Determination of Circular Arc Length and Midpoint by Hough Transform

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ABSTRACT

We propose a method to detect circular arcs, including the determinations of their centers, radii, lengths, and midpoints. The center, radius of a circular arc can easily be determined using circular Hough transform. But, the determinations of the arc midpoint and length are more complicated. Theoretical analysis of the centroid of the near - peak Hough transform data is presented. Then, the arc midpoint and length can be extracted from it.

INTRODUCTION

The Hough transform can be used to detect circular arcs (see Fig. 1) by choosing center and radius as parameters. The location of a Hough transform peak indicates the contour (a circle) where an arc lies upon. Thus, center and radius are acquired. However, the location of the Hough transform peak does not identify the midpoint and length of the arc. Although the arc length is equal to the number of votes at the Hough transform peak, it is invalid under noisy environment. If the input digital picture is affected by noise, some votes of the Hough transform peak will spread into its neighboring accumulators and result in erroneous estimation of the arc length. The information about midpoint and length can be extracted from the near - peak Hough transform data.

Fig. 1 The arc detection problem

Similar research on line position (midpoint of a line segment) determination had been made by Jeffery Richards and David P. Casasent [1]. They use the average slope of the near - peak Hough transform data to approximate the slope of the midpoint. Then, midpoint of the line segment is calculated using the estimated slope and the position of peak. Research on line length determination had been made by M. W. Akhtar and M. Atiquzzaman [2]. Due to the discretisation of the Hough transform parameters, the votes of the Hough transform peak spread into its neighboring accumulators. They analyze the distribution of votes near the peak to estimate the length of the line segment. But, direct extensions of these method to circular arc do not exist, especially when the arc length is longer than a semicircle.

DETECTION OF CIRCLES USING HOUGH TRANSFORM

The equation for a circle is given by

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

where (x_0, y_0) is the center, r is radius.

Choosing center and radius as the parameters of Hough transform, the locus of parameters voted by an edge point on image space will be a right circular cone (see Fig. 2). If there is a circle in image space, all right circular cones voted by points lie on it will intersect at a common point in parameter space. Coordinates of this common point are parameters of the equation for the circle in image space.

Hough transform is able to detect partial shapes. By treating circular arc as partial shape of a circle, the method for circle detection can be used to detect circular arcs. The coordinates of a peak in the parameter space indicate center and radius of the detected arc.

Fig. 2 Circular Hough transform voted by an edge point

DETERMINATIONS OF THE ARC MIDPOINT AND LENGTH

The midpoint and length of a circular arc are unknowable from the position of the Hough transform peak. In this section, we define the near - peak Hough transform data and extract information about the arc midpoint and length from it.

Definition of the near - peak Hough transform data (see Fig. 3)

The near - peak Hough transform data is defined as the Hough transform values of those accumulators lying on $r = r_0$ plane (the plane where the peak lies upon) and having distances from the peak within a threshold, say δ , which need not be defined strictly. The domain of (x, y) is thus a disk with radius δ .

Fig. 3 The near - peak Hough transform data
(only votes of A and B are shown)

Centroid of the near - peak Hough transform data

The circular Hough transform of an edge point S on $r = r_0$ plane is shown in Fig. 4. The arc PQ is the votes of S in the near - peak region and is symmetric about the line ST, where the point T is the peak position. The angle ϕ can be calculated using r_0 and δ

$$\phi = \cos^{-1} \left(\frac{2 r_0^2 - \delta^2}{2 r_0^2} \right)$$

The centroid of the arc PQ lies on the line segment ST and the distance from the centroid to S is

$$d_S = \frac{1}{2\phi} \int_{-\phi}^{\phi} r_0 \cos \phi \, d\phi = r_0 \frac{\sin \phi}{\phi}$$

Thus, all points on the arc AB shown in Fig. 5 have their centroids on the arc CD with the same weight, where the arc CD has a radius of $r_0 - d_S$. Therefore, the centroid of the near - peak data voted by the arc AB lies on the line segment TE, where the point T is the peak position and E is the midpoint of the arc AB. The distance from the centroid to the point T is

$$d = (r_0 - d_S) \frac{\sin \psi}{\psi} = r_0 \left(1 - \frac{\sin \phi}{\phi} \right) \frac{\sin \psi}{\psi}$$

where ψ is half of the angle ATB.

Fig. 4 The circular Hough transform of an edge point

Fig. 5 The centroid of the near - peak data

The midpoint and length of the arc

The centroid of the near - peak data is calculated in advance. Then, the midpoint and length of the arc can be calculated by the following way:

<1> The midpoint

According to the above derivation, the centroid of the near - peak data lies on the line segment connecting the Hough transform peak and the arc midpoint. Thus, the arc midpoint is in the direction from the peak position toward the centroid. The distance from the peak position to the midpoint is r_0 .

<2> The length

The last equation derived in B can be rewrote as

$$\frac{\sin \psi}{\psi} = \frac{d}{r_0 \left(1 - \frac{\sin \phi}{\phi} \right)}$$

The angle ψ has unique solution within reasonable range ($0, \pi$). Thus, the arc length can be calculated by

$$L = 2 \psi r_0$$

SIMULATIONS

The digital picture of a circle is shown in Fig. 6 (a). The arc plotted in Fig. 6 (b) is a partial shape of the circle plotted in Fig. 6 (a). The circular Hough transform of the arc on $r = r_0$ plane, where the peak lies upon, is plotted in Fig. 6 (c). The peak position of the circular Hough transform indicates the center of the arc, which is marked by a star (*) in Fig. 6 (b). The near - peak Hough transform data is shown in Fig. 6 (d), where the disk radius is chosen equal to the radius of the circle (r_0). The centroid of the near - peak Hough transform data is then calculated. The arc midpoint and length can be determined by the centroid using method proposed in the previous section. The arc midpoint is marked by a plus (+) in Fig. 6 (b) and two end points of the arc are marked by crosses (x). An additional simulation is shown in Fig. 7.

Fig.6(a) A circle

Fig.6(b) The input arc and the detecting result

Fig.6(c) The Hough transform data

Fig.6(d) The near-peak Hough transform data

Fig.7(a) A circle

Fig.7(b) The input arc
and the detecting result

Fig.7(c) The Hough transform data

Fig.7(d) The near-peak
Hough transform data

CONCLUSION

Circular Hough transform is used to detect the existences of arcs together with the contours they lie upon. The midpoint and length of an arc are extracted from the near - peak Hough transform data via the calculation of its centroid. Simulation results indicate that the method we proposed is quite useful.

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