Network Flow Optimization Model for Basin-Scale Water Supply Planning

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Abstract: This paper develops a generalized network flow optimization model for long-term supply-demand analysis for basin-wide water resources planning. A set of nodes and arcs are used to form the network, and the decision variables are reservoir storage and water supply for public and agricultural uses. The objective function to be minimized is formed by summing the products of the decision variables multiplied by their corresponding cost coefficients. The constraints of the model include continuity equations, reservoir operation rule curves, reduced water supply due to water shortage, and evaporation losses from reservoirs. The formulated network model has losses in the transshipment and is solved by an efficient embedded generalized network solver (EMNET). The developed model is applied to a river basin located in the northern part of Taiwan. To analyze the characteristics of the model, the results obtained from the optimization model are compared with those obtained from a well-calibrated simulation model. The comparison shows that the water shortage computed by the optimization model is smaller than that computed by the simulation model. The developed model is then used to analyze future water supply-demand conditions for the area.

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Introduction

Traditionally, both simulation and optimization models are used for basin-wide water resources planning and management [see reviews by Yeh (1985) and Wurbs (1993)]. Simulation models analyze the results from various stipulated alternatives and then check for feasibility and optimality. It is inherently a trial-and-error approach and optimality is not assured. The major advantages of using a simulation model are that the model is easy to operate and it generally requires minimum computer time and storage. On the other hand, optimization models minimize or maximize a given objective function subject to the imposed set of constraints; thus the solution obtained is feasible and optimal. However, for large-scale water resources systems, this approach may become infeasible because of the dimensionality problem. In recent years, improvements in both computer capability and algorithm efficiency make the optimization models more attractive.

Originally, simulation models were common for water resources planning and management. Maass et al. (1962) used simulation to evaluate the benefit for basin-wide water resources planning. Hufschmidt and Fiering (1966) used simulation to study the planning problem of the Rhine River system under multiple-reservoir and multiple-objective conditions. The U.S. Corps of Engineers developed HEC-5 (HEC 1979) for flood control and

water conservation. Kuo and Young (1985) used stochastic simulation to derive joint operation policies for the Shimen and Feitui reservoirs. Delft Hydraulics (1991) developed a simulation model, RIBSIM, to analyze the long-term supply-demand planning in Taiwan. Chow (1995) formulated the simulation model into a network model for water resources management for the Tsenwen and Kaoping watersheds in southern Taiwan. Many researchers used linear programming to solve water resources planning and management problems. Loucks and Dorfman (1975) developed chance-constrained linear programming models to evaluate linear decision rules for reservoir planning and operation.

Certain types of the water resources planning and management problems formulated in terms of linear programming can also be formulated into network flow models. The most commonly used method to solve a pure network model, a network in which there is no gain or loss in the transshipment, is the out-of-kilter algorithm. These types of models include ACRES (Sigvaldason 1976), MODSIM3 (Labadie et al. 1986), WASP (Kuczera and Diment 1988), CRAM (Brendecke et al. 1989), DWRSIM (Chung et al. 1989), and KCOM (Andrews et al. 1992). Pure network models can also be solved by using the primal simplex algorithm (Kuczera 1989). Kuczera used decomposition along with the primal simplex algorithm to solve a generalized network model, although computational efficiency was sacrificed. McBride (1985) used the network simplex method along with matrix partition to develop EMNET, which can be used to solve an embedded generalized network model.

Sun et al. (1995) compared the efficiency of MINOS (Murtagh and Saunders 1987) and EMNET in analyzing the long-term supply-demand planning problem for the Metropolitan Water District of Southern California and concluded that the latter was 11 to 117 times faster than the former. Sun et al. (1996) demonstrated the use of EMNET to solve a water distribution system with losses. Yeh and Yang (1997) formulated the water resources man-

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agement problem in the Chianan area in Taiwan into a network model and solved it by EMNET.

The purpose of this paper is to formulate the long-term planning problem for a large-scale water resources system into a generalized network model. The formulation takes into account the reservoir operation rule-curves, reduced water supply due to water shortage, and evaporation losses from reservoirs. The developed model assumes that the statistics of the past flow process as recorded in the data will repeat in the future. The methodology is applied to two river basins within the greater Tanshui River basin, located in the northern part of Taiwan.

Network Formulation of Basin-Wide System

Construction of Network Framework

A basin-wide supply-demand system is usually constructed by combining various conveyance, storage, diversion, and pump facilities as well as the demand and supply points. Conveyance facilities include natural rivers, canals, and pressure conduits. Storage facilities include reservoirs and detention ponds. Demand points include cities, industrial areas, agricultural lands, and minimum requirements for environmental purposes. To proceed with the network formulation, nodes are introduced to represent reservoirs, diversions, public demand points, agricultural demand points, minimum requirement points, and source points. In addition, arcs are used to represent river reaches, pipelines, reservoir storage zones, and paths that link all water uses. Each arc is subject to an upper and lower bound. The framework of the network is then constructed by connecting the nodes and arcs according to their physical locations in the system.

Network Formulation of Reservoir Operation Rule Curves

Reservoir operation can be categorized into normal operation and flood-mitigation operation. During the normal operation period, rule-curves are usually used. For most reservoirs in Taiwan, three rule-curves are designated to separate the reservoir storage into four zones throughout the year, as shown in Fig. 1. The curves are called upper limit, lower limit, and extreme lower limit. The operational policy is then defined based upon the storage status at the beginning of each time period.

To incorporate the rule-curves and their associated operational policy into the network framework, four arcs are introduced to connect the two adjacent time periods t and $t+1$ of the same reservoir node, as can be seen in Fig. 2. The arcs $X_{R,1,t}$, $X_{R,2,t}$, $X_{R,3,t}$, and $X_{R,4,t}$ represent the four zones. Each arc has its own inflow and outflow multiplier, upper and lower bound, and unit cost coefficient. By properly assigning the values for the upper and lower bounds and the unit cost coefficients for the four arcs, the rule-curves and their associated operational policy can be completely described. As can be seen in Table 1, the upper bounds of the four arcs are ΔV_1 , ΔV_2 , ΔV_3 , and ΔV_4 , the lower bounds are all zero, and the unit cost coefficients are α_1 , α_2 , α_3 , and α_4 . Then, the four zones of the rule-curves can be described by the following equations:

$$\Delta V_1 = V_1 - V_2 \quad (1a)$$

$$\Delta V_2 = V_2 - V_3 \quad (1b)$$

$$\Delta V_3 = V_3 - V_4 \quad (1c)$$

$$\Delta V_4 = V_4 \quad (1d)$$

Fig. 1. General zonation of reservoir capacity

where $\Delta V_1 \sim \Delta V_4$ = upper limit of zones 1, 2, 3, and 4; V_1 = reservoir capacity; V_2 = upper limit of the rule curves; V_3 = lower limit of the rule curves; and V_4 = extreme lower limit of the rule curves.

To conform to the fact that inflow into a reservoir is stored from the lowest (fourth) zone, first the unit cost coefficients of the four arcs $\alpha_1 \sim \alpha_4$ need to follow the following relationship:

$$\alpha_4 < \alpha_3 < \alpha_2 < \alpha_1 \quad (2)$$

As can also be seen from Table 1, if no loss occurred in the reservoir storage arc i , then the inflow and outflow multipliers γ_i and λ_i of the arcs are -1 and 1 , respectively. The negative value denotes that water in the arc is flowing into a node, whereas the positive value denotes that water in the arc is flowing out of a node.

Network Formulation of Hedging Rule

When reservoir storage is below the extreme low limit during a drought period, the demand cannot be fully met and is usually subject to reduction. This rule is called hedging (Yeh and Yang 1997). The network formulation of the hedging rule is similar to that of the rule curves. As can be seen from Fig. 3, the demand from a demand point is separated into two zones by the reduced demand D_2 . The volumes of the two zones are ΔD_1 and ΔD_2 , respectively. The network framework of the hedging rule can be

Fig. 2. Arcs connecting reservoir zones between consecutive time-steps

Table 1. Parameters of Arcs Connecting Reservoir Zones between Consecutive Time-Steps

Arc (<i>i</i>)	Parameters				
	Inflow Multiplier (γ_i)	Outflow Multiplier (λ_i)	Lower Bound	Upper Bound	Unit Cost
1	-1	1	0	ΔV_1	α_1
2	-1	1	0	ΔV_2	α_2
3	-1	1	0	ΔV_3	α_3
4	-1	1	0	ΔV_4	α_4

represented by Fig. 4. Two arcs, $X_{D,1,t}$ and $X_{D,2,t}$, are used to convey water from ΔD_1 and ΔD_2 to the demand point during time period t . As can be seen in Table 2, the upper bounds of the two arcs are ΔD_1 and ΔD_2 , the lower bounds are all zero, and the unit cost coefficients are β_1 and β_2 . Then the two zones of the hedging rule can be described by the following equations:

$$\Delta D_1 = D_1 - D_2 \quad (3a)$$

$$\Delta D_2 = D_2 \quad (3b)$$

where ΔD_1 , ΔD_2 =volumes of the two zones that separate the total demand; D_1 =total demand; and D_2 =reduced demand.

Since the reduced demand D_2 has to be supplied and satisfied first, the cost coefficient of the arc $X_{D,2,t}$ has to be less than that of $X_{D,1,t}$, that is,

$$\beta_2 < \beta_1 \quad (4)$$

As can also be seen from Table 2, the inflow and outflow multipliers γ_i and λ_i are -1 and 1, respectively, if no loss occurred in the arc i connecting to a demand node.

To consider the rule-curves as well as the hedging rule, the unit cost coefficients of the arcs introduced in the formulation of the rule-curves and the hedging rule have to be reordered. Assuming that the supply to a demand point is reduced, if the storage in

Fig. 3. General zonation of demand at demand node

Fig. 4. Arcs connecting diversion node and demand node under reduced water supply conditions

the reservoir is below the extreme lower limit, the unit cost coefficients have to follow the following order:

$$\beta_2 < \alpha_4 < \beta_1 < \alpha_3 < \alpha_2 < \alpha_1 \quad (5)$$

Network Formulation of Reservoir Evaporation Loss

When evaporation loss in a reservoir is considered, the formulated network problem is no longer a pure-network problem but a generalized-network problem (Sun et al. 1996). To formulate the generalized-network problem, the outflow multiplier λ_i will be in the range between 0 and 1. In other words, if the loss rate is l , then $\lambda_i = 1 - l$.

Evaporation loss is usually proportionate to the surface area, which, in turn, is a nonlinear function of the reservoir storage. To maintain linearity of the formulation, the nonlinear function is piecewisely linearized. Assume that the four zones of the reservoir storage are adopted and the corresponding proportionality coefficients of surface area to storage are m_1 , m_2 , m_3 , and m_4 . Letting the corresponding surface area values of storage values V_1 , V_2 , V_3 , and V_4 be A_1 , A_2 , A_3 , and A_4 , as well as allowing the corresponding surface area values of the upper bounds in the four zones be ΔA_1 , ΔA_2 , ΔA_3 , and ΔA_4 , one has $m_i = \Delta A_i / \Delta V_i$, $i = 1, 2, 3, 4$.

If the evaporation rate E_t (in depth) is known in a given time period t , then the ratio of evaporation to storage volume in each zone i of the reservoir is $e_{R,i,t} = 1 - E_t m_i$. The outflow multipliers λ_1 , λ_2 , λ_3 , and λ_4 of the four arcs connecting the reservoir node at period t and period $t + 1$ can easily be determined. Assume that the storage volume of the reservoir and its associated surface area

Table 2. Parameters of Arcs Connecting Diversion Node and Demand Node under Reduced Water Supply Conditions

Arc (<i>i</i>)	Parameters				
	Inflow Multiplier (γ_i)	Outflow Multiplier (λ_i)	Lower Bound	Upper Bound	Unit Cost
1	-1	1	0	ΔD_1	β_1
2	-1	1	0	ΔD_2	β_2

Table 3. Parameters of Arcs Connecting Reservoir Zones between Consecutive Time-Steps Subject to Evaporation Loss

Arc (<i>i</i>)	Parameters				
	Inflow Multiplier (γ_i)	Outflow Multiplier (λ_i)	Lower Bound	Upper Bound	Unit Cost
1	-1	$1 - E_t m_1$	0	ΔV_1	α_1
2	-1	$1 - E_t m_2$	0	ΔV_2	α_2
3	-1	$1 - E_t m_3$	0	ΔV_3	α_3
4	-1	$1 - E_t m_4$	0	ΔV_4	α_4

at time period t are V_t and A_t and assume that the storage volume is reduced to V'_t when the evaporation loss $E_t A_t$ is subtracted. Then one has

$$V'_t = V_t - E_t A_t \quad (6a)$$

By adopting the four zones of the reservoir storage and assuming that

$$V_2 \leq V_t \leq V_1 \quad (6b)$$

$$V_t = V_2 + \Delta V_{1t} \quad (6c)$$

$$A_t = A_2 + \Delta A_{1t} \quad (6d)$$

one has

$$\begin{aligned} V'_t &= (\Delta V_4 + \Delta V_3 + \Delta V_2 + \Delta V_{1t}) - E_t (\Delta A_4 + \Delta A_3 + \Delta A_2 + \Delta A_{1t}) \\ &= (\Delta V_4 - E_t \Delta A_4) + (\Delta V_3 - E_t \Delta A_3) + (\Delta V_2 - E_t \Delta A_2) \\ &\quad + (\Delta V_{1t} - E_t \Delta A_{1t}) \\ &= (1 - E_t m_4) \Delta V_4 + (1 - E_t m_3) \Delta V_3 + (1 - E_t m_2) \Delta V_2 \\ &\quad + (1 - E_t m_1) \Delta V_{1t} \\ &= (1 - E_t m_4) X_{R,4,t} + (1 - E_t m_3) X_{R,3,t} \\ &\quad + (1 - E_t m_2) X_{R,2,t} + (1 - E_t m_1) X_{R,1,t} \\ &= \sum (1 - e_{R,i,t}) X_{R,i,t} = \sum \lambda_i X_{R,i,t} \end{aligned} \quad (6e)$$

Therefore, as can be seen from Table 3, the inflow multipliers γ_1 , γ_2 , γ_3 , and γ_4 as well as the outflow multipliers λ_1 , λ_2 , λ_3 , and λ_4 of the four arcs can be determined as follows:

$$\gamma_1 = -1; \quad \lambda_1 = 1 - E_t m_1 \quad (7a)$$

$$\gamma_2 = -1; \quad \lambda_2 = 1 - E_t m_2 \quad (7b)$$

$$\gamma_3 = -1; \quad \lambda_3 = 1 - E_t m_3 \quad (7c)$$

$$\gamma_4 = -1; \quad \lambda_4 = 1 - E_t m_4 \quad (7d)$$

It is obvious that Eq. (6e) is valid for all four zones.

Formulation of Network Model

Objective

The standard form of the objective function of a network model is

$$\text{minimize} \quad \sum_{(i,j) \in A} C_{(i,j)} X_{(i,j)} \quad (8a)$$

where $X_{(i,j)}$ =decision variable associated with arc (i,j) that denotes flow rate leaving from node i and entering into node j ; $C_{(i,j)}$ =unit cost coefficient of $X_{(i,j)}$ associated with arc (i,j) ; and A =set of all arcs.

In this study, the objective function is to minimize the total cost of water transshipment; that is,

$$\begin{aligned} \text{minimize} \quad & \sum_t \left\{ \sum_R \sum_i C_{R,i,t} X_{S,i,t} + \sum_P \sum_i C_{P,i,t} X_{P,i,t} \right. \\ & \left. + \sum_F \sum_i C_{F,i,t} X_{F,i,t} \right\} \end{aligned} \quad (8b)$$

where $X_{R,i,t}$ =decision variable associated with a reservoir storage arc that denotes the ending storage in zone i of reservoir node R at time period t ; $X_{P,i,t}$ =decision variable associated with an arc i that conveys public water supply to zone i of demand node P at time period t ; $X_{F,i,t}$ =decision variable associated with an arc i that conveys agricultural water use to zone i of demand node F at time period t ; $C_{R,i,t}$ =unit cost coefficient associated with $X_{R,i,t}$; $C_{P,i,t}$ =unit cost coefficient associated with $X_{P,i,t}$; and $C_{F,i,t}$ =unit cost coefficient associated with $X_{F,i,t}$.

Constraints

The standard form of constraints in a network model is

$$\sum_{(i,j) \in A} a_{(i,k)} X_{(i,k)} + \sum_{(k,i) \in A} a_{(k,i)} X_{(k,i)} = b_k \quad (9)$$

where $X_{(i,k)}$ =decision variable associated with arc (i,k) that denotes flow rate leaving node i and entering node k ; $X_{(k,j)}$ =decision variable associated with arc (k,j) that denotes flow rate leaving node k and entering node j ; $a_{(i,k)}$ =multiplier of $X_{(i,k)}$ in a constraint associated with node k ; $a_{(k,j)}$ =multiplier of $X_{(k,j)}$ in a constraint associated with node k ; and b_k =right-hand side of a constraint associated with node k .

It is obvious that the continuity requirement at each node poses a constraint. The formulation of the constraints associated with the six types of nodes considered in this paper is described below.

Reservoir Nodes

The constraints associated with a reservoir node R account for continuity in both spatial and temporal domains. The rule curves and the hedging rule also must be satisfied. The constraints can be expressed as follows:

$$\begin{aligned} -\sum X_{R,i,1} + \sum X_{I,i,1} - \sum X_{O,i,1} &= -(1 - e_{R,0}) S_0 \\ &\quad (\text{for time period } t=1) \end{aligned} \quad (10)$$

$$\begin{aligned} \sum (1 - e_{R,i,t}) X_{R,i,t-1} - \sum X_{R,i,t} + \sum X_{I,i,t} - \sum X_{O,i,t} &= 0 \\ &\quad (\text{for time period } 2 \leq t < T) \end{aligned} \quad (11)$$

$$\begin{aligned} \sum (1 - e_{R,i,t}) X_{R,i,T-1} + \sum X_{I,i,T} - \sum X_{O,i,T} &= S_0 \\ &\quad (\text{for time period } t=T) \end{aligned} \quad (12)$$

$$0 < X_{R,i,t} \leq \Delta V_{R,i,t} \quad (13)$$

where $X_{R,i,(t-1)}$ =decision variable associated with a reservoir storage arc that denotes the ending storage in zone i of reservoir R at time period $t-1$; $X_{R,i,t}$ =decision variable associated with a

reservoir storage arc that denotes the ending storage in zone i of reservoir R at time period t ; $X_{I,i,t}$ =decision variable associated with an arc that denotes flow rate entering reservoir node R at time period t ; $X_{O,i,t}$ =decision variable associated with an arc that denotes flow rate leaving reservoir node R at time period t ; $e_{R,i,t}$ =ratio of evaporation to storage in zone i of reservoir R at time t ; S_0 =initial storage of reservoir R ; $e_{R,0}$ =ratio of evaporation to initial storage of reservoir R ; T =total number of time periods; and $\Delta V_{R,i,t}$ =upper bound of $X_{R,i,t}$.

Demand Nodes for Public Water Supply

The constraint at a demand node P for public water supply with a hedging rule is

$$0 < X_{P,i,t} \leq \Delta D_{P,i,t} \quad (14)$$

where $X_{P,i,t}$ =decision variable associated with an arc that conveys public water supply to zone i of demand node P at time period t , and $\Delta D_{P,i,t}$ =upper bound of $X_{P,i,t}$.

Demand Nodes for Agricultural Water Use

The constraint at a demand node F for agricultural water use with a hedging rule is

$$0 < X_{F,i,t} \leq \Delta D_{F,i,t} \quad (15)$$

where $X_{F,i,t}$ =decision variable associated with an arc that conveys agricultural water use to zone i of demand node F at time period t , and $\Delta D_{F,i,t}$ =upper bound of $X_{F,i,t}$.

Nodes for Water Diversion

The constraints associated with a diversion node D are

$$\sum X_{I,i,t} - \sum X_{O,i,t} - \sum X_{D,i,t} = 0 \quad (16)$$

$$0 < X_{D,i,t} \leq U_{D,i,t} \quad (17)$$

where $X_{I,i,t}$ =decision variable associated with an arc that denotes flow rate entering node D at time t ; $X_{O,i,t}$ =decision variable associated with an arc which denotes flow rate leaving node D at time t ; $X_{D,i,t}$ =decision variable associated with an arc that denotes flow rate diverted from diversion node D at time t ; and $U_{D,i,t}$ =capacity limitation of diverted flow rate associated with $X_{D,i,t}$.

Source Nodes

The constraint associated with a source node S is

$$\sum X_{I,i,t} - \sum X_{O,i,t} = - \sum \text{IN}_{S,i,t} \quad (18)$$

where $X_{I,i,t}$ =decision variable associated with an arc that denotes flow rate entering the source node S at time t ; $X_{O,i,t}$ =decision variable associated with an arc that denotes flow rate leaving source node S at time t ; and $\text{IN}_{S,i,t}$ =flow rate from external source i entering source node S at time t .

Junction Nodes

The constraint associated with a junction node is

$$\sum X_{I,i,t} - \sum X_{O,i,t} = 0 \quad (19)$$

where $X_{I,i,t}$ =decision variable associated with an arc that denotes flow rate entering the junction node at time t , and $X_{O,i,t}$ =decision variable associated with an arc that denotes flow rate leaving the junction node at time t .

We now summarize the above-described formulation procedure for the proposed long-term, supply-demand analysis for n years using a 10-day time period:

$$\begin{aligned} \text{minimize } & \sum_{y=1}^n \sum_{t=36 \times (y-1)+1}^{36 \times y} \left\{ \sum_{i \in A_R} C_{i,t} X_{i,t} + \sum_{i \in A_P} C_{i,t} X_{i,t} \right. \\ & \left. + \sum_{i \in A_F} C_{i,t} X_{i,t} \right\} \end{aligned} \quad (20)$$

Subject to

$$\sum_{h_i=j} (1 - e_{i,t-1}) X_{i,t-1} - \sum_{h_i=j} X_{i,t} + \sum_{g_i=j} X_{i,t} - \sum_{f_i=j} X_{i,t} = 0 \quad j \in N_R \quad (21)$$

$$\sum_{g_i=j} X_{i,t} - \sum_{f_i=j} X_{i,t} = - \sum_{s_i=j} \text{IN}_{i,t} \quad j \in N_S \quad (22)$$

$$\sum_{g_i=j} X_{i,t} - \sum_{f_i=j} X_{i,t} = 0 \quad j \in N_G \quad (23)$$

$$0 \leq X_{i,t} \leq \Delta V_{i,t} \quad h_i=j \quad j \in N_R \quad (24)$$

$$0 \leq X_{i,t} \leq \Delta D_{j,t} \quad g_i=j \quad j \in N_P \quad (25)$$

$$0 \leq X_{i,t} \leq \Delta D_{j,t} \quad g_i=j \quad j \in N_F \quad (26)$$

$$0 \leq X_{i,t} \leq U_{i,t} \quad f_i=j \quad j \in N_D \quad (27)$$

$$t \in (36 \times (y-1) + 1) \sim (36 \times y)$$

where n =number of years under planning; y =yearly index; t =time-step index; $e_{i,t-1}$ =ratio of evaporation to reservoir storage arc i at time $t-1$; $X_{i,t}$ =decision variable associated with arc i at time t ; $C_{i,t}$ =unit cost coefficient associated with $X_{i,t}$; $\text{IN}_{i,t}$ =flow rate entering source node $s_i=j$ from external source i at time t ; $\Delta V_{i,t}$ =upper bound of reservoir storage arc i at time t ; $\Delta D_{j,t}$ =upper bound of public-water-supply or agricultural-water-use arc i with head node $h_i=j$; $U_{i,t}$ =capacity limitation of diverted flow rate in arc i at time t ; f_i =tail node of arc i ; g_i =head node of arc i ; h_i =reservoir node of storage arc i ; s_i =source node which receives source i ; A_R =set of all reservoir storage arcs; A_P =set of all public-water-supply arcs; A_F =set of all agricultural-water-use arcs; N_R =set of all reservoir nodes; N_S =set of all source nodes; N_G =set of all junction nodes; N_P =set of all demand nodes for public water supply; N_F =set of all demand nodes for agricultural water use; and N_D =set of all diversion nodes.

Note that Eq. (21) takes the following form for time-steps 1 and $T=36 \times y$:

$$- \sum_{h_i=j} X_{i,1} + \sum_{g_i=j} X_{i,1} - \sum_{f_i=j} X_{i,1} = (1 - e_{j,0}) S_{j,0} \quad j \in N_R \quad (28a)$$

$$\sum_{h_i=j} (1 - e_{i,T-1}) X_{i,T-1} - \sum_{h_i=j} X_{i,T} + \sum_{g_i=j} X_{i,T} - \sum_{f_i=j} X_{i,T} = S_{j,0} \quad j \in N_R \quad (28b)$$

where $S_{j,0}$ =initial storage of reservoir j , and $e_{j,0}$ =ratio of evaporation to initial storage of reservoir j .

Model Application

The developed model has been applied to a portion of the greater Tanshui River basin, located in the northern part of Taiwan (Fig.

Fig. 5. Greater Tanshui River Basin

5). The greater basin consists of three major river basins: the Tahan River, Hsintien River, and Keelung River basins. Although the Tahan River and Hsintien River basins are hydrologically connected, the Keelung River basin is independent and hence not considered in the joint operation. Therefore, in the rest of this paper the term Tanshui River basin applies only to the Tahan River and Hsintien River basins.

Tahan River Basin

The Tahan River originates from Mt. Pinten and joins the Hsintien River at Chiang chutsui, which is the starting point of the

Tanshui River. The length of the Tahan River is 126 km with a drainage area of 1,163 km². There are four major tributaries in the Tahan River basin: Yufong creek, Shankwan creek, Shanshia creek, and Heng creek. The Shihmen reservoir, the most important hydraulic structure in the Tahan River basin, is located downstream of Yufong and Shankwan creeks and upstream of Shanshia and Heng creeks. The Shihmen reservoir, completed in 1964, has an effective storage capacity of 252×10^6 m³. Approximately 20×10^6 m³ of the storage is reserved for flood control during the typhoon season (Hsu et al. 1995). The Shihmen reservoir was built primarily for irrigation and water supply along the Tahan River. The reservoir is currently managed by the Water Resources

Department of the Ministry of Economic Affairs and has been operating according to the M-5 rule-curves, as depicted in Fig. 1. The capacity of the spillway and the spill-tunnel are 10,000 cm/s and 2,400 cm/s, respectively. The capacity of the outlet is 34 cm/s. The M-5 rule-curves separate the reservoir capacity into four zones by the upper limit, the lower limit, and the extreme lower limit. The annual operational policy and its associated hedging rules can be found in WRPC (1994).

The Yuanshan weir, another major facility in the Tahan River basin, was built primarily for diversion of water to the Panhsin filtration plant. The capacity of the diversion is 11.57 cm/s.

Hsintien River Basin

The Hsintien River originates from Mt. Chilan and joins the Tahan River at Chiang chutsui. The length of the Hsintien River is 82 km, with a drainage area of 916 km². There are three major tributaries in the Hsintien River: Peishih creek, Nanshih creek, and Gingmei creek. The Feitsui reservoir, the most important hydraulic structure in the Hsintien River basin, is located on the Peishih creek. The Feitsui reservoir, completed in 1985, has an effective capacity of 327×10^6 m³ and was built primarily to provide domestic water supply to the Taipei Metropolitan area. The Feitsui reservoir is currently under the jurisdiction of the Taipei City government and has been operating according to its designed rule-curves. The capacity of the spillway, the spill-tunnel, and the spill sluiceway are 5,850 cm/s, 1,500 cm/s, and 2,520 cm/s, respectively. The rule-curves separate the reservoir capacity into four zones by the upper limit, the lower limit, and the extreme lower limit. The annual operational policy and its associated hedging rules can be found in WRPC (1994).

The Chihtan weir and the Chintan weir are the other major facilities in the Hsintien River basin and are used for water diversion. The Chihtan weir is located downstream of the confluence of the Peishih and the Nanshih creeks. It is used to divert water to the Chihtan filtration plant. The diversion capacity is 31.25 cm/s. The Chintan weir is used to divert water to the Konkwan filtration plant. The diversion capacity is 12.56 cm/s.

Water Use

Public water supply and agricultural water use are the two major types of water use in the study area. The public water supply users include the districts of Taipei and Panhsin-Shihmen. Although Taipei users are supplied by the Feitsui reservoir, the Panhsin-Shihmen users are supplied by the Shihmen reservoir. The agricultural water users include Shihmen, Taoyuan, and the upper and lower Tahan River basins. All agricultural water users are supplied by the Shihmen reservoir.

Currently, the Hsintien system supports 15×10^4 m³/day to the public water supply user of Panhsin. This support will be discontinued by 2008 and substituted by the supply from two major hydraulic facilities—namely, the Shanshia weir and the Chihtan transmountain diversion tunnel. The Shanshia weir will be completed by 2001 with a design capacity of 50×10^4 m³/day, and the Chihtan transmountain diversion tunnel will be completed by 2008 with a design capacity of 100×10^4 m³/day.

Optimization Model for Study Area

The network framework of the study area can be constructed based upon the above-mentioned information using 40 nodes and 68 arcs. The time period used in the model is 10 days, with a few

Table 4. Cost Coefficients Associated with Types of Arc Groups

Type of arc group	Cost coefficient
Reservoir storage zone 1	−10
Reservoir storage zone 2	−20
Reservoir storage zone 3	−38
Reservoir storage zone 4	−44
Public water supply 1	−42
Public water supply 2	−48
Irrigation diversion 1	−40
Irrigation diversion 2	−46

exceptions at the end of some calendar months. Traditionally, a month is divided into three time periods. The first two time periods always contain 10 days, whereas the last time period varies from a minimum of eight days to a maximum of 11 days, depending upon the particular calendar month under consideration (Hsu 1995). In this paper, we will refer to the time period as the 10-day period, recognizing the fact that it may not be exactly 10 days. Let T be the total number of time periods; then the total number of constraints is $(40 \times T)$.

Input Data of Study Area

The input data required to conduct the long-term supply-demand analysis for the study area are listed below.

Inflows

Twenty-three years of historical streamflows (1971–1993) have been collected at the Shihmen and Feitsui reservoirs and four stations located at Nanshih creek, Yufong creek, Shanshia creek, and Heng creek. The seven inflows to the six source nodes are then estimated by using the relationships suggested by the Water Resources Planning Commission of Taiwan (WRPC 1994).

Demands

Two demand nodes are used to represent the two public water supply users in the study area. The annual demands of the two users are estimated as 222.6×10^4 m³/day and 128.6×10^4 m³/day, respectively, under the current conditions. The annual demands are also estimated as 240.5×10^4 m³/day and 194.5×10^4 m³/day, respectively, by 2011 (WRPC 1994).

Four demand nodes are used to represent the four agricultural water users in the study area. Due to the fact that a large portion of the agricultural land has been changed to industrial use, which has the highest priority of water right, it is assumed that agricultural water use will not increase in the future. The total annual demands of the four users are estimated as 132.0×10^4 m³/day under the current conditions (WRPC 1994).

Other Input Information

Other required input information that can be found in WRPC (1994) includes the area-volume-elevation curves, the annual operation policies and monthly evaporation rates of reservoirs, the 10-day distribution of agricultural water use, and the 10-day distribution of public water supply use. The cost coefficients associated with the decision variables are categorized into eight types and are listed in Table 4.

Numerical Results

Three case studies are conducted to demonstrate model application.

Case Study 1

The purpose of this case study is to indicate that, for planning purposes, the developed model, as expected, out-performs the RIBSIM model for not only minimizing the shortage but also following the operation and hedging rules. The RIBSIM model was jointly developed by Delft Hydraulics (1991) of the Netherlands and the Water Resources Planning Commission (WRPC) of Taiwan and has been well calibrated for the study area. Although the RIBSIM model does not strictly follow the rule-curves, there are two reasons for selecting the RIBSIM model for comparison purposes: (1) the RIBSIM model was originally developed for the study area and is well known in Taiwan; (2) the RIBSIM model gives reasonably good results (WRB 1999). The hydrologic data of 1993, a drought year, and the predicted demand of 2001, which is $1967.11 \times 10^6 \text{ m}^3$, are used. To ensure that the comparison is made on the same basis, a single-time-period optimization is used. In the single-time-period optimization, we formulate the network model for each 10-day period and solve it independently. The computed ending storage volumes of the Feitsui and Shihmen reservoirs in each time period serve as the initial storage volumes of the following time period. In other words, we are using optimization to solve the simulation problem in a manner that is consistent with the RIBSIM model. The total shortages computed by both models are $203.97 \times 10^6 \text{ m}^3$ and $139.04 \times 10^6 \text{ m}^3$, respectively. As can be expected, the developed model gives less water shortage.

Case Study 2

The purpose of this case study is to show that assigning the cost coefficients is crucial when applying the developed model to handle the rule-curves and the hedging rules. The results obtained from the developed model using both single-time-period and multiple-time-period optimization are also compared. The hydrologic data of 1978, a drought year, and the predicted demand of 2001 are used. The rule-curves of the Feitsui and Shihmen reservoirs along with their hedging rules are considered. In the case of multiple-time-period optimization, the initial storage volumes of the first time period and the ending storage volumes of the 36th period in the two reservoirs are identical to the ending storage volumes of 1977.

To consider the carry-over effect of storage, it was observed by Yeh and Yang (1997) that the unit cost coefficients associated with the storage arcs need to be modified. Supposing that a unit of flow is assigned to the arc at an early time period, a unit benefit (negative of cost), say $-\alpha_4$, will be introduced recursively at each time period before this unit of water is released from the reservoir. Therefore, the actual unit benefit for flow in a storage arc is larger than the original one. If a unit of flow in a supply arc is favorably compared to that in a storage arc, the unit benefit for the supply arc should be larger than that for the total benefit, including future benefit that the storage can bring to the system. At the beginning of the analysis the actual unit benefit for the flow in the first storage arc should be 36 times its original unit benefit for an analysis with 36 time periods. Therefore, to preserve the order of flow, fulfillment β_2 should be less than $36\alpha_4$. Adopting the same concept, the unit cost coefficients should have values such that

$$\beta_2 < 36\alpha_4 < \beta_1 < 36\alpha_3 < 36\alpha_2 < 36\alpha_1 \quad (29)$$

In Eq. (29), $36\alpha_1$, $36\alpha_2$, $36\alpha_3$, and $36\alpha_4$ are used to account for the accumulative effects of the carry-over of storage during the thirty-six 10-day periods in one year.

The total shortages computed by both approaches are 41.74

Table 5. Sources and Filtration Plant Capacities for Public Water Supply under Current Conditions and in Years 2001 and 2011 (10^4 cm/day)

Watershed	Name of plant and source		Current conditions		Year 2011	
	Filtration Plant	Source	Plant Capacity	Source Capacity	Plant Capacity	Source Capacity
Hsintien River	Chihtan	Chihtan weir	165.0	270.0	270.0	270.0
	Konkwan	Chintan weir	108.5	168.5	108.5	168.5
Tahan River	Longtan	Shihmen res.	5.0	5.0	5.0	5.0
	Pincheng	Shihmen res.	30.0	30.0	60.0	68.0
	Tanan	Shihmen res.	30.0	25.0	50.0	25.0
		Yuanshan weir		25.0		25.0
	Shihmen	Shihmen res.	12.0	12.0	12.0	12.0
	Panhsin	Yuanshan weir	60.0	160.0	120.0	160.0
		Shanshia weir		—		50.0
	Kanyuan	Chihtan diversion	—	—	100.0	100.0

$\times 10^6 \text{ m}^3$ and 0, respectively. As can be expected, the multiple-time-period approach is more cost-effective and less conservative, due to the carry-over effect.

Case Study 3

The purpose of this case study is to show the efficiency of the developed model when it is applied to the Tanshui River basin—a real and large-scale system. The hydrologic data from 1971 to 1993 and the predicted demand with a moderate growth are used. The rule curves of the Feitsui and Shihmen reservoirs along with their hedging rules are considered. The initial storage volumes of the first time period in the two reservoirs are identical to the ending storage volumes of 1970 and the ending storage volumes of the (36 $\times$ 23)th period are set to be identical to the initial storage volumes of the first time period. To account for the effect of the carry-over of storage, cost coefficients are modified (Yeh and Yang 1997). The number of decision variables and constraints are 56,304 and 33,120, respectively. Each run of the optimization model takes about 11 s of CPU time on an HP9000/710 workstation.

Two alternatives that the governmental agency proposed (WRB 1999) are evaluated in this case study: current as well as projected conditions and planned construction of facilities for 2011. The major facilities under current conditions consist of Feitsui reservoir, Shihmen reservoir, Chihtan weir, Chihtan weir, and Yuanshan weir. Shanshia weir and Chihtan diversion are planned to activate by 2001 and 2008, respectively. Table 5 shows the sources and capacities of the filtration plants for public water supply under current conditions and target year 2011.

While Hsu (1995) proposed a generalized shortage index (GSI), the shortage index (SI) that was introduced by the U.S. Army Hydrologic Engineering Center (HEC 1966, 1975) is modi-

Table 6. Results of Supply-Demand Analysis for Case Study 3

Watershed		Hsintien River	Tahan River					Entire region
User		Taipei public water supply	Panhsin-Shihmen public water supply	Shihmen irrigation	Taoyuan irrigation	Upper Tahan River irrigation	Lower Tahan River irrigation	-
Total demand ($10^6 m^3$)	Current	18687.27	10797.53	2938.47	6909.10	285.38	946.58	40601.95
	Year 2011	20186.57	16328.82					47595.64
Total shortage ($10^6 m^3$)	Current	0	139.69	65.81	151.72	7.27	8.37	369.86
	Year 2011	0	0	9.84	16.90	0.27	1.46	29.10
Shortage rate (%)	Current	0	1.27	2.24	2.20	2.55	0.89	0.91
	Year 2011	0	0	0.34	0.25	0.27	0.16	0.01
MSI index	Current	0	0.357	0.598	0.712	0.919	0.150	0.160
	Year 2011	0	0	0.067	0.054	0.055	0.022	0.004
Annual water supply potential for MSI=1.0 ($10^4 cmd$)	Current	365.4	349.6					724.9
	Year 2011	443.2	472.1					915.3

fied and used to indicate the degree of water shortage. The shortage index can be defined by the following:

$$SI = \frac{100}{n} \sum_{i=1}^n \left(\frac{AS_i}{AD_i} \right)^2 \quad (30)$$

where SI=shortage index; AS_i =annual shortage in i th year; AD_i =annual demand in i th year; and n =number of years under planning.

This index has been used in the United States as an estimate of water shortage in connection with reservoir construction, and it also implies that the socioeconomic impact of deficit is proportional to the square of the deficit ratio. Moreover, because the SI index is the average of the annual deficit rate squared, the deficit frequency and intensity are incorporated in the index (Hsu 1995).

To characterize the extremely uneven distribution of the hydrologic conditions in Taiwan, a modified shortage index (MSI) is used and can be defined as the following:

$$MSI = \frac{100}{n'} \sum_{i=1}^{n'} \left(\frac{TS_i}{TD_i} \right)^2 \quad (31)$$

where MSI=modified shortage index; TS_i =shortage in i th period; TD_i =demand in i th period; and n' =number of 10-day periods.

In this paper, a water deficit corresponding to MSI=1.0 is considered to be tolerable and is used to determine the annual yield of the system, which is defined as the summation of the

demand at all demand nodes under the condition that MSI=1.0. In reality, a trial-and-error approach needs to be carried out to obtain the annual yield, although we use the following algorithm to accelerate the search.

Step 1: Determine the upper bound of the yield by assuming that there is a reservoir with an unlimited capacity in the system and all the streamflow occurred during the simulation periods can be consumed.

Step 2: Determine the lower bound of the yield by assuming that there is no storage facility in the system and that there is no carry-over between the simulation periods.

Step 3: Determine the yield for a given system with known facilities by the following procedures:

1. Set the assigned demand equal to the average of the upper and lower bounds;
2. Compute the demand at all demand nodes;
3. Run the optimization model;
4. Compute the MSI index M_1
 - a. If M_1 is greater than 1.0, set the upper bound equal to the assigned demand, then go to 5;
 - b. If M_1 is less than 1.0, set the lower bound equal to the assigned demand, then go to 5;
5. Repeat 1;
6. Repeat 2 and 3;
7. Compute the MSI index M_2
 - a. If M_2 is approximately equal to 1.0, then go to 8;
 - b. Otherwise, carry out the linear interpolation using M_1 and

M_2 and their associated demands to obtain the new assigned demand such that $MSI=1.0$. Then assign M_2 to M_1 and go to 6; and

8. If converged, the assigned demand is the yield of the system.

The upper and lower bounds can easily be determined. For example, assume that the streamflows of the five periods are 3, 5, 12, 2, and 8 units. For a reservoir with an unlimited capacity, the demand without any shortage is 6 units. The upper bound can then be computed as 6.67 units (i.e., $100/5[5 \times (6 - 6.67/6.67)^2] = 1.0$). If there is no storage, the lower bound can be computed as 2.58 units (i.e., $100/5[0 + 0 + 0 + (2 - 2.58/2.58)^2 + 0] = 1.0$). A spreadsheet is needed to compute the upper and lower bounds for a real problem, such as the given case study. Using the above-outlined algorithm, convergence takes place in at most five iterations.

The corresponding results obtained from the two alternatives are shown in Table 6. As can be seen from Table 6, the total demands under the current conditions for the Hsintien River system, the Tanhan River system, and the entire study area during the 23 years are $18,687.27 \times 10^6 \text{ m}^3$, $21,914.68 \times 10^6 \text{ m}^3$, and $40,601.68 \times 10^6 \text{ m}^3$. The total shortages are 0, $369.86 \times 10^6 \text{ m}^3$ and $369.86 \times 10^6 \text{ m}^3$. The MSI indices are 0 and 0.160 for the Hsintien River system and the entire study area. The annual water supply potentials for $MSI=1.0$ are also computed as $365.4 \times 10^4 \text{ cmd}$, $349.6 \times 10^4 \text{ cmd}$, and $724.9 \times 10^4 \text{ cmd}$ for the Hsintien River system, the Tanhan River system, and the entire study area. In other words, under the current conditions, the demand can be satisfied for $MSI=1.0$.

As can also be seen from Table 6, the total demands by 2011 for the Hsintien River system, the Tanhan River system, and the entire study area during the 23 years are $20,186.57 \times 10^6 \text{ m}^3$, $27,409.07 \times 10^6 \text{ m}^3$, and $47,595.64 \times 10^6 \text{ m}^3$. The total shortages are 0, $0.01 \times 10^6 \text{ m}^3$ and $0.01 \times 10^6 \text{ m}^3$. The MSI indices are 0 and 0.23 for the Hsintien River system and the entire study area. The annual water supply potentials for $MSI=1.0$ are also computed as $443.2 \times 10^4 \text{ m}^3/\text{day}$, $472.1 \times 10^4 \text{ m}^3/\text{day}$, and $915.3 \times 10^4 \text{ m}^3/\text{day}$ for the Hsintien River system, the Tanhan River system, and the entire study area. In other words, the demand can be satisfied for $MSI=1.0$ by 2011 if the construction of the Shanshia weir and the Chihtan diversion tunnel can be completed according to the planned schedule.

By comparing the annual water supply potentials for $MSI=1.0$ in Table 6, one can see that the annual water supply potentials are all largely increased for the Hsintien River system, the Tanhan River system, and the entire study area. For example, the annual water supply potential increases from $724.9 \times 10^4 \text{ m}^3/\text{day}$ under current conditions to $915.3 \times 10^4 \text{ m}^3/\text{day}$ by 2011, while the MSI index decreases from 0.16 to 0.004 for the entire study area. This is because the capacities of the filtration plants are increased and the Chihtan diversion is tunnel expected to activate by 2011.

Conclusions

A generalized network optimization model has been developed for basin-wide, long-term, supply-demand analysis. An efficient embedded generalized network solver (EMNENT) was used to solve the formulated model. The following contributions have been made in this paper: (1) Zoned reservoir operation rules coupled with the hedging rules have been incorporated and formulated in the model; (2) evaporation loss from a reservoir has been incorporated and formulated in the model; (3) an algorithm has been developed to efficiently determine the water supply po-

tential; (4) the developed model has been applied to a real and large-scale problem for supply-demand analysis.

Several suggestions can be made for future study: (1) use the proposed network formulation to determine the optimal operation rules and the optimal design capacity of a reservoir; (2) modify the proposed network formulation to incorporate hydropower and typhoon release; (3) analyze the water supply potential of the greater Tanshui River basin, considering the construction of the proposed Pinglin reservoir located upstream of the Feitsui reservoir.

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