Nonlinear Control for Flexible Manipulators

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Abstract

This work focuses on the control of flexible manipulators. Due to the complicated dynamics of a flexible manipulator, which does not satisfy the so-called "matching condition", many control strategies which succeed in the conventional rigid robot control cannot be directly used in the flexible robot control problems. In this work, a generalized computed torque control scheme has been proposed to solve this kind of control problem. Furthermore, to cope with the model uncertainty, an adaptive control scheme has also been proposed. Simulation and experimental results have demonstrated the effectiveness of the proposed controller.

1 Introduction

In the recent years, much attention has been paid to the modeling and control of flexible manipulators. For high structural stiffness, conventional rigid robot are often built to be heavy and bulky. Thus, some drawbacks, such as high power-consumption, low motion speed, actuators with high capacity, and low payload ratio, may appear. To remedy these problems, the links of the robot are made lighter. Therefore the structural flexibility can no longer be ignored, especially when the motion speed of the robot is high. In some applications, such as space robotic systems and vehicles, mechanisms made of lightweight materials are often employed to help launch, and hence are subject to high degree of flexibility, and encounter unavoidable mechanical vibration.

Many attempts have been made to suppress the vibration inherent in the flexible systems over the past decade. Due to the nature of distributed systems, the dynamics of the flexible manipulators are much more complicated than those of the rigid ones. And, the number of control inputs for a flexible manipulator is normally less than its motional degrees of freedom. Thus, many control strategies which succeed in the rigid robot control can not be directly applied to this kind of system. So far, it seems still to be an open problem for the control of flexible manipulators. Therefore, this paper will concentrate on dealing with this problem.

In the literature, there have been many control schemes proposed for flexible manipulators. In [1], [2], linear system approaches were used to control a single-link flexible robot. In there, LQG and the stable factorization technique have been used for the controller design. Yurkovitch et al. [5], [6] used acceleration feedback for the endpoint position control of a

flexible robot arm. The results have been compared with those obtained by using LQG approach, and better performance has been shown. In [3], [4], some self-tuning type adaptive control schemes have been proposed. In these schemes, either ARMA model or linear state space model was used for the control law design, but the nonlinear coupling effects are ignored.

Recently, the perturbation technique has attracted widespread attention. Using the technique, the dynamics of a flexible robot can be decomposed into two subsystems, namely, a slow subsystem and a fast subsystem. The slow subsystem corresponds to the rigid body movement and the fast subsystem is mainly to account for the elastic modes. Usually, the composite control schemes are designed to deal with this kind of formulation. ([12], [13])

In [7], [8], the nonlinear control schemes, such as those using computed torque, inverse dynamics, and feedback linearization, have been proposed for the control of multilink flexible robot arms. Seering et al. [10], [11] used the so-called input-preshaping technique, which applies a set of impulse train to convolve with the input command so as to generate new torque profile in order to suppress the vibration modes of the flexible robot arm. Recently, some researchers [14], [15] start to use fuzzy control methodology for the control of flexible robot arms.

In this paper, a generalized computed torque control scheme has been proposed for the flexible manipulator control. To cope with the model uncertainty, an adaptive control has also been presented. In IRL (Intelligent Robot Lab.) at NTU (National Taiwan University), a two-link flexible manipulator has been constructed to demonstrate the performance of the proposed controller, as shown in Fig.(a). The simulation and experimental results realistically demonstrate the effectiveness of the controller.

This paper is organized as follows: section 2 proposes a model-based nonlinear control law and an adaptive type control law for a general flexible manipulator. In section 3, some simulation results and experimental results are shown to demonstrate the control performance. Finally, some conclusions are given in section 4.

2 Problem Formulation and Controller Design

In this section, two control schemes will be developed for the control of a general multi-link flexible manipulator. The first

controller is a generalization of the computed-torque method which is used in conventional rigid robot control. In this model-based control scheme, we have assumed that the model parameters of the flexible manipulator are perfectly known. Under this assumption, the tracking errors of the joint positions, the velocities, and the vibration modes will be exponentially vanishing. Based on the design procedure of the first controller, the second controller which assumes an adaptive scheme is then developed to cope with the parametric uncertainty.

2.1 Model-Based Nonlinear Controller

For convenience, the dynamic equations of a flexible manipulator are expressed in the following:

$$M(q)\ddot{q} + h(q, \dot{q})\dot{q} + Kq = Q \tau, \quad (1)$$

where

$$Q = \begin{bmatrix} I_{n \times n} \\ 0_{m \times n} \end{bmatrix},$$

q represents the generalized coordinates, and τ is the applied torque.

Equivalently, (1) can be rewritten as

$$\begin{bmatrix} M_{rr} & M_{rf} \\ M_{fr} & M_{ff} \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{q}_f \end{bmatrix} + \begin{bmatrix} h_{rr} & h_{rf} \\ h_{fr} & h_{ff} \end{bmatrix} \begin{bmatrix} \dot{q}_r \\ \dot{q}_f \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_{vf} \end{bmatrix} \begin{bmatrix} q_r \\ q_f \end{bmatrix} = \begin{bmatrix} \tau_r \\ 0 \end{bmatrix} \quad (2)$$

where the subscript 'r' denotes the rigid-mode part and 'f' denotes the flexible-mode part. Assume that there are totally n rigid modes and m flexible modes for this manipulator system. Let the desired trajectory, its first-, and second-order derivatives be denoted as $q_{rd}(t)$, $\dot{q}_{rd}(t)$, and $\ddot{q}_{rd}(t)$, respectively, and define the following error signals as:

$$e = \begin{bmatrix} e_r \\ e_f \end{bmatrix} = \begin{bmatrix} q_d - q_r \\ -q_f \end{bmatrix}, \quad (3)$$

and

$$s = \dot{e} + \lambda e = \begin{bmatrix} \dot{e}_r + \lambda_r e_r \\ \dot{e}_f + \lambda_f e_f \end{bmatrix} = \begin{bmatrix} s_r \\ s_f \end{bmatrix}, \quad (4)$$

where

$$\lambda = \begin{bmatrix} \lambda_r & 0 \\ 0 & \lambda_f \end{bmatrix}$$

is a positive definite matrix.

Then, the dynamics of the newly defined signals s_r , s_f can be derived as:

$$\begin{bmatrix} M_{rr} & M_{rf} \\ M_{fr} & M_{ff} \end{bmatrix} \begin{bmatrix} \dot{s}_r \\ \dot{s}_f \end{bmatrix} + \begin{bmatrix} h_{rr} & h_{rf} \\ h_{fr} & h_{ff} \end{bmatrix} \begin{bmatrix} s_r \\ s_f \end{bmatrix} + \begin{bmatrix} K_{vr} & 0 \\ 0 & K_{vf} \end{bmatrix} \begin{bmatrix} s_r \\ s_f \end{bmatrix} = \begin{bmatrix} \tau_m + K_{vr}s_r - \tau \\ \tau_o \end{bmatrix} \quad (5)$$

where

$$\begin{aligned} \tau_m &= M_{rr}(\ddot{q}_{rd} + \lambda_r \dot{e}_r) + M_{rf}(\lambda_f \dot{e}_f) \\ &\quad + h_{rr}(\dot{q}_{rd} + \lambda_r e_r) + h_{rf} \lambda_f e_f \end{aligned}$$

$$\begin{aligned} \tau_a &= M_{fr}(\ddot{q}_{rd} + \lambda_r \dot{e}_r) + M_{ff}(\lambda_f \dot{e}_f) \\ &\quad + h_{fr}(\dot{q}_{rd} + \lambda_r e_r) + h_{ff} \lambda_f e_f + K_{vf}s_f - K_{pf}e_f \end{aligned}$$

Now, if we choose the following control law

$$\tau = K_{vr}s_r + \tau_m + \tau_f, \quad (6)$$

where

$$\tau_f = s_r(s_f^T \tau_a) / \|s_r\|^2,$$

we can obtain the following cleaner form of the error dynamics (5) as:

$$\begin{bmatrix} M_{rr} & M_{rf} \\ M_{fr} & M_{ff} \end{bmatrix} \begin{bmatrix} \dot{s}_r \\ \dot{s}_f \end{bmatrix} + \begin{bmatrix} h_{rr} & h_{rf} \\ h_{fr} & h_{ff} \end{bmatrix} \begin{bmatrix} s_r \\ s_f \end{bmatrix} + \begin{bmatrix} K_{vr} & 0 \\ 0 & K_{vf} \end{bmatrix} \begin{bmatrix} s_r \\ s_f \end{bmatrix} = \begin{bmatrix} -\tau_f \\ -\tau_a \end{bmatrix} \quad (7)$$

which can be further rewritten in a more compact form as:

$$M(q)\dot{s} + h(q, \dot{q})s + K_v s = \begin{bmatrix} -\tau_f \\ -\tau_a \end{bmatrix} \quad (8)$$

In the following, it is shown that the error dynamics in equation (7) which results from the control law (6) is exponentially stable. This is first stated in the following theorem.

Theorem 1 Consider a general multilink flexible manipulator whose dynamic model is described by (1) or (2). Suppose the control objective is to have the joints track the desired trajectories while simultaneously damping out the flexible modes. Then, the control law proposed in (6) can achieve the objective exponentially in time t , i.e., $e \rightarrow 0$ exponentially as $t \rightarrow \infty$.

Proof :

Let us choose the Lyapunov function

$$V(t) = \frac{1}{2} s^T M(q) s \quad (9)$$

and take the time derivative of $V(t)$ to get

$$\begin{aligned} \dot{V} &= s^T M(q)\dot{s} + \frac{1}{2} s^T \dot{M}(q)s \\ &= s^T (-h(q, \dot{q})s - K_v s + \begin{bmatrix} -\tau_f \\ -\tau_a \end{bmatrix}) \\ &\quad + \frac{1}{2} s^T \dot{M}(q)s \\ &= -s^T K_v s, \end{aligned} \quad (10)$$

in which clearly

$$\dot{V}(t) < 0 \quad \text{whenever} \quad s \neq 0$$

since K_v is positive definite. Then, Lyapunov theory readily implies that $s \in L_\infty$ and, hence, $e \in L_\infty$ directly from equation (4).

On the other hand,

$$\dot{V} \leq -\sigma \|s\|^2$$

and

$$\frac{\lambda_m}{2} \|s\|^2 \leq V \leq \frac{\lambda_M}{2} \|s\|^2,$$

where σ is the minimum eigenvalue of K_v , λ_m is the minimum eigenvalue of $M(q)$, and λ_M is the maximum eigenvalue of $M(q)$ for all possible q . By a little algebraic manipulation, it is easy to find that

$$\dot{V} \leq -\gamma V, \quad (11)$$

where $\gamma = \min(\sigma, \lambda_m)$, or after integration,

$$V(t) \leq V(0) - \int_0^t \gamma V(t) dt \quad (12)$$

From Bellman-Grownwall Lemma, we then have

$$V(t) \leq V(0) \exp(-\gamma t) \quad (13)$$

which then leads to

$$\|s\| \leq \frac{2\sqrt{V(0)}}{\lambda_m} \exp(-\frac{\gamma}{2}t) \quad (14)$$

As a result, we can conclude that

$$s \rightarrow 0 \text{ exponentially as } t \rightarrow \infty$$

and hence

$$e \rightarrow 0 \text{ exponentially as } t \rightarrow \infty$$

Note that in the above proof we have used the property $(\dot{M}(q) - 2h(q, \dot{q}))$ is skew-symmetric. This concludes our proof. $\square$

Remark 1 :

One should note that in the control law proposed in (6), namely,

$$\tau_f = s_r(s_f^T \tau_a) / \|s_r\|^2,$$

s_r may have chance to be extremely close to zero and, therefore, the control force defined above may no longer be numerically feasible. As a consequence, the control law can be modified slightly as:

$$\tau_f = s_r(s_f^T \tau_a) / (\|s_r\|^2 + \epsilon)$$

for practical implementation, where ϵ is a small positive number. By this modified control law, it is not hard to verify that the tracking errors as well as the flexible modes will remain as small as ϵ , though no longer converge to zero.

Remark 2 :

For better grasp of the control law developed in (6), we note that each term in the following synthesized force:

$$\tau = K_v r s_r + \tau_m + \tau_f$$

can be associated with an interpretation as described below:

- τ_m : denotes the computed torque to produce the desired motion trajectory;
- τ_f : is used to cancel the vibration effects which result from the motion of the manipulator;
- $K_v r s_r$: represents the PD type control action.

2.2 Nonlinear Adaptive Control Law

The dynamic equations of a flexible manipulator can be verified to possess the well-known linear-in-parameter property. Let us define

$$W_1 \Theta_1 = \begin{matrix} M_{rr}(\ddot{q}_{rd} + \lambda_r \dot{e}_r) + M_{rf}(\lambda_f \dot{e}_f) \\ h_{rr}(\dot{q}_{rd} + \lambda_r e_r) + h_{rf} \lambda_f e_f, \end{matrix}$$

$$W_2 \Theta_2 = \begin{matrix} M_{fr}(\ddot{q}_{rd} + \lambda_r \dot{e}_r) + M_{ff}(\lambda_f \dot{e}_f) \\ h_{fr}(\dot{q}_{rd} + \lambda_r e_r) + h_{ff} \lambda_f e_f - K_{pf} e_f \end{matrix}$$

where

- W_1, W_2 are $n \times r_1$ and $m \times r_2$ regressor matrices, respectively, for some appropriate $r_1, r_2 > 0$;
- Θ_1, Θ_2 are $r_1 \times 1$ and $r_2 \times 1$ vectors of unknown constant parameters, respectively.

If the system parameters in the control law (6) as stated in the previous section are replaced by their estimates, that is,

$$\begin{aligned} \tau &= W_1 \hat{\Theta}_1 + K_v r s_r + s_r(s_f^T \tau_a) / \|s_r\|^2 \\ \tau_a &= W_2 \hat{\Theta}_2 + K_v f s_f \end{aligned} \quad (15)$$

where, equivalently,

$$W_1 \hat{\Theta}_1 = \begin{matrix} \hat{M}_{rr}(\ddot{q}_{rd} + \lambda_r \dot{e}_r) + \hat{M}_{rf}(\lambda_f \dot{e}_f) \\ + \hat{h}_{rr}(\dot{q}_{rd} + \lambda_r e_r) + \hat{h}_{rf} \lambda_f e_f, \end{matrix}$$

$$W_2 \hat{\Theta}_2 = \begin{matrix} \hat{M}_{fr}(\ddot{q}_{rd} + \lambda_r \dot{e}_r) + \hat{M}_{ff}(\lambda_f \dot{e}_f) \\ + \hat{h}_{fr}(\dot{q}_{rd} + \lambda_r e_r) + \hat{h}_{ff} \lambda_f e_f - \hat{K}_{pf} e_f \end{matrix}$$

and $\hat{\Theta}_1, \hat{\Theta}_2$ are the estimates of Θ_1 and Θ_2 , respectively, then the error dynamics can be obtained as

$$\begin{aligned} &\begin{bmatrix} M_{rr} & M_{rf} \\ M_{fr} & M_{ff} \end{bmatrix} \begin{bmatrix} \dot{s}_r \\ \dot{s}_f \end{bmatrix} + \begin{bmatrix} h_{rr} & h_{rf} \\ h_{fr} & h_{ff} \end{bmatrix} \begin{bmatrix} s_r \\ s_f \end{bmatrix} \\ &+ \begin{bmatrix} K_{vr} & 0 \\ 0 & K_{vf} \end{bmatrix} \begin{bmatrix} s_r \\ s_f \end{bmatrix} = \begin{bmatrix} W_1 \hat{\Theta}_1 - s_r s_f^T \tau_a / \|s_r\|^2 \\ W_2 \hat{\Theta}_2 + K_v f s_f \end{bmatrix} \end{aligned}$$

or more compactly as:

$$M(q)\dot{s} + h(q, \dot{q})s + K_v s = \begin{bmatrix} W_1 \hat{\Theta}_1 - s_r s_f^T \tau_a / \|s_r\|^2 \\ W_2 \hat{\Theta}_2 + K_v f s_f \end{bmatrix} \quad (16)$$

where $\tilde{\Theta}_1 = \Theta_1 - \hat{\Theta}_1$ and $\tilde{\Theta}_2 = \Theta_2 - \hat{\Theta}_2$. Now, in addition we choose the adaptation law as follows:

$$\begin{aligned} \dot{\hat{\Theta}}_1 &= -K_1 W_1^T s_r \\ \dot{\hat{\Theta}}_2 &= -K_2 W_2^T s_f \end{aligned} \quad (17)$$

where K_1, K_2 are some positive definite gain matrices.

In the following we will show that the error dynamics (16) along with the adaptation scheme (17) are asymptotically stable. This is exactly stated in the following theorem.

Theorem 2 Consider the same system as described in Theorem 1, except that the system parameters are currently not precisely known. Then, by applying the control law (15) and the adaptation law (17), we can achieve the same objective asymptotically in time t .

Proof:

Choose a Lyapunov function as

$$V = \frac{1}{2} s^T M(q) s + \frac{1}{2} \tilde{\Theta}_1^T K_1^{-1} \tilde{\Theta}_1 + \frac{1}{2} \tilde{\Theta}_2^T K_2^{-1} \tilde{\Theta}_2$$

and take the time derivative of V to get

$$\begin{aligned} \dot{V} &= s^T M(q) \dot{s} + \frac{1}{2} s^T \dot{M}(q) s + \dot{\tilde{\Theta}}_1^T K_1^{-1} \tilde{\Theta}_1 + \dot{\tilde{\Theta}}_2^T K_2^{-1} \tilde{\Theta}_2 \\ &= s^T (-h(q, \dot{q}) s - K_v s + \begin{bmatrix} W_1 \tilde{\Theta}_1 - s_r s_f^T \tau_a / \|s_r\|^2 \\ W_2 \tilde{\Theta}_2 + K_{vf} s_f \end{bmatrix}) \\ &\quad + \frac{1}{2} s^T \dot{M}(q) s - s_r^T W_1 \tilde{\Theta}_1 - s_f^T W_2 \tilde{\Theta}_2 \\ &= -s^T K_v s, \end{aligned} \quad (18)$$

in which clearly

$$\dot{V} < 0, \quad \text{whenever } s \neq 0,$$

which then implies that

$$s, \tilde{\Theta}_1, \tilde{\Theta}_2 \in L_\infty$$

by Lyapunov theory.

Since

$$\dot{V} = -s^T K_v s \leq -\sigma \|s\|^2, \quad (19)$$

where σ is the minimum eigenvalue of K_v , by integrating (19), we can find that

$$-\int_0^\infty \dot{V} dt = V(0) - V(\infty) < \infty \quad (20)$$

or equivalently that

$$\int_0^\infty \|s\|^2 dt < \infty \quad (21)$$

i.e., $s \in L_2$. From the error dynamics, we can further conclude that $\dot{s} \in L_\infty$. Then, as a consequence of Babalatt's lemma, we readily obtain that

$$s \longrightarrow 0 \text{ asymptotically as } t \longrightarrow \infty$$

and, hence,

$$e \longrightarrow 0 \text{ asymptotically as } t \longrightarrow \infty$$

This concludes the proof. $\square$

3 Computer Simulation and Experimental Results

In this section, the approaches to obtaining the natural frequencies experimentally are first described briefly, and then some simulation and experimental results are shown in the following subsections.

Two methods are used to identify the characteristic frequencies of the flexible modes. The first method is to take the fast Fourier transform (FFT) of the impulse response for

mode	calculation	1-st method	2-nd method
1	1.4	1.5	1.5
2	21.1	22.3	22.9

Table 1: Characteristic frequencies for link 1

mode	calculation	1-st method	2-nd method
1	3.5	3.5	3.9
2	21.9	21.4	21.9

Table 2: Characteristic frequencies for link 2

a given impulse command. This is done by using a hammer to produce an impact on the link which has been mounted on the motor shaft. The end-point acceleration data are collected by the accelerometer sensor which is attached at the end of the beam. The collected data are then fed into FFT analyzer to obtain the characteristic frequencies.

The second method of identifying the characteristic frequencies is to take FFT of those elastic state variables, which are obtained through strain measurements for an impulse command. These results are summarized in Table 1. and Table 2.

3.1 Results of Computer Simulation

For the time being, we have accomplished single-link control simulation and implementation. The control law mentioned in Section 2 is tested on the second link. The parameters of the second link are repeated in Table 3 and the inertia of the hub at joint 2 is 0.015 Kgm.

beam width	0.08 (m)
beam length	0.50 (m)
beam thickness	0.001 (m)
flexural rigidity (EI)	0.46 (Nm ²)
mass / unit length	0.222 (Kg/m)

Table 3: Link 2 parameters

The following control gains are used for the simulation,

$$K_v = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \lambda = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

and $\epsilon = 0.5$.

The desired trajectory $q_d(t)$ takes up a fifth-order polynomial that is defined by:

$$q_d(t) = q_i + d\tau(t) \quad (22)$$

where q_i is the initial position, d is the displacement, and $\tau(t)$ is a function of time. Here, we choose $q_i = 0$ and $d = 1.57$

(rad). The desired positions of the flexible modes are set to be zero. To obtain a trajectory that is smooth and continuous in position, velocity and acceleration, we choose:

$$r(t) = 6\left(\frac{t}{T_m}\right)^5 - 15\left(\frac{t}{T_m}\right)^4 + 10\left(\frac{t}{T_m}\right)^3 \quad (23)$$

where $T_m = 0.3\text{sec}$ is the expected duration of the motion.

The computer simulation results for the trajectory are shown in Figure 1 to Figure 2. The joint position response is given in Figure 1, The responses of the flexible modes are plotted in Figure 2. We can see the satisfactory results from Figure 1.

3.2 Experimental Results

The experiments are made under the same conditions described in previous section. The sampling rate for the control law implementation is chosen as 500 Hz.

Figure 3 to Figure 4 show the experimental results for the fifth-order polynomial trajectory, and Figure 5 to Figure 6 show the experimental results in adaptive case. From the output torque profile, Figure 7, we can observe that there are some high frequency components resided in the output torque signal. This phenomenon is mainly due to the noise of the joint velocity signal, which arises from the usage of difference method for calculation of the joint speed. On the other hand, the induced noise of the strain measurements made the situation worse.

4 Conclusion

When performing the state measurements, we find that there is some induced noise in the flexible modes when the motor rotates, and this phenomenon results in some undesirable effects. Some unbalanced behaviour are found when the motor moves in different directions, such as different overshoot and settling time. This problem has been solved by properly arranging shielding and grounding of the signal wire.

For avoiding the observation spillover problem, we have used a second-order analog filter in the strain gauge amplifier and a first-order digital filter in the state measurement calculation. The cut-off frequencies of these filters need to be determined carefully, or the closed-loop system is apt to become unstable.

In this experiment, we find that there always exists some trade-off between the joint tracking performance and the regulation effects of the flexible modes. Intuitively, since for a flexible manipulator system, the number of the control inputs is always less than the degrees of freedom of motion, so the existence of this kind of trade-off should be expected. Also, theoretically, we have proved the tracking and regulation performance of the control scheme proposed in Section 2. Experiment on full the 2-link flexible manipulator is still under way at the present stage.

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Fig.3. Joint Position Response(Experiment)

Fig.4. First Flexible Mode(Experiment)

Fig.5. Joint Position Response-Adaptive(Experiment)

Fig.6. First Flexible Mode-Adaptive(Experiment)

Fig.7. Actuation Force(Experiment)