A NONCOHERENT SEQUENTIAL PN CODE ACQUISITION SCHEME IN DS/SS RECEIVERS

Jia-Chin Lin and Lin-Shan Lee

Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.

Abstract—A noncoherent sequential PN code acquisition technique is proposed in this paper. Noncoherent detection is employed to handle the seriously noisy environments, while the carrier frequency offset and data modulation effects can be simultaneously taken care of. To realize the sequential acquisition, the out-of-phase and in-phase sequences are properly modeled to avoid the significantly high error probabilities caused by the conventionally widely used zero sequence model. Extensive computer simulation results indicated that the proposed PN code acquisition outperforms the fixed-dwell-time counterparts by roughly 2–6 dB with much lower false alarm and miss probabilities, and the superiority of the proposed technique becomes more significant at lower SNR conditions.

I. INTRODUCTION

Substantial efforts have been made in the code acquisition problem [1], [2], [3], [4], [5], but most of the analyses for the mean acquisition time were only in the context of fixed-sample-size (FSS, or fixed-dwell-time) techniques, in which the correlation duration is fixed. Although the FSS PN acquisition techniques are the simplest and the most well studied, it is inefficient in terms of the acquisition time, since it always takes a long enough fixed correlation duration before the rejection or acceptance of the synchronization state can be decided. Because the code acquisition usually takes very long time, it is highly desired to speed up the code acquisition processes for efficient DS/SS communications. The removal of the FSS requirement is therefore very attractive. The PN code acquisition in DS/SS can be considered as a detection problem testing between two simple hypotheses, i.e., the received and locally generated PN sequences are aligned in phase or not. It is well-known that the sequential probability ratio test (SPRT) [6] is very efficient in obtaining a decision between two

simple hypotheses in the sense that it requires the minimum average detection time for given error probabilities (i.e., false alarm/miss probabilities) [7]. By modeling the acquisition problem as testing between two hypotheses, several sequential test techniques under coherent demodulation environments were proposed and analyzed [8], [9] using different out-of-phase sequence models. However, it is practically almost impossible to achieve coherent demodulation, because the signal-to-noise ratio (SNR) before despreading is very low. Several sequential detectors or sequential probability ratio tests (SPRT) designed for PN code acquisition under noncoherent demodulation environments have also been discussed [1], [6], [10], [11], but under the assumption that the out-of-phase sequence could be modeled as a zero sequence. However, such assumption may not be very practical as well, because it is almost impossible to design a front-end filter with narrow enough bandwidth on the one hand to reject completely the residual cross-correlation between the incoming and locally generated PN sequences under the out-of-phase condition so that the out-of-phase sequence vanishes; but with wide enough bandwidth on the other hand to accommodate the data modulation and frequency offset effects so that the in-phase sequence can be kept from any damage caused by the data modulation and frequency offset effects. As a result, the upper bound of the cross-correlation between any two non-synchronized PN sequences was employed in some analyses to model the out-of-phase sequence to avoid any performance degradation occurring with the conventional SPRT based on the zero sequence model [8], [9].

In this paper, an SPRT-based acquisition technique is derived with the upper bound of the cross-correlation sequence as the out-of-phase sequence model, with the modified in-phase sequence and under noncoherent demodulation en-

vironments. In addition, the data modulation and carrier frequency offset effects can be simultaneously coped with. A sliding correlator is employed in the search of the acquisition process, and so the mean acquisition time is proportional to the average number of the received PN code chips required for each synchronization test. Simulation results show that the proposed technique outperforms the FSS counterparts by roughly 2 – 6 dB in terms of SNR in order to achieve given error probabilities. Furthermore, such superiority even improves with lower error probabilities and/or lower SNR's.

II. CONVENTIONAL SEQUENTIAL ACQUISITION TECHNIQUES

In a M-PSK DS/SS communication system, the information-bearing signal can be expressed as

$$d(t) = \sum_{i=-\infty}^{\infty} d_i \Pi\left(\frac{t-iT}{T}\right) \quad (1)$$

where d_i is the i -th M-PSK data symbol, T is the symbol duration and the above signal $d(t)$ is then spread by a PN waveform,

$$p(t) = \sum_{j=-\infty}^{\infty} c_j \Pi\left(\frac{t-jT_c}{T_c}\right), \quad (2)$$

where $c_j \in \{-1, 1\}$ is the j -th PN code value, $c_j = c_{j+N}$ for all j , N is the period of the PN sequence, and T_c is the code chip duration. Without loss of generality, here we assume that the processing gain is the PN code period, N . The resulting signal is then used to modulate a carrier, which is further corrupted by AWGN. The complex representation of the signal received at the front-end of the acquisition receiver is

$$r(t) = \sqrt{\frac{E_c}{T_c}} d(t - \tau T_c) p(t - \tau T_c) e^{j(\omega_c t + \theta(t))} + n'(t), \quad (3)$$

where E_c is the received chip energy, τ is the received code phase offset which is assumed to be an integer, $\theta(t)$ is the carrier phase error, $\theta(t) = 2\pi\Delta f t + \theta_0$, introduced by the carrier frequency offset Δf , and $n'(t)$ is a complex white Gaussian noise with two-sided power spectral density $N_0/2$. The baseband representation of the conventional noncoherent acquisition method based

Fig. 1. The baseband representation of the conventional noncoherent sequential acquisition for DSSS communication.

on SPRT is shown in Fig. 1. The received signal is first down-converted by means of a noncoherent local carrier, $e^{j\omega_c t}$, and then passed through a lowpass filter LPF0. To minimize the noise variance, LPF0 has a bandwidth $B = \frac{1}{2T_c}$ (Hz). The resulting signal is then cross-correlated with the local code sequence $p(t - \hat{\tau}T_c)$, where $\hat{\tau}$ is the local PN code phase offset. The cross-correlation signal is passed through a second lowpass filter LPF1, whose bandwidth W (Hz) is assumed to be wide enough to accommodate the data modulation. The output of LPF1 is then fed into an envelope detector. The envelope samples S_k are obtained by sampling the output of the envelope detector at a rate low enough (say, $\frac{1}{mT_c}$) such that the samples can be considered to be independent. Finally, the envelope samples enter the likelihood ratio calculator and the sequential detection logic. The detection of the SPRT is then directly derived from the statistics of the envelope samples of the cross-correlation function filtered by LPF1.

In the above structure, the lowpass filter LPF1 may be very difficult to design. The bandwidth of this filter has to be wide enough to accommodate the frequency offset and data modulation effects, or to prevent the in-phase sequence from any damages caused by such effects. But if its bandwidth is too wide, say $W \approx \frac{1}{2T_c}$, the cross-correlation of the incoming and the local sequences under the out-of-phase condition can not be suppressed at all. This makes it impossible to distinguish the envelopes of the cross-correlation under in-phase and out-of-phase conditions and leads to a significantly high false alarm probability. As a result, the bandwidth W of

this filter has to be narrow enough as well, say $W \ll \frac{1}{2T_c}$, to reject the residual cross-correlation under the out-of-phase conditions, because the likelihood ratio calculator and the sequential detection logic are designed based on the assumption that the out-of-phase sequence can be modeled as a zero sequence here. On the other hand, the SPRT algorithm is derived based on another assumption that the samples entering the likelihood ratio calculator are spaced sufficiently, say $m'T_c$ where $m' \gg 1$, and can be considered independent, but with that assumption the carrier frequency offset and data modulation effects may possibly change the statistics of the in-phase sequence significantly and destroy the functions of SPRT, which may lead to a high probability of miss. Therefore, it may be practically very difficult to actually design and implement the conventional noncoherent sequential acquisition scheme.

III. PROPOSED NONCOHERENT SEQUENTIAL ACQUISITION TECHNIQUE

The noncoherent sequential acquisition method proposed in this paper is shown in Fig. 2. The received signal is also down-converted first by means of a noncoherent local carrier, $e^{j\omega_c t}$, passed through a lowpass filter (LPF0) and then sampled at the chip rate to generate the base-band received sample stream r_k

$$r_k = \sqrt{\frac{E_c}{T_c}} d((k-\tau)T_c) p((k-\tau)T_c) e^{j\theta_k} + n_k, \quad (4)$$

where $\theta_k = 2\pi\Delta f \cdot kT_c + \theta_0$, and n_k is the noise sample. To minimize the noise variance, the bandwidth of the lowpass filter LPF0 is set at $B = \frac{1}{2T_c}$ (Hz), and it can be shown that the noise samples $\{n_k\}$ are independent and identically distributed (i.i.d.) random variables with zero mean and variance $\sigma_0^2 = \frac{N_0}{2T_c}$ on each dimension. For simplicity and without loss of generality, we let $\frac{E_c}{T_c} = 1$. The resulting sequence, $\{r_k\}$, is cross-correlated with the local code sequence $p((k-\hat{\tau})T_c)$, where $\hat{\tau}$ is the local PN code phase offset and it is an integer, and then the samples $\{X'_k\}$ are generated as below,

$$X'_k = Y'_k d((k-\tau)T_c) e^{j\theta_k} + N'_k, \quad (5)$$

Fig. 2. Block diagram of the proposed noncoherent sequential acquisition for direct-sequence spread-spectrum communication.

where $Y'_k = \sqrt{\frac{E_c}{T_c}} p((k-\tau)T_c) p((k-\hat{\tau})T_c)$ and $N'_k = n_k \cdot p((k-\hat{\tau})T_c)$. In the above equation, the sequence $\{Y'_k\}$ provides the desired information to detect whether the received and the locally generated PN sequences are synchronized. It is corrupted by the data modulation effect, $d((k-\tau)T_c)$, and the carrier frequency offset effect, $e^{j\theta_k}$. It can be shown that the noise term N'_k is a complex Gaussian random variable with zero mean and the same variance as n_k .

The cross-correlation samples, X'_k , are integrated/dumped (I/D-ed) for each m (non-overlapped) samples to generate the I/D-ed cross-correlation samples X_i

$$X_i = \sum_{k=im}^{(i+1)m-1} X'_k = Y_i e^{j\theta_0} + N_i, \quad (6)$$

where $Y_i = \sum_{k=im}^{(i+1)m-1} Y'_k e^{j(2\pi\Delta f k T_c)}$ and $N_i = \sum_{k=im}^{(i+1)m-1} N'_k$. The acquisition scheme considered here is a sliding correlator structure with serial search. The I/D-ed cross-correlation samples X_i enter the likelihood ratio calculator one by one, and they are then used to test whether the phase of the local PN sequence is synchronized with that of the received sequence. If they are not synchronized, the phase of the local PN generator is shifted by δT_c and the above test process is re-started and so on until they are synchronized. The step size δ for each phase update can be usually 1, $\frac{1}{2}$, or some other fractional value, depending on the pull-in range of the following tracking subsystem. Here it actually suffices to use $\delta = 1$ in order to avoid any possibility of lost code phase states caused by updating the code phase, since the pull-in range is usually

$T_c/2$. To test the synchronization of the two sequences, assume hypothesis H_0 is the case when the two sequences are not aligned, i.e., $\tau \neq \hat{\tau}$, and hypothesis H_1 is the case when $\tau = \hat{\tau}$. The test statistic, $\lambda(w)$, to be compared with some thresholds to decide between H_0 and H_1 is given by:

$$\lambda(w) = \sum_{i=1}^w h(X_i), \quad (7)$$

where $h(\cdot)$ is some monotonic function, and w is a random variable, called the dwell time and determined by the detection method. The stopping time (or sample number) of a test is defined by

$$W = \min\{w : \lambda(w) \notin (A, B)\}, \quad (8)$$

where A and B are the threshold levels used to decide between H_0 and H_1 . W is a random variable which denotes the minimum number w of the I/D-ed cross-correlation samples, X_i , required to decide between H_0 or H_1 . So, the required chip number to make a decision can be also obtained by mW . By contrast with a fixed-sample-size (FSS) test where any decision is always made after some fixed number of samples have entered the synchronization tester, the sequential probability ratio test (SPRT) exploits an "active" correlator, where the correlation and the following synchronization test are performed sequentially on a "sample-by-sample" basis.

We denote the false alarm probability (i.e., H_1 claimed while H_0 is true) by α and the miss probability (i.e., H_0 claimed while H_1 is true) by $1 - \beta$ (i.e., the detection probability is β). For calculation of the mean acquisition time and for simplicity, it is assumed that the initial phase offset between the received and the local PN sequences is some integer multiple of T_c , and it is uniformly distributed over the range of the whole period (i.e., from 0 to $(N - 1)T_c$). It has been shown that the average acquisition time using a sliding correlator is proportional to the average number of chips, $m\bar{W}$, required for each synchronization test [4], [8]. So, to reduce the code acquisition time is equivalent to reducing the average chip number required for each synchronization test.

Based on the model of the partial correlation of PN sequences [8], the acquisition problem can be considered as the testing of a composite hypothesis

H_0 against a simple alternative H_1 by means of the envelopes of the I/D-ed cross-correlation samples, X_i . So, the desired information sequence, $\{Y_i\}$, in Eq. (6) can be modeled as a constant being either $\hat{Y}^1$ for H_1 or $\hat{Y}^0$ for H_0 , i.e.,

$$Y_i = \begin{cases} \hat{Y}^1 & = f_1, & H_1 & (\tau = \hat{\tau}) \\ \hat{Y}^0 & \leq f_0 \cdot m, & H_0 & (\tau \neq \hat{\tau}), \end{cases} \quad (9)$$

where $f_1 = \|\sum_{k=1}^m e^{j(2\pi\Delta f k T_c)}\|$ is a factor introduced by the carrier frequency offset and it approaches to m as m decreases, and $f_0 = 0.5$ is used here because it corresponds to the worst-case correlation under synchronization (i.e., f_0 is the PN code cross-correlation value computed at the boundaries of the lock range of the code tracking subsystem) for the case whenever the chip timing is unknown and the local PN generator is updated by T_c when each H_0 claimed. Based on the model of Y_i and after some manipulation, the envelope samples, $\|X_i\|$, of the I/D-ed correlation samples can be described as i.i.d. Rician random variables with parameter either $\hat{Y}^1$ for H_1 or $\hat{Y}^0$ for H_0 . The log-likelihood ratio for the i -th envelope sample, $\|X_i\|$, is then

$$Z_i = \frac{(\hat{Y}^0)^2 - (\hat{Y}^1)^2}{2m\sigma_0^2} + \log(I_0(\frac{\hat{Y}^1\|X_i\|}{m\sigma_0^2})) - \log(I_0(\frac{\hat{Y}^0\|X_i\|}{m\sigma_0^2})), \quad (10)$$

and we will consider the sequential test using the statistic below,

$$\lambda(w) = \sum_{i=1}^w Z_i. \quad (11)$$

i.e., $h(X_i)$ in Eq. (7) is taken as Z_i here. The SPRT is then described by:

$$\lambda(w) \begin{cases} \geq B & \Rightarrow H_1 & (\tau = \hat{\tau}) \\ \leq A & \Rightarrow H_0 & (\tau \neq \hat{\tau}) \\ \in (A, B) & \Rightarrow \text{take another sample,} \end{cases} \quad (12)$$

where the threshold levels, A and B , should be chosen so that the test can achieve the false alarm probability α and the miss probability $1 - \beta$ [6]. The sample number W of the test has been defined in Eq. (8), and its mean value has been used for performance evaluation.

IV. SIMULATION RESULTS

Computer simulation results illustrating the performance of the proposed noncoherent sequential PN code acquisition technique are presented in this section. The false alarm probability, miss probability and average chip numbers required for each synchronization test are evaluated by Monte Carlo methods.

For several reasons, the baseband received signal may be affected by a finite carrier frequency offset Δf comparable to the symbol rate $\frac{1}{T_c}$. Δf is in general much smaller than the chip rate $\frac{1}{T_c}$, but may turn out to be of the same order of magnitude as $\frac{1}{T_c}$ in some voice transmission networks and even a few times larger for low-bit-rate applications [13].

Shown in Fig. 3 are the false alarm and miss probabilities of the proposed and conventional sequential acquisition techniques with various values of the frequency offset and LPF1 bandwidth for $\frac{\Delta f}{1/T_c} = 0.025$ in case the desired error probability (false alarm and miss probabilities) is 10^{-2} . The false alarm and miss probabilities with $m' = 4$ for the conventional SPRT and $m = 4$ for the proposed SPRT algorithm are plotted in Fig. 3(1a) & (1b), while those for $m' = 8$ and $m = 8$ are shown in Fig. 3(2a) & (2b). From Fig. 3(1a), the false alarm probabilities of the conventional SPRT are significantly higher than the desired false alarm probability for a wide range of signal-to-noise ratios, because when $m' = 4$ the bandwidth, $W = \frac{1}{2m'T_c}$, of LPF1 is too wide to reject the residual cross-correlation under the out-of-phase conditions, but the out-of-phase sequences were incorrectly modeled as a zero sequence. The miss probabilities in Fig. 3(1b) are also relatively high for some range of signal-to-noise ratios. For a larger m' value or a narrower LPF1 bandwidth, say $m' = 8$ as shown in Fig. 3(2a), the false alarm probability of the conventional SPRT algorithm is reduced, because the narrower bandwidth of LPF1 can reject more residual cross-correlation. However, such narrow bandwidth may lead to the accumulation of the carrier frequency offset effects and change the statistics of the in-phase sequence. This is why much higher miss probabilities were obtained as shown in Fig. 3(2b).

On the other hand, as can be seen from Fig. 3, the false alarm and miss probabilities of the proposed SPRT algorithm are well under or around the desired value of 10^{-2} . It is clear in Fig. 3 that for the conventional SPRT algorithm when m' is smaller, the false alarm probabilities are very high and they decrease as m' increases; while the miss probabilities are very high when m' is larger, and they decrease as m' decreases. These results indicate that no matter how to choose a m' value or the LPF1 bandwidth, either a high false alarm probability or a high miss probability will be inevitable with the conventional sequential acquisition technique. But such a problem is well taken care of in the proposed technique due to proper modeling of the in-phase and out-of-phase sequences.

Finally, the average required chip numbers versus SNR is shown in Fig. 4 in the case of a carrier frequency offset $\frac{\Delta f}{1/T_c} = 10^{-3}$ for different error probabilities (10^{-2} to 10^{-5}) and m values. From Fig. 4, the proposed technique is shown to yield significant improvement (up to roughly 2 – 6 dB) in SNR. Furthermore, such improvement increases as the input SNR decreases. It should be remarked that the optimum threshold setting for FSS must be incorporated in the evaluation of the average required chip numbers for each SNR value, while the thresholds, A & B , of SPRT are not functions of SNR values. In other words, for a given SNR value and a given chip number, there exists an optimal threshold to achieve the desired error probabilities for FSS. This optimal threshold has been employed in all the above simulation results.

V. CONCLUSION

In this paper, a noncoherent sequential PN code acquisition technique is proposed. The out-of-phase sequence has been modeled as the upper bound of the cross-correlation between any two non-synchronized sequences, and the in-phase sequence has been modified by a factor caused by the carrier frequency offset effect to avoid high error probabilities occurring with the conventional SPRT-based acquisition techniques. The proposed technique significantly outperforms its FSS counterparts and simultaneously overcomes the carrier frequency offset and data modulation

effects.

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Fig. 3. The false alarm and miss probabilities when $\frac{\Delta f}{1/T_c} = 0.025$ and desired error probability 10^{-2} : (1a) false alarm probabilities and (1b) miss probabilities for $m' = 4$, (2a) false alarm probabilities and (2b) miss probabilities for $m' = 8$.

Fig. 4. The average required chip numbers for a synchronization test versus SNR when $\frac{\Delta f}{1/T_c} = 10^{-3}$. (a) $\alpha = 1 - \beta = 10^{-2}$, (b) $\alpha = 1 - \beta = 10^{-3}$, (c) $\alpha = 1 - \beta = 10^{-4}$, (d) $\alpha = 1 - \beta = 10^{-5}$.