On the Adaptive Control of Space Robots

Jong-Hann Jean¹ and Li-Chen Fu^{1,2}

Department of Electrical Engineering¹
Department of Computer Science and Information Engineering²
National Taiwan University, Taiwan, R. O. C.

Abstract

It is shown in this paper that, for a variety of reasons, conventional adaptive control schemes can not be generally applied to the control of space robots. Therefore, in this paper, a robust adaptive control scheme that can be successfully applied to various applications in control of space robots is presented. By using a special lagrangian formalism, a globally valid dynamic model of space robots is derived and its dynamic properties which are useful for adaptive controller design are investigated. A unified framework is then presented to formulate the various applications as mentioned, based on which, the robust adaptive control scheme is thus derived. Since the framework is general, the proposed scheme can be actually applied to wider applications. To demonstrate the proposed scheme, the design process is applied to typical applications such as joint space control and Cartesian space control of space robots.

1. INTRODUCTION

Robot manipulators with their bases being mounted on spacecrafts have attracted great attention in the future space missions [1,2] such as the construction, repair and maintenance of satellites and space stations. At the same time, the dynamics and control of space robots called much researchers' interest [3-14] since it poses some problems that are not found in terrestrial robots. For example, due to the dynamic coupling between the robot and the spacecraft, the motion of the robot will disturb the position and the attitude of the spacecraft and, hence, will induce the drift of the robot base. This phenomenon makes the kinematics and the dynamics of a space robot more complicated than those of a terrestrial robot.

Generally, the life span of a spacecraft depends on the amount of the reaction jet fuel it carries for station keeping and attitude control. Therefore, during the operation of manipulators, the reaction jets are turned off to minimize fuel consumption. Moreover, if the operation duration is so short that the effects due to sun-radiation moment and gravity-gradient moment are negligible, then a space robot is only subject to internal forces. Conservation of linear momentum and angular momentum thus provides possible reduction of the dynamics.

Without use of reaction jets, there are two strategies which are currently employed to control space robots. The first strategy [3,4] calls for momentum wheels in controlling the attitude but not the translation of a spacecraft. Since the fuel of momentum wheels is recoverable, this strategy will not shorten the useful life span of the spacecraft. In the second strategy [5-9][12-14], the spacecraft is completely not controlled and, hence, it will move freely in response to the motion of the manipulators.

It has been remarked by Papadopoulos and Duhowsky [8] that nearly any control algorithm that can be used for fixed-based manipulators can also be applied for the space robot systems. Although this remark indeed applies to a variety of control schemes, unfortunately, it is not generally applicable to the current adaptive control schemes [15-21] mainly for two reasons. First, due to the reduction of the system dynamics by conservation laws, the linear-in-parameter property may no longer hold [8]. Therefore, those adaptive control schemes requiring the linear-in-parameter property can not be directly applied in this situation. Second, due to the coupling between kinematics and dynamics, the kinematics can not be determined by only the knowledge of the kinematic structure of the manipulator [8-10]. As a result, conventional Cartesian adaptive control schemes will fail with only the knowledge of kinematic parameters. Nevertheless, in some specific applications or with additional knowledge of dynamic parameters, current adaptive control techniques [22,23] can still be applied.

In this paper, we address the problem of adaptive controller design that can be generally applied to various applications in control of space robots irrespective of which control strategy mentioned above is used. First, using the lagrangian formalism in its invariant form [24], we derive a globally valid (i.e., valid in the whole configuration space) dynamic model which is applicable whether momentum wheels are used to control the attitude of the spacecraft or not. In comparison with the current literature, the proposed dynamic model provides a more general point of view to realize the two

strategies and is useful for controller design in either strategy. For this model, we also investigate some important dynamic properties which are useful for conventional adaptive controller design. Next, we present a unified framework to formulate applications in control of space robots, under which a robust adaptive control scheme is then derived. The main idea in controller design is the following: (i) decompose the dynamic uncertainties into two parts, where one part is linear in system parameters and the other part is upper bounded by a known bound; (ii) combine a robust scheme that handles the latter kind of uncertainties with an adaptive scheme which can take care of the former kind. Finally, to illustrate the proposed scheme, we apply the design process to joint space control and Cartesian space control of space robots in both strategies, respectively.

The paper is organized as follows: In section 2, the dynamic model of space robots is derived. In section 3, some useful dynamic properties and the design process of the robust adaptive control scheme are presented. In section 4, applications of the proposed scheme are given. Conclusions and some supplementary discussions are presented in section 5.

2. DYNAMIC MODEL

Consider a space robot system, as shown in figure 1, consisting of an n -link manipulator mounted on a spacecraft with m momentum wheels. The system is assumed free-floating but not subject to gravitational force and other external forces. All joints of the manipulator are assumed revolute. Each momentum wheel is assumed symmetrical with respect to its axis of rotation. Hereafter, the spacecraft with the wheels locked is referred as body 0 and the i -th link of the manipulator is referred as body i . Body 0 is well-determined owing to the symmetry of momentum wheels.

On the spacecraft, a coordinate frame $\{B_0\}$ is fixed at the center of mass of body 0. On the i -th link of the manipulator, a frame $\{B_i\}$ is assigned following the Denavit-Hartenberg convention [25]. The frame $\{B_i\}$, $i = 0, \dots, n$, are uniquely determined by the pairs (R_i, p_i) where R_i denotes the rotation matrix of the frame with respect to a preselected inertia frame $\{B_f\}$ and p_i is the position vector of the origin of the frame with respect to $\{B_f\}$.

It can be shown [26] that the kinetic energy of the space robot mentioned above can be written as

$$T = \frac{1}{2} \sum_{i=0}^n (\omega_i^T I_i \omega_i + m_i \dot{r}_i^T \dot{r}_i) + \frac{1}{2} \sum_{i=1}^m \frac{l_i^2}{j_i} \quad (2.1.1)$$

$$I_0 = I_{lock} - \sum_{i=1}^m j_i b_i b_i^T \quad (2.1.2)$$

$$l_i = j_i (b_i^T \omega_0 + \dot{\vartheta}_i)$$

where

- ω_i : the angular velocity of body i ;
- I_i , $i = 1, \dots, n$: the inertia matrix of the i -th link;
- m_i : the mass of body i ;
- r_i : the center of mass of body i or its coordinate with respect to $\{B_f\}$;
- I_{lock} : the inertia matrix of body 0;
- b_i : the axis of rotation of the i -th wheel;
- j_i : the moment of inertia of the i -th wheel about b_i ;
- l_i : the angular momentum of the i -th wheel about b_i ;
- ϑ_i : the angle variable of the i -th wheel.

(All the vectors and the inertia matrices mentioned above are defined with respect to $\{B_f\}$)

In the following, we will represent the kinetic energy in terms of the joint variables q and $\dot{q}$ and the spacecraft's angular velocity ω_0 . By the kinematics of the space robot, we have, for $1 \leq i \leq n$,

$$r_i = r_0 + \sum_{j=0}^{i-1} R_j p_{j+1}^T + R_i r_i^T \quad (2.2.1)$$

$$\dot{R}_i = \dot{\omega}_i R_i \quad (2.2.2)$$

$$\dot{r}_i = \dot{r}_0 + \sum_{j=0}^{i-1} \dot{\omega}_j R_j l_{j+1}^i + \dot{\omega}_i R_i r_i^i \quad (2.2.3)$$

$$\begin{aligned} \omega_i &= \omega_{i-1} + a_i \dot{q}_i \\ &= \omega_0 + A_i(q) \dot{q} \end{aligned} \quad (2.2.4)$$

In the above equations, p_{i+1}^i and r_i^i are the coordinates of the points p_{i+1} and r_i relative to $\{B_i\}$; ω denotes a skew-symmetric matrix such that, for any vector $b \in R^3$, $\omega b = \omega \times b$; a_i is the axis of rotation of the i -th joint; q_i is the angle of the i -th joint; $q = [q_1 \dots q_n]^T$ and $A_i(q) = [a_1 \dots a_i \ 0_{3 \times (n-i)}]$.

Since there is no external force acting on the system, conservation of linear and angular momentum can be obtained. Furthermore, if the linear momentum is initially zero, the conservation of linear momentum yields

$$\begin{aligned} m_T r_c &= \sum_{i=0}^n m_i r_i \\ &= m_T r_0 + \sum_{i=1}^n m_i (R_i r_i^i + \sum_{j=0}^{i-1} R_j l_{j+1}^i) \\ &= \text{const.} \end{aligned} \quad (2.3)$$

where m_T is the total mass and r_c is the center of mass of the space robot system.

Using (2.3) to eliminate the position variable r_0 in (2.2.2), we obtain

$$\begin{aligned} r_i &= r_c + \sum_{j=0}^n R_j (l_{j+1}^i - c_j r_j^j - \alpha_{j+1} l_{j+1}^i) \\ &= r_c + \sum_{j=0}^n v_{ij} \end{aligned} \quad (2.4)$$

where $c_j = \frac{m_j}{m_T}$, $\alpha_j = \sum_{k=j}^n c_k$ and $l_{j+1}^j = \begin{cases} l_{j+1}^j, & \text{if } j > i \\ l_j^j, & \text{if } j = i \\ 0, & \text{if } j < i \end{cases}$.

It is noteworthy that the vectors v_{ij} , $i, j = 0, \dots, n$, are exactly *barycentric vectors* defined in [8]. From (2.4), the kinetic energy can be written as

$$\begin{aligned} T &= \frac{1}{2} \sum_{i=0}^n \sum_{j=0}^n \omega_i^T D_{ij} \omega_j + \frac{1}{2} \sum_{i=1}^n l_i^2 \dot{q}_i^2 \\ &= \frac{1}{2} [\dot{q}^T \omega_0^T] M(q) \begin{bmatrix} \dot{q} \\ \omega_0^0 \end{bmatrix} + \frac{1}{2} \sum_{i=1}^n l_i^2 \dot{q}_i^2 \end{aligned} \quad (2.5)$$

$$M(q) = \begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix}$$

$$l_i = j_i (b_i^0 \omega_0^0 + \dot{q})$$

where ω_0^0 is the absolute angular velocity of the spacecraft expressed in $\{B_0\}$ and b_i^0 is the vector b_i expressed in $\{B_0\}$. The definitions of the matrices D_{ij} , $i, j = 0, \dots, n$, and M_{ij} , $i, j = 1, 2$, are given as follows: For $i, j = 0, \dots, n$,

$$D_{ij} = \begin{cases} -\sum_{k=0}^n m_k v_{ik} v_{jk}, & \text{if } i < j \\ l_i - \sum_{k=0}^n m_k v_{ik} v_{jk}, & \text{if } i = j \\ D_{ji}, & \text{if } i > j \end{cases}$$

$$M_{11} = \sum_{i=0}^n \sum_{j=0}^n A_i^T D_{ij} A_j$$

$$M_{21} = \sum_{i=0}^n \sum_{j=0}^n R_0^T D_{ij} A_j, \quad M_{12} = M_{21}^T$$

$$M_{22} = \sum_{i=0}^n \sum_{j=0}^n R_0^T D_{ij} R_0$$

In the following, by use of Lagrange equation, the equations of motion are presented. Before we proceed, it is noteworthy that ω_0^0 is not a generalized velocity since it is not a time derivative of any generalized coordinate. As a result, in deriving the Lagrange equation, it is not meaningful to take the partial derivative of the kinetic energy (2.5) with respect to ω_0^0 . To solve this problem, one can represent ω_0^0 in terms of time derivatives of some local coordinates (e.g. Euler angles) and then apply the Lagrange equation in those coordinates. However, the drawback is that such model is only locally

valid. As an alternative, by using the lagrangian formalism in its invariant form [24], we can adopt a globally defined coordinate in deriving the Lagrange equation to obtain a globally valid dynamic model. The resultant dynamic model is summarized as follows and the detailed derivations can be found in [26].

$$\begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \dot{\omega}_0^0 \end{bmatrix} + \begin{bmatrix} C_{11}(q, \dot{q}, \omega_0^0) & C_{12}(q, \dot{q}, \omega_0^0) \\ C_{21}(q, \dot{q}, \omega_0^0) & C_{22}(q, \dot{q}, \omega_0^0) \end{bmatrix} \begin{bmatrix} \dot{q} \\ \omega_0^0 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (2.6.1)$$

or written more concisely as

$$M(q) \begin{bmatrix} \ddot{q} \\ \dot{\omega}_0^0 \end{bmatrix} + C(q, \dot{q}, \omega_0^0) \begin{bmatrix} \dot{q} \\ \omega_0^0 \end{bmatrix} = \tau \quad (2.6.2)$$

$$\tau_2 = - \sum_{i=1}^m \tau_{wi} b_i^0 \quad (2.6.3)$$

$$\tau_{wi} = \frac{d}{dt} l_i \quad (2.6.4)$$

where τ_1 is the vector of driving torques at manipulator's joints, τ_{wi} is the actuation torque on the i -th wheel, $i = 1, \dots, m$, and the definitions of C_{ij} , $i, j = 1, 2$, are given in appendix.

As mentioned in the introduction, there are two strategies in control during the operation of the manipulator. Specifically, we refer these two strategies as *turn-on of momentum wheels* and *turn-off of momentum wheels*. It is noteworthy that the dynamic equations in (2.6) apply to both strategies except that the control variables in (2.6) are different in the two strategies. This difference and its effects are discussed as follows.

• Turn-On of Momentum Wheels:

In this case, momentum wheels are activated to directly control the attitude of the spacecraft. As a result, the control variables are τ_1 and τ_{wi} , $i = 1, \dots, m$. An intuitive question is what is the least number of wheels so as to control the spacecraft to be pointed to an arbitrary attitude? This question has been widely discussed ([27] and references therein). Here, by use of similar arguments in [27], it can be shown that no smooth control law can be found to arbitrarily control the attitude of the spacecraft if $m < 3$. Therefore, throughout this paper we assume that there are at least three momentum wheels with their axes of rotation being linear independent.

• Turn-Off of Momentum Wheels:

In this case, the control variables are only the driving torques at robot's joints, i.e., $\tau_2 = 0$. Since the actuation of the momentum wheels, τ_{wi} , $i = 1, \dots, m$, are turned off, the angular momentum of the wheels about their axes of rotation, l_i , $i = 1, \dots, m$, are conserved. In this situation, it can be found from (2.7) that the velocities ω_0^0 and $\dot{q}$ are dependent. Therefore, (2.7) can be viewed as a constraint on the system. On the other hand, this can be interpreted as that the attitude of the spacecraft can be controlled through the momentum transfer from the motion of the manipulator.

The dependence relationship between ω_0^0 and $\dot{q}$ can be simplified by properly controlling the initial momentum. There are two possible approaches for this. In the first one, the momentum l_i , $i = 1, \dots, m$, and the overall angular momentum of the system are set to zero so that the all-time constraint (2.7) becomes

$$M_{22} \omega_0^0 + M_{21} \dot{q} = 0, \quad l_i = 0$$

In the second one, the momentum wheels are locked and ω_0^0 and $\dot{q}$ are set to zero initially. After a simple algebraic manipulation, the constraint (2.7) become

$$M'_{22} \omega_0^0 + M_{11} \dot{q} = 0$$

where M'_{22} is obtained by replacing l_0 in the definition of M_{22} with l_{lock} .

3. ADAPTIVE CONTROLLER DESIGN

To derive adaptive control schemes for a variety of applications in a general setting, we present the controller design process in two steps. First, we present a unified framework to formulate applications in control of space robots. More specifically, we associate a particular application with a new coordinate of interest and a velocity transformation between the new coordinate and the original one. Then, we construct a new dynamic model

associated with the new coordinate and explore dynamic properties useful in controller design. Second, we design an adaptive robust control scheme under the generally formulated dynamical model.

3.1 DYNAMIC REFORMULATION AND USEFUL DYNAMIC PROPERTIES

We first summarize some important dynamic properties of a space robot system. The proof of these properties can be found in [26].

- (P1) The matrix $M(q)$ is symmetric and positive definite.
- (P2) The matrix $\dot{M}(q) - 2C(q, \dot{q}, \omega_0^0)$ is skew-symmetric.
- (P3) The dynamic equation (2.6.1) or (2.6.2) is linear in a suitably selected set of dynamic parameters of the system and, hence, the following decomposition will hold

$$M(q)\ddot{u} + C(q, \dot{q}, \omega_0^0)\dot{u} = Y^T(q, \dot{q}, \omega_0^0, \dot{u}, \ddot{u})\eta$$

where $Y(q, \dot{q}, \omega_0^0, \dot{u}, \ddot{u})$ is a known function matrix and the vector η denotes the set of system parameters.

- (P4) There are positive constants $\alpha_i, i = 1, \dots, 4$, such that

$$\|M(q)\| \leq \alpha_1, \quad \|C(q, \dot{q}, \omega_0^0)\| \leq \alpha_2\|\dot{q}\| + \alpha_3\|\dot{q}\| + \alpha_4\|\omega_0^0\|$$

Remark: Although these properties are similar to those of a terrestrial robot, some of them (e.g. (P2)) can not be immediately implied by the widely recognized proof [16,17] for a terrestrial robot. This is owing to that ω_0^0 is not a generalized velocity.

In many robot applications, it is more convenient to describe the dynamics in a particularly chosen coordinate instead of the joint coordinate. This may be either due to the nature of tasks (e.g. tasks specified in Cartesian space) or reduction of dynamics induced by constraints. In the following, the dynamic model is reformulated in terms of a new coordinate of interest and some dynamic properties are investigated for the new model.

Assume the relation between the new coordinate and the original one can be described by a general velocity transformation written as

$$\begin{bmatrix} \dot{q} \\ \dot{\omega}_0^0 \end{bmatrix} = E(q, q_0)\dot{\xi} + v(q, q_0) \quad (3.1)$$

or written more concisely,

$$\begin{bmatrix} \dot{q} \\ \dot{\omega}_0^0 \end{bmatrix} = E(q_e)\dot{\xi} + v(q_e)$$

In the transformation, $q_0 \in R^3$ denotes a local representation (e.g. Euler angles) of the rotation matrix R_0 ; $q_e = [q^T q_0^T]^T$; $\xi \in R^k$ is the new coordinate of interest; $E(q_e) : O \rightarrow R^{(n+3) \times k}$; $v(q_e) : O \rightarrow R^{n+3}$; O , a bounded open set in R^{n+3} , is the region in which the transformation is valid. The matrix $E(q_e)$ is full column rank for all $q_e \in O$. The velocity transformation is further required to satisfy the following assumption.

- (H1) All elements of the function matrix E and the vector v are bounded functions of q_e and their time derivatives are bounded functions of q_e and $\dot{q}_e$.

To obtain the dynamics about the new state ξ , we substitute the transformation (3.1) into (2.6.2) and multiply E^T to the transformed equation. As a result, the following equation is obtained.

$$\bar{M}(q_e)\ddot{\xi} + \bar{C}(q_e, \dot{q}_e)\dot{\xi} + \bar{v}(q_e, \dot{q}_e) = E^T \tau \quad (3.2)$$

where

$$\begin{aligned} \bar{M}(q_e) &= E^T M E, \\ \bar{C}(q_e, \dot{q}_e) &= E^T M \dot{E} + E^T C E, \\ \bar{v} &= E^T (M \dot{v} + C v) \end{aligned} \quad (3.3)$$

The following proposition shows that, with some additional assumption on the velocity transformation (3.1), the transformed dynamic equation (3.2) have similar properties as (P1)-(P3).

Proposition 1: The transformed dynamic equation (3.2) has the following properties.

- (P5) The matrix $\bar{M}$ is symmetric and positive definite.
- (P6) The matrix $\bar{M} - 2\bar{C}$ is skew-symmetric.
- For the transformation (3.1), we assume that
- (H2) Each element of the matrix E and of the vector v are either completely known or linear in some unknown constant parameters. As a result, the following property holds.

- (P7) The dynamic equation (3.2) is linear in a set of system parameters and, similarly, the following decomposition will hold.

$$\bar{M}\ddot{\xi} + \bar{C}\dot{\xi} + \bar{v} = \bar{Y}^T(q_e, \dot{q}_e, \dot{\xi}, \ddot{\xi})\zeta$$

Proof: First, since E is full column rank, by the property (P1), (P5) obviously holds.

Second, directly expanding the matrix $\bar{M} - 2\bar{C}$ yields

$$\begin{aligned} \bar{M} - 2\bar{C} &= E^T M E + E^T \dot{M} E + E^T M \dot{E} - 2(E^T M \dot{E} + E^T C E) \\ &= E^T M E - E^T M \dot{E} + E^T (\dot{M} - 2C) E \end{aligned}$$

From above equation and the property (P2), it is easy to observe that (P6) holds.

Finally, if the elements of E and v are completely known, then (P7) holds with

$$\bar{Y} = E^T Y'(q_e, \dot{q}_e, \dot{\xi}, \ddot{\xi}), \quad \zeta = \eta$$

where Y' is the function matrix Y after substituting the transformations $\dot{u} = E\dot{v} + v$ and $\ddot{u} = E\ddot{v} + \dot{v} + \dot{E}\dot{v}$. On the other hand, if these elements are linear in some unknown constant parameters, it is easy to observe that, after some reparametrizations, (P7) still holds. $\square$

It is noteworthy that, in general, the function matrix E and the vector v may not satisfy the assumption (H2). Therefore, the linear-in-parameter property (P7) may not hold and, hence, adaptive control schemes based on this property can not be applied in this situation. However, if the dynamics contains a linear-in-parameter part such that the remaining part is upper bounded by a known bound, then robust control schemes can be combined with adaptive schemes to handle such kind of uncertainties. This suggests the following decompositions.

$$\begin{aligned} \bar{M} &= M_*(q_e) + \Delta M(q_e), \quad \bar{C} = C_*(q_e, \dot{q}_e) + \Delta C(q_e, \dot{q}_e) \\ \bar{v} &= v_*(q_e) + \Delta v(q_e), \quad E = E_*(q_e) + \Delta E(q_e) \end{aligned} \quad (3.4)$$

Assume that each element of M_* , C_* , and v_* is linear in the system parameters and the matrix E_* is completely known and also full column rank. ΔM , ΔC , Δv , and ΔE denote the parts not linear in parameters. For these parts, we further assume

- (H3) The norms of ΔM , ΔC , ΔE , and Δv are bounded by functions of their arguments, i.e., there are positive functions $\delta M(q_e)$, $\delta C(q_e, \dot{q}_e)$, $\delta E(q_e)$, and $\delta v(q_e)$ such that

$$\begin{aligned} \|\Delta M(q_e)\| &\leq \delta M(q_e), \quad \|\Delta C(q_e, \dot{q}_e)\| \leq \delta C(q_e, \dot{q}_e) \\ \|\Delta E(q_e)\| &\leq \delta E(q_e), \quad \|\Delta v(q_e)\| \leq \delta v(q_e) \end{aligned}$$

3.2 ROBUST ADAPTIVE CONTROL SCHEME

First, the control objective is stated as follows:

Control Objective:

Given a desired trajectory $\xi_d(t)$ which is bounded and has bounded derivatives up to sufficiently large order, and with some or none of the knowledge about system parameters; determine an adaptive robust control law such that the coordinate of interest, ξ , tracks the desired trajectory asymptotically, i.e.,

$$\tilde{\xi} = \xi - \xi_d \rightarrow 0 \quad \text{as } t \rightarrow \infty$$

The following assumptions are essential to the design of a robust adaptive control scheme.

- (A1) The uncertainty bounds δM , δC , δE , and δv are known *a priori*.
- (A2) The measurements of the signals q_e , $\dot{q}_e$, ξ , and $\dot{\xi}$ are available.

Now, we are ready to derive the adaptive robust control scheme. Define the auxiliary signals ξ_r and s as

$$\dot{\xi}_r = \dot{\xi}_d - k_r \tilde{\xi}, \quad s = \tilde{\xi} - \dot{\xi}_r \quad (3.5)$$

where k_r is a positive constant. Design the control signal τ such that

$$E_*^T \tau = \bar{M}_* \ddot{\xi}_r + \bar{C}_* \dot{\xi}_r + v_* - K_D s + \nu \quad (3.6)$$

where ν is an auxiliary control signal to be designed and $\bar{M}_*$, $\bar{C}_*$, and v_* denote the estimates of M_* , C_* , and v_* respectively. By using the linear-in-parameter property, (3.6) can be similarly written as

$$E_*^T \tau = Y_*^T(q_e, \dot{q}_e, \dot{\xi}_r, \ddot{\xi}_r)\theta - K_D s + \nu \quad (3.7)$$

where $\hat{\theta}$ denotes an estimate of θ which is a properly selected set of system parameters and Y_* is a completely known function matrix.

Applying the control signals (3.6) to (3.2), we have the error equation as

$$\begin{aligned} \bar{M}\dot{s} + \bar{C}s + K_D s &= Y_*^T \hat{\theta} + \nu - \Delta M \ddot{\xi}_r - \Delta C \dot{\xi}_r - \Delta v + \Delta E^T \tau \\ &= Y_*^T \hat{\theta} + \nu + \Delta w(q_r, \dot{q}_r, \ddot{\xi}_r, \tau) \end{aligned} \quad (3.8)$$

where $\bar{\theta} = \hat{\theta} - \theta$. The auxiliary control signal ν is designed as

$$\begin{aligned} \nu &= -sgn(s)\delta w \\ \delta w &= \delta M[\ddot{\xi}_r] + \delta C[\dot{\xi}_r] + \delta v + \delta E[\tau]_1 \\ sgn(s) &= \begin{bmatrix} sgn(s_1) \\ \vdots \\ sgn(s_k) \end{bmatrix} \end{aligned} \quad (3.9)$$

whereas the parameter adaptation law is selected as

$$\dot{\hat{\theta}} = -\Gamma Y_*^T s \quad (3.10)$$

where Γ is a positive-definite matrix.

Remark: In (3.4), if the matrices M_* , C_* , v_* are completely known, (For example, they are evaluated by using a fixed estimate of the vector θ), then there is no need to update the parameters. As a result, the control law (3.6) or (3.7) becomes a pure robust control scheme.

The main result of this section is stated as follows.

Theorem 2: In addition to assumptions (A1) and (A2), we further assume (A3) δ_E is small enough such that the matrix $E_*^T + \delta E \cdot \Pi$ is full row rank, where Π is any $k \times (n+3)$ matrix containing only 1 and -1.

(A4) The state q_r will not leave the region O so that the velocity transformation (3.1) is always valid.

Then, the control objective is achieved provided the control law (3.7), (3.9) and (3.10) is applied.

Proof: Define the positive function

$$V(t) = \frac{1}{2} s^T \bar{M}(q_r) s + \frac{1}{2} \bar{\theta}^T \Gamma^{-1} \bar{\theta}$$

Taking the time derivative of $V(t)$ along trajectories of (3.8)–(3.10), we have

$$\begin{aligned} \frac{d}{dt} V(t) &= s^T \left(\frac{1}{2} \dot{\bar{M}} - \bar{C} \right) s - s^T K_D s \\ &\quad + s^T (Y_*^T \hat{\theta} + \nu + \Delta w) + \bar{\theta}^T \Gamma^{-1} \dot{\bar{\theta}} \\ &= -s^T K_D s + \sum_{i=1}^k (s_i \Delta w_i - |s_i| \delta w) \leq 0 \end{aligned}$$

in which the property (P6) is used. As a result, all signals in $V(t)$ are bounded and, furthermore, s is square-integrable. From the definition of s , we have

$$s = \dot{\xi} + k_r \ddot{\xi}$$

It is well-known [28] that the output of a strict proper stable system with the input square-integrable will converge to zero asymptotically. Hence, the tracking error $\ddot{\xi}$ converges to zero asymptotically. This, in addition, implies the boundedness of ξ and $\dot{\xi}$.

In the following, the boundedness of the control signals τ is investigated. From above discussions, the boundedness of ξ , $\dot{\xi}$, s , and $\bar{\theta}$ is guaranteed. By virtue of (A4) and (H1), the boundedness of q_r , $\dot{q}_r$, $\omega_{q_0}^0$, $\dot{q}_0$ are guaranteed. Furthermore, from (3.3), (3.4), (3.6), and (3.7), the norm of the function matrix Y_* is bounded. It is easily observed that the equation (3.7) can be rewritten as

$$E_*^T \tau + \delta E sgn(s) sgn(\tau)^T \tau = Y_*^T \hat{\theta} - K_D s - (\delta M[\ddot{\xi}_r] + \delta C[\dot{\xi}_r] + \delta v) sgn(s)$$

Owing to the above discussions and (H3), the right hand side of the above equation is bounded. Since $sgn(s)sgn(\tau)^T$ is a $k \times (n+3)$ matrix containing only 1 and -1, by (A3), the matrix $E_*^T + \delta E sgn(s)sgn(\tau)^T$ is full row rank. As a result, bounded solutions for τ can be solved to satisfy (3.7). $\square$

4. APPLICATIONS OF THE ROBUST ADAPTIVE SCHEME

In this section, we show how to apply the adaptive control scheme derived in section 3 to the four cases: (i) control in joint space when momentum wheels are turn on; (ii) control in Cartesian space when momentum

wheels are turn on; (iii) control in joint space when momentum wheels are turn off; (iv) control in Cartesian space when momentum wheels are turn off. Specifically, in each case we will show the selection of the new coordinate together with the associated velocity transformation and discuss the validness of the assumptions (A1)–(A4) and the realization of the driving torque τ .

4.1 JOINT SPACE CONTROL WHEN MOMENTUM WHEELS ARE TURNED ON

In this case, both the joint angles of the manipulator and the attitude of the spacecraft are controlled. Accordingly, the coordinate ξ and the velocity transformation are

$$\begin{aligned} \xi &= q_r = \begin{bmatrix} q \\ q_0 \end{bmatrix} \\ \begin{bmatrix} \dot{q} \\ \omega_{q_0}^0 \end{bmatrix} &= \begin{bmatrix} I & 0 \\ 0 & N(q_0) \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{q}_0 \end{bmatrix} \end{aligned} \quad (4.1)$$

where $N(q_0)$ is a known function matrix and is nonsingular for all q_0 in its domain of definition.

From the above equation, it is easily observed that $E(q_r)$ is completely known and $v(q_r) = 0$. As a result, there is no uncertainty which is not linear in parameters, i.e., $\Delta M = \Delta C = 0$, and $\Delta E = 0$. Moreover, the signals q_r and $\dot{q}_r$ are, in practice, basic measurements of a space robot system. Accordingly, the assumptions (A1)–(A3) are satisfied. Since q_r is directly controlled, (A4) can be easily satisfied.

Finally, the control torque τ is determined as

$$\tau = \begin{bmatrix} I & 0 \\ 0 & N(q_0)^T \end{bmatrix}^{-1} (M_* \ddot{\xi}_r + C_* \dot{\xi}_r - K_D s) \quad (4.2)$$

Note that the control law (4.2) is indeed realizable since the spacecraft has at least three momentum wheels with their axes of rotation linearly independent.

4.2 CARTESIAN SPACE CONTROL WHEN MOMENTUM WHEELS ARE TURNED ON

In this case, the Cartesian position of the end-effector together with the attitude of the spacecraft are controlled. Therefore, the coordinate of interest is

$$\xi = \begin{bmatrix} x \\ q_0 \end{bmatrix} \quad (4.3)$$

where $x \in R^6$ denotes the Cartesian coordinate (i.e., position and orientation) of a frame attached on the end-effector. In the following, only the case $n = 6$ is considered.

It has been shown [4] that the kinematics of a space manipulator is equivalent to the one of an inertially fixed manipulator called *virtual manipulator*. As a result, the following kinematic relationship can be obtained.

$$x = h(q_0, q) = h(q_r), \quad \dot{x} = J_0(q_r) \dot{q}_0 + J(q_r) \dot{q} \quad (4.4)$$

where $J_0 = \frac{\partial h}{\partial q_0}$ and $J = \frac{\partial h}{\partial q}$. It is noteworthy that, on contrast with the kinematics of a terrestrial robot, the kinematics (4.4) contains not only kinematic parameters (e.g. link length, link twist, link offset and etc.) but also the dynamic parameters m_i , r_i^i , $i = 1, \dots, n$. This is due to reduction of the dynamics by conservation of linear momentum. In case that all kinematic parameters together with the dynamic parameters m_i , r_i^i , $i = 1, \dots, n$, are known, then h , J , and J_0 are computable and, hence, the signals x and $\dot{x}$ can be obtained through (4.4). Thus (A1)–(A3) are immediately satisfied. If the bounded open set O is chosen such that $J(q_r)$ is invertible, then the velocity transformation

$$\begin{bmatrix} \dot{q} \\ \omega_{q_0}^0 \end{bmatrix} = \begin{bmatrix} J^{-1} & J^{-1} J_0 \\ 0 & N(q_0) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{q}_0 \end{bmatrix} \quad (4.5)$$

is well-defined. By suitably planning the trajectory, (A4) can be satisfied. The control torque is determined as

$$\tau = \begin{bmatrix} J^{-T} & 0 \\ J_0^T J^{-T} & N(q_0)^T \end{bmatrix}^{-1} (M_* \ddot{\xi}_r + C_* \dot{\xi}_r - K_D s) \quad (4.6)$$

In practice, it is more difficult to accurately measure the dynamic parameters r_i^i , $i = 1, \dots, n$, than to measure kinematic parameters or the dynamic parameters m_i , $i = 1, \dots, n$. Therefore, if the knowledge about r_i^i ,

$i = 1, \dots, n$, is not exact, then the control law (4.6) may fail to achieve the control objective. However, if $r_i^1, i = 1, \dots, n$, can be determined up to a compact region D , then the matrices J and J_0 can be decomposed as

$$J = J_* + \Delta J, \quad J_0 = J_{0*} + \Delta J_0$$

where J_* and J_{0*} are evaluated at a point $(r_1^1, \dots, r_n^1) \in D$. If we define the bounded open set O as

$$O = \{q_c \mid |\det(J(q_c))| > \sigma > 0, \forall (r_1^1, \dots, r_n^1) \in D\},$$

then we have

$$E_* = \begin{bmatrix} J_*^{-1} & J_*^{-1} J_{0*} \\ 0 & N(q_0) \end{bmatrix}, \quad \Delta E = E - E_* \quad (4.7)$$

The bound δE is determined as

$$\delta E = \sup \{ \|\Delta E\| \mid \forall q_c \in O, (r_1^1, \dots, r_n^1) \in D \}$$

and the bounds δM and δC can be determined by using (3.3) and the property (P4). Therefore, (A1) is satisfied. It is noteworthy that the signals x and $\dot{x}$ can not be computed through (4.4). Therefore, to fulfill the assumption (A2), extra sensors are needed. Furthermore, the uncertainty region D has to be sufficiently small to satisfy assumption (A3). To restrict q_c in the region O , the trajectory must be carefully planned. Finally, in this situation, the control torque τ is determined as

$$\tau = E_*^{-T} (\dot{M}_* \dot{\xi}_r + \dot{C}_* \dot{\xi}_r + v_* - K_D s + \nu) \quad (4.8)$$

4.3 JOINT SPACE CONTROL WHEN MOMENTUM WHEELS ARE TURNED OFF

In this case, only the joint angles q of the manipulator are directly controlled, i.e., $\xi = q$. Due to the conservation of angular momentum (2.7), we have the velocity transformation

$$\begin{bmatrix} \dot{q} \\ \omega_0^0 \end{bmatrix} = \begin{bmatrix} I \\ -M_{22}^{-1} M_{21} \end{bmatrix} \dot{q} + \begin{bmatrix} 0 \\ -M_{22}^{-1} (\sum_{i=1}^m l_i b_i^0 - c^0) \end{bmatrix} \quad (4.9)$$

where $c^0 = R_0^T c$ is the angular momentum of the overall system expressed in $\{B_0\}$. It is easy to check that, in this case, (3.3) is written as

$$\begin{aligned} \bar{M} &= M_{11} - M_{12} M_{22}^{-1} M_{21} \\ \bar{C} &= C_{11} - M_{12} M_{22}^{-1} C_{21} - C_{12} M_{22}^{-1} M_{21} + M_{12} M_{22}^{-1} C_{22} M_{22}^{-1} M_{21} \\ \bar{v} &= C_{12} M_{22}^{-1} v_1 + M_{12} M_{22}^{-1} C_{22} M_{22}^{-1} v_1 \end{aligned} \quad (4.10)$$

where $v_1 = \sum_{i=1}^m l_i b_i^0 - c^0$. Since the right hand sides of the equations in (4.10) contain the inverse of M_{22} , the dynamics is not linear in parameters. Therefore, decompose M_{22}^{-1} as

$$M_{22}^{-1} = W_{22*} + \Delta W_{22}$$

where each element of W_{22*} is linear in parameters and $\|\Delta W_{22}\| \leq \delta W_{22}$. By using the property (P1) and (P4), a possible solution to the decomposition is to select W_{22*} as a positive diagonal matrix. After a proper decomposition of M_{22}^{-1} , the matrices M_* , C_* , E_* and the vector v_* are obtained by replacing M_{22}^{-1} in (4.9) and (4.10) with W_{22*} . By using the property (P4), the bounds δM , δC , and δv can be obtained accordingly. Now, it is clear that (A1) and (A2) are satisfied. It is noteworthy that in this case the assumption (A3) is not needed because $\tau_2 = 0$ and

$$\Delta E = \begin{bmatrix} 0 \\ -M_{22}^{-1} M_{21} + W_{22*} M_{21} \end{bmatrix}$$

and, hence, $\Delta E^T \tau = 0$. As observed from (4.9), the velocity transformation is globally valid and, hence, (A4) is trivially satisfied.

In this case, the control law (3.6) can be rewritten as

$$E_* \tau = \tau_1 = \bar{M}_* \dot{\xi}_r + \bar{C}_* \dot{\xi}_r + \bar{v}_* - K_D s + \nu \quad (4.11)$$

in which the auxiliary signal ν is modified as

$$\nu = -\text{sgn}(s) (\delta M \dot{\xi}_r + \delta C \dot{\xi}_r + \delta v)$$

because there is no uncertainty on $\Delta E^T \tau$.

4.4 CARTESIAN SPACE CONTROL WHEN MOMENTUM WHEELS ARE TURNED OFF

In this case, only the Cartesian position of the end-effector is controlled, i.e., $\xi = x$. By virtue of (4.4) and (4.9), we have the velocity transformation,

$$\begin{bmatrix} \dot{x} \\ \omega_0^0 \end{bmatrix} = \begin{bmatrix} I & -J^{-1} J_0 N^{-1} \\ 0 & I + M_{22}^{-1} M_{21} J^{-1} J_0 N^{-1} \end{bmatrix}^{-1} \left(\begin{bmatrix} I \\ -M_{22}^{-1} M_{21} \end{bmatrix} J^{-1} \dot{x} + \begin{bmatrix} 0 \\ -M_{22}^{-1} v_1 \end{bmatrix} \right) \quad (4.12)$$

Remark: Note that, as compared with case (ii), for the velocity transformation to be valid, additional configurations, in which the matrix $I + M_{22}^{-1} M_{21} J^{-1} J_0 N^{-1}$ is singular, should be avoided. Since these configurations can not be predicted by the kinematic structure of the manipulator, they are called *dynamic singularities* [8].

It can be found that in above transformation all kinematic parameters and dynamic parameters are involved. Let η denote the vector consisting of all kinematic and dynamic parameters. If the vector η can be determined up to a compact region D , then E_* and v_* are estimated as

$$\begin{aligned} E_* &= \begin{bmatrix} I - J_*^{-1} J_{0*} N^{-1} (I + M_{22*}^{-1} M_{21*} J_*^{-1} J_{0*} N^{-1})^{-1} M_{22*}^{-1} M_{21*} \\ -(I + M_{22*}^{-1} M_{21*} J_*^{-1} J_{0*} N^{-1})^{-1} M_{22*}^{-1} M_{21*} \end{bmatrix} J_*^{-1} \\ v_* &= \begin{bmatrix} I & J_*^{-1} J_{0*} N^{-1} \\ 0 & I - M_{22*}^{-1} M_{21*} J_*^{-1} J_{0*} N^{-1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -M_{22*}^{-1} v_1 \end{bmatrix} \end{aligned}$$

where J_* , J_{0*} , M_{22*} , and M_{21*} are evaluated at a point in D . Define the bounded open set O as

$$O = \{q_c \mid |\det(J)| > \sigma_1 > 0, \\ |\det(I + M_{22}^{-1} M_{21} J^{-1} J_0 N^{-1})| > \sigma_2 > 0, \forall \eta \in D\}$$

and compute δE as

$$\delta E = \sup \{ \|\Delta E\| \mid \forall q_c \in O, \eta \in D \}$$

Again from the property (P4), (A1) can be satisfied. To fulfill (A2), the two methods mentioned in case (ii) can be used depending on whether the knowledge of $r_i^1, i = 1, \dots, n$, is available. Furthermore, the uncertainty region D has to be sufficiently small to satisfy assumption (A3). The trajectory must be carefully planned to satisfy (A4).

The control law (3.6) can be rewritten as

$$\begin{aligned} E_*^T \tau &= (I - J_*^{-1} J_{0*} N^{-1} (I + M_{22*}^{-1} M_{21*} J_*^{-1} J_{0*} N^{-1})^{-1} M_{22*}^{-1} M_{21*}) J_*^{-1} \tau_1 \\ &= (\bar{M}_* \dot{\xi}_r + \bar{C}_* \dot{\xi}_r + \bar{v}_* - K_D s + \nu) \end{aligned} \quad (4.13)$$

It is proved in the appendix that the matrix

$$(I - J_*^{-1} J_{0*} N^{-1} (I + M_{22*}^{-1} M_{21*} J_*^{-1} J_{0*} N^{-1})^{-1} M_{22*}^{-1} M_{21*}) J_*^{-1}$$

is nonsingular for all $q_c \in O$. As a result, τ_1 can be realized from (4.13).

5. CONCLUSIONS AND DISCUSSIONS

In this paper, we have shown that, due to either the dynamics reduction by conservation laws or the coupling between kinematics and dynamics, conventional adaptive control schemes generally can not be applied to control of space robots. Therefore, we present, in a unified framework, an adaptive robust control scheme which can be successfully applied to general applications in control of space robots.

To show the generality of the scheme, we have applied it to joint space control and Cartesian space control of the system employing two different strategies, namely, to turn on momentum wheels and to turn off momentum wheels. In addition, the proposed scheme can also be applied to constrained motion control in the presence of uncertainties on constraint surface. In fact, nearly any motion control problem can be formulated by the unified framework proposed in section 3.1, and, hence, can be solved by the proposed control scheme.

APPENDIX

Definitions of C_{ij} : We directly give the definition of $C(q, \dot{q}, \omega_0^0)$. Let c_{ij} , denote the (i, j) entry of the matrix C . For $i, j = 1, \dots, n$,

$$c_{ij} = \frac{1}{2} \sum_{k=1}^n \frac{\partial m_{ij}}{\partial q_k} \dot{q}_k + \frac{1}{2} \left(\frac{\partial m^i}{\partial q_j} - \frac{\partial m^j}{\partial q_i} \right) \left[\dot{q} \right]$$

where $\hat{\theta}$ denotes an estimate of θ which is a properly selected set of system parameters and Y_* is a completely known function matrix.

Applying the control signals (3.6) to (3.2), we have the error equation as

$$\begin{aligned}\bar{M}\dot{s} + \bar{C}s + K_D s &= Y_*^T \hat{\theta} + \nu - \Delta M \ddot{\xi}_r - \Delta C \dot{\xi}_r - \Delta v + \Delta E^T \tau \\ &= Y_*^T \hat{\theta} + \nu + \Delta w(q_r, \dot{q}_r, \ddot{\xi}_r, \dot{\xi}_r, \tau)\end{aligned}\quad (3.8)$$

where $\hat{\theta} = \hat{\theta} - \theta$. The auxiliary control signal ν is designed as

$$\nu = -\text{sgn}(s)\delta w \quad (3.9)$$

$$\delta w = \delta M |\ddot{\xi}_r| + \delta C |\dot{\xi}_r| + \delta v + \delta E \|\tau\|_1$$

$$\text{sgn}(s) = \begin{bmatrix} \text{sgn}(s_1) \\ \vdots \\ \text{sgn}(s_k) \end{bmatrix}$$

whereas the parameter adaptation law is selected as

$$\dot{\hat{\theta}} = -\Gamma Y_*^T s \quad (3.10)$$

where Γ is a positive-definite matrix.

Remark: In (3.4), if the matrices M_* , C_* , v_* are completely known, (For example, they are evaluated by using a fixed estimate of the vector θ), then there is no need to update the parameters. As a result, the control law (3.6) or (3.7) becomes a pure robust control scheme.

The main result of this section is stated as follows.

Theorem 2: In addition to assumptions (A1) and (A2), we further assume (A3) δ_E is small enough such that the matrix $E_*^T + \delta E \cdot \Pi$ is full row rank, where Π is any $k \times (n+3)$ matrix containing only 1 and -1. (A4) The state q_e will not leave the region O so that the velocity transformation (3.1) is always valid.

Then, the control objective is achieved provided the control law (3.7), (3.9) and (3.10) is applied.

Proof: Define the positive function

$$V(t) = \frac{1}{2} s^T \bar{M}(q_r) s + \frac{1}{2} \hat{\theta}^T \Gamma^{-1} \hat{\theta}$$

Taking the time derivative of $V(t)$ along trajectories of (3.8)-(3.10), we have

$$\begin{aligned}\frac{d}{dt} V(t) &= s^T \left(\frac{1}{2} \dot{\bar{M}} - \bar{C} \right) s - s^T K_D s \\ &\quad + s^T (Y_*^T \hat{\theta} + \nu + \Delta w) + \hat{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} \\ &= -s^T K_D s + \sum_{i=1}^k (s_i \Delta w_i - |s_i| \delta w) \leq 0\end{aligned}$$

in which the property (P6) is used. As a result, all signals in $V(t)$ are bounded and, furthermore, s is square-integrable. From the definition of s , we have

$$s = \dot{\xi} + k_r \ddot{\xi}$$

It is well-known [28] that the output of a strict proper stable system with the input square-integrable will converge to zero asymptotically. Hence, the tracking error ξ converges to zero asymptotically. This, in addition, implies the boundedness of ξ and $\dot{\xi}$.

In the following, the boundedness of the control signals τ is investigated. From above discussions, the boundedness of ξ , $\dot{\xi}$, s , and $\hat{\theta}$ is guaranteed. By virtue of (A4) and (H1), the boundedness of q_r , $\dot{q}_r$, ω_0^0 , $\dot{q}_0$ are guaranteed. Furthermore, from (3.3), (3.4), (3.6), and (3.7), the norm of the function matrix Y_* is bounded. It is easily observed that the equation (3.7) can be rewritten as

$$E_*^T \tau + \delta E \text{sgn}(s) \text{sgn}(\tau)^T \tau = Y_*^T \hat{\theta} - K_D s - (\delta M |\ddot{\xi}_r| + \delta C |\dot{\xi}_r| + \delta v) \text{sgn}(s)$$

Owing to the above discussions and (H3), the right hand side of the above equation is bounded. Since $\text{sgn}(s) \text{sgn}(\tau)^T$ is a $k \times (n+3)$ matrix containing only 1 and -1, by (A3), the matrix $E_*^T + \delta E \text{sgn}(s) \text{sgn}(\tau)^T$ is full row rank. As a result, bounded solutions for τ can be solved to satisfy (3.7). $\square$

4. APPLICATIONS OF THE ROBUST ADAPTIVE SCHEME

In this section, we show how to apply the adaptive control scheme derived in section 3 to the four cases: (i) control in joint space when momentum wheels are turn on; (ii) control in Cartesian space when momentum

wheels are turn on; (iii) control in joint space when momentum wheels are turn off; (iv) control in Cartesian space when momentum wheels are turn off. Specifically, in each case we will show the selection of the new coordinate together with the associated velocity transformation and discuss the validity of the assumptions (A1)-(A4) and the realization of the driving torque τ .

4.1 JOINT SPACE CONTROL WHEN MOMENTUM WHEELS ARE TURNED ON

In this case, both the joint angles of the manipulator and the attitude of the spacecraft are controlled. Accordingly, the coordinate ξ and the velocity transformation are

$$\begin{aligned}\xi &= q_e = \begin{bmatrix} q \\ q_0 \end{bmatrix} \\ \begin{bmatrix} \dot{q} \\ \dot{q}_0 \end{bmatrix} &= \begin{bmatrix} I & 0 \\ 0 & N(q_0) \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{q}_0 \end{bmatrix}\end{aligned}\quad (4.1)$$

where $N(q_0)$ is a known function matrix and is nonsingular for all q_0 in its domain of definition.

From the above equation, it is easily observed that $E(q_e)$ is completely known and $v(q_e) = 0$. As a result, there is no uncertainty which is not linear in parameters, i.e., $\Delta M = \Delta C = 0$, and $\Delta E = 0$. Moreover, the signals q_e and $\dot{q}_e$ are, in practice, basic measurements of a space robot system. Accordingly, the assumptions (A1)-(A3) are satisfied. Since q_e is directly controlled, (A4) can be easily satisfied.

Finally, the control torque τ is determined as

$$\tau = \begin{bmatrix} I & 0 \\ 0 & N(q_0)^T \end{bmatrix}^{-1} (M_* \ddot{\xi}_r + C_* \dot{\xi}_r - K_D s) \quad (4.2)$$

Note that the control law (4.2) is indeed realizable since the spacecraft has at least three momentum wheels with their axes of rotation linearly independent.

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In this case, the Cartesian position of the end-effector together with the attitude of the spacecraft are controlled. Therefore, the coordinate of interest is

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where $x \in R^6$ denotes the Cartesian coordinate (i.e., position and orientation) of a frame attached on the end-effector. In the following, only the case $n = 6$ is considered.

It has been shown [4] that the kinematics of a space manipulator is equivalent to the one of an inertially fixed manipulator called *virtual manipulator*. As a result, the following kinematic relationship can be obtained.

$$x = h(q_0, q) = h(q_e), \quad \dot{x} = J_0(q_0) \dot{q}_0 + J(q_e) \dot{q} \quad (4.4)$$

where $J_0 = \frac{\partial h}{\partial q_0}$ and $J = \frac{\partial h}{\partial q}$. It is noteworthy that, on contrast with the kinematics of a terrestrial robot, the kinematics (4.4) contains not only kinematic parameters (e.g. link length, link twist, link offset and etc.) but also the dynamic parameters m_i , r_i^T , $i = 1, \dots, n$. This is due to reduction of the dynamics by conservation of linear momentum. In case that all kinematic parameters together with the dynamic parameters m_i , r_i^T , $i = 1, \dots, n$, are known, then h , J , and J_0 are computable and, hence, the signals x and $\dot{x}$ can be obtained through (4.4). Thus (A1)-(A3) are immediately satisfied. If the bounded open set O is chosen such that $J(q_e)$ is invertible, then the velocity transformation

$$\begin{bmatrix} \dot{q} \\ \dot{q}_0 \end{bmatrix} = \begin{bmatrix} J^{-1} & J^{-1} J_0 \\ 0 & N(q_0) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{q}_0 \end{bmatrix} \quad (4.5)$$

is well-defined. By suitably planning the trajectory, (A4) can be satisfied. The control torque is determined as

$$\tau = \begin{bmatrix} J^{-T} & 0 \\ J_0^T J^{-T} & N(q_0)^T \end{bmatrix}^{-1} (M_* \ddot{\xi}_r + C_* \dot{\xi}_r - K_D s) \quad (4.6)$$

In practice, it is more difficult to accurately measure the dynamic parameters r_i^T , $i = 1, \dots, n$, than to measure kinematic parameters or the dynamic parameters m_i , $i = 1, \dots, n$. Therefore, if the knowledge about r_i^T ,