Parasitic-capacitance-insensitive current-mode filters using operational transresistance amplifiers

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Abstract: A parasitic-capacitance-insensitive MOSFET-C integrator and differentiator using operational transresistance amplifiers are proposed and experimental results demonstrated. The bandwidths of these circuits are also independent of their gains. The required capacitances are smaller than those in previous work. Two new configurations of universal current-mode biquad filters based on these circuits are presented. A second-order bandpass filter constructed using universal biquad filters and another third-order Chebyshev lowpass filter with 0.1 dB passband ripple are breadboarded and the measured results presented.

1 Introduction

The continuous-time filter techniques have been receiving considerable attention in MOS VLSI technology because the characteristics of these filters can be stabilised against fabrication tolerances and temperature variations [2]. In certain applications it has been demonstrated that continuous-time filters offer advantages over other types of IC filter, such as switched-capacitor and digital filters [2]. Also, it is well known that the bandwidth of a traditional operational amplifier depends on the closed-loop voltage gain. The attempt to overcome this problem has led to renewed interest in circuits that operate in the current mode [1-4]. Circuits using current-processing techniques have improved dynamic capability and a particular feature of these designs is their ability to provide a constant bandwidth that is virtually independent of the closed-loop gain. In this paper, we use an amplifier termed an operational transresistance amplifier (OTRA) to implement a current-mode integrator and differentiator. Since the OTRA has one output terminal with low impedance and two input terminals that are virtually grounded, most effects of parasitic capacitances disappears and the remainder can be compensated without adding any extra components. Also, since the input terminals of these circuits are virtually grounded, they are cascable. To verify the functions of our proposed circuits, experiments on these circuits have been carried out and the resulting measurements are presented.

2 Circuit descriptions

The circuit symbol and a practical circuit of the OTRA are described in Fig. 1a and b, whose terminal characteristics can be described as [5]

$$\begin{bmatrix} V+ \\ V- \\ V_o \\ I_d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Z_o \parallel Z_L & -Z_o \parallel Z_L & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_o \\ V_d \end{bmatrix} \quad (1)$$

Obviously both input terminals of an OTRA are virtually grounded and the output voltage is the difference of two input currents multiplied by transimpedance $Z_o \parallel Z_L$ (Z_o is

Fig. 1 Operational transresistance amplifier

a Symbol
b Circuit

the total effective output impedance of the two-output current mirrors and Z_L is the external load impedance). In most applications the operational transresistance

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amplifiers together with passive networks can be employed to construct signal-processing circuits. Negative feedback through passive elements forces the currents I_1 and I_2 to be equal whenever the magnitude of $Z_o \parallel Z_L$ approaches infinity. Therefore if suitable passive networks are selected, many useful transfer functions can be realised.

The channel current i_d of the MOSFET biased in the triode region can be expressed as [2]

$$i_d = F(v_D, V_G) - F(v_S, V_G) \quad (2)$$

where

$$F(v, V_G) = 2K(V_G - V_B - V_{FB} - \phi_p)v - K(v - V_B)^2 - \frac{4}{3}K\gamma(v - V_B + \phi_p)^{3/2}$$

$$K = \frac{W}{2L} \mu_s C_{ox}$$

$$\gamma = \frac{\sqrt{(2qN_A \epsilon_s)}}{C_{ox}}$$

W and L are the channel width and length, respectively, μ_s is the effective channel mobility, V_{FB} is the flatband voltage, N_A is the substrate doping concentration, C_{ox} is the oxide capacitance per unit area, ϕ_p is the approximate surface potential at strong inversion, V_G and V_B are the gate and substrate voltages, respectively.

We use a capacitor, an OTRA and two MOSFETs to realise the MOSFET-C integrator and differentiator. The proposed MOSFET-C integrator and differentiator are shown in Figs. 2 and 3. The relationship between the

Fig. 2 Proposed integrator based on single OTRA

Fig. 3 Proposed differentiator based on single OTRA

output currents I_3 , I_4 and input currents I_1 , I_2 of the integrator is

$$\begin{aligned} I_3(s) - I_4(s) &= 2K(V_{GA} - V_{GB})V_x(s) \\ &= 2K(V_{GA} - V_{GB}) \frac{I_1(s) - I_2(s)}{sC + 1/Z_o(s)} \end{aligned} \quad (3)$$

or

$$\frac{I_3(s) - I_4(s)}{I_1(s) - I_2(s)} = \frac{2K(V_{GA} - V_{GB})}{sC + 1/Z_o(s)} \quad (4)$$

$$\simeq \frac{2K}{sC} (V_{GA} - V_{GB}) \text{ as } |Z_o(s)| \rightarrow \infty \quad (5)$$

To analyse the frequency-domain characteristics of this integrator $Z_o(s)$ can be represented as $N(s)/D(s)$ and all roots of both polynomials $N(s)$ and $D(s)$ are located in the left half of the s -plane. Eqn. 4 can therefore be rewritten as

$$\frac{I_3(s) - I_4(s)}{I_1(s) - I_2(s)} = \frac{2K(V_{GA} - V_{GB})N(s)}{sCN(s) + D(s)} \quad (6)$$

Since the capacitance C is fixed and the current gain of this integrator can be varied by adjusting the voltage difference $V_{GA} - V_{GB}$, the poles and zeros of the proposed integrator are always fixed. Thus the bandwidth of the MOSFET-C integrator is nearly constant and independent of the current gain.

In a similar way the current transfer function of the MOSFET-C differentiator can be derived as

$$\begin{aligned} \frac{I_3(s)}{I_1(s) - I_2(s)} &= \frac{sC}{2K(V_{GA} - V_{GB}) + 1/Z_o(s)} \\ &= \frac{sCN(s)}{2K(V_{GA} - V_{GB})N(s) + D(s)} \end{aligned} \quad (7)$$

$$\simeq \frac{sC}{2K(V_{GA} - V_{GB})} \text{ as } |Z_o(s)| \rightarrow \infty \quad (8)$$

This MOSFET-C differentiator also has a bandwidth independent of the current gain as long as the voltage difference $V_{GA} - V_{GB}$ is kept constant.

We then use the proposed integrator and differentiator to implement some configurations of universal biquad filters and higher-order filters. The transfer function of the universal biquad filter shown as Fig. 4 can be

Fig. 4 Universal biquad filter using the integrators shown in Fig. 2

$$\begin{aligned} A_1 &= V_{GA1}, \dots \\ B_1 &= V_{GB1}, \dots \end{aligned}$$

obtained as

$$\frac{I_o(s)}{I_m(s)} = \frac{H_1(s)}{s^2 + \frac{2K(V_{GA2} - V_{GB2})}{C}s + \left(\frac{2K(V_{GA1} - V_{GB1})}{C}\right)^2} \quad (9)$$

Table 1: List of second-order filters derived from Fig. 4

S1	S2	S3	S4	V_{GA3}	V_{GB3}	V_{GA4}	V_{GB4}	$H_1(s)$	I_o/I_{in}
1	off	off	off	V_{GA3}	V_{GB3}	V_{SS}	V_{SS}	$2K(V_{GA3} - V_{GB3})s/C$	BP
2	on	off	off	V_{SS}	V_{SS}	V_{SS}	V_{SS}	s^2	HP
3	off	on	off	V_{SS}	V_{SS}	V_{SS}	V_{SS}	$-s^2$	HP
4	off	off	off	V_{SS}	V_{SS}	V_{GA4}	V_{GB4}	$4K^2(V_{GA4} - V_{GB4})(V_{GA1} - V_{GB1})/C^2$	LP
5	off	off	on	V_{SS}	V_{SS}	V_{SS}	V_{SS}	$2K(V_{GA1} - V_{GB1})s/C$	BP
6	off	off	on	V_{SS}	V_{SS}	V_{SS}	V_{SS}	$-2K(V_{GA1} - V_{GB1})s/C$	BP
7	on	off	off	V_{SS}	V_{SS}	V_{GA1}	V_{GB1}	$s^2 + (2K(V_{GA1} - V_{GB1})/C)^2$	NF
8	on	off	on	V_{SS}	V_{SS}	V_{GA1}	V_{GB1}	$s^2 - 2K(V_{GA1} - V_{GB1})s/C + (2K(V_{GA1} - V_{GB1})/C)^2$	AP

V_{SS} = most negative power supply; HP = highpass filter; BP = bandpass filter; LP = lowpass filter; NF = notch filter; AP = allpass filter

the numerator $H_1(s)$ of the transfer function is dependent on the connection patterns of the switches shown in Fig. 4. For example, if a bandpass filtering function is required the switches and gate voltages of MOSFETs shown in Fig. 4 must be connected as described in the first row of Table 1. All of the switches must be turned off, the gate voltages V_{GA3} and V_{GB3} is used to bias the MOSFETs in the triode region, and the MOSFETs with gate voltage V_{SS} will operate in the cutoff region. The other possible options, which can yield useful second-order filtering functions, are also summarised in Table 1. The centre frequency ω_o and quality factor Q of this universal biquad filter can be derived as

$$\omega_o = \frac{2K(V_{GA1} - V_{GB1})}{C} \quad (10)$$

$$Q = \frac{(V_{GA1} - V_{GB1})}{(V_{GA2} - V_{GB2})} \quad (11)$$

The transfer function of the differentiator-based universal biquad filter shown in Fig. 5 can be expressed as

$$\frac{I_o(s)}{I_{in}(s)} = \frac{H_2(s)}{s^2 + \frac{2K(V_{GA1} - V_{GB1})C_2}{C_1^2}s + \left(\frac{2K(V_{GA1} - V_{GB1})}{C_1}\right)^2} \quad (12)$$

with

$$\omega_o = \frac{2K(V_{GA1} - V_{GB1})}{C_1} \quad (13)$$

$$Q = \frac{C_1}{C_2} \quad (14)$$

where $H_2(s)$ also depends on the connection patterns of switches and MOSFETs. The useful second-order filtering functions that can be realised with this biquad filter

Fig. 5 Universal biquad filter using the differentiators shown in Fig. 3

$A_1 = V_{GA1}, \dots$
 $B_1 = V_{GB1}, \dots$

are listed in Table 2. Furthermore, using the leapfrog synthesis method, a third-order lowpass Chebyshev filter shown in Fig. 6 can also be obtained.

Fig. 6 Third-order Chebyshev lowpass filter using integrators shown in Fig. 2

$A = V_{GA}$
 $B = V_{GB}$

3 Effects of parasitic capacitances

To investigate the effects of parasitic capacitance in the proposed integrator, we redraw this integrator including

Table 2: List of second-order filters derived from Fig. 5

S1	S2	S3	S4	V_{GA2}	V_{GB2}	V_{GA3}	V_{GB3}	$H_2(s)$	I_o/I_{in}
1	off	off	off	V_{GA2}	V_{GB2}	V_{SS}	V_{SS}	$2K(V_{GA2} - V_{GB2})s/C_1$	BP
2	on	off	off	V_{SS}	V_{SS}	V_{SS}	V_{SS}	s^2	HP
3	off	on	off	V_{SS}	V_{SS}	V_{SS}	V_{SS}	$-s^2$	HP
4	off	off	off	V_{SS}	V_{SS}	V_{GA3}	V_{GB3}	$4K^2(V_{GA1} - V_{GB1})(V_{GA3} - V_{GB3})/C_1^2$	LP
5	off	off	on	V_{SS}	V_{SS}	V_{SS}	V_{SS}	$2K(V_{GA1} - V_{GB1})s/C_1$	BP
6	off	off	on	V_{SS}	V_{SS}	V_{SS}	V_{SS}	$-2K(V_{GA1} - V_{GB1})s/C_1$	BP
7	on	off	off	V_{SS}	V_{SS}	V_{GA1}	V_{GB1}	$s^2 + 4K^2(V_{GA1} - V_{GB1})/C_1^2$	NF
8	on	off	on	V_{SS}	V_{SS}	V_{GA1}	V_{GB1}	$s^2 - 2K(V_{GA1} - V_{GB1})s/C_1 + 4K^2(V_{GA1} - V_{GB1})/C_1^2$	AP

V_{SS} = most negative power supply; HP = highpass filter; BP = bandpass filter; LP = lowpass filter; NF = notch filter; AP = allpass filter

parasitic capacitances as shown in Fig. 7. Due to the virtually grounded input and low-impedance output characteristics of an OTRA, capacitances C_{p9} , C_{p10} and C_{p11}

Fig. 7 Proposed integrator with all parasitic capacitances

Fig. 8 Proposed differentiator with all parasitic capacitances

have little effect. Next, since the voltage sources V_{GA} and V_{GB} are DC, the effect of C_{p5} , C_{p6} , C_{p7} and C_{p8} will also be negligible. Then the voltage V_x can be obtained as

$$V_x(s) = \frac{I_1(s) - I_2(s)}{s(C + C_{p4} - C_{p3}) + 1/Z_o(s)} \quad (15)$$

The relationship between the input and output currents I_1 , I_2 , I_3 and I_4 is

$$\begin{aligned} I_3(s) - I_4(s) &= (2K(V_{GA} - V_{GB}) + s(C_{p1} - C_{p2}))V_x(s) \\ &= \frac{(2K(V_{GA} - V_{GB}) + s(C_{p1} - C_{p2}))}{s(C + C_{p4} - C_{p3}) + 1/Z_o(s)} \\ &\quad \times (I_1(s) - I_2(s)) \end{aligned} \quad (16)$$

The capacitances C_{p1} and C_{p2} are drain-to-source capacitances of the two NMOS transistors in Fig. 7. Since the aspect ratios and drain-to-source voltages of both NMOS transistors are identical, capacitances C_{p1} and C_{p2} are approximately equal. With $|Z_o(s)|$ approaching

infinity, the current gain function is simplified to

$$\frac{I_3(s) - I_4(s)}{I_1(s) - I_2(s)} = \frac{2K(V_{GA} - V_{GB})}{s(C + C_{p4} - C_{p3})} \quad (18)$$

The gain tolerance of this integrator caused by capacitances C_{p3} and C_{p4} can be compensated through adjusting the voltages V_{GA} and V_{GB} or the use of some predistortion technique without adding any extra components.

The proposed differentiator with parasitic capacitances is shown in Fig. 8. For similar reasons, the capacitances C_{p4} , C_{p5} , C_{p6} , C_{p7} , C_{p8} , C_{p9} and C_{p10} have negligible effect, and the resultant current transfer function of the differentiator can be expressed as

$$\frac{I_3(s)}{I_1(s) - I_2(s)} = \frac{s(C + C_{p3})}{2K(V_{GA} - V_{GB})} \quad (19)$$

This circuit can also be compensated by adjusting V_{GA} and V_{GB} or through the use of some predistortion technique without additional devices.

If the layouts of all integrators and differentiators are similar, the gain tolerances are nearly equal and the required compensating adjustments of voltages V_{GA} and V_{GB} are also identical. This is convenient when considering the compensation aspects of circuit design.

4 Harmonic distortion analysis

Using the level-3 MOSFET model of SPICE, the dependence of the mobility on V_{GS} and the threshold voltage on V_{DS} can be derived as [6, 7]

$$\mu_s = \frac{\mu}{1 + \theta(V_{GS} - V_{TH})} \quad (20)$$

$$\begin{aligned} V_{TH} &= V_{FB} + 2\phi_p - \sigma v_{DS} + \gamma F_s \sqrt{(2\phi_p - V_{BS})} \\ &\quad + F_n(2\phi_p - V_{BS}) \end{aligned} \quad (21)$$

F_s and F_n are the parameters of short-channel and narrow-width MOSFETs. Analysis results (Section 8.1) show that only second-order harmonics exist in the proposed integrator and the total harmonic distortion can be expressed as

$$\begin{aligned} THD &\approx HD_2 \\ &= \frac{\frac{\theta}{2} \left(\frac{1 + F_B}{2} - 2\sigma \right) \frac{I_{p-p}}{\omega C}}{1 + 2\theta \left[V_{FB} + 2\phi_p + \gamma F_s \sqrt{(2\phi_p - V_{BS})} \right.} \\ &\quad \left. + F_n(2\phi_p - V_{BS}) - \frac{V_{GA} + V_{GB}}{2} \right] \end{aligned} \quad (22)$$

where I_{p-p} is the peak-to-peak amplitude of the input current. The total harmonic distortion of the proposed differentiator has also been derived and is given in Section 8.2.

5 Experimental results

We use the circuit shown in Fig. 9 to implement an OTRA. The commercial IC AD844AN and CA3600E are

employed to construct all the experimental circuits. To investigate the effect of parasitic capacitances, the proposed integrator with parasitic capacitances shown in

Fig. 9 OTRA constructed with current feedback op-amps

Fig. 7 is breadboarded. We have added external capacitors to simulate parasitic capacitances C_{p11} , C_{p9} , C_{p5} and C_{p7} and selected all 'parasitic' capacitances to be 1 nF. The measured gain and phase response of our integrator without and with parasitic capacitors is shown in Figs. 10 and 11, respectively. Curves A-E are the meas-

Fig. 10 Comparisons between measured gain responses of proposed integrators with and without adding capacitors to simulate 'parasitic' capacitances

Fig. 11 Comparisons between measured phase responses of proposed integrators with and without adding capacitors to simulate 'parasitic' capacitances

ured results of the proposed integrator, with no 'parasitic' capacitor and one of C_{p11} , C_{p9} , C_{p5} and C_{p7} included one at a time, respectively. The maximum gain difference is seen to be only 0.4 dB.

The second-order bandpass filter derived from Fig. 4 is breadboarded and its experimental results are given in Fig. 12. To obtain a bandpass filtering function

Fig. 12 Measured amplitude and phase responses of bandpass filter derived from Fig. 4

with centre frequency at 91.3 kHz and quality factor $Q = 1$, we turned off all switches shown in Fig. 4 and set $V_{GA1} = V_{GA2} = V_{GA3} = 7.6$ V, $V_{GB1} = V_{GB2} = V_{GB3} = 5.4$ V, $V_{GA4} = V_{GB4} = V_{SS}$ and all capacitors to be 2 nF. The gain of the bandpass filter at centre frequency is 1.98 dB, which is greater than the theoretical value (0 dB). This gain tolerance is caused mainly by the mismatch of MOSFETs. All of the filters summarised in Tables 1 and 2 are tunable by varying the gate voltages of NMOS transistors.

The third-order Chebyshev lowpass filter with 0.1 dB passband ripple as shown in Fig. 6 is also tested. The bandwidth of this lowpass filter is about 190 kHz. The capacitor values chosen are $C_1 = 1.43$ nF, $C_2 = 1.59$ nF, $C_3 = 1.43$ nF and $V_{GA} = 8.6$ V, $V_{GB} = 5.9$ V. The experimental results of the gain and phase responses are shown in Fig. 13. To verify the theoretical results of harmonic

Fig. 13 Measured amplitude and phase responses of third-order Chebyshev lowpass filter shown in Fig. 6

distortions, the harmonic components in the output of the proposed integrator have also been measured. We select $I_1 - I_2 = 500 \cos(2\pi 60 \times 10^3 t) \mu A$ and sense the

output current $I_3 - I_4$ using a 10 k Ω resistor; the output of the integrator is then measured and shown in Fig. 14. Obviously, only the second-order harmonic component is

Fig. 14 Spectrum analyser display of harmonic components at output of proposed integrator with $V_{GA} = 7.9$ V and $V_{GB} = 6.9$ V; only second-order harmonic components at 118.2 kHz is observable

detectable and is 44 dB lower than the fundamental signal.

6 Conclusion

New parasitic-capacitance-insensitive current-mode filters using operational transresistance amplifiers have been proposed. Since the OTRA has virtually grounded inputs and low impedance output, most effects of the parasitic capacitances disappear and the remainder can be easily compensated without adding any extra elements. All the proposed circuits are cascable and have almost constant bandwidths, thanks to the low input impedance and the use of current feedback. These filters are also tunable by varying the gate voltages of the MOS transistors and are therefore suitable in many signal processing applications.

7 References

- 1 CZARNUL, Z.: 'Novel MOS resistive circuit for synthesis of fully integrated continuous-time filters', *IEEE Trans.*, 1986, **CAS-33**, pp. 718-721
- 2 BANU, M., and TSIVIDIS, Y.: 'Continuous-time MOSFET-C filters in VLSI', *IEEE Trans.*, 1986, **CAS-33**, pp. 125-139
- 3 SEDRA, A., and SMITH, K.C.: 'A second generation current conveyor and its applications', *IEEE Trans.*, 1970, **CT-17**, pp. 132-134
- 4 TOUMAZOU, C., LIDGEY, F.J., and HAIGH, D.G. (Eds.): 'Analogue IC design: the current-mode approach' (Peter Peregrinus, 1990)
- 5 CHEN, J.J., TSAO, H.W., and CHEN, C.C.: 'Operational transresistance amplifier using CMOS technology', *Electron. Lett.*, 1992, **28**, (22), pp. 2087-2088
- 6 ANTOGNETTI, P., and MASSOBRI, G.: 'Semiconductor device modelling with Spice' (McGraw-Hill, Singapore, 1988), pp. 184-207
- 7 HSPICE User's Manual, version H92. Meta-Software, Inc.
- 8 ISMAIL, M., SMITH, S.V., and BEALE, R.G.: 'A new MOSFET-C universal filter structure for VLSI', *IEEE J. Solid State Circuits*, 1988, **23**, pp. 183-194
- 9 TOUMAZOU, C., LIDGEY, F.J., and CHEUNG, P.Y.K.: 'Current-mode analogue signal processing circuits - A overview of recent developments'. Proceedings of IEEE international symposium on Circuits and systems, 1980, pp. 1572-1575

- 10 ALLEN, P.E., and HOLBERG, D.R.: 'CMOS analog circuit design'. HRW, Inc., Orlando, FL, 1987
- 11 TAKAGI, S., FUJII, N., and YANAGISAWA, T.: 'A canonical continuous-time MOSFET-capacitor filter with high linearity'. Proceedings of IEEE international symposium on Circuits and systems, 1988, pp. 2177-2180
- 12 ALLEN, P.E., and TERRY, M.B.: 'The use of current amplifiers for high performance voltage applications', *IEEE J. Solid State Circuits*, 1980, **17**, pp. 155-162

8 Appendix

8.1 Analysis of harmonic distortions in proposed integrator

The difference of the output currents I_3 and I_4 as shown in Fig. 2 can be obtained as

$$i_3 - i_4 \approx \frac{W}{2L} \mu C_{ox} (V_{GA} - V_{GB}) \times \left(1 + 20V_{TH} - \theta(V_{GA} + V_{GB}) + \frac{(1 + F_B)\theta}{2} v_x \right) v_x$$

with $\theta(V_{GS} - V_{TH}) \gg 1$ (23)

If $i_1 - i_2 = I_{p-p} \cos \omega t$ and $|Z_o(j\omega)|$ approaches infinity, the voltage v_x will be

$$\begin{aligned} v_x(t) &= \mathcal{L}^{-1}\{V_x(s)\} \\ &= \mathcal{L}^{-1}\left\{\frac{I_{p-p}}{(s^2 + \omega^2)C}\right\} \\ &= \frac{I_{p-p}}{\omega C} \sin \omega t \end{aligned} \quad (24)$$

Since

$$\mathcal{L}^{-1}\{V_x(s) * V_x(s)\} = (v_x(t))^2 = \frac{I_{p-p}^2}{(\omega C)^2} \sin^2 \omega t \quad (25)$$

$$\begin{aligned} i_3(t) - i_4(t) &= \frac{W}{2L} \mu C_{ox} (V_{GA} - V_{GB}) \\ &\times \left(\left(1 + 20 \left(V_{FB} + 2\phi_p + \gamma F_{s\sqrt{2}} (2\phi_p - V_{BS}) \right. \right. \right. \\ &\left. \left. \left. + F_n(2\phi_p - V_{BS}) - \frac{V_{GA} + V_{GB}}{2} \right) \right) \frac{I_{p-p} \sin \omega t}{\omega C} \right. \\ &\left. + \theta \left(\frac{1 + F_B}{2} - 2\sigma \right) \frac{I_{p-p}^2 \sin^2 \omega t}{(\omega C)^2} \right) \\ &= \frac{W}{2L} \mu C_{ox} (V_{GA} - V_{GB}) \left(\left(\frac{1 + F_B}{2} - 2\sigma \right) \frac{\theta I_{p-p}^2}{2(\omega C)^2} \right. \\ &\left. + \left(1 + 20 \left(V_{FB} + 2\phi_p + \gamma F_{s\sqrt{2}} (2\phi_p - V_{BS}) \right. \right. \right. \\ &\left. \left. \left. + F_n(2\phi_p - V_{BS}) - \frac{V_{GA} + V_{GB}}{2} \right) \right) \frac{I_{p-p} \sin \omega t}{\omega C} \right. \\ &\left. - \frac{\theta}{2} \left(\frac{1 + F_B}{2} - 2\sigma \right) \frac{I_{p-p}^2 \cos 2\omega t}{(\omega C)^2} \right) \end{aligned} \quad (26)$$

Obviously, only the second-order harmonic component is present and the total harmonic distortion can be

expressed as

$$THD \approx HD_2$$

$$= \frac{\frac{\theta}{2} \left(\frac{1+F_B}{2} - 2\sigma \right) \frac{I_{p-p}}{\omega C}}{1 + 2\theta \left[V_{FB} + 2\phi_p + \gamma F_s \sqrt{(2\phi_p - V_{BS})} + F_n(2\phi_p - V_{BS}) - \frac{V_{GA} + V_{GB}}{2} \right]} \quad (27)$$

8.2 Analysis of harmonic distortions in proposed differentiator

For the proposed differentiator shown as Fig. 3 the relationship between V_x , I_1 and I_2 can be written as

$$I_1(s) - I_2(s) = \frac{V_x(s)}{Z_o(s)} + \frac{W}{2L} \mu C_{ox} (V_{GA} - V_{GB}) \times \left(\left(1 + 2\theta \left(V_{FB} + 2\phi_p + \gamma F_s \sqrt{(2\phi_p - V_{BS})} + F_n(2\phi_p - V_{BS}) - \frac{V_{GA} + V_{GB}}{2} \right) \right) V_x(s) + \theta \left(\frac{1+F_B}{2} - 2\sigma \right) \{ V_x(s) * V_x(s) \} \right) \quad (28)$$

If $|Z_o(s)|$ approaches infinity, eqn. 28 can be rewritten as

$$A \{ V_x(s) * V_x(s) \} - BV_x(s) + D(s) = 0 \quad (29)$$

with

$$A = \frac{W}{2L} \mu C_{ox} \theta \left(-\frac{1+F_B}{2} + 2\sigma \right) (V_{GA} - V_{GB})$$

$$B = \frac{W}{2L} \mu C_{ox} (V_{GA} - V_{GB}) \times \left(1 + 2\theta \left(V_{FB} + 2\phi_p + \gamma F_s \sqrt{(2\phi_p - V_{BS})} + F_n(2\phi_p - V_{BS}) - \frac{V_{GA} + V_{GB}}{2} \right) \right)$$

$$D(s) = I_1(s) - I_2(s)$$

In the ideal case, v_x must be equal to $D(t)/B$. The solution to eqn. 29 is therefore uniquely given by

$$v_x = \mathcal{L}^{-1} \{ V_x \} = \frac{B - \sqrt{(B^2 - 4AD(t))}}{2A} \quad (30)$$

The output current I_3 can then be obtained as

$$i_3(t) = C \frac{dv_x}{dt}$$

$$= C \frac{d}{dt} \left\{ \frac{D(t)}{B} + \frac{AD^2(t)}{B^3} + \frac{2A^2D^3(t)}{B^5} + \frac{5A^3D^4(t)}{B^7} + \frac{14A^4D^5(t)}{B^9} + \text{high-order term} \right\} \quad (31)$$

If $D(t) = i_1(t) - i_2(t) = I_{p-p} \cos \omega t$, the harmonic components of $i_3(t)$ can be derived as

$$i_3(t) = C \frac{d}{dt} \left(\frac{I_{p-p} \cos \omega t}{B} + \frac{AI_{p-p}^2 \cos^2 \omega t}{B^3} + \frac{2A^2I_{p-p}^3 \cos^3 \omega t}{B^5} + \dots \right)$$

$$= \omega C \left(-\left(\frac{I_{p-p}}{B} + \frac{3A^2I_{p-p}^3}{2B^5} \right) \sin \omega t - \frac{AI_{p-p}^2}{B^3} \times \sin(2\omega t) - \frac{3A^2I_{p-p}^3}{2B^5} \sin(3\omega t) + \dots \right) \quad (32)$$

The total harmonic distortion of the proposed differentiator can be evaluated using

$$THD \approx \sqrt{[(HD_2)^2 + (HD_3)^2 + \dots]} \quad (33)$$

where

$$HD_2 \approx \left| \frac{\frac{AI_{p-p}}{B^2}}{1 + \frac{3A^2I_{p-p}^2}{2B^4}} \right| \quad HD_3 \approx \left| \frac{\frac{3A^2I_{p-p}^2}{2B^4}}{1 + \frac{3A^2I_{p-p}^2}{2B^4}} \right|, \dots$$