INCOHERENT MICROWAVE IMAGING USING FREQUENCY DIVERSITY TECHNIQUE

Tah-Hsiung Chu and We-Kung Deng
Electrical Engineering Department
National Taiwan University
Taipei, Taiwan, Republic of China

I. Introduction

The use of Van Cittert-Zernike theorem to reconstruct the image of a three-dimensional spatially incoherent source from the cross-spectral density function observed in the far field has been described by Carter and Wolf [1]. In this paper the analytical and numerical results of microwave tomographic image reconstruction of a three-dimensional incoherent source using frequency diversity technique and projection-slice theorem [2] are presented.

II. Theoretical analysis

The incoherent microwave imaging system has found applications in stellar interferometry and radio astronomy. The aperture synthesized technique used in the interferometer is usually performed either directly by physically moving the receiving elements or indirectly by earth rotation, and hence is time consuming. The frequency diversity technique has been demonstrated as a cost-effective tool in active coherent microwave imaging system [3]. The analysis given in this section is to derive formulation of aperture synthesis using frequency diversity technique in an incoherent microwave imaging system.

Considering a 3-D spatially incoherent source $i_P(\vec{r}', f)$, shown in Fig.1, the far field cross-spectral density function is given as [1]

$$W(\vec{r}_1, \vec{r}_2, f) = \frac{e^{j2\pi f(r_1 - r_2)/c}}{r_1 r_2} \int i_P(\vec{r}', f) e^{-j2\pi f(\hat{r}_1 - \hat{r}_2) \cdot \vec{r}'/c} d^3 r' \quad (1)$$

and this is known as the three-dimensional Van Cittert-Zernike theorem. In eq.(1), f is the frequency of radiation selectively received at observation points P1 and P2 through two synchronized tunable narrow-band filters, and $f = f_c + \Delta f$ where f_c and Δf are the center frequency and bandwidth of the filter used. Fourier inversion of eq.(1) and assuming $\Delta f \ll f_c$, the left side of eq.(1) becomes the mutual coherence function

$$\Gamma(\vec{r}_1, \vec{r}_2, f) = e^{j2\pi f_c \tau} \Gamma(\vec{r}_1, \vec{r}_2, 0) \quad (2)$$

whereas the right side is found to be

$$\frac{e^{j2\pi f_c [(r_1 - r_2)/c + \tau]}}{r_1 r_2} \iint i_P(\vec{r}', f) d^3 r' e^{-j2\pi f_c (\hat{r}_1 - \hat{r}_2) \cdot \vec{r}'/c} \quad (3)$$

by assuming $2\pi\Delta f(r_1-r_2)/c \ll 1$ and $2\pi\Delta f\bar{r}_1' \cdot (\hat{r}_1 - \hat{r}_2)/c \ll 1$.
Therefore

$$\Gamma(\bar{r}_1, \bar{r}_2, 0) = \frac{e^{j2\pi\bar{k}(r_1-r_2)/c}}{r_1 r_2} \int I(\bar{r}_1') e^{-j2\pi\bar{k}(\hat{r}_1 - \hat{r}_2) \cdot \bar{r}_1'/c} d^3\bar{r}_1' \quad (4)$$

where $I(\bar{r}_1') = \int i_P(\bar{r}_1', f) df$ is the total intensity of the incoherent source. By defining $\bar{k} = 2\pi f_c (\hat{r}_1 - \hat{r}_2)/c$, eq.(4) becomes

$$\Gamma(\bar{r}_1, \bar{r}_2, 0) = \frac{e^{j2\pi\bar{k}(r_1-r_2)/c}}{r_1 r_2} \int I(\bar{r}_1') e^{-j\bar{k} \cdot \bar{r}_1'} d^3\bar{r}_1' \quad (5)$$

Equation (5) shows that the range dependent phase term vanishes as locations of P1 and P2 are on a spherical surface with the incoherent source at the center of coordinate, and a line in the $\bar{k}$ -space along $\hat{r}_1 - \hat{r}_2$ direction can be synthesized by recording the far field mutual coherence function as the frequency selected by filters stepped from f_1 to f_2 (or k stepped from k_1 to k_2) as illustrated in Fig.2. Therefore, with P1 fixed and P2 scanned within region A on the spherical surface, a cylindrical volume in the $\bar{k}$ -space can be synthesized by properly choosing the stepped frequency range f_1 and f_2 for each P2 location.

With $\bar{k}$ -space volume available, tomographic reconstruction of the source intensity $I(\bar{r}_1')$ can then be obtained at selected $z=z_0$ plane based on the projection-slice theorem as

$$I(x', y', z'=z_0) = \iint \Gamma(\bar{r}_1, \bar{r}_2, 0) e^{j\bar{k}_z z_0} d\bar{k}_x d\bar{k}_y e^{j(\bar{k}_x x' + \bar{k}_y y')} d\bar{k}_x d\bar{k}_y \quad (6)$$

where $\bar{r}_1' = x'\hat{x} + y'\hat{y} + z'\hat{z}$, and $\bar{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$ are the position vectors used in eq.(5).

III. Numerical results

Figure 3 is a 3-D incoherent source used in the simulation study consisting of three discrete points located at $z=0m$ and $z=1m$ planes. Mutual coherence function at observation points P1 and P2 is then calculated with P1 fixed at $z=10m$ and P2 scanned within the region A confined between $z_2=4.7m$ and $z_4=9.7m$ as shown in Fig.2(a). By properly choosing the filter stepped frequency in the range (1.7-9.75)GHz, a cylindrical volume in the $\bar{k}$ -space can be synthesized within the region $\sqrt{k_x^2 + k_y^2} : (8\pi \sim 16\pi) \text{rad/m}$, and $k_z : (-6\pi \sim -2\pi) \text{rad/m}$ as shown in Fig.2(b).

Figures 4(a) and 5(a) are the projections of the resulting mutual coherence function in the $\bar{k}$ -space with $z_0=0m$ and $1m$ respectively. The tomographic images obtained from 2-D Fourier inversion are given in Fig.4(b) and 5(b). They are shown in good agreement with the geometry of the simulated incoherent source.

The results presented here demonstrate that a large $\bar{k}$ -space volume of the far field mutual coherence function

can be synthesized by an incoherent microwave imaging system using frequency diversity technique, and the tomographic image of a 3-D incoherent source can then be reconstructed based on the projection-slice theorem.

Reference

- [1] W.H.Carter, E.Wolf, "Correlation theory of wavefields generated by fluctuating, three-dimensional, primary, scalar sources", Optic Acta, vol.28, pp.227-244, 1981.
- [2] G.T.Herman, Image Reconstruction from Projections, Academic Press, 1980.
- [3] N.H.Farhat, T.H.Chu et al, "Microwave diversity imaging with near optical resolution", 1986 IEEE AP/URSI Symposium, Phila. June 1986.

Fig.1 Geometry of the microwave imaging system.

Fig.2 (a) Arrangement of the incoherent microwave imaging system using frequency diversity technique and (b) the accessed k-space volume.

Fig.3 Geometry of the simulated discrete 3-D incoherent source.

Fig.4 Simulation results of (a) projection of the mutual coherence function and (b) its reconstructed image at $z=0\text{m}$ plane.

Fig.5 Simulation results of (a) projection of the mutual coherence function and (b) its reconstructed image at $z=1\text{m}$ plane.