A COMPUTER PROGRAM FOR DESIGNING LINEAR PHASE FIR DIGITAL FILTERS BY EIGEN-APPROACH

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ABSTRACT

This paper presents a computer program for designing a large class of FIR linear phase digital filters by eigen-approach. The unique advantage of this approach over the McClellan-Parks algorithm is that it is general enough to incorporate both time and frequency domain constraints, this program not only can design standard bandpass filters, differentiators, Hilbert transformers, and also suitable for time-constrained FIR filter design such as Nyquist filters, partial response FIR filters, Mth band filters and higher order differentiators. The simplicity and generality of this eigen-design algorithm make this program useful for a wide variety of design applications.

INTRODUCTION

In the past fifteen years, the most popular program for designing linear phase FIR digital filters is that developed by McClellan and Parks [1]. It can design multi-bandpass filters, differentiators and Hilbert transformers etc. very efficiently. But the McClellan-Parks algorithm are emphasized on finding the optimal frequency response without any time constraint on the filter coefficients, and it is very difficult to incorporate both the time- and frequency-domain constraints to design some important filters in communication applications such as pulse shaping filters, multirate Mth band filters and Nyquist filters with zero intersymbol interference [2].

Recently Vaidyanathan and Nguyen have introduced the eigenfilter approach for designing linear phase FIR digital filters [3]. By minimizing a quadratic measure of the error in the frequency band, an eigenvector of appropriate matrix is computed to get the filter coefficients. The unique advantage of this approach over the McClellan-Parks algorithm is that it is general enough to incorporate both time and frequency domain constraints; also the design time

is comparable to that of McClellan-Parks algorithm. We have extended this approach to design multiple band-pass filters, higher order differentiators [4] and Hilbert transformers [5] by eigenfilter, this method is not only simple and fast, but also optimal in the least square sense [3]. Comparison with the McClellan-Parks algorithm for minimax equiripple filters shows that both are optimal in the sense of different minimum norms of the error function, but much better performance is obtained with our approach in most of the frequency band except in the narrowband region near the cutoff edge [4] [5].

This paper presents a general purpose computer program for designing a wide class of FIR linear phase digital filters by eigen-approach. This program can design Nyquist filters, Mth band filters, class 1 partial frequency response filters [6] and higher order differentiators in addition to those McClellan-Parks program can design. It takes the interactive form such that the users give the input data according to the hint, and if the input data is not correct it will give warning and demand another input again. Moreover, the program also allows to get the quantized filter coefficients and the quantized error, if necessary. The simplicity and generality of this eigen-design algorithm make this program useful for a wide variety of design applications.

DESIGN ALGORITHM

A typical FIR digital filter is characterized by a transfer function $H(Z)$, i.e.

$$H(Z) = \sum_{n=0}^{N-1} h(n)Z^{-n} \quad (1)$$

where N is the length of the filter impulse response sequence $\{h(n)\}$, its frequency response is presented by [1]

$$H(e^{j\omega}) = H'(w)e^{j[\frac{L}{2} - (\frac{N-1}{2})w]} \quad (2)$$

where $H'(w)$ is a real valued function

which is the value of the amplitude response, and L is equal to 0 or 1. The least-squares approach to this filter design problem is to formulate an object function

$$E = \frac{1}{2} \int_0^1 [D(w) - H'(w)]^2 dw \quad (3)$$

where $D(w)$ is the desired frequency response we wish to approximate and c represents the band region $0 \leq w \leq \pi$, but excluding the transition band. Now we wish to reformulate Eq.(3) such that it has the following form

$$E = A^t Q A \quad (4)$$

where t is the vector transpose operation, A is a column vector containing the coefficients of $H'(w)$, and Q is a real, symmetric and positive-definite matrix depending upon the design requirement. Once matrix Q is found, the eigenvector corresponding to the smallest eigenvalue of matrix Q is the desired solution. Due to the limited space, the design detail of multi-band filters, Nyquist filters, Hilbert transforms and differentiators can be referred to the References 3,4,5.

PROGRAM DESCRIPTION

This program is written in interactive form such that the users only input data according to the program's instruction, sometimes it will give the users the warning statements to revise the input data set; The eigen-filter design algorithm consists of an input section, generation of matrix Q for eigenformulation, compute the eigenvector A corresponding to the smallest eigenvalue of matrix Q , obtain the filter impulse response $h(n)$ from eigenvector A , and program output the filter's coefficients with/without quantization, the four subroutines QG1, QG2, QG3 and QG4 are used separately to calculate the elements $q(n,m)$ of matrix Q according to the chosen type of filter.

The input which describes the filter specifications consists of the following.

- 1) The type of the filter (ITYPE):
 - a) Multiple passband/stopband (ITYPE=1): Subroutine QG1.
 - b) Nyquist filter (ITYPE=2), Mth-band filter (ITYPE=3) and Class 1 partial response filter (ITYPE=4): Subroutine QG2.
 - c) Hilbert transformer (ITYPE=5): Subroutine QG3.
 - d) Low/High order differentiator (ITYPE=6): Subroutine QG4.
- 2) The filter length: $3 \leq NL \leq 128$.
- 3) The frequency bands, specified by upper and lower cutoff frequency (EDGE array) up to a maximum of 10 frequency bands: $1 \leq IBAND \leq 10$.
- 4) The desired frequency response (DES array) in each band.

- 5) A positive weight constant (WEI array) in each band.
- 6) The intersymbol duration needs to be specified first for the cases of Nyquist, M-th band and Class 1 partial response filters: ITYPE = 2,3,4 and $2 \leq MIN$.

The program output will list the filter's impulse response coefficients (H array, 32 bit precision on VAX 11/780 computer), also it includes the option to quantize the impulse response with fewer number of bits ($4 \leq IBIT \leq 16$: including sign bit) and prints out the quantized impulse response coefficients (HE array), the quantization error of the filters coefficient is stored in E array for the user's reference ($E(\cdot) = HE(\cdot) - H(\cdot)$).

DESIGN EXAMPLES

This section presents some specific examples designed by this package; In Fig.1, we show a three-passband filter with length $N=57$ and 3 different filter gain passband responses. Fig.2 is a FIR Nyquist filter with length $N=39$, $f_p=0.1$ and $M=4$, where M is the intersymbol duration, in this figure we also plotted its impulse response in which every other 4th filter coefficient except the central one is zero due to its time constraints.

Fig.3(a) shows a length $N=32$, full-band Hilbert transformer with cutoff frequencies $f_{p1}=0.04$, $f_{p2}=0.5$, in this example we compare the error curves of the eigen-approach (solid line) and McClellan-Parks algorithm (dotted line) in Fig.3(b), the passband ripple is much smaller in the eigen-approach than that in McClellan-Parks algorithm.

A fifth-order full-band differentiator with length $N=32$ is designed, compare Fig.4(a) with Fig.4(b) the performance of the eigen-method is much smoother and less deviation than the McClellan-Parks algorithm.

For the computation of the elements $q(n,m)$ in matrix Q , since Q is a real, symmetric and positive-definite matrix, i.e. $q(n,m)=q(m,n)$, then only half the elements of $q(n,m)$ need to be calculated for computation savings. The design time of the eigenfilter approach is mostly spent in the computation of the smallest eigenvector, however this computation can be done efficiently. The design time for the eigenfilter approach is comparable to that of the McClellan-Parks algorithms for filter length less than 100. By extensive computer simulations, we should avoid to choose the reference frequency w_0 at the band edge in our eigenfilter design [4]. A better choice is to choose the reference frequency at the center of

the band [4], which is being automatically set in our eigenfilter design package.

CONCLUSIONS

This paper presents a computer program for designing linear phase FIR digital filters by the eigen-approach, it's capable of designing a wide variety of digital filters such as multi-band filters (including one band, low-pass, high-pass, band-pass and band-stop filters), transmission filters (such as Nyquist filters, Mth band filters and Class 1 partial response filters), Hilbert transformers and low/high order differentiators. The simplicity and generality of this eigen-design algorithm make this program useful for a lot of design applications.

REFERENCES

- [1] J.H. McClellan, T.W. Parks and L.R. Rabiner, "A computer program for designing optimum FIR linear phase digital filters", IEEE Trans. Audio Electroacoust, Vol.AU-21, pp.506-526, Dec. 1973.
- [2] K. Nakayama and T. Mizukami, "A new IIR Nyquist filter with zero intersymbol interference and its frequency response approximation," IEEE Trans. Circuits Syst. Vol.CAS-29, pp.23-34, Jan. 1982.
- [3] P.P. Vaidyanathan and T.Q. Nguyen, "Eigenfilters: a new approach to least squares FIR filter design and applications including Nyquist filters", IEEE Trans. Circuits and Systems, Vol.CAS-34, pp.11-23, Jan. 1987.
- [4] S.C. Pei and J.J. Shyu, "Eigenfilter design of higher order digital differentiators", to be published in IEEE Trans. Acoustics, Speech, Signal Proc., April 1989.
- [5] S.C. Pei and J.J. Shyu, "Design of FIR Hilbert transformers and differentiators by eigenfilter," to be published in IEEE Trans. Circuits and Systems, PP. 1457-1461, Nov. 1988.
- [6] J.K. Liang, R.J.P. DeFigueiredo and F.C. Lu, "Design of optimal Nyquist, partial response, Nth band and non-uniform tap spacing FIR digital filters using linear programming techniques", IEEE Trans. Circuits and Systems, Vol.CAS-32, No.4, pp.386-392, April 1985.

Fig.1 Magnitude response for an N=57 three-passband filter with different gain in each passband.

Fig.2(a) The impulse response and (b) Log magnitude response for an N=39 Nyquist filter with $f_p=0.1$ and $M=4$.

Fig.3(a) Magnitude response for an N=32 full-band Hilbert transformer.

Fig.4(a) Magnitude response for an N=32 full-band 5th order differentiator by eigen-approach.

Fig.3(b) The error curves for an N=32 full-band Hilbert transformer (Solid line: Eigen-approach, dotted line: McClellan-Parks algorithm).

Fig.4(b) Magnitude response for an N=32 full-band 5th order differentiator by McClellan-Parks algorithm.