ADAPTIVE BEAMFORMING USING CYCLIC SIGNALS IN THE PRESENCE OF CYCLE FREQUENCY ERROR*

Ju-Hong Lee[†] and Yung-Ting Lee

Room 517, 2nd Building, Department of Electrical Engineering
National Taiwan University Taipei, Taiwan, R.O.C.

Abstract

A well-known property exhibited by most man-made signals is that they generate spectral lines when they are passed through a nonlinear transform[1]. For instance, BPSK signal generates spectral lines at multiples of the symbol rate and the twice of the carrier frequency due to cyclostationary property. The Self-COherent REstoral (SCORE) algorithms presented by [2] have shown to give the optimal solution for the signal extraction problem using cyclostationary statistics. However, they suffer performance degradation in the presence of cycle frequency error. This paper considers the behavior of the SCORE algorithms in the presence of cycle frequency error. Based on the theoretical analysis, we present an efficient method to alleviate the performance degradation. Simulation results are presented for showing the effectiveness of the proposed method.

I. The SCORE Algorithms

Consider an adaptive beamforming using an antenna array excited by a SOI (Signal Of Interest), J interferers, and background noise. If the

inverse bandwidth of the receiver is small with respect to the electrical distance between the array elements, then the received data vector $\mathbf{x}(t)$ is given by

$$\begin{aligned}\mathbf{x}(t) &= s(t)\mathbf{S}_d + \sum_{j=1}^J s_j(t)\mathbf{S}_j + \mathbf{n}(t) \\ &= s(t)\mathbf{S}_d + \mathbf{i}(t)\end{aligned}\quad (1)$$

where $\mathbf{S}_d$ and $\mathbf{S}_j$ denote the SOI and interference aperture vectors, respectively. Assume that $s(t)$ is self-coherent at α which is one of the cycle frequencies of $s(t)$, and that $\mathbf{i}(t)$ is not self-coherent at α and is temporally uncorrelated with $s(t)$. The array output is given by

$$y(t) = \mathbf{W}^H \mathbf{x}(t) \quad (2)$$

From (1), we note that the extraction of the SOI can be accomplished by constructing a suitable cost function on $y(t)$ and optimizing it with respect to the weight vector $\mathbf{W}$. In the LS-SCORE algorithm, the following cost function is used[2].

$$F_{sc}(\mathbf{W}; \mathbf{c}) = \langle |y(t) - r(t)|^2 \rangle_T \quad (3)$$

where the reference signal $r(t)$ is given by

$$r(t) = \mathbf{c}^H \mathbf{x}^{(*)}(t - \tau) e^{j2\pi\alpha t} \quad (4)$$

and $\langle \cdot \rangle_T$ denotes the time average over $[0, T]$.

The solution of minimizing (3) is given by

$$\mathbf{W}_{sc} = \hat{\mathbf{R}}_{xx}^{-1} \hat{\mathbf{R}}_{xr}(\alpha) \quad (5)$$

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[†]Author for All Correspondence.

where $\hat{\mathbf{R}}_{xx}$ and $\hat{\mathbf{R}}_{xr}(\alpha)$ are the sample autocorrelation matrix of $\mathbf{x}(t)$ and the cross-correlation vector of $\mathbf{x}(t)$ and $r(t)$ computed over $[0, T]$, respectively. It is shown in [2] that (5) converges to the maximum-SINR solution when T approaches infinite for any vector of $\mathbf{c}$ as long as $\mathbf{c}^H \mathbf{S}_d \neq 0$. Due to the fact that $r(t)$ contains the interference, the LS-SCORE algorithm converges slowly. To alleviate this difficulty, Gardner [2] also proposed to find the optimal weight vector $\mathbf{W}_{opt}$ and the optimal control vector $\mathbf{c}_{opt}$ by maximizing the correlation coefficient between $y(t)$ and $r(t)$.

$$\hat{F}_{sc}(\mathbf{W}; \mathbf{c}) = \frac{\mathbf{W}^H \hat{\mathbf{R}}_{xu} \mathbf{c}}{(\mathbf{W}^H \hat{\mathbf{R}}_{xx} \mathbf{W})(\mathbf{c}^H \hat{\mathbf{R}}_{uu} \mathbf{c})} \quad (6)$$

where $\mathbf{u}(t) = \mathbf{x}^{(*)}(t - \tau)e^{j2\pi\alpha t}$ is the control signal. The solutions of maximizing (6) are given by

$$\begin{aligned} \mathbf{W}_{opt} &\propto \hat{\mathbf{R}}_{xx}^{-1} \hat{\mathbf{R}}_{xu} \mathbf{c}_{opt} = \hat{\mathbf{R}}_{xx}^{-1} \hat{\mathbf{R}}_{xr} \\ \mathbf{c}_{opt} &\propto \hat{\mathbf{R}}_{uu}^{-1} \hat{\mathbf{R}}_{ux} \mathbf{W}_{opt} = \hat{\mathbf{R}}_{uu}^{-1} \hat{\mathbf{R}}_{uy} \end{aligned} \quad (7)$$

where $\hat{\mathbf{R}}_{xr} = \hat{\mathbf{R}}_{xu} \mathbf{c}_{opt}$ and $\hat{\mathbf{R}}_{xy} = \hat{\mathbf{R}}_{ux} \mathbf{W}_{opt}$. It is shown [2] that $\mathbf{W}_{opt}$ also maximizes the output SINR as $T \rightarrow \infty$. Simulation results show that the performance of Cross-SCORE is much better than LS-SCORE, especially in the presence of strong interference.

II. Performance Analysis

From (1), we obtain (by letting $f = \alpha$)

$$\begin{aligned} \mathbf{R}_{xx}^f &= \langle \mathbf{x}(t + \frac{\tau}{2}) \mathbf{x}^H(t - \frac{\tau}{2}) e^{-j2\pi f t} \rangle_{\infty} \\ &= \mathbf{R}_d^f \mathbf{S}_d \mathbf{S}_d^H + \sum_{j=1}^J \mathbf{R}_{s_j}^f \mathbf{S}_j \mathbf{S}_j^H + \mathbf{R}_n^f \end{aligned} \quad (8)$$

where

$$\mathbf{R}_a^f = \langle a(t + \frac{\tau}{2}) a^*(t - \frac{\tau}{2}) e^{-j2\pi f t} \rangle_{\infty} \quad (9)$$

From the property of Fourier transform, the sample cross-correlation vector can be represented by

$$\hat{\mathbf{R}}_{xr}(f) = \mathbf{R}_{xr}(f) \otimes \text{SINC}(fT) \quad (10)$$

where $\otimes$ denotes the convolution operation and $\text{SINC}(fT) = \frac{\sin(\pi f T)}{(\pi f T)}$. Moreover, due to the fact that cycle frequencies are discrete, the cyclic autocorrelation function of the SOI can be written as

$$R_s^f(\tau) = \sum_n d_n(\tau) \delta(f - \alpha_n) \quad (11)$$

From (8), the cross-correlation vector can be expressed by

$$\begin{aligned} \mathbf{R}_{xr}(f) &= \mathbf{R}_{xx}^f \mathbf{c} e^{-j\pi f \tau} \\ &= k_d R_d^f \mathbf{S}_d + \sum_{j=1}^J k_j R_{s_j}^f \mathbf{S}_j + \mathbf{k}_n \end{aligned} \quad (12)$$

where $k_d = \mathbf{S}_d^H \mathbf{c} e^{-j\pi f \tau}$, $k_j = \mathbf{S}_j^H \mathbf{c} e^{-j\pi f \tau}$ and $\mathbf{k}_n = \mathbf{R}_n^f(\tau) \mathbf{c} e^{-j\pi f \tau}$. Substituting (12) into (10) yields (Actually, $\hat{\mathbf{R}}_{xr}(f)$ includes the cyclic cross correlations between the SOI, interference, and noise. However, they are negligible due to large T .)

$$\begin{aligned} \hat{\mathbf{R}}_{xr}(f) &= k_d \left\{ \sum_n d_n(\tau) \text{SINC}((f - \alpha_n)T) \right\} \mathbf{S}_d \\ &+ \sum_{j,m} k_j \{ \rho_{j,m}(\tau) \text{SINC}((f - \beta_{j,m})T) \} \mathbf{S}_j \\ &+ \hat{\mathbf{k}}_{nn} \otimes \text{SINC}(fT) \end{aligned} \quad (13)$$

where non-cyclostationary interference and noise are contained in $\hat{\mathbf{k}}_{nn}$. Without loss of generality, we set the known cycle frequency $\alpha = \alpha_1$. If α and $\beta_{j,m}$ are well separated and due to the fact that the value of $\text{SINC}(fT)$ is small for large fT then the effect of interference and noise in (13) is small. When $f = \alpha + \Delta\alpha$, where $\Delta\alpha$ is the amount of deviation, (13) can be written by

$$\hat{\mathbf{R}}_{xr}(f) \approx k_d \{ d_1(\tau) \text{SINC}(\Delta\alpha T) \}$$

$$\begin{aligned}
& + \sum_{n \neq 1} d_n(\tau) \text{SINC}(\tilde{\alpha}_n T) \mathbf{S}_d \\
& + \sum_{j,m} k_j \{\rho_{j,m}(\tau) \text{SINC}(\tilde{\beta}_{j,m} T)\} \mathbf{S}_j \\
& + \tilde{\mathbf{k}}_n \otimes \text{SINC}((\alpha + \Delta\alpha)T) \quad (14)
\end{aligned}$$

where $\tilde{\alpha}_n = \alpha - \alpha_n + \Delta\alpha$ and $\tilde{\beta}_{j,m} = \alpha - \beta_{j,m} + \Delta\alpha$. Due to the fact that $\text{SINC}(\Delta\alpha T) = 0$ when $T = T_n = \frac{\pi n}{\Delta\alpha}$, $n = 1, 2, \dots$, $\hat{\mathbf{R}}_{xr}(f) \cong k_d d_1(\tau) \text{SINC}(\Delta\alpha T) \mathbf{S}_d$ as T is far from T_n . However, when T is near to T_n , the components of $\mathbf{S}_j$ in (14) are not negligible and thus the performance of the SCORE algorithms becomes poor. From (7), we note that the difference between LS-SCORE and Cross-SCORE is that Cross-SCORE adjusts adaptively the control vector $\mathbf{c}$ and thus the behavior of the Cross-SCORE algorithm is similar to the LS-SCORE algorithm when cycle frequency is error.

III. An Efficient Method

Consider the following new reference signal in the LS-SCORE algorithm

$$\tilde{r}(t) = \{\gamma + \text{SINC}(\kappa t)\} c^H \mathbf{x}^{(*)}(t - \tau) e^{j2\pi\omega t} \quad (15)$$

where both γ and κ are two constants. Accordingly, we also obtain the new control signal $\tilde{u}(t) = \{\gamma + \text{SINC}(\kappa t)\} \mathbf{x}^{(*)}(t - \tau) e^{j2\pi\omega t}$ in the Cross-SCORE algorithm. From (15), we obtain

$$\begin{aligned}
\mathbf{R}_{x\tilde{r}}(f) &= \gamma \mathbf{R}_{xr}(f) + \frac{1}{\kappa} \text{rect}\left(\frac{f}{\kappa}\right) \otimes \mathbf{R}_{xr}(f) \\
&= k_d \left(\gamma R_s^f + \frac{1}{\kappa} \text{rect}\left(\frac{f}{\kappa}\right) \otimes R_s^f \right) \mathbf{S}_d \\
&+ \sum_{j=1}^J k_j \left(\gamma R_{s_j}^f + \frac{1}{\kappa} \text{rect}\left(\frac{f}{\kappa}\right) \otimes R_{s_j}^f \right) \mathbf{S}_j \\
&+ \left(\gamma \mathbf{k}_n + \frac{1}{\kappa} \text{rect}\left(\frac{f}{\kappa}\right) \otimes \mathbf{k}_n \right) \quad (16)
\end{aligned}$$

where $\text{rect}(\frac{f}{\kappa})$ equals 1 when $-\frac{\kappa}{2} \leq f \leq \frac{\kappa}{2}$ and equals 0, elsewhere. The corresponding sample

cross-correlation vector when $f = \alpha + \Delta\alpha$ is also given by

$$\begin{aligned}
\hat{\mathbf{R}}_{x\tilde{r}}(f) &= k_d \{d_1(\tau) \Omega(\Delta\alpha T) \\
&+ \sum_{n \neq 1} d_n(\tau) \Omega(\tilde{\alpha}_n T)\} \mathbf{S}_d \\
&+ \sum_{j,m} k_j \{\rho_{j,m}(\tau) \Omega(\tilde{\beta}_{j,m} T)\} \mathbf{S}_j \\
&+ \mathbf{k}_n \otimes \Omega((\alpha + \Delta\alpha)T) \quad (17)
\end{aligned}$$

where $\Omega(fT) = \gamma \text{SINC}(fT) + \frac{1}{\kappa} \text{rect}(\frac{f}{\kappa}) \otimes \text{SINC}(fT)$. Under the conditions that $\Delta\alpha = 0$ and $\Delta\alpha \neq 0$, $\mathbf{R}_{x\tilde{r}}(f)$ is proportional to $\mathbf{S}_d$. The result is due to the fact that from (16), $\text{rect}(\frac{f}{\kappa}) \otimes R_s^f = \sum d_n(\tau) \text{rect}(\frac{f - \alpha_n}{\kappa})$ is not equal to 0 as long as $\Delta\alpha \leq \frac{\kappa}{2}$. Moreover, owing to $\text{rect}(\frac{\Delta\alpha}{\kappa}) \otimes \text{SINC}(\Delta\alpha T)$ and $\text{SINC}(\Delta\alpha T)$ in (17) are not equal to 0 at the same time, $\hat{\mathbf{R}}_{x\tilde{r}}$ is not seriously effected by $\mathbf{S}_j$ when T is near to $T_n = \frac{\pi n}{\Delta\alpha}$. As a result, using $\tilde{r}(t)$ in the LS-SCORE algorithm and $\tilde{u}(t)$ in the Cross-SCORE algorithm make these algorithms robust against the effect of cycle frequency error. We note that $\text{rect}(\frac{f}{\kappa}) \otimes \text{SINC}(\tilde{\beta}_{j,m} T)$ in (17) enhances the components of $\mathbf{S}_j$ in $\hat{\mathbf{R}}_{x\tilde{r}}(f)$ and leads to performance degradation. However, choosing larger γ can alleviate this difficulty. Furthermore, the components of $\mathbf{S}_j$ in (17) become larger as κ or γ increases. Therefore, by choosing the values of γ and κ adequately, we can alleviate the effect of cycle frequency error on the performance of the SCORE algorithms.

IV. Simulation and Observation

Consider an adaptive beamformer using a 21 uniform linear array with interelement spacing = half wavelength of the desired signal. All of the inputs are BPSK. The carrier frequency of the desired signal is set to 1. The noise is spatially white. Moreover, we set $\gamma = 0.3$, $\kappa = 0.02$,

and the sampling interval $T_s = 0.2$. Two cases in which J equals 0 and 2 are considered. For each case, $f = 2$ and $f = 2.01$ (i.e., cycle frequency error = 0.01) are used. The vector $\mathbf{c}$ in the LS-Score algorithm is fixed and given by $\mathbf{c} = [1, 0, 0, \dots, 0]^T$. From (14), the coefficient of S_d is proportional to $\text{SINC}(\Delta\alpha T)$ when $J = 0$. Owing to $\text{SINC}(\Delta\alpha T) = 0$ when $T = \frac{4\pi n}{\Delta\alpha}$, $n = 1, 2, \dots$, there are nulls at about $500n$ snapshots which are shown in Figure 1(b) when using the LS-Score and Cross-Score algorithms. In the case of $J = 2$, nulls still appear at about $500n$ snapshots which are shown in Figure 2(b). For both cases, the proposed method improves the performance of the Score algorithms in the presence of cycle frequency error. Moreover, we note that an acceptable performance can be achieved by choosing appropriate γ and κ .

References

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Fig. 1: $J=0$. (a): $f=2$, (b): $f=2.01$.

'-' line: LS-Score using $\hat{r}(t)$. 'x' line: Cross-Score using $\hat{u}(t)$.
'.' line: LS-Score. '+' line: Cross-Score.

Fig. 2: $J=2$. (a): $f=2$, (b): $f=2.01$.

'-' line: LS-Score using $\hat{r}(t)$. 'x' line: Cross-Score using $\hat{u}(t)$.
'.' line: LS-Score. '+' line: Cross-Score.