial x} = U\frac{\partial\gamma}{\partial x} + \frac{\partial}{\partial t}\left[(1-n_0)\left(\frac{\partial\varphi_d}{\partial y} - \frac{\partial\omega_d}{\partial x}\right) + n_0\left(\frac{\partial\varphi_D}{\partial y} - \frac{\partial\omega_D}{\partial x}\right) \right. \\ \left. - (1-n_0)\frac{\partial\gamma}{\partial x}\left(\frac{\partial\varphi_d}{\partial y} - \frac{\partial\omega_d}{\partial x}\right) - n_0\frac{\partial\gamma}{\partial x}\left(\frac{\partial\varphi_D}{\partial y} - \frac{\partial\omega_D}{\partial x}\right)\right]. \end{aligned} \quad (26)$$

The continuity of effective stress of the solid, i.e. $\underline{n}_2 \cdot \underline{\tau} = 0$, gives

$$G(2\varphi_{d,xy} + \omega_{d,yy} - \omega_{d,xx}) - \frac{\partial\gamma}{\partial x}[(2G + \lambda)\varphi_{d,xx} + 2G\omega_{d,yx} + \lambda\varphi_{d,yy}] = 0 \quad (27)$$

$$(2G + \lambda)\varphi_{d,yy} + 2G\omega_{d,xy} + \lambda\varphi_{d,xx} - \frac{\partial\gamma}{\partial x}G(2\varphi_{d,xy} + \omega_{d,yy} - \omega_{d,xx}) = 0 \quad (28)$$

At the far field of the porous bed, i.e. $y \rightarrow -\infty$, the boundary conditions consist of vanishing displacement vectors, i.e.

$$\lim_{y \rightarrow -\infty} \{\phi_1, \varphi_d, \varphi_D, \omega_d\} = 0 \quad (29)$$

and no perturbation is propagated to the upstream and downstream boundaries, i.e.

$$\begin{aligned} \lim_{x \rightarrow \pm\infty} \frac{\partial}{\partial n} \{\varphi_d, \varphi_D, \omega_d\} &= 0 \\ \lim_{x \rightarrow \pm\infty} \begin{cases} \frac{\partial \phi_1}{\partial n} \pm ik\phi_1 = 0, & \text{periodic wave problem} \\ \frac{\partial \phi_1}{\partial n} = 0, & \text{transient wave problem} \end{cases} \end{aligned} \quad (30)$$

In addition, the kinematic boundary condition at the porous bed interface results in

$$\frac{\partial \gamma}{\partial t} = \frac{\partial}{\partial t} \left[\frac{\partial \varphi_d}{\partial y} - \frac{\partial \omega_d}{\partial x} - \frac{\partial \gamma}{\partial x} \left(\frac{\partial \varphi_d}{\partial x} + \frac{\partial \omega_d}{\partial y} \right) \right] \quad (31)$$

which allows to determine the surface elevation of the porous bed, γ .

The governing equations (2), (18)–(21) with perturbed pressure in porous bed, Equation (22), surface elevations ζ and γ , Equations (24) and (31), together with other boundary conditions, Equations (23), (25)–(30), and a properly ‘warm-up’ initial condition form a well-posed problem.

2.2. Non-dimensionalization through boundary layer correction

When the poroelastic bed is composed of hard material, the inertial terms of the governing equations (first two terms on the right-hand side of Equations (18)–(21)) are unimportant and they can be neglected (see Reference [24]). The deformation is also negligible. However, when the poroelastic material is soft, the inertial terms are important, and the second dilatational wave travelling through the porous bed has a much shorter wavelength than that of the wave travelling through homogeneous water (see Reference [17]). Thus there exists a small boundary layer near the interface, inside the soft poroelastic bed. Because the effect of the boundary layer will render errors of the partial derivative in the vertical direction for the second longitudinal wave, the traditional estimation of the deformable bed by Stokes expansion based on ε_1 will fail [17]. Hsieh *et al.* [20] proposes a boundary layer correction approach based on multiple length scales to overcome this difficulty. Since we are interested in bed form deformation in this study, the following discussion will concentrate on soft poroelastic material only.

The coefficients of the governing equations, Equations (18)–(21), range too far to be solved directly by numerical computation so that non-dimensionalization is needed to reduce the difference of the magnitude of each term. We define

$$x = Lx^*, \quad y = Ly^* \quad (32)$$

$$\phi_1 = L\sqrt{gL}\phi_1^*, \quad t_1 = \sqrt{L/g}t_1^* \quad (33)$$

$$\varphi_d = L^2\varphi_d^*, \quad \omega_d = L^2\omega_d^* \quad (34)$$

$$\varphi_D = L^2\varphi_D^*, \quad \omega_D = L^2\omega_D^* \quad (35)$$

$$P_2 = -\frac{K}{n_0}P_2^*, \quad t_2 = \sqrt{L/g}t_2^* \quad (36)$$

In which, all symbols with an asterisk stand for the dimensionless variables. The order of t_2^* is found to be $O(1)$. Equations (2) and (18)–(22) can be rewritten as

$$\hat{\alpha}_1 \phi_{1,\xi\xi}^* + 2\hat{\alpha}_2 \phi_{1,\xi\eta}^* + \hat{\alpha}_3 \phi_{1,\eta\eta}^* + \hat{\alpha}_4 \phi_{1,\xi}^* + \hat{\alpha}_5 \phi_{1,\eta}^* = 0 \quad (37)$$

$$\begin{aligned} \frac{(2G + \lambda)}{K} \nabla_*^2 \varphi_d^* - (1 - n_0) P_2^* &= \frac{\rho_{11} g L}{K} \frac{\partial^2 \varphi_d^*}{\partial t^{*2}} + \frac{\rho_{12} g L}{K} \frac{\partial^2 \varphi_D^*}{\partial t^{*2}} \\ &+ b \sqrt{\frac{g}{L}} \frac{L^2}{K} \left(\frac{\partial \varphi_d^*}{\partial t^*} - \frac{\partial \varphi_D^*}{\partial t^*} \right) \end{aligned} \quad (38)$$

$$-n_0 P_2^* = \frac{\rho_{12} g L}{K} \frac{\partial^2 \varphi_d^*}{\partial t^{*2}} + \frac{\rho_{22} g L}{K} \frac{\partial^2 \varphi_D^*}{\partial t^{*2}} - b \sqrt{\frac{g}{L}} \frac{L^2}{K} \left(\frac{\partial \varphi_d^*}{\partial t^*} - \frac{\partial \varphi_D^*}{\partial t^*} \right) \quad (39)$$

$$\frac{G}{K} \nabla_*^2 \omega_d^* = \frac{\rho_{11} g L}{K} \frac{\partial^2 \omega_d^*}{\partial t^{*2}} + \frac{\rho_{12} g L}{K} \frac{\partial^2 \omega_D^*}{\partial t^{*2}} + b \sqrt{\frac{g}{L}} \frac{L^2}{K} \left(\frac{\partial \omega_d^*}{\partial t^*} - \frac{\partial \omega_D^*}{\partial t^*} \right) \quad (40)$$

$$0 = \frac{\rho_{12} g L}{K} \frac{\partial^2 \omega_d^*}{\partial t^{*2}} + \frac{\rho_{22} g L}{K} \frac{\partial^2 \omega_D^*}{\partial t^{*2}} - b \sqrt{\frac{g}{L}} \frac{L^2}{K} \left(\frac{\partial \omega_d^*}{\partial t^*} - \frac{\partial \omega_D^*}{\partial t^*} \right) \quad (41)$$

$$\frac{\partial P_2^*}{\partial t^*} = -\frac{1}{n_0} \frac{\partial}{\partial t^*} [(1 - n_0) \nabla_*^2 \varphi_d^* + n_0 \nabla_*^2 \varphi_D^*] \quad (42)$$

in which, the coefficients $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4$ and $\hat{\alpha}_5$ are referred in Appendix A.

Adding Equation (38) to Equation (39) to eliminate Darcy's terms, we get

$$\frac{(2G + \lambda)}{K} \nabla_*^2 \varphi_d^* - P_2^* = \frac{(\rho_{11} + \rho_{12}) g L}{K} \frac{\partial^2 \varphi_d^*}{\partial t^{*2}} + \frac{(\rho_{12} + \rho_{22}) g L}{K} \frac{\partial^2 \varphi_D^*}{\partial t^{*2}} \quad (43)$$

and similarly adding Equation (40) to Equation (41), we have

$$-\frac{G}{K} \nabla_*^2 \omega_d^* = \frac{(\rho_{11} + \rho_{12}) g L}{K} \frac{\partial^2 \omega_d^*}{\partial t^{*2}} + \frac{(\rho_{12} + \rho_{22}) g L}{K} \frac{\partial^2 \omega_D^*}{\partial t^{*2}} \quad (44)$$

The huge coefficient b of Darcy's terms in Equation (39) is not suitable for numerical computation. However, for soft poroelastic material the magnitude of the second longitudinal wave is rather small and the displacement potentials of solid and fluid are very close, i.e. $\varphi_d^* \cong \varphi_D^*$ and $\omega_d^* \cong \omega_D^*$, by the boundary layer correction proposed by Hsieh *et al.* [20]. Since Darcy's terms are mainly due to the second longitudinal wave, we find that when the porous material is soft, Darcy's terms are negligible. We therefore can simplify Equations (39) and (41), respectively, to

$$-n_0 \frac{K}{b L^2} \sqrt{\frac{L}{g}} P_2^* = \frac{\rho_{12}}{b} \sqrt{\frac{g}{L}} \frac{\partial^2 \varphi_d^*}{\partial t^{*2}} + \frac{\rho_{22}}{b} \sqrt{\frac{g}{L}} \frac{\partial^2 \varphi_D^*}{\partial t^{*2}} \quad (45)$$

$$0 = \frac{\rho_{12}}{b} \sqrt{\frac{g}{L}} \frac{\partial^2 \omega_d^*}{\partial t^{*2}} + \frac{\rho_{22}}{b} \sqrt{\frac{g}{L}} \frac{\partial^2 \omega_D^*}{\partial t^{*2}} \quad (46)$$

Note that Equations (39), (41), (43), (44) and (42) are the full non-dimensional governing equations for the soft porous bed, while Equations (43)–(46) and (42) are the simplified non-dimensional governing equations for the soft porous bed. The non-dimensional boundary

conditions are as follows:

1. Kinematic free surface boundary condition

$$\begin{aligned} \frac{\partial \phi_1^*}{\partial y^*} = & -\frac{1}{g} \left[\frac{\partial^2 \phi_1^*}{\partial t^{*2}} + 2 \left(U^* + \frac{\partial \phi_1^*}{\partial x^*} \right) \frac{\partial^2 \phi_1^*}{\partial x^* \partial t^*} + \left(U^* + \frac{\partial \phi_1^*}{\partial x^*} \right)^2 \frac{\partial^2 \phi_1^*}{\partial x^{*2}} \right. \\ & \left. + \frac{\partial \phi_1^*}{\partial y^*} \frac{\partial^2 \phi_1^*}{\partial y^{*2}} + \frac{\partial \phi_1^*}{\partial y^*} \left(U^* + \frac{\partial \phi_1^*}{\partial x^*} \right) \frac{\partial^2 \phi_1^*}{\partial x^* \partial y^*} \right]. \end{aligned} \quad (47)$$

2. Dynamic free surface boundary condition

$$\frac{\partial \phi_1^*}{\partial t^*} + \frac{1}{2} \left[2U^* \frac{\partial \phi_1^*}{\partial x^*} + \left(\frac{\partial \phi_1^*}{\partial x^*} \right)^2 + \left(\frac{\partial \phi_1^*}{\partial y^*} \right)^2 \right] + \zeta^* = 0 \quad (48)$$

3. Continuity of flux at water/porous bed interface

$$\begin{aligned} \frac{\partial \phi_1^*}{\partial y^*} - \frac{\partial \gamma^*}{\partial x^*} \frac{\partial \phi_1^*}{\partial x^*} = & U^* \frac{\partial \gamma^*}{\partial x^*} + \frac{\partial}{\partial t^*} \left[(1 - n_0) \left(\frac{\partial \varphi_d^*}{\partial y^*} - \frac{\partial \omega_d^*}{\partial x^*} \right) \right. \\ & + n_0 \left(\frac{\partial \varphi_D^*}{\partial y^*} - \frac{\partial \omega_D^*}{\partial x^*} \right) - (1 - n_0) \frac{\partial \gamma^*}{\partial x^*} \left(\frac{\partial \varphi_d^*}{\partial y^*} - \frac{\partial \omega_d^*}{\partial x^*} \right) \\ & \left. - n_0 \frac{\partial \gamma^*}{\partial x^*} \left(\frac{\partial \varphi_D^*}{\partial y^*} - \frac{\partial \omega_D^*}{\partial x^*} \right) \right]. \end{aligned} \quad (49)$$

4. Continuity of effective stresses (i.e. $\tau_{xy} = 0$ and $\tau_{yy} = 0$) at water/porous bed interface

$$\begin{aligned} \frac{G}{K} (2\varphi_{d,x^*y^*}^* + \omega_{d,y^*y^*}^* - \omega_{d,x^*x^*}^*) \\ - \frac{\partial \gamma^*}{\partial x^*} \left[\frac{(2G + \lambda)}{K} \varphi_{d,x^*x^*}^* + \frac{2G}{K} \omega_{d,y^*x^*}^* + \frac{\lambda}{K} \varphi_{d,y^*y^*}^* \right] = 0 \end{aligned} \quad (50)$$

$$\begin{aligned} \frac{(2G + \lambda)}{K} \varphi_{d,y^*y^*}^* - \frac{2G}{K} \omega_{d,x^*y^*}^* + \frac{\lambda}{K} \varphi_{d,x^*x^*}^* \\ - \frac{\partial \gamma^*}{\partial x^*} \frac{G}{K} (2\varphi_{d,x^*y^*}^* + \omega_{d,y^*y^*}^* - \omega_{d,x^*x^*}^*) = 0 \end{aligned} \quad (51)$$

5. Continuity of pressure at water/porous bed interface

$$P_2^* K + \rho_f g L \left\{ \frac{\partial \phi_1^*}{\partial t^*} + \frac{1}{2} \left[2U^* \frac{\partial \phi_1^*}{\partial x^*} + \left(\frac{\partial \phi_1^*}{\partial x^*} \right)^2 + \left(\frac{\partial \phi_1^*}{\partial y^*} \right)^2 \right] \right\} = 0 \quad (52)$$

6. Far field of porous bed

$$\lim_{y^* \rightarrow -\infty} \{\phi_1^*, \varphi_d^*, \varphi_D^*, \omega_d^*\} = 0 \quad (53)$$

7. Upstream/downstream boundary

$$\lim_{x^* \rightarrow \pm \infty} \frac{\partial}{\partial x^*} \{\varphi_d^*, \varphi_D^*, \omega_d^*\} = 0$$

$$\lim_{x^* \rightarrow \pm \infty} \begin{cases} \frac{\partial \phi_1^*}{\partial x^*} \pm ik_0 \phi_1^* = 0, & \text{periodic wave problem} \\ \frac{\partial \phi_1^*}{\partial x^*} = 0, & \text{transient wave problem} \end{cases} \quad (54)$$

In addition, the non-dimensional kinematic boundary condition at the bed surface is

$$\frac{\partial \gamma^*}{\partial t^*} = \frac{\partial}{\partial t^*} \left[\frac{\partial \varphi_d^*}{\partial y^*} - \frac{\partial \omega_d^*}{\partial x^*} - \frac{\partial \gamma^*}{\partial x^*} \left(\frac{\partial \varphi_d^*}{\partial x^*} + \frac{\partial \omega_d^*}{\partial y^*} \right) \right] \quad (55)$$

3. COMPUTATION AND DISCUSSION

To estimate the variation of channel bed accurately and effectively, the proposed numerical computation adopts the boundary-fitted co-ordinate system of Thompson *et al.* [25] to transform the physical domain into the computational domain by substitution of the Dirichlet boundary conditions. Taking the Laplace equations

$$\xi_{xx} + \xi_{yy} = 0 \quad (56)$$

$$\eta_{xx} + \eta_{yy} = 0 \quad (57)$$

as the grid generator, the orthogonality of grids on the boundaries can be maintained. Besides the orthogonal boundary-fitted co-ordinate transformations, the new equations are expressed in semi-differential form using the finite difference method. Time and space are discretized separately. The time scale is discretized by the first-order backward difference scheme while the space domain is discretized using the central difference method. The computation is divided into channel flow region and porous bed region. These two regions are carried out separately and sequentially, and they are linked by the boundary conditions at the interface. At the moving boundary, the interpolation adopts a cubic spline to keep the shape of porous bed and regenerate the grids to make sure the orthogonality condition is maintained at each time step. The expressions of the independent variables by the finite difference method are written as:

$$F_{i,j}^n = \frac{5}{2} F_{i+1,j}^{n-1} - \frac{4}{2} F_{i+2,j}^{n-1} + \frac{1}{2} F_{i+3,j}^{n-1} \quad (58)$$

$$F_{i,j}^n = \frac{5}{2} F_{i-1,j}^{n-1} - \frac{4}{2} F_{i-2,j}^{n-1} + \frac{1}{2} F_{i-3,j}^{n-1} \quad (59)$$

where F is the independent variable for the channel flow or the porous bed, the superscript n refers to the time step and the subscripts i, j are the grid co-ordinates.

In order to verify our formulation, this numerical model without simplification (Equations (37), (39), (41)–(44), along with the boundary conditions equations (47), (49)–(54) and the surface elevation equations (48), (55)) is applied to simulate the problem of a simple periodic wave progression in combination with a constant current. Taking the current

Figure 2. Refined grid near the interface of the computational domain.

Figure 3. Comparison of computed profiles of free surface (ζ) and bed form (γ) with exact solution.

$U = 0.3$ m/s, water depth $h = 3$ m, time to warm up $t = 60$ s, time step $\Delta t = 0.1$ s for channel flow, but 0.01 s for porous bed, and the thickness of channel bed = 3 m, the computed results of free surface and bed form are compared with the analytic solution proposed by Hsieh *et al.* [20]. Figure 2 shows that the meshes are taken as 10 by 10 grids for region 1, but they are much finer close to the interface with region 2. Figure 3 shows good agreement for the free surface and bed form with the solution of Hsieh *et al.* [20]. Furthermore, the results for the displacement potentials of both the dilatational and rotational waves at selected sections (indicated by the downward arrows in Figure 4) also agree very well with the Hsieh *et al.* [20] analytical solutions.

Figure 4. Comparison of computed displacement potentials of dilatational wave (φ_d) and rotational wave (ω_d) with exact solution.

Figure 5. Schematic diagram of a trench at the channel bed.

Figure 6. Transient variation of free surface and a trench at bed form, full formulation ($\Delta t = 1$ s, $U = 0.3$ m/s).

For the first application, the transient response of a small trench at the channel bed, an important problem of river bed dredging, is simulated. Figure 5 is the schematic diagram of a small trench that is analysed. Figure 6 shows the transient transport of the trench by adopting the original complete formulation (Equations (37), (39), (41), (43), (44) with boundary conditions (47)–(54)). Figure 6 shows that the fluctuation of water surface is very large and continuously increases while the bed form remains unchanged. From the results in Figure 6, we find that although small meshes are used near the interface and the time step is small, the numerical model without simplification is still very unstable and will diverge after a very short period of time, say 10 s. This is because that the dilatational waves for water and solid inside the soft porous bed are almost the same, hence Darcy's terms multiplied by a very large coefficient b (the third terms on the right-hand side of Equations (39) and (41)) are impossible to estimate accurately even after non-dimensionalization. Referring to Hsieh *et al.* [20], it is noticeable that when the porous bed is soft, the wavelength of the second dilatational wave near the interface is much shorter than that in the homogeneous water, which makes convergence problematic. Thus, the traditional approach of refining meshes near the interface is unsuccessful. However,

Figure 7. Transient variation of free surface and a trench at bed form, simplified formulation ($\Delta t = 60$ s, $U = 0.3$ m/s).

Figure 8. Schematic diagram of channel bed with a downward step.

Figure 9. Transient variation of free surface and a downward-step channel bed, simplified formulation ($\Delta t = 60$ s, $U = 0.3$ m/s).

the magnitude of the second longitudinal wave is rather small and requires only the so-called boundary layer correction (see Reference [20]). Since Darcy's terms are mainly due to the second longitudinal wave, i.e. the relative velocity of the irrotational part, we find that when the porous material is soft, Darcy's terms are negligible. After neglecting Darcy's terms, Equations (39) and (41) turn out to be Equations (45) and (46). This simplified model is stable, larger time step is allowable, and the meshes near the interface no longer need to be refined.

Indeed, this becomes apparent when the same problem as in Figure 6 is analysed using the simplified procedure. (The finite difference scheme is described in Appendix A.) The results are shown in Figure 7. The time step in Figure 7 is 60 s instead of 1 s in Figure 6. After about 12 min, the channel bed begins to deform.

A second application involves the channel bed with a downward step under a uniform flow, as shown in Figure 8. This is an important problem for bridge pier protection applications when river beds are dredged. The transient variations of the free surface and the channel bed are shown in Figure 9, which indicates the deformation of the head-cutting of channel bed together with the upstream going water wave.

5. CONCLUSION

The transient response of poroelastic bed form due to stream flow and non-linear water wave are investigated numerically. The theory of potential flow is applied to channel flow while Biot's theory of poroelasticity [21] is adopted to deal with the deformable porous bed. The numerical model without simplification can simulate the periodic wave/current problem with a simple or regular geometry only. But it is easy to diverge within very short time for the transient problem with irregular geometry. The transient problem with irregular geometry always needs small grids near the bed/water interface as well as very small time steps. Even then, convergence is not assured. The present work establishes a simplified numerical model based on the boundary layer concept proposed by Hsieh *et al.* [20] by neglecting Darcy's terms for a soft poroelastic bed. This makes the study of transient deformations of a channel bed with a trench or a downward step feasible.

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APPENDIX A: FINITE DIFFERENCE FORMULATIONS

Governing equations:

Region 1:

$$\hat{\alpha}_1 \phi_{1,\xi\xi}^{*n} + 2\hat{\alpha}_2 \phi_{1,\xi\eta}^{*n} + \hat{\alpha}_3 \phi_{1,\eta\eta}^{*n} + \hat{\alpha}_4 \phi_{1,\xi}^{*n} + \hat{\alpha}_5 \phi_{1,\eta}^{*n} = 0 \quad (\text{A1})$$

where

$$\hat{\alpha}_1 = \xi_{x^*}^2 + \xi_{y^*}^2 \quad (\text{A2})$$

$$\hat{\alpha}_2 = \xi_{x^*} \eta_{x^*} + \xi_{y^*} \eta_{y^*} \quad (\text{A3})$$

$$\hat{\alpha}_3 = \eta_{x^*}^2 + \eta_{y^*}^2 \quad (\text{A4})$$

$$\hat{\alpha}_4 = \xi_{x^*} \eta_{x^*} + \xi_{y^*} \eta_{y^*} \quad (\text{A5})$$

$$\hat{\alpha}_5 = \eta_{x^*} \eta_{y^*} \quad (\text{A6})$$

Region 2:

$$\begin{aligned} & \frac{2G + \lambda}{K} (\hat{\alpha}_1 \phi_{d,\xi\xi}^{*n} + 2\hat{\alpha}_2 \phi_{d,\xi\eta}^{*n} + \hat{\alpha}_3 \phi_{d,\eta\eta}^{*n} + \hat{\alpha}_4 \phi_{d,\xi}^{*n} + \hat{\alpha}_5 \phi_{d,\eta}^{*n}) - P_2^{*n} \\ &= \Omega_1 \frac{\phi_d^{*n} - 2\phi_d^{*n-1} + \phi_d^{*n-2}}{\Delta t^{*2}} + \Omega_2 \frac{\phi_D^{*n} - 2\phi_D^{*n-1} + \phi_D^{*n-2}}{\Delta t^{*2}} \end{aligned} \quad (\text{A7})$$

$$-\Omega_3 P_2^{*n} = \Omega_4 \frac{\phi_d^{*n} - 2\phi_d^{*n-1} + \phi_d^{*n-2}}{\Delta t^{*2}} + \Omega_5 \frac{\phi_D^{*n} - 2\phi_D^{*n-1} + \phi_D^{*n-2}}{\Delta t^{*2}} \quad (\text{A8})$$

$$\begin{aligned} & \frac{G}{K} (\hat{\alpha}_1 \omega_{d,\xi\xi}^{*n} + 2\hat{\alpha}_2 \omega_{d,\xi\eta}^{*n} + \hat{\alpha}_3 \omega_{d,\eta\eta}^{*n} + \hat{\alpha}_4 \omega_{d,\xi}^{*n} + \hat{\alpha}_5 \omega_{d,\eta}^{*n}) \\ &= \Omega_1 \frac{\omega_d^{*n} - 2\omega_d^{*n-1} + \omega_d^{*n-2}}{\Delta t^{*2}} + \Omega_2 \frac{\omega_D^{*n} - 2\omega_D^{*n-1} + \omega_D^{*n-2}}{\Delta t^{*2}} \end{aligned} \quad (\text{A9})$$

$$0 = \Omega_4 \frac{\omega_d^{*n} - 2\omega_d^{*n-1} + \omega_d^{*n-2}}{\Delta t^{*2}} + \Omega_5 \frac{\omega_D^{*n} - 2\omega_D^{*n-1} + \omega_D^{*n-2}}{\Delta t^{*2}} \quad (\text{A10})$$

$$\begin{aligned} & \frac{P_2^{*n} - 2P_2^{*n-1} + P_2^{*n-2}}{\Delta t^{*2}} \\ &= \frac{1}{n_0} \frac{1}{\Delta t^*} \{ (1 - n_0) [(\hat{\alpha}_1 \phi_{d,\xi\xi}^{*n} + 2\hat{\alpha}_2 \phi_{d,\xi\eta}^{*n} + \hat{\alpha}_3 \phi_{d,\eta\eta}^{*n} + \hat{\alpha}_4 \phi_{d,\xi}^{*n} + \hat{\alpha}_5 \phi_{d,\eta}^{*n}) \\ & \quad - (\hat{\alpha}_1 \phi_{d,\xi\xi}^{*n-1} + 2\hat{\alpha}_2 \phi_{d,\xi\eta}^{*n-1} + \hat{\alpha}_3 \phi_{d,\eta\eta}^{*n-1} + \hat{\alpha}_4 \phi_{d,\xi}^{*n-1} + \hat{\alpha}_5 \phi_{d,\eta}^{*n-1})] \\ & \quad + n_0 [(\hat{\alpha}_1 \phi_{D,\xi\xi}^{*n} + 2\hat{\alpha}_2 \phi_{D,\xi\eta}^{*n} + \hat{\alpha}_3 \phi_{D,\eta\eta}^{*n} + \hat{\alpha}_4 \phi_{D,\xi}^{*n} + \hat{\alpha}_5 \phi_{D,\eta}^{*n}) \\ & \quad - (\hat{\alpha}_1 \phi_{D,\xi\xi}^{*n-1} + 2\hat{\alpha}_2 \phi_{D,\xi\eta}^{*n-1} + \hat{\alpha}_3 \phi_{D,\eta\eta}^{*n-1} + \hat{\alpha}_4 \phi_{D,\xi}^{*n-1} + \hat{\alpha}_5 \phi_{D,\eta}^{*n-1})] \} \end{aligned} \quad (\text{A11})$$

where

$$\Omega_1 = \frac{(\rho_{11} + \rho_{12})gL}{K} \quad (\text{A12})$$

$$\Omega_2 = \frac{(\rho_{12} + \rho_{22})gL}{K} \quad (\text{A13})$$

$$\Omega_3 = n_0 \frac{K}{bL^2} \sqrt{\frac{L}{g}} \quad (\text{A14})$$

$$\Omega_4 = \frac{\rho_{12}}{b} \sqrt{\frac{g}{L}} \quad (\text{A15})$$

$$\Omega_5 = \frac{\rho_{22}}{b} \sqrt{\frac{g}{L}} \quad (\text{A16})$$

Boundary conditions at the free surface:

(a) *Kinematic boundary conditions:*

$$\begin{aligned} & (\phi_{1,\xi}^{*n} \zeta_{y^*} + \phi_{1,\eta}^{*n} \eta_{y^*}) \\ &= \frac{-1}{g} \left\{ \frac{\phi_1^{*n} - 2\phi_1^{*n-1} + \phi_1^{*n-2}}{\Delta t^{*2}} + 2(U^* + \phi_{1,\xi}^{*n-1} \zeta_{x^*} + \phi_{1,\eta}^{*n-1} \eta_{x^*}) \right. \\ & \quad \left(\frac{\phi_{1,\xi}^{*n} \zeta_{x^*} + \phi_{1,\eta}^{*n} - \phi_{1,\xi}^{*n-1} - \phi_{1,\eta}^{*n-1} \eta_x}{\Delta t^*} \right) \\ & \quad + (\phi_{1,\xi}^{*n} \zeta_{x^*}^2 + 2\phi_{1,\xi\eta}^{*n} \zeta_{x^*} \eta_{x^*} + \phi_{1,\eta\eta}^{*n} \eta_{x^*}^2 + \phi_{1,\xi}^{*n} \zeta_{x^*x^*} + \phi_{1,\eta}^{*n} \eta_{x^*x^*}) \\ & \quad (U^* + \phi_{1,\eta}^{*n-1} \zeta_{x^*} + \phi_{1,\eta}^{*n-1} \eta_{x^*})^2 + (\phi_{1,\xi}^{*n-1} \zeta_{y^*} \phi_{1,\eta}^{*n-1} \eta_{y^*}) \\ & \quad \left(\frac{\phi_{1,\xi}^{*n-1} \zeta_{y^*} + \phi_{1,\eta}^{*n-1} \eta_{y^*} - \phi_{1,\xi}^{*n-2} \zeta_{y^*} - \phi_{1,\eta}^{*n-2} \eta_{y^*}}{\Delta t^*} \right) \\ & \quad + (\phi_{1,\xi}^{*n-1} \zeta_{y^*} + \phi_{1,\eta}^{*n-1} \eta_{y^*})(U^* + \phi_{1,\xi}^{*n-1} \zeta_{x^*} + \phi_{1,\eta}^{*n-1} \eta_{x^*}) \\ & \quad [\phi_{1,\xi\xi}^{*n-1} \zeta_{x^*} \zeta_{y^*} + \phi_{1,\xi\eta}^{*n-1} (\zeta_{y^*} \eta_{x^*} + \zeta_{x^*} \eta_{y^*}) \\ & \quad + \phi_{1,\eta\eta}^{*n-1} \eta_{x^*} \eta_{y^*} + \phi_{1,\xi}^{*n-1} \zeta_{x^*y^*} + \phi_{1,\eta}^{*n-1} \eta_{x^*y^*}] \} \end{aligned} \quad (\text{A17})$$

(b) *Dynamic boundary condition:*

$$\begin{aligned} & \frac{\phi_1^{*n} - \phi_1^{*n-1}}{\Delta t^*} + \frac{1}{2} [2U^* (\phi_{1,\xi}^{*n} \zeta_{x^*} + \phi_{1,\eta}^{*n} \eta_{x^*}) + (\phi_{1,\xi}^{*n} \zeta_{x^*} + \phi_{1,\eta}^{*n} \eta_{x^*})^2 \\ & + (\phi_{1,\xi}^{*n} \zeta_{y^*} + \phi_{1,\eta}^{*n} \eta_{y^*})^2] + \zeta^n = 0 \end{aligned} \quad (\text{A18})$$

Boundary conditions at the porous bed interface:

(a) *Continuity of flux:*

$$\begin{aligned} & (\phi_{1,\xi}^{*n} \zeta_{y^*} + \phi_{1,\eta}^{*n} \eta_{y^*}) - \frac{y_{\xi}^{*n-1}}{x_{\xi}^{*n-1}} (\phi_{1,\xi}^{*n} \zeta_{x^*} + \phi_{1,\eta}^{*n} \eta_{x^*}) \\ &= U^* \frac{y_{\xi}^{*n-1}}{x_{\xi}^{*n-1}} + \frac{1}{\Delta t^*} \left\{ \left[(1 - n_0) \left(1 - \frac{y_{\xi}^{*n-1}}{x_{\xi}^{*n-1}} \right) (\phi_{d,\xi}^{*n} \zeta_{y^*} + \phi_{d,\eta}^{*n} \eta_{y^*} - \omega_{d,\xi}^{*n} \zeta_{x^*} - \omega_{d,\eta}^{*n} \eta_{x^*}) \right. \right. \end{aligned}$$

$$\begin{aligned}
& + n_0 \left(1 - \frac{y_{\xi}^{*n-1}}{x_{\xi}^{*n-1}} \right) (\varphi_{D,\xi}^{*n} \xi_{y^*} + \varphi_{D,\eta}^{*n} \eta_{y^*} - \omega_{D,\xi}^{*n} \xi_{x^*} - \omega_{D,\eta}^{*n} \eta_{x^*}) \\
& - \left[(1 - n_0) \left(1 - \frac{y_{\xi}^{*n-2}}{x_{\xi}^{*n-2}} \right) (\varphi_{D,\xi}^{*n-1} \xi_{y^*} + \varphi_{D,\eta}^{*n-1} \eta_{y^*} - \omega_{D,\xi}^{*n-1} \xi_{x^*} - \omega_{D,\eta}^{*n-1} \eta_{x^*}) \right. \\
& \left. + n_0 \left(1 - \frac{y_{\xi}^{*n-2}}{x_{\xi}^{*n-2}} \right) (\varphi_{D,\xi}^{*n-1} \xi_{y^*} + \varphi_{D,\eta}^{*n-1} \eta_{y^*} - \omega_{D,\xi}^{*n-1} \xi_{x^*} - \omega_{D,\eta}^{*n-1} \eta_{x^*}) \right] \}. \quad (A19)
\end{aligned}$$

(b) *Continuity of effective stress:*

$$\begin{aligned}
& \frac{G}{K} \{ 2[\varphi_{d,\xi\xi}^{*n} \xi_{x^*} \xi_{y^*} + \varphi_{d,\xi\eta}^{*n} (\xi_{x^*} \eta_{y^*} + \xi_{y^*} \eta_{x^*}) \\
& + \varphi_{d,\eta\eta}^{*n} \eta_{x^*} \eta_{y^*} + \varphi_{d,\xi}^{*n} \xi_{x^*} y^* + \varphi_{d,\eta}^{*n} \eta_{x^*} y^*] \\
& + [\omega_{d,\xi\xi}^{*n} \xi_{y^*}^2 + 2\omega_{d,\xi\eta}^{*n} \xi_{y^*} \eta_{y^*} + \omega_{d,\eta\eta}^{*n} \eta_{y^*}^2 + \omega_{d,\xi}^{*n} \xi_{y^*} y^* + \omega_{d,\eta}^{*n} \eta_{y^*} y^*] \\
& + [\omega_{d,\xi\xi}^{*n} \xi_{x^*}^2 + 2\omega_{d,\xi\eta}^{*n} \xi_{x^*} \eta_{x^*} + \omega_{d,\eta\eta}^{*n} \eta_{x^*}^2 + \omega_{d,\xi}^{*n} \xi_{x^*} x^* + \omega_{d,\eta}^{*n} \eta_{x^*} x^*] \} \\
& - \frac{y_{\xi}^{*n-1}}{x_{\xi}^{*n-1}} \left\{ \frac{2G + \lambda}{K} [\varphi_{d,\xi\xi}^{*n} \xi_{x^*}^2 + 2\varphi_{d,\xi\eta}^{*n} \xi_{x^*} \eta_{x^*} + \varphi_{d,\eta\eta}^{*n} \eta_{x^*}^2 + \varphi_{d,\xi}^{*n} \xi_{x^*} x^* + \varphi_{d,\eta}^{*n} \eta_{x^*} x^*] \right. \\
& + 2\frac{G}{K} [\omega_{d,\xi\xi}^{*n} \xi_{x^*} \xi_{y^*} + \omega_{d,\xi\eta}^{*n} (\xi_{x^*} \eta_{y^*} + \xi_{y^*} \eta_{x^*}) \\
& + \omega_{d,\eta\eta}^{*n} \eta_{x^*} \eta_{y^*} + \omega_{d,\xi}^{*n} \xi_{x^*} y^* + \omega_{d,\eta}^{*n} \eta_{x^*} y^*] \\
& \left. + \frac{\lambda}{K} [\varphi_{d,\xi\xi}^{*n} \xi_{y^*}^2 + 2\varphi_{d,\xi\eta}^{*n} \xi_{y^*} \eta_{y^*} + \varphi_{d,\eta\eta}^{*n} \eta_{y^*}^2 + \varphi_{d,\xi}^{*n} \xi_{y^*} y^* + \varphi_{d,\eta}^{*n} \eta_{y^*} y^*] \right\} = 0 \quad (A20)
\end{aligned}$$

$$\begin{aligned}
& \left\{ \frac{2G + \lambda}{K} [\varphi_{d,\xi\xi}^{*n} \xi_{y^*}^2 + 2\varphi_{d,\xi\eta}^{*n} \xi_{y^*} \eta_{y^*} + \varphi_{d,\eta\eta}^{*n} \eta_{y^*}^2 + \varphi_{d,\xi}^{*n} \xi_{y^*} y^* + \varphi_{d,\eta}^{*n} \eta_{y^*} y^*] \right. \\
& - 2\frac{G}{K} [\omega_{d,\xi\xi}^{*n} \xi_{x^*} \xi_{y^*} + \omega_{d,\xi\eta}^{*n} (\xi_{x^*} \eta_{y^*} + \xi_{y^*} \eta_{x^*}) \\
& + \omega_{d,\eta\eta}^{*n} \eta_{x^*} \eta_{y^*} + \omega_{d,\xi}^{*n} \xi_{x^*} y^* + \omega_{d,\eta}^{*n} \eta_{x^*} y^*] \\
& \left. + \frac{\lambda}{K} [\varphi_{d,\xi\xi}^{*n} \xi_{x^*}^2 + 2\varphi_{d,\xi\eta}^{*n} \xi_{x^*} \eta_{x^*} + \varphi_{d,\eta\eta}^{*n} \eta_{x^*}^2 + \varphi_{d,\xi}^{*n} \xi_{x^*} x^* + \varphi_{d,\eta}^{*n} \eta_{x^*} x^*] \right\} \\
& - \frac{y_{\xi}^{*n-1}}{x_{\xi}^{*n-1}} \frac{G}{K} \{ 2[\varphi_{d,\xi\xi}^{*n} \xi_{x^*} \xi_{y^*} + \varphi_{d,\xi\eta}^{*n} (\xi_{x^*} \eta_{y^*} + \xi_{y^*} \eta_{x^*}) \\
& + \varphi_{d,\eta\eta}^{*n} \eta_{x^*} \eta_{y^*} + \varphi_{d,\xi}^{*n} \xi_{x^*} y^* + \varphi_{d,\eta}^{*n} \eta_{x^*} y^*] \\
& + [\omega_{d,\xi\xi}^{*n} \xi_{y^*}^2 + 2\omega_{d,\xi\eta}^{*n} \xi_{y^*} \eta_{y^*} + \omega_{d,\eta\eta}^{*n} \eta_{y^*}^2 + \omega_{d,\xi}^{*n} \xi_{y^*} y^* + \omega_{d,\eta}^{*n} \eta_{y^*} y^*] \\
& + [\omega_{d,\xi\xi}^{*n} \xi_{x^*}^2 + 2\omega_{d,\xi\eta}^{*n} \xi_{x^*} \eta_{x^*} + \omega_{d,\eta\eta}^{*n} \eta_{x^*}^2 + \omega_{d,\xi}^{*n} \xi_{x^*} x^* + \omega_{d,\eta}^{*n} \eta_{x^*} x^*] \} = 0 \quad (A21)
\end{aligned}$$

(c) *Continuity of pressure:*

$$P_2^{*n}K + \rho_f gL \left\{ \frac{\phi_1^{*n} - \phi_1^{*n-1}}{\Delta t^*} + \frac{1}{2} [2U^*(\phi_{1,\xi}^{*n} \zeta_{x^*} + \phi_{1,\eta}^{*n} \eta_{x^*}) + (\phi_{1,\xi}^{*n} \zeta_{x^*} + \phi_{1,\eta}^{*n} \eta_{x^*})^2 + (\phi_{1,\xi}^{*n} \zeta_{y^*} + \phi_{1,\eta}^{*n} \eta_{y^*})^2] \right\} = 0 \quad (\text{A22})$$

(d) *Far field of porous bed:*

$$\phi_1^* = \varphi_d^* = \varphi_D^* = \omega_d^* = \omega_D^* = 0 \quad (\text{A23})$$

(e) *Upstream boundary:*

$$\phi_{1i,j}^{*n} = \frac{5}{2}\phi_{1i+1,j}^{*n-1} - \frac{4}{2}\phi_{1i+2,j}^{*n-1} + \frac{1}{2}\phi_{1i+3,j}^{*n-1} \quad (\text{A24})$$

$$5\varphi_{di+1,j}^{*n} - 4\varphi_{di+2,j}^{*n} + \varphi_{di+3,j}^{*n} = 0 \quad (\text{A25})$$

$$5\varphi_{Di+1,j}^{*n} - 4\varphi_{Di+2,j}^{*n} + \varphi_{Di+3,j}^{*n} = 0 \quad (\text{A26})$$

$$5\omega_{di+1,j}^{*n} - 4\omega_{di+2,j}^{*n} + \omega_{di+3,j}^{*n} = 0 \quad (\text{A27})$$

(f) *Downstream boundary:*

$$\phi_{1i,j}^{*n} = \frac{5}{2}\phi_{1i-1,j}^{*n-1} - \frac{4}{2}\phi_{1i-2,j}^{*n-1} + \frac{1}{2}\phi_{1i-3,j}^{*n-1} \quad (\text{A28})$$

$$5\varphi_{di-1,j}^{*n} - 4\varphi_{di-2,j}^{*n} + \varphi_{di-3,j}^{*n} = 0 \quad (\text{A29})$$

$$5\varphi_{Di-1,j}^{*n} - 4\varphi_{Di-2,j}^{*n} + \varphi_{Di-3,j}^{*n} = 0 \quad (\text{A30})$$

$$5\omega_{di-1,j}^{*n} - 4\omega_{di-2,j}^{*n} + \omega_{di-3,j}^{*n} = 0 \quad (\text{A31})$$

After the solutions are obtained, the variation of bed form can be found as

$$\begin{aligned} \gamma^{*n} = & \gamma^{*n-1} + \left[(\varphi_{d,\xi}^{*n} \zeta_{y^*} + \varphi_{d,\eta}^{*n} \eta_{y^*} - \omega_{d,\xi}^{*n} \zeta_{x^*} - \omega_{d,\eta}^{*n} \eta_{x^*}) \right. \\ & - \frac{y_{\xi}^{*n-1}}{x_{\xi}^{*n-1}} (\varphi_{d,\xi}^{*n} \zeta_{x^*} + \varphi_{d,\eta}^{*n} \eta_{x^*} + \omega_{d,\xi}^{*n} \zeta_{y^*} + \omega_{d,\eta}^{*n} \eta_{y^*}) \\ & - \left[(\varphi_{d,\xi}^{*n-1} \zeta_{y^*} + \varphi_{d,\eta}^{*n-1} \eta_{y^*} - \omega_{d,\xi}^{*n-1} \zeta_{x^*} - \omega_{d,\eta}^{*n-1} \eta_{x^*}) \right. \\ & \left. \left. - \frac{y_{\xi}^{*n-2}}{x_{\xi}^{*n-2}} (\varphi_{d,\xi}^{*n-1} \zeta_{x^*} + \varphi_{d,\eta}^{*n-1} \eta_{x^*} + \omega_{d,\xi}^{*n-1} \zeta_{y^*} + \omega_{d,\eta}^{*n-1} \eta_{y^*}) \right] \right]. \quad (\text{A32}) \end{aligned}$$

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