Nonlinear Observer-Based Tracking Control for Induction Motors

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Abstract

In this paper, we propose a nonlinear observer-based controller for induction motors. Via the use of skew-symmetrical property of induction motors, a two-stage design and technique is applied to construct an observer-based controller for velocity and position tracking control. To demonstrate the effectiveness of the proposed scheme, the scheme is applied to the tracking control of robot manipulators. Simulation results have verified the effective performance of the controller.

To implement the proposed scheme for experiment, a voltage-control type of drive system has been set up, which employs two personal computers, PC 486 and PC 286. One is adopted to perform the calculation of the control law and the other is to perform the function of pulse width modulation (PWM) and the generation of gating pulses. Satisfactory experimental studies are also shown in this paper.

Nomenclature :

$v_{as}, v_{bs}, v_{cs} (v_{ar}, v_{br}, v_{cr})$: stator (rotor) phase voltages
 $i_{as}, i_{bs}, i_{cs} (i_{ar}, i_{br}, i_{cr})$: stator (rotor) phase currents
 $v_{ds} (v_{qs})$: d-axis (q-axis) stator voltage
 $i_{ds} (i_{qs})$: d-axis (q-axis) stator current
 $\Phi_{dr} (\Phi_{qr})$: d-axis (q-axis) rotor flux
 θ_s : position of rotor flux vector
 θ_r : rotor angle
 θ_{sl} : slip angle
 ω_s : stator angular frequency
 ω_r : rotor angular speed
 ω_{sl} : slip angular speed
 $L_{ls} (L_{lr})$: leakage inductances of stator (rotor) windings
 $L_{ms} (L_{mr})$: magnetizing inductances of stator (rotor) windings
 L_{sr} : mutual inductances between stator and rotor windings
 $R_s (R_r)$: stator (rotor) resistance
 $L_s (L_r)$: stator (rotor) self-inductance
 M : stator/rotor mutual inductance
 p : number of pole pairs
 J : rotor inertia
 D : damping coefficient
 K_T : torque constant ($= 3pM/2L_r$)
 T_L : disturbance torque
 σ : leakage coefficient ($= 1 - M^2/L_s L_r$)
 $c : 1/\sigma L_s$
 $a_1 : c(R_s + M^2 R_r/L_r^2)$
 $a_2 : cM R_r/L_r^2$
 $a_3 : pcM/L_r$
 $a_4 : R_r/L_r$
 $a_5 : MR_r/L_r$

1 Introduction

The induction motor has a wide variety of applications as the electromechanical actuator because of its ruggedness, low maintenance, and low cost. Advances in power electronics and microprocessor technology make it feasible to use the induction motors in place of dc and synchronous motors in a wide range of servo applications. However it is very difficult to achieve high performance with an induction motor, which is caused by the nonlinear coupling terms of the dynamics comparing the electrical part and the mechanical torque part.

In a separate-excitation dc motor, the electromechanical arrangement provides decoupling between magnetic effects in the stator and in the rotor windings. This decoupling is reflected in the torque dynamical equation. The dc motor can be linearly operated simply by keeping the stator current constant.

However, for an induction motor, the stator windings produce an associated flux, and indirectly create a rotor flux as well. Angular separation between the fluxes results from the time delay inherent in the rotor circuit. As the stator current is the only mechanism for torque generation, the stator and rotor effects are tightly coupled. Thus, the motor machine cannot be operated as a linear device unless some kind of nonlinear feedback is applied.

To overcome the problems mentioned above in practical implementations, the field orientation control [12] is a widely used approach. This method uses a nonlinear feedback to obtain the approximate decoupling between the stator and rotor, and cause the dynamics of an induction motor to behave like those of a separate-excitation dc motor. When the full-state feedback is used, it is called direct field orientation control [8, 9]. But it requires accurate flux sensors which are expensive and suffer drawbacks such as temperature sensitivity and its induced changes to the motor structure. Another approach is to use observers for flux in place of the previous sensors. This method is called indirect field orientation control. Its stability, robustness, and related control properties are, however, not fully studied yet, although the field orientation control performs well in practice.

In the past few years, the advances in nonlinear control theory have had a notable impacts on the field of induction motor control. Input-output decoupling control was presented in [4] and [3] using geometric techniques. In [4], a simplified model is used, i.e., only the electromagnetic part is considered and the speed is assumed to be a slowly varying parameter. Exact decoupling in the control of electric torque and flux amplitude using the amplitude and frequency of the voltage supply as inputs are achieved by a static state feedback. On the other hand, the results in [21, 22] are based on the adaptive control theories of linear systems so that linearized models of induction motors are used for controller design and stability analysis. Along the same line, the work in [18] proposed an adaptive tracking control scheme based on the nonlinear model. Moreover, Marino [5, 6] proposed a direct adaptive controller for speed regulation in which the motor model is input-output decoupled by a feedback-linearizing controller but with the load torque and rotor resistance being adapted in time. The results of that is the assurance of asymptotic convergence of the system states and the estimated parameters.

In aforementioned schemes [4, 5, 6, 18], the full states must be available. So practical implementations will require the flux observers, and hence a good deal of research effort has been directed towards the design of nonlinear controllers that employ flux observers. In [5, 6], the controller was combined with the flux observer [10]. However, no proof of stability was given for the resulting closed-loop system.

In [3], the observer-based adaptive controller was designed and the closed-loop stability was proved under several restrictive assumptions. In [15], closed-loop stability is also proved by using design technique for nonlinear feedback control [16]. But the singularity of the control laws remains as in [3, 4, 5, 6, 18, 15]. The globally stable controller for regulating torque and flux amplitudes was presented in [17]. Using the skew-symmetric properties of the model of induction motors, a controller without singularity is designed by using a fourth-order observer, but only torque tracking and flux regulation are achieved.

In this paper, we propose a nonlinear observer-based controller for induction motors. Via the use of skew-symmetric properties of the model [17], the two-stage design technique [18] is applied to construct an observer-based controller for velocity and position tracking control. To demonstrate the effectiveness of the proposed scheme, an application to the tracking control of robot manipulators is presented. Simulation results show that the tracking is satisfactory and both the input voltage signals as well as the current signals remain within the acceptable limits.

In order to implement the proposed scheme for experiment, a voltage-control type of drive system [13, 14] has been set up, which employs two personal computers, namely, PC 486 and PC 286. One is adopted to perform the calculation of the control laws and the other is to perform the function of pulse width modulation (PWM) and generation of gating pulses.

The layout of this paper is as follows. The nonlinear model of the induction motors is shown in section 2. Such a model is then rearranged for the controller design, in which torque load is taken into more realistic considerations. In section 3, the controller is proposed and the main results of this paper are derived as well. The simulation results are shown in section 4, and experimental setup as well as the results are presented in section 5. Finally, the conclusions are provided in section 6.

2 Model of Induction Motors

With the notations in [1] [3], the dynamics of a balanced 3-phase Y-connected that has symmetric, and linear magnetic circuits can be described as a fifth-order state-space dynamical system. Let the motor have p poles, then, after a coordinate transform onto the d-q coordinate frame rotating synchronously with an angular speed ω_s , the dynamical system can be represented as follows:

$$\dot{i}_{ds} = -a_1 i_{ds} + \omega_s i_{qs} + a_2 \Phi_{dr} + a_3 \omega_r \Phi_{qr} + c V_{ds} \quad (1)$$

$$\dot{i}_{qs} = -\omega_s i_{ds} - a_1 i_{qs} - a_3 \omega_r \Phi_{dr} + a_2 \Phi_{qr} + c V_{qs} \quad (2)$$

$$\dot{\Phi}_{dr} = -a_4 \Phi_{dr} + a_5 i_{ds} + (\omega_s - p\omega_r) \Phi_{qr} \quad (3)$$

$$\dot{\Phi}_{qr} = -a_4 \Phi_{qr} + a_5 i_{qs} - (\omega_s - p\omega_r) \Phi_{dr} \quad (4)$$

$$\dot{\omega}_r = (-D\omega_r + T_e - T_L)/J \quad (5)$$

$$T_e = k_T(\Phi_{dr} i_{qs} - \Phi_{qr} i_{ds}) \quad (6)$$

where V_{ds}, V_{qs}, ω_s are control inputs and T_e is the generated torque. The definitions of the notations and symbols are given in the Nomenclature.

It can be seen shortly that the dynamical model described by equations (1)-(6) possesses a skew-symmetric property, which will be used in the design of the controller. In order to manifest this property, we first rearrange the dynamical equations by using more compact notations as follows.

Denoting $x^T = [x_1 \ x_2 \ x_3 \ x_4] = [i_{ds} \ i_{qs} \ \Phi_{dr} \ \Phi_{qr}]$, $X^T = [x^T \ x_5]$, $u_1 = V_{ds}$, $u_2 = V_{qs}$, $u_3 = \omega_s$, we can rewrite equations (1)-(6) as

$$\dot{x}_1 = -a_1 x_1 + u_3 x_2 + a_2 x_3 + a_3 x_5 x_4 + c u_1 \quad (7)$$

$$\dot{x}_2 = -u_3 x_1 - a_1 x_2 - a_3 x_5 x_3 + a_2 x_4 + c u_2 \quad (8)$$

$$\dot{x}_3 = -a_4 x_3 + a_5 x_1 + (u_3 - p x_5) x_4 \quad (9)$$

$$\dot{x}_4 = -a_4 x_4 + a_5 x_2 - (u_3 - p x_5) x_3 \quad (10)$$

$$\dot{x}_5 = (-D x_5 + T_e - T_L)/J \quad (11)$$

$$T_e = k_T(x_3 x_2 - x_4 x_1) \quad (12)$$

Note that the equations (7)-(10) can be further rewritten in a more compact symbolic form as

$$\dot{x} = Ax + Bu, \quad (13)$$

where

$$A = A_0 + A_1 + A_2, \quad u^T = [u_1 \ u_2],$$

with

$$A_0 \triangleq \begin{bmatrix} -a_1 & 0 & 0 & 0 \\ 0 & -a_1 & 0 & 0 \\ 0 & 0 & -a_4 & 0 \\ 0 & 0 & 0 & -a_4 \end{bmatrix},$$

$$A_1 \triangleq \begin{bmatrix} 0 & u_3 & -a_5 & 0 \\ -u_3 & 0 & 0 & -a_5 \\ a_5 & 0 & 0 & (u_3 - p x_5) \\ 0 & a_5 & -(u_3 - p x_5) & 0 \end{bmatrix},$$

$$A_2 \triangleq \begin{bmatrix} 0 & 0 & (a_2 + a_5) & p a_3 x_5 \\ 0 & 0 & -p a_3 x_5 & (a_2 + a_5) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$B \triangleq \begin{bmatrix} c & 0 \\ 0 & c \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

It should be noted that A_1 is a skew-symmetric matrix, i.e., $A_1^T = -A_1$.

In the following, we assume that the torque load is a known function of the rotor speed. This assumption is more realistic than the one with constant load [15, 17, 18]. It is well-known that the bearings and lots of viscous forces vary linearly with the speed, while the large-scale fluid systems such as pumps and fans have loads that typically vary as the square of the speed [19]. Hence, the torque load is assumed to be in the form of

$$T_L = \mu_0 + \mu_1 x_5 + \mu_2 x_5^2, \quad (14)$$

where μ_0, μ_1, μ_2 are some known constants.

3 Control of Induction Motors

Given the dynamic model of an induction motor described in the previous section, the control problem is stated subjected to some prior assumptions about the system in the following.

Assumptions :

- (A1) The motor parameters are known in advance.
- (A2) Rotor speed x_5 and stator currents x_1, x_2 are measurable.
- (A3) The desired rotor speed $\omega_d \in C^2$, i.e., $\omega_d^{(i)}$, the i -th derivative of ω_d , $i = 1, 2$, exist and are continuous.

Control Objective :

Given a desired rotor speed trajectory $\omega_d(t)$, determine the control inputs u_1, u_2, u_3 such that the rotor speed ω_r can track the desired trajectory exponentially in time t , i.e.,

$$x_5 \rightarrow x_{5d}, \quad \text{as } t \rightarrow \infty.$$

assuming that the signals x_1, x_2 , and x_5 are available.

Furthermore, the rotor fluxes are also regulated according to the desired commands, i.e.,

$$\begin{aligned} \Phi_{dr} &\rightarrow \beta, & \text{as } t \rightarrow \infty. \\ \Phi_{qr} &\rightarrow 0, & \text{as } t \rightarrow \infty. \end{aligned}$$

In the following, the proposed controller is motivated by the two-stage analysis first presented in [18]. The model of an induction motor can be viewed as a cascade of two subsystem shown in Fig.1, or in other words a two-stage system.

3.1 Control Strategy

3.1.1 The First Stage

Referring to equation (11), i.e.,

$$J \dot{x}_5 + D x_5 + T_L = T_e, \quad (15)$$

the output signal in this stage is ω_r and the generated torque T_e is viewed as the input. If we want the output signal x_5 to track x_{5d} , a sufficient condition is to have the input signal T_e be equal to the desired torque T_d as follows :

$$T_d = J \dot{x}_{5d} + k_1 x_{5d} + \mu_0 + (\mu_1 + D - k_1) x_5 + \mu_2 x_5^2,$$

where T_L is replaced by equation (14). Let $e_5 \triangleq x_5 - x_{5d}$, we then have

$$J \dot{e}_5 + k_1 e_5 = T_e - T_d. \quad (16)$$

Since T_e is not the actual exogenous input, the right hand side of (16) can not be made identically zero.

3.1.2 The Second Stage

In this stage, we just take the equations (7)-(10) into consideration. Here, the generated torque T_e is viewed as the output, and the input signals are u_1, u_2 , and u_3 . It is easy to show that as $x_4 = x_{4d} = 0$, $x_3 = x_{3d} = \beta$, $x_2 = x_{2d} = \frac{T_d}{k_T \beta}$, the generated torque will coincide with T_d from (12).

Let $e = x - x_d$, where $x_d^T = [x_{1d} \ x_{2d} \ x_{3d} \ x_{4d}]$, $x^T = [x_1 \ x_2 \ x_3 \ x_4]$, $\hat{x}^T = [\hat{x}_1 \ \hat{x}_2 \ \hat{x}_3 \ \hat{x}_4]$, so that $\hat{x}^T = x^T - \hat{x}^T = [\hat{x}_1 \ \hat{x}_2 \ \hat{x}_3 \ \hat{x}_4] = [x_1 - \hat{x}_1 \ x_2 - \hat{x}_2 \ x_3 - \hat{x}_3 \ x_4 - \hat{x}_4]$, where $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4$ are the estimates of the flux states x_1, x_2, x_3, x_4 . From equation (13), we have

$$\dot{e} = (A_0 + A_1 + A_2)e - \dot{x}_d + (A_0 + A_1 + A_2)x_d + Bu \quad (17)$$

which motivates us to design the control inputs u_1, u_2, u_3 , and x_{1d} as :

$$u_1 = \begin{bmatrix} \dot{\hat{x}}_{1d} + a_1 x_{1d} - u_3 x_{2d} + a_5 x_{3d} - (a_2 + a_5) x_{3d} \\ - (a_2 + a_5)(\hat{x}_3 - x_{3d}) - a_3 x_5(\hat{x}_4 - x_{4d}) \\ - k_2(x_1 - x_{1d}) + v_1 \end{bmatrix} / c \quad (18)$$

$$u_2 = \begin{bmatrix} \dot{\hat{x}}_{2d} + a_1 x_{2d} + u_3 x_{1d} + a_3 x_5 x_{3d} \\ + a_3 x_5(\hat{x}_3 - x_{3d}) - (a_2 + a_5)(\hat{x}_4 - x_{4d}) \\ - k_3(x_2 - x_{2d}) + v_2 \end{bmatrix} / c \quad (19)$$

$$u_3 = p x_5 + \frac{a_5}{\beta} x_{2d} - \frac{1}{\beta} v_4 \quad (20)$$

$$x_{1d} = \frac{a_4}{a_5} \beta + \frac{1}{a_5} v_3 \quad (21)$$

so that

$$\dot{e} = (A_0 + A_1 - K)e + R(t, \hat{z}) + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}, \quad (22)$$

where

$$R(t, \hat{z}) = \begin{bmatrix} (a_2 + a_5)\hat{x}_3 + a_3 x_5 \hat{x}_4 - (\hat{x}_{1d} - \dot{\hat{x}}_{1d}) \\ - a_3 x_5 \hat{x}_3 + (a_2 + a_5)\hat{x}_4 - (\hat{x}_{2d} - \dot{\hat{x}}_{2d}) \\ 0 \\ 0 \end{bmatrix}. \quad (23)$$

$$K = \text{diag}[k_2, k_3, 0, 0] \quad (24)$$

with k_2, k_3 being feedback control gains, and $v_i, i = 1, 2, 3, 4$, are extra control terms used for cancelling the coupling terms in the subsequent closed loop stability analysis.

3.1.3 Observer Design

In order to estimate the flux signals, we design a nonlinear observer as follows :

$$\dot{\hat{x}}_3 = -a_4 \hat{x}_3 + a_5 x_1 + (u_3 - p x_5) \hat{x}_4 + \xi_{11} + \xi_{21} \quad (25)$$

$$\dot{\hat{x}}_4 = -a_4 \hat{x}_4 + a_5 x_2 - (u_3 - p x_5) \hat{x}_3 + \xi_{12} + \xi_{22} \quad (26)$$

where ξ_{ij} , $1 \leq i, j \leq 2$, are some suitable functions to be specified in the following. Then, the estimation errors satisfy

$$\dot{\tilde{z}} = -a_4 I_2 \tilde{z} - (u_3 - p x_5) J_2 \tilde{z} + E, \quad (27)$$

where

$$I_2 \triangleq \text{diag}[1, 1],$$

$$J_2 \triangleq \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

$$E \triangleq - \begin{bmatrix} \xi_{11} + \xi_{21} \\ \xi_{12} + \xi_{22} \end{bmatrix}.$$

Here, ξ_{21} , ξ_{22} are chosen for the purpose of canceling the coupling terms in equation (23), namely, $R(t, \tilde{z})$, whereas ξ_{11} , ξ_{12} are designed for canceling the coupling terms in the overall closed-loop stability analysis.

For simplicity, we first define some notations in the following:

$$\begin{aligned} v_3 &\triangleq -k_T x_{2d} e_5 \\ \eta_0 &\triangleq \frac{k_T}{J} (\mu_1 + D + 2\mu_2 x_5 - k_1) x_2 \\ \eta_1 &\triangleq \frac{k_T}{J} (\mu_1 + D + 2\mu_2 x_5 - k_1) x_1 \\ \eta_2 &\triangleq \frac{k_T}{J} x_2 \\ \eta_3 &\triangleq \frac{k_T}{J} x_1 \\ T_A &\triangleq J \tilde{x}_{5d} + k_1 \tilde{x}_{5d} + \frac{1}{J} (\mu_1 + D + 2\mu_2 x_5 - k_1) (-D x_5 - T_L) \\ T_B &\triangleq \eta_0 x_3 - \eta_1 x_4 \\ T_C &\triangleq \frac{1}{J} (-D x_5 - T_L) - \tilde{x}_{5d} \\ T_D &\triangleq \eta_2 x_3 - \eta_3 x_4 \\ \hat{x}_{1d} &\triangleq -\frac{1}{a_5 \beta} [e_5 T_A + T_d T_C + (e_5 \eta_0 + T_d \eta_2) \tilde{x}_3 - (e_5 \eta_1 + T_d \eta_3) \tilde{x}_4] \\ \hat{x}_{2d} &\triangleq \frac{T_A + \eta_0 \tilde{x}_3 - \eta_1 \tilde{x}_4}{k_T \beta} \\ f_0 &\triangleq a_2 + a_5 + \frac{e_5 \eta_0 + T_d \eta_2}{a_5 \beta} \\ f_1 &\triangleq -(a_3 x_5 + \frac{1}{k_T \beta} \eta_0 - k_T e_5) \\ f_2 &\triangleq a_3 x_5 - k_T e_5 - \frac{a_5 \eta_1 + T_d \eta_3}{a_5 \beta} \\ f_3 &\triangleq a_2 + a_5 + \frac{1}{k_T \beta} \eta_1 \end{aligned}$$

Theorem 1 Consider an induction motor whose dynamics are governed by equations (1)-(6) under the assumptions (A1)-(A3). Then, the control objective can be achieved provided the observer-based controller is designed according to (18)-(21) with

$$v_1 = k_T \tilde{x}_4 e_5 \quad (28)$$

$$v_2 = -k_T \tilde{x}_3 e_5 \quad (29)$$

$$v_4 = k_T x_{1d} e_5, \quad (30)$$

where $\tilde{x}_3, \tilde{x}_4$ are obtained from the following nonlinear observer

$$\dot{\hat{x}}_3 = -a_4 \hat{x}_3 + a_5 x_1 + (u_3 - p x_5) \hat{x}_4 + f_0 e_1 + f_1 e_2 \quad (31)$$

$$\dot{\hat{x}}_4 = -a_4 \hat{x}_4 + a_5 x_2 - (u_3 - p x_5) \hat{x}_3 + f_2 e_1 + f_3 e_2 \quad (32)$$

Proof : Choose the Lyapunov function as

$$V = \frac{1}{2} (e^T e + \tilde{z}^T \tilde{z} + J e_5^2) \quad (33)$$

From equations (16)-(22)-(27), the derivative of V is obtained as follows :

$$\begin{aligned} \dot{V} &= e^T (A_0 + K) e + \tilde{z}^T (-a_4 I_2 \tilde{z} - k_1 e_5^2 + e^T R(t, \tilde{z}) + e_1 v_1 + e_2 v_2 \\ &\quad + e_3 v_3 + e_4 v_4 - (\xi_{11} + \xi_{21}) \tilde{x}_3 - (\xi_{12} + \xi_{22}) \tilde{x}_4 + e_5 (T_c - T_d) \end{aligned}$$

First of all,

$$e_5 (T_c - T_d) = k_T e_5 x_3 e_2 - k_T e_5 x_4 e_1 + k_T e_5 (e_3 x_{2d} - e_4 x_{1d}).$$

Next, let

$$\xi_{11} = k_T e_5 e_2, \quad \xi_{12} = -k_T e_5 e_1,$$

which then yields that

$$\begin{aligned} \dot{V} &= -\alpha_0 \|e\|^2 - a_4 \|\tilde{z}\|^2 - k_1 e_5^2 + e_3 v_3 + e_4 v_4 + k_T e_5 (e_3 x_{2d} - e_4 x_{1d}) \\ &\quad + e^T R(t, \tilde{z}) - \xi_{21} \tilde{x}_3 - \xi_{22} \tilde{x}_4, \end{aligned} \quad (34)$$

where

$$\alpha_0 = \min\{a_1 + k_2, a_2 + k_3, a_4, a_4\}$$

Furthermore, by substituting the definitions of v_3 and v_4 into (34), we obtain

$$\dot{V} \leq -\alpha_0 \|e\|^2 - a_4 \|\tilde{z}\|^2 - k_1 e_5^2 + e^T R(t, \tilde{z}) - \xi_{21} \tilde{x}_3 - \xi_{22} \tilde{x}_4 \quad (35)$$

It should be noted that $\hat{T}_d$ which needs to be used for implementing $\hat{x}_{2d}$ and $\hat{x}_{1d}$ includes $\tilde{x}_5$. This is the reason why we design $\hat{x}_{2d}$ and $\hat{x}_{1d}$ instead of $\tilde{x}_{2d}$ and $\tilde{x}_{1d}$. Accordingly, more detailed expressions for $\hat{x}_{1d}$ and $\hat{x}_{2d}$ are derived in the following.

To find $\hat{x}_{1d}$ and $\hat{x}_{2d}$, we first derive

$$\dot{T}_d = J \tilde{x}_{5d} + k_1 \tilde{x}_{5d} + (\mu_1 + D - k_1) \tilde{x}_5 + 2\mu_2 x_5 \tilde{x}_5 \quad (36)$$

and then substitute equations (11)-(12) into the above to yield

$$\dot{T}_d = T_A + T_B \quad (37)$$

Since

$$\dot{e}_5 = T_C + T_D, \quad (38)$$

we thus have

$$\begin{aligned} \hat{x}_{1d} &= -\frac{1}{a_5 \beta} (e_5 T_A + e_5 T_B + T_d T_C + T_d T_D) \\ &= -\frac{1}{a_5 \beta} [e_5 T_A + T_d T_C + (e_5 \eta_0 + T_d \eta_2) \tilde{x}_3 - (e_5 \eta_1 + T_d \eta_3) \tilde{x}_4] \\ \hat{x}_{2d} &= \frac{T_A + \eta_0 \tilde{x}_3 - \eta_1 \tilde{x}_4}{k_T \beta} \end{aligned}$$

Note that $\hat{x}_{1d}$ and $\hat{x}_{2d}$ are equal to $\tilde{x}_{1d}$ and $\tilde{x}_{2d}$, respectively, except that x_3, x_4 are replaced by $\tilde{x}_3, \tilde{x}_4$, then, we can easily find that

$$\hat{x}_{1d} - \tilde{x}_{1d} = -\frac{[(e_5 \eta_0 + T_d \eta_2) \tilde{x}_3 - (e_5 \eta_1 + T_d \eta_3) \tilde{x}_4]}{a_5 \beta} \quad (39)$$

$$\hat{x}_{2d} - \tilde{x}_{2d} = \frac{1}{k_T \beta} (\eta_0 \tilde{x}_3 - \eta_1 \tilde{x}_4) \quad (40)$$

To cancel the term involving $R(t, \tilde{z})$ in equation (35), ξ_{21} and ξ_{22} are, hence, designed as

$$\begin{aligned} \xi_{21} &= e_1 (a_2 + a_5 + \frac{e_5 \eta_0 + T_d \eta_2}{a_5 \beta}) - e_2 (a_3 x_5 + \frac{1}{k_T \beta} \eta_0) \\ \xi_{22} &= e_1 (a_5 x_5 - \frac{e_5 \eta_1 + T_d \eta_3}{a_5 \beta}) + e_2 (a_2 + a_5 + \frac{1}{k_T \beta} \eta_1) \end{aligned}$$

Finally, we have

$$\dot{V} \leq -\alpha_0 \|e\|^2 - a_4 \|\tilde{z}\|^2 - k_1 e_5^2$$

which thus concludes our proof by following a standard Lyapunov analysis [23]. $\square$

3.2 Application to a Single-Link Manipulator

Previously, we have proposed a nonlinear observer-based tracking control scheme for induction motors with a known desired function of the rotor speed. However, the proposed scheme can be widely applied to the high performance control of systems using induction motors as actuators. As an illustrative example, we will apply this scheme to the tracking control of robot manipulators actuated by induction motors. For simplicity, here we will only consider a single-link manipulator as shown in Fig.2 which is driven by an induction motor.

The modeling of the mechanical part of the manipulator system can be described as:

$$J \ddot{\theta} + D \dot{\theta} = T_e - T_L, \quad (41)$$

where

$$T_L = I \ddot{\theta} + B \dot{\theta} + m g l \sin \theta,$$

so that

$$(J + I) \ddot{\theta} + (B + D) \dot{\theta} + m g l \sin \theta = T_e.$$

Note that I, m, l stand for the moment of inertia, the mass, and the length of the link, respectively, B, g represent the viscous frictional coefficient and gravity constant, respectively, and θ is the angle of rotation of the link.

Similarly, we design the control input of the first stage as

$$T_d = (J + I) \ddot{\theta}_d + (B + D) \dot{\theta}_d + m g l \sin \theta - k_\theta (\theta - \theta_d) - (k_\omega - (B + D)) (\dot{\theta} - \dot{\theta}_d), \quad (42)$$

where k_θ, k_ω are some appropriate constants. Denoting $e_\theta = \theta - \theta_d$, $e_\omega = \dot{\theta} - \dot{\theta}_d$, and $e_m^T = [e_\theta \ e_\omega]$, we then have

$$\dot{e}_m = A_m e_m + B_m (T_c - T_d), \quad (43)$$

where

$$A_m = \begin{bmatrix} 0 & 1 \\ -\frac{k_\theta}{J+I} & -\frac{k_\omega}{J+I} \end{bmatrix}$$

$$B_m = \begin{bmatrix} 0 \\ \frac{1}{J+I} \end{bmatrix}.$$

In the second stage, the design methodology is the same as has been described earlier. Thus, for any $Q > 0$, there exists $P > 0$ such that $P^T A_m + A_m P = -Q$.

In the same way, we define some notations in the following :

$$\begin{aligned}\eta_0 &\triangleq \frac{[k_w - (B + D)]k_T x_2}{J + I} \\ \eta_1 &\triangleq \frac{[k_w - (B + D)]k_T x_1}{J + I} \\ \eta_2 &\triangleq \frac{k_T x_2}{J + I} \\ \eta_3 &\triangleq \frac{k_T x_1}{J + I} \\ T_A &\triangleq (J + I)\theta_d^{(3)} + k_w \ddot{\theta}_d + k_\theta \dot{\theta}_d + mgl\theta \cos \theta - k_\theta \dot{\theta} - (k_w - (B + D))(T_C + \ddot{\theta}_d) \\ T_B &\triangleq -(k_w - (B + D))(\eta_2 x_3 - \eta_3 x_4) \\ \hat{T}_B &\triangleq -(k_w - (B + D))(\eta_2 \hat{x}_3 - \eta_3 \hat{x}_4) \\ T_C &\triangleq \frac{-(B + D) - mgl \sin \theta}{J + I} - \ddot{\theta}_d \\ T_D &\triangleq \eta_2 x_3 - \eta_3 x_4 \\ \hat{T}_D &\triangleq \eta_2 \hat{x}_3 - \eta_3 \hat{x}_4 \\ g_0 &\triangleq [0 \ \eta_2] \\ g_1 &\triangleq [0 \ -\eta_3] \\ g_2 &\triangleq [\dot{e}_\theta \ T_C] \\ \hat{x}_{1d} &\triangleq -\frac{1}{a_5 \beta} [(g_0 \hat{x}_3 + g_1 \hat{x}_4) P B_m T_d + e_m^T P B_m (T_A + \hat{T}_B)] \\ \hat{x}_{2d} &\triangleq \frac{T_A - (k_w - (B + D))(\eta_2 \hat{x}_3 - \eta_3 \hat{x}_4)}{k_T \beta} \\ f_0 &\triangleq e_1(a_2 + a_5) + \frac{e_1}{a_5 \beta} (g_0 P B_m T_d - e_m^T P B_m \eta_2) - e_2 a_3 \dot{\theta} \\ &\quad + \frac{k_w - (B + D)}{k_T \beta} \eta_2 e_2 + k_T e_m^T P B_m e_2 \\ f_1 &\triangleq a_5 e_1^2 \dot{\theta} + \frac{e_1}{a_5 \beta} (g_1 P B_m T_d + e_m^T P B_m \eta_3) + e_2(a_2 + a_5) \\ &\quad - \frac{k_w - (B + D)}{k_T \beta} \eta_3 e_2 - k_T e_m^T P B_m e_1\end{aligned}$$

Theorem 2 Consider a single link manipulator actuated by an induction motor whose dynamics are governed by (1) and (6). Let the desired positional trajectory be $\theta_d(t)$, whose $\theta_d^{(i)}(t)$, $i = 1, 2, 3$ are continuous. Based on the assumptions (A1) and (A2), the objective of position and velocity tracking, i.e.,

$$\theta(t) \rightarrow \theta_d(t), \quad \dot{\theta}(t) \rightarrow \dot{\theta}_d(t) \quad \text{exponentially as } t \rightarrow \infty$$

are achieved provided that the following nonlinear observer-based controller is designed according to (18)-(21) and (42) with

$$\begin{aligned}v_1 &= k_T e_m^T P B_m \hat{x}_4 \\ v_2 &= -k_T e_m^T P B_m \hat{x}_3 \\ v_3 &= -k_T e_m^T P B_m x_{2d} \\ v_4 &= k_T e_m^T P B_m x_{1d}\end{aligned}$$

where $\hat{x}_3, \hat{x}_4$ are estimated by the following nonlinear observer

$$\begin{aligned}\dot{\hat{x}}_3 &= -a_4 \hat{x}_3 + a_2 x_1 + (u_3 - p x_3) \hat{x}_4 + f_0 \\ \dot{\hat{x}}_4 &= -a_4 \hat{x}_4 + a_2 x_2 - (u_3 - p x_3) \hat{x}_3 + f_1\end{aligned}$$

with

$$P A_m^T + A_m P = -Q.$$

Proof : Choose a Lyapunov function as

$$V = \frac{1}{2} (e^T e + \hat{z}^T \hat{z} + e_m^T P e_m).$$

From equations (18)-(21)(42), the derivative of V is obtained as follows :

$$\begin{aligned}\dot{V} &= e^T (A_0 + K) e + \hat{z}^T (-a_4 I_2) \hat{z} - e_m^T Q e_m + e_m^T P B_m (T_A - T_d) + R(t, \hat{z}) \\ &\quad + e_1 v_1 + e_2 v_2 + e_3 v_3 + e_4 v_4 - (\xi_{11} + \xi_{21}) \hat{x}_3 - (\xi_{12} + \xi_{22}) \hat{x}_4\end{aligned}$$

Following the results of Theorem 1, ξ_{11} , ξ_{12} , ξ_{21} , and ξ_{22} are designed as

$$\begin{aligned}\xi_{11} &= k_T e_m^T P B_m e_2 \\ \xi_{12} &= -k_T e_m^T P B_m e_1 \\ \xi_{21} &= e_1(a_2 + a_5) + \frac{e_1}{a_5 \beta} (g_0 P B_m T_d - e_m^T P B_m \eta_2) - e_2 a_3 \dot{\theta} \\ &\quad + \frac{k_w - (B + D)}{k_T \beta} \eta_2 e_2 \\ \xi_{22} &= a_5 e_1^2 \dot{\theta} + \frac{e_1}{a_5 \beta} (g_1 P B_m T_d + e_m^T P B_m \eta_3) + e_2(a_2 + a_5) \\ &\quad - \frac{k_w - (B + D)}{k_T \beta} \eta_3 e_2\end{aligned}$$

So, after some algebraic manipulation, we also have

$$\dot{V} \leq -\alpha_0 \|e\|^2 - \alpha_4 \|\hat{z}\|^2 - e_m^T Q e_m.$$

which then completes our proof. $\square$

4 Simulation Results

In this section, the preceding observer-based control scheme is applied to simulation studies of the velocity tracking of the induction motor and the tracking control of a single-link manipulator actuated by an induction motor. The numerical values of the six-pole squirrel-cage induction motor are $R_s = 3.745 \Omega$, $R_r = 3.583 \Omega$, $L_s = 163.3 \text{ mH}$, $L_r = 163.3 \text{ mH}$, $M = 154.67 \text{ mH}$, $J = 0.05 \text{ kgm}^2$, and $D = 0.065 \text{ kgm}^2 \text{ s}^{-1}$.

First the control scheme is tested for load $T_L = \mu_0 + \mu_1 x_3 + \mu_2 x_3^2$, where $\mu_0 = 3.3$, $\mu_1 = 0.25$, $\mu_2 = 0.004$, and the response for sine wave tracking are shown in Fig.3(a)-(b).

Secondly, a single-link manipulator dynamics are used to demonstrate the proposed control law. Here, the numerical values of the manipulator are $I = 0.0284 \text{ kgm}^2$, $B = 0.001 \text{ kgm}^2 \text{ s}^{-1}$, $m = 0.3 \text{ kg}$, $g = 9.8 \text{ m/s}^2$, and $l = 0.5 \text{ m}$. The desired trajectory is chosen as $\theta_d = \sin 3t$. Fig.4(a)-(b) show the performance for tracking control of this manipulator.

5 Experimental Results

The proposed controller is tested in a system with a squirrel-cage motor rated one house power (HP) with an optical encoder attached to its shaft, a PWM transistor inverter, and two personal computers (PC486 with 16 bits A/D D/A card and PC286) communicating through 8255 cards. The configuration of the system is shown in Fig.5.

5.1 Rotor position and current detection

The rotor position is detected by a two pulse (with complementary outputs) optical encoder with two phases set apart by 90° . The resolution is 1024 pulses per revolution. The pulses go through a digital circuit, which accumulates the pulses with 12 bit counter. The corresponding speed is calculated by PC486.

Because of the balanced three phases, i.e., $i_{as} + i_{bs} + i_{cs} = 0$, two currents are adequate for the coordinate transformation. Currents are measured with Hall effect transducers (LB-10GA) and low-pass filters.

5.2 Control Algorithm Processing

The control algorithm is performed by PC486. In each period of 2 ms, the program computes the following tasks: rotor position and current sampling, speed calculation, coordinate transformation, control algorithm execution, data fetching to the common memory for PC286, and informing PC286.

5.3 Results

The squirrel-cage motor used in the experiment is manufactured by ELMA MOTORS CO. with delta-connected stator. The parameters of the motor are shown below:

Rated power = 0.75 kW
Rated current = 4 A
Rated voltage = 220 V
Rated frequency = 60 Hz
Rated speed = 1120 rpm
poles = 6
 $R_s = 3.745 \Omega$
 $R_r = 3.583 \Omega$
 $L_s = 163.3 \text{ mH}$
 $L_r = 163.3 \text{ mH}$
 $L_m = 154.67 \text{ mH}$

Fig.6 shows the velocity response for 200 rpm.

6 Conclusions

From the skew-symmetric property of induction motors, the observer-based controller is proposed. Without the measurement of fluxes, velocity and position tracking can be achieved under the assumptions that the system parameters are known.

The simulation and experiment results have shown the effectiveness of the proposed controller in combination with the flux observer. The future theoretical work is to enhance the robustness of the controller and/or to add parameter estimation with an adaptive observer.

References

- [1] Paul C. Krause, *Analysis of Electric Machinery*, McGraw-Hill, 1986.
- [2] B. K. Bose, *Power Electronics and AC Drives*, Prentice-Hall, 1986.
- [3] Dong-Il Kim, In-Joong Ha and Myoung-Sam Ko, "Control of Induction Motors via Feedback Linearization with Input-Output Decoupling," *Int. J. Control*, Vol. 51, No. 4, pp. 863-883, 1990.
- [4] Alessandro De Luca and Giovanni Ulivi, "Design of an Exact Nonlinear Controller for Induction Motors," *IEEE Trans. on Automatic Control*, Vol. 34, No. 12, December 1989.
- [5] R. Marino, S. Pereda, P. Valigi, "Adaptive Partial Feedback Linearization of Induction Motors," *Proc. 29th CDC*, pp. 3313-3318, 1990.
- [6] R. Marino, S. Pereda, P. Valigi, "Adaptive Input-Output Linearizing Control of Induction Motors," *IEEE Trans. Automat. Contr.*, Vol. 38, No. 2, pp. 208-221, Feb. 1993.
- [7] W. Leonhard, "Microcomputer Control of High Dynamic Performance AC-Drives-A Survey," *Automatica*, Vol. 22, No. 1, pp. 1-19, 1986.
- [8] Allan B. Plunkett, "Direct Flux and Torque Regulation in a PWM Inverter-Induction Motor Drive," *IEEE Trans. on Industry Applications*, Vol. IA-13, No. 2, March/April 1977.
- [9] Thomas A. Lipo and Kwong C. Chang, "A New Approach to Flux and Torque-Sensing in Induction Machines," *IEEE Trans. on Industry Applications*, Vol. IA-22, No. 4, July/August 1986.
- [10] George C. Verghese and Seth R. Sanders, "Observers for Flux Estimation in Induction Machines," *IEEE Trans. on Industrial Electronics*, Vol. 35, No. 1, February 1988.
- [11] A. Bellini, G. Figalli and G. Ulivi, "Analysis and Design of a Microcomputer-based Observer for an Induction Machine," *Automatica*, Vol. 24, No. 4, pp. 549-555, 1988.
- [12] Rupperecht Gabriel, Werber Leonhard, and Craigj. Nordby, "Field-Oriented Control of a Standard AC Motor Using Microprocessors," *IEEE Trans. on Industry Applications*, Vol. IA-16, No. 2, March/April 1980.
- [13] Zheng Kaung Wu and Elias G. Strangas, "Feed Forward Field Orientation Control of an Induction Motor Using a PWM Voltage Source Inverter and Standardized Single-Board Computers," *IEEE Trans. Industrial Electronics*, Vol. 35, No. 1, February 1988.
- [14] Kenji Kubo, Masahiko Watanabe, Tsutomu Ohmae, Kenzo Kamiyama, "A Fully Digitalized Speed Regulator using Multimicroprocessor System for Induction Motor Drives," *IEEE Trans. on Industry Applications*, Vol. IA-21, No. 4, pp. 1001-1008, July/Aug. 1985.
- [15] I. Kanellakopoulos, P. T. Krein, and F. Disilvestro, "Nonlinear Flux-Observer Based Control of Induction Motors," *Proc. 1992 ACC*, pp. 1700-1704.
- [16] I. Kanellakopoulos, P. V. Kokotovic, and A. S. Morse, "A Toolkit for Nonlinear Feedback Design," *Systems and Control Letters*, Vol. 18, No. 2, pp. 83-92, 1992.
- [17] Romeo Ortega, and Carlos Canudas, and Seleme I. Seleme, "Nonlinear Control of Induction Motors Torque Tracking with Unknown Load Disturbance," *Proc. 1992 ACC*, pp. 206-210.
- [18] Jong-Hann Jean and Li-Chen Fu, "A new Adaptive Control Scheme for Induction Motors," *15th National Symposium on Automatic Control*, pp. 593-601, Taiwan, R.O.C., 1991.
- [19] Ned Mohan, Tore M. Undeland, William P. Robbins, *Power Electronics : converters, applications and design*, Wiley, 1989.
- [20] Joachim Holtz, "Pulsewidth Modulation - A Survey," *IEEE Trans. on Industrial Electronics*, Vol. 39, No. 5, December, 1992, pp.410-420.
- [21] C. M. Liaw, C.T. Pan, and Y. C. Chen, "An Adaptive controller for current-fed induction motor," *IEEE Trans. on Aerosp. Elect. Syst.*, Vol. 24, No. 3, pp. 250-262, 1988.
- [22] C. C. Chan, W. S. Leung and C. W. Ng, "Adaptive Decoupling Control of Induction Motor Drives," *IEEE Ind. Electron.*, Vol. IE-37, No. 1, pp. 41-47, Feb. 1990.
- [23] M. Vidyasagar, *Nonlinear System Analysis*, Prentice-Hall, 1992.
- [24] I. Kanellakopoulos and P. T. Krein, "Integral-Action Nonlinear Control of Induction Motors," *Proc. of IFAC*, Vol. 7, pp. 251, 1993.

Fig. 1. Two-stage Model for Induction Motor

Fig. 2. A Single-Link Manipulator

Fig. 3(a). Velocity Response for Sin Wave Tracking

Fig. 3(b). Flux Response

Fig. 4(a). Position Response for Single-Link Manipulator

Fig. 4(b). Flux Response

Fig. 5. Configuration for Experimental system

Fig. 6. Velocity Response (experimental result)