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COMMENT

VECTOR HARTLEY TRANSFORM

Introduction: The vector Hartley transform algorithm was recently proposed for efficient transformation of real data by Villaseñor and Bracewell.¹ It maps a very long one-dimensional input into a multidimensional array by the following index mapping:²

$$H(k) = \frac{1}{N} \sum_{n=0}^{N-1} X(n) \text{cas}(2\pi nk/N) \quad (1)$$

for $n, k = 0, 1, \dots, N-1$, where $\text{cas } \theta = \cos \theta + \sin \theta$. Eqn. 1 is rewritten as

$$H(k_1, k_2) = \frac{1}{N} \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} X(n_1, n_2) \times \text{cas} \left[2\pi \left(\frac{n_1 k_1}{N} + \frac{n_2 k_2}{N} \right) \right] \quad (2)$$

where $N = N_1 N_2$ and the indices are mapped from one to two dimensions according to

$$n = n_1 + n_2 N_1$$

$$n_1 = 0, \dots, N_1 - 1 \quad n_2 = 0, \dots, N_2 - 1 \quad (3)$$

$$k = k_1 + k_2 N_1$$

$$k_1 = 0, \dots, N_1 - 1 \quad k_2 = 0, \dots, N_2 - 1 \quad (4)$$

Although this index mapping is general enough for both prime and non-prime factor decompositions, it does, however, result in a complicated expression; also, multiplying each element of the intermediate array by an appropriate factor is necessary (see eqn. 8 in Reference 1). These operations are very time-consuming, and will slow down the transformation speed. This 'Comment' is to point out that these multiplications are unnecessary, and can be eliminated by a proper prime-factor mapping technique.^{3,4}

Prime factor mapping: Consider a composite number $N = N_1 N_2$, where N_1 and N_2 are relatively prime. We can convert a length- N discrete Hartley transform (DHT) of eqn. 1,

$$H(k) = \frac{1}{N} \sum_{n=0}^{N-1} X(n) \text{cas}(2\pi nk/N)$$

into a multidimensional form:

$$H(k_1, k_2) = \frac{1}{N} \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} X(n_1, n_2) \times \text{cas} \left[2\pi \left(\frac{n_1 k_1}{N_1} + \frac{n_2 k_2}{N_2} \right) \right] \quad (5)$$

where

$$n = N_2 n_1 + N_1 n_2 \quad (\text{mod } N) \quad (6)$$

$$k = N_2^{-1} k_1 + N_1^{-1} k_2 \quad (\text{mod } N) \quad (7)$$

with

$$N_2 N_2^{-1} = 1 \quad (\text{mod } N_1)$$

$$N_1 N_1^{-1} = 1 \quad (\text{mod } N_2)$$

for $n_1, k_1 = 0, 1, \dots, N_1 - 1$ and $n_2, k_2 = 0, 1, \dots, N_2 - 1$.

Note that eqn. 5 is much simpler than eqn. 2, and the above intermediate multiplications are eliminated by this prime factor mapping.

By direct analogy with the two-dimensional FFT, one takes the one-dimensional DHTs of the rows one by one, and then transforms the columns. The temporary output $T(k_1, k_2)$ is of the form⁵

$$T(k_1, k_2) = \frac{1}{N} \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} X(n_1, n_2) \times \text{cas}(2\pi n_1 k_1 / N_1) \text{cas}(2\pi n_2 k_2 / N_2) \quad (8)$$

Eqn. 8 is not the two-dimensional Hartley transform; however, the result can be converted to the desired two-dimensional Hartley transform of eqn. 5 as follows.

Since

$$2 \text{cas}(\alpha + \beta) = \text{cas } \alpha \text{cas } \beta + \text{cas } \alpha \text{cas}(-\beta) + \text{cas}(-\alpha) \text{cas } \beta - \text{cas}(-\alpha) \text{cas}(-\beta) \quad (9)$$

The desired Hartley transform $H(k_1, k_2)$ can be expressed as a sum of four temporary transforms:

$$2H(k_1, k_2) = T(k_1, k_2) + T(k_1, N_2 - k_2) + T(N_1 - k_1, k_2) - T(N_1 - k_1, N_2 - k_2) \quad (10)$$

Finally, we use the index mapping (eqns. 6 and 7) to recover $H(k)$ from $H(k_1, k_2)$.

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REPLY

The point of our earlier letter was to show a Hartley transform application to simultaneous parallel processing of vector data. The use of prime factor decomposition is well known in Fourier transformation as providing very efficient operation, and the same may be expected for the Hartley transform. The reduction of the argument of the cas function from three to two terms is clearly very helpful. However, the gain in speed is dependent on the prime factors that are chosen, and needs to be checked in a given environment by overall timing. When one has 365 data values it is not immediately obvious whether packing with zeros to a length of 512 will be inferior in speed to decomposing into the prime factors 5 times 73, or to packing with some other number of zeros to reach an auspicious length that factors into a product of different primes. Possibly the time spent choosing one's primes should be considered as a term in the overall equation. One should also remember that operations modulo some prime incur costly division, where operations modulo 2 are natural to a binary computer.

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7th December 1989

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EFFECTS OF ZINC DOPING IN DFB LASERS EMITTING AT 1.3 AND 1.55 μm

Indexing terms: Semiconductor lasers, Semiconductor doping, Optical communications

We report on the performance of GaInAsP/InP DFB lasers with Zn-doped active region. At both 1.3 and 1.55 μm we achieved a large bandwidth owing to the increased differential gain. The carrier lifetime of the lasers decreased with doping level. The decreased carrier lifetime had the effect of reducing the pattern effect, and we could obtain a clear eye opening with 4 Gbit/s NRZ modulation.

Introduction: There is increasing demand for lasers for use in multi-Gbit/s optical communication systems. DFB lasers exhibiting a clear eye opening will soon be required for such systems. However, because of the limited bandwidth and comparatively long carrier lifetime, DFB lasers often exhibit a pattern effect, and it is very difficult to get a clear eye opening under multi-Gbit/s modulation. It has been reported in Fabry-Perot (FP) lasers that Zn doping of the active layer has the effect of increasing the differential gain and reducing the carrier lifetime,¹ and then we can expect improvement in high-speed performance by Zn doping. We have demonstrated bandwidth expansion by Zn doping in a 1.3 μm DFB laser.² Here we have further investigated the effects of Zn doping, both for 1.3 and 1.55 μm DFB lasers, and confirm a drastic improvement in the eye opening in 1.3 μm DFB lasers.

Structure and fabrication: We doped Zn directly into the active region of all-LPE-grown GaInAsP/InP FBH-DFB lasers.³ We also doped Zn heavily into the cladding layer, to prevent outdiffusion of Zn from the active region, and doped Te heavily into the guiding layer to maintain the position of the pn junction. A narrow mesa of 5 μm width was introduced to reduce parasitic capacitance.⁴ The width and thickness of the active layer are 1 and 0.15 μm , respectively, and the cavity length was 300 μm . The κL was designed to be in the range

Fig. 1 Pulsed threshold current against doping level
● $\lambda = 1.3 \mu\text{m}$ ○ $\lambda = 1.55 \mu\text{m}$

0.5–1.0.³ One end of the cavity was coated with SiN to achieve a reflectivity of below 1%.

Fig. 1 shows the dependence of the pulsed threshold current at room temperature on the doping level. The threshold currents increase with doping level. Because of this increase in the threshold current, the doping level must be adjusted to a level at which we can obtain a compromise between improvement in the high-speed performance and deterioration in the I/L characteristics. Fig. 2 shows the dependence of the carrier lifetime on the doping level. The carrier lifetime was evaluated from the 3 dB cutoff frequency in the small-signal response under LED mode operation. There is a pronounced difference between the 1.3 and 1.55 μm DFB lasers. Because of the large carrier lifetime in the undoped 1.3 μm DFB laser, the reduction in the carrier lifetime for increasing doping level is steep. On the other hand, the carrier lifetime of the undoped 1.55 μm laser is comparatively short, and the reduction by doping is not prominent.

Fig. 2 Carrier lifetime against doping level
● $\lambda = 1.3 \mu\text{m}$ ○ $\lambda = 1.55 \mu\text{m}$

We evaluated the dependence of the peak gain of the Zn-doped FP laser on the carrier density. The peak gains of lasers were measured by the Hakki-Paoli method,⁵ and the carrier densities were evaluated from the bias currents and carrier lifetimes. We confirm that the differential gain of the 1.3 μm laser at a doping level of $4.5 \times 10^{18} \text{ cm}^{-3}$ is 1.6 times that of the undoped laser, and that of the 1.55 μm laser at a doping level of $3 \times 10^{18} \text{ cm}^{-3}$ is 1.3 times that of the undoped laser. The increases in the differential gain contribute to bandwidth expansion. Fig. 3 shows the 3 dB bandwidth against the doping level of the lasers. The wide bandwidth was obtained at a higher doping level owing to the increase in relaxation

Fig. 3 3 dB bandwidth in small-signal response against doping level
● $\lambda = 1.3 \mu\text{m}$ ○ $\lambda = 1.55 \mu\text{m}$