DISPERSION ANALYSIS OF MICROSTRIP LINES WITH NONRECTANGULAR CROSS-SECTIONS

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ABSTRACT

A volume integral equation formulation incorporated with the Galerkin's method of moments is proposed to compute the conductor loss of multilayered microstrip lines. The effects of the conductor thickness, the strip cross-sectional shape, and the proximity of the ground planes on the dispersion relation are examined. Results show that, due to skin effect, the conductor loss is larger for the case with longer strip cross-sectional contour. It is also found that the conductor loss is less sensitive to the strip thickness as the latter is larger than three skin depths. Moving the ground planes closer to the conductor strip will result in larger loss, as was observed in this work.

I. INTRODUCTION

In high-density, high-speed microwave integrated circuits, the conductor and dielectric losses in microstrip-type interconnects and line structures are of particular concern. As the operating frequency is getting higher, they should be taken into account to accurately predict the electromagnetic behaviors of the circuits. Conventionally, to compute the conductor loss, a perturbation approach has usually been used [1]. The conductor loss is treated as a perturbation in the lossless case, and is evaluated by using the surface resistance and the surface current. Other approaches, such as the integral equation method that is derived using the equivalent surface impedance in the boundary condition [2], or as the approximate method proposed by Wheeler [3] that is based on the incremental inductance rule, are also used to calculate the conductor loss. All these methods assumed that the strip thickness is larger than the skin depth. In practical application, however, the advanced fabrication process may result in conductors that are very thin, so that conventional perturbation method could lose its accuracy in computing the dissipation of the microstrip lines. In [4], Kiang proposed an volume integral

equation formulation using the dyadic Green's function to solve for the dispersion relation of the single and the coupled conductor strips. The microstrips considered in this technique can be very thin and their cross sections can be of arbitrary shape.

In this work, a volume integral equation formulation is employed to analyze the multilayered microstrip lines. The dyadic Green's function for the planar layered media is derived from a multi-sectioned transmission-line network analog [5]. In the formulation, the number of the background dielectric layers is assumed arbitrary, and the dielectrics can be lossy. In addition, the microstrip lines of arbitrary cross-section can reside in different layers. For numerical solution, the method of moments is implemented. To do that, the cross-section of the conductor strip is first divided into small cells. Then, the eigenmode electric fields on the cross section are represented by a set of pulse basis functions. Following Galerkin's procedure, the same set of basis functions is chosen to test the integral equation. The moment method procedure will result in a matrix equation, from which both the phase and attenuation constant are solved. In this research, the multilayered structures with conductor thickness comparable to the skin depth are of primary interest. The dispersion characteristics of conductor strips having rectangular, trapezoidal, and fish-eyed cross-sections are studied. The effects of the conductor thickness and the proximity of the ground planes on the dispersion relation are also examined.

II. THEORY AND ANALYSIS

Consider the configuration of the multilayered structure with conducting strips embedded in layers ℓ and m , as depicted in Fig. 1. Both the dielectrics and the conductors are assumed to be lossy in nature. To solve the problem, the spectral Green's function for the planar multilayered

medium is first developed. Then, assuming uniformity along the propagation direction y for the whole structure, and an $e^{-j\eta y}$ time dependence, the electric fields in layer ℓ can be expressed, in terms of the dyadic Green's function, as

$$\vec{E}(\vec{r}) = \int_{V_\ell} dV' \vec{G}(\vec{r} | \vec{r}') \cdot \vec{J}_{eq}(\vec{r}'), \quad (1)$$

or

$$\begin{aligned} \vec{E}(\vec{r}) &= \int_{V_\ell} dV' \left[\vec{G}^{PV}(\vec{r} | \vec{r}') + \frac{j}{\omega \epsilon_0 \epsilon_{r\ell}} \delta(\vec{r} - \vec{r}') \hat{z} \hat{z} \right] \cdot \vec{J}_{eq}(\vec{r}') \\ &= \hat{z} \frac{j}{\omega \epsilon_0 \epsilon_{r\ell}} \Delta \sigma(\vec{r}) \vec{E}_z(\vec{r}) + \int_{V_\ell} dV' \vec{G}^{PV}(\vec{r} | \vec{r}') \cdot \vec{J}_{eq}(\vec{r}') \end{aligned} \quad (2)$$

where $\epsilon_{r\ell} = \epsilon'_{r\ell} - j(\sigma_\ell / \omega \epsilon_0)$ is the complex-valued relative permittivity of layer ℓ , $\vec{G}^{PV}(\vec{r} | \vec{r}')$ is the principal-value part of the dyadic Green's function, and $\frac{j}{\omega \epsilon_0 \epsilon_{r\ell}} \delta(\vec{r} - \vec{r}') \hat{z} \hat{z}$ is the source dyadic. Equation (2) is the needed volume integral equation, in which V_ℓ is the volume occupied by the conductors. Define the Fourier transform pair as

$$\vec{G}^{PV}(\vec{r} | \vec{r}') = \frac{1}{4\pi^2} \int \int_{-\infty}^{\infty} d\vec{k}_\rho e^{-j\vec{k}_\rho \cdot (\vec{\rho} - \vec{\rho}')} \vec{G}^{PV}(\vec{k}_\rho; z | z'), \quad (3)$$

where $\vec{r} = \vec{\rho} + \hat{z}z$, $\vec{\rho} = \hat{x}x + \hat{y}y$, and $\vec{k}_\rho = \hat{x}k_x + \hat{y}k_y$. Then (2) can be rewritten as

$$\begin{aligned} \vec{E}(\vec{r}) &= \hat{z} \frac{j}{\omega \epsilon_0 \epsilon_{r\ell}} \Delta \sigma(\vec{r}) \vec{E}_z(\vec{r}) \\ &= \frac{1}{4\pi^2} \int_{V_\ell} dV' \int \int_{-\infty}^{\infty} d\vec{k}_\rho e^{-j\vec{k}_\rho \cdot (\vec{\rho} - \vec{\rho}')} \vec{G}^{PV}(\vec{k}_\rho; z | z') \Delta \sigma(\vec{r}') \cdot \vec{E}(\vec{r}'). \end{aligned} \quad (4)$$

For a wave propagating along the axial direction of the conducting strips, the electric field can be expressed as

$$\vec{E}(\vec{r}) = \vec{E}(\vec{r}_s) e^{-j\eta y}, \quad (5)$$

where $\vec{r}_s = \hat{x}x + \hat{z}z$, and η is the propagation constant of the eigenmode that is yet to be determined. Thus, (4) becomes

$$\begin{aligned} \vec{E}(\vec{r}_s) e^{-j\eta y} &= \hat{z} \frac{j}{\omega \epsilon_0 \epsilon_{r\ell}} \Delta \sigma_s(\vec{r}_s) \vec{E}_z(\vec{r}_s) e^{-j\eta y} \\ &= \frac{1}{4\pi^2} \int_{S_\ell} dS' \left\{ \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dk_y e^{-j\vec{k}_y \cdot (\vec{y} - \vec{y}')} \int_{-\infty}^{\infty} dk_x e^{-j\vec{k}_x \cdot (\vec{x} - \vec{x}')} \right. \\ &\quad \left. \vec{G}^{PV}(\vec{k}_x, k_y; z | z') \Delta \sigma(\vec{r}') \cdot \vec{E}(\vec{r}') e^{-j\eta y'} \right\}. \end{aligned} \quad (6)$$

where S_ℓ is the cross-sectional area of the conductor strips in layer ℓ . Note that

$$\int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dk_y e^{-j\vec{k}_y \cdot (\vec{y} - \vec{y}')} e^{-j\eta y'} \vec{G}^{PV}(\vec{k}_x, k_y; z | z')$$

$$\begin{aligned} &= 2\pi \int_{-\infty}^{\infty} dk_y e^{-j\vec{k}_y \cdot \vec{y}} \vec{G}^{PV}(\vec{k}_x, k_y; z | z') \delta(k_y - \eta) \\ &= 2\pi e^{-j\eta y} \vec{G}^{PV}(\vec{k}_x, \eta; z | z'). \end{aligned} \quad (7)$$

Hence, by using (7) in (6) and dropping the factor $e^{-j\eta y}$ we obtain

$$\begin{aligned} \vec{E}(\vec{r}_s) &= \hat{z} \frac{j}{\omega \epsilon_0 \epsilon_{r\ell}} \Delta \sigma(\vec{r}_s) \vec{E}_z(\vec{r}_s) \\ &= \frac{1}{2\pi} \int_{S_\ell} dS' \int_{-\infty}^{\infty} dk_x e^{-j\vec{k}_x \cdot (\vec{x} - \vec{x}')} \vec{G}^{PV}(\vec{k}_x, \eta; z | z') \Delta \sigma(\vec{r}') \cdot \vec{E}(\vec{r}'). \end{aligned} \quad (8)$$

Equation (8) is the final form of the integral equation for the multilayered structure with conductor strips in only one layer. For the case of multiconductors in different layers, the integral equation is similar to that of (1) (or (8)), and can be derived accordingly.

To obtain numerical solution, the integral equation (8) is solved using Galerkin's method of moments. The eigenmode electric field on the conductor cross-section S is represented by a set of pulse basis functions as

$$\vec{E}(\vec{\rho}) = \sum_{n=1}^N (\hat{x}a_{nx} + \hat{y}a_{ny} + \hat{z}a_{nz}) P\left(\frac{x - x_n}{\Delta x_n}\right) P\left(\frac{z - z_n}{\Delta z_n}\right) \quad (9)$$

where

$$P(t) = \begin{cases} 1, & |t| \leq 1/2, \\ 0, & \text{otherwise,} \end{cases} \quad (10)$$

and Δx_n is the length of the n th segment in the x direction. Δz_n is that of the n th segment in the z direction, with $1 \leq n, m \leq N$. The center coordinate of the (n, m) cell is denoted as (x_n, z_m) . Then the same set of basis functions is chosen as the testing functions to construct a matrix equation, from which both the phase constant and the attenuation constant are solved.

In this study, the multilayered microstrip structures with conductor thickness on the order of the skin depth are of primary interest. The approach introduced above will be used to analyze such structures. Computed results and the pertaining discussions are presented in the following section.

III. RESULTS AND DISCUSSION

In this section, the computed results of some typical sample structures of Fig. 1 are presented. To validate the outlined approach, we first show, in Fig. 2, the attenuation constant of a microstrip line of width 258 versus the thickness to skin depth ratio, t/δ . The results from [6] are also plotted for comparison, and good agreement was observed. Next, we examine the effect of cross-sectional shape on conductor loss. In Fig. 3, we present the dispersion relation of the attenuation constant of a stripline for three different conductor cross-sections. With the area of the cross-section

fixed, the fish-eyed case, which has relatively larger cross-sectional contour, has higher conductor loss. This behavior is seen to be more pronounced in high frequency region due to skin effect. In addition to the cross-sectional shape effects, we are also interested to know the dependence of the dispersion behavior on the dielectric constants of the background media. In Fig. 4, we plot the attenuation constant of a stripline embedded in the interface of two dielectric layers. With ϵ_1 fixed, we see that larger ϵ_2 will cause higher conductor loss (see Fig. 4(a)). This is due to the fact that in this case the field between the strip line and the upper shielding conducting plane is stronger, and hence will also be the case for the field inside the conductor region near the upper strip surface. It is also seen in Fig. 4(b) that increasing ϵ_2 will give larger phase constant. Finally, we present in Fig. 5 the effects of strip thickness and the proximity of the ground planes on conductor loss. Results reveal that when the strip thickness is increased beyond three skin depths, the conductor loss is approaching a constant value. Also, the loss will increase if the ground planes are moved closer to the conducting strips.

IV. CONCLUSIONS

In this work, the multilayered microstrip lines with conductor thickness comparable to the skin depth are studied using a volume integral equation formulation incorporated with the Galerkin's method of moments. Results show that, due to the skin effect, the conductor loss is larger for the case with longer strip cross-sectional contour. The conductor loss is less sensitive to the strip thickness as the later is larger than three skin depths. The study of the effects on the conductor loss due to relative permittivity of the background medium reveals that higher dielectric constant will cause larger loss. Also, decreasing the separation between the ground planes and the conductor strip will increase the conductor loss.

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Fig. 1 Conductors of arbitrary cross section embedded in layered medium.

Fig. 2 Attenuation constant for a microstrip of width 25δ plotted versus the thickness to skin depth ratio, t/δ . ($f = 1$ GHz, $w = 62.5 \mu\text{m}$, $\delta = 2.5 \mu\text{m}$, $\epsilon = 11\epsilon_0$, $\sigma = 4 \times 10^7 \text{ S/m}$, $\tan \delta = 2.5 \times 10^{-4}$)

Fig. 3 Dispersion relation of striplines with the same cross-sectional area but different shapes. For rectangular case, $w_1 = w_2 = 100 \mu\text{m}$, $t_1 = t_2 = 12.5 \mu\text{m}$, for trapezoidal case, $w_1 = 75 \mu\text{m}$, $w_2 = 125 \mu\text{m}$, $t_1 = 25 \mu\text{m}$, $t_2 = 0 \mu\text{m}$, and for fish-eyed case, $w_1 = 75 \mu\text{m}$, $w_2 = 125 \mu\text{m}$, $t_1 = t_2 = 12.5 \mu\text{m}$. Also $\epsilon = 10 \epsilon_0$, $\sigma = 5.92 \times 10^7 \text{ S/m}$, $h = 600 \mu\text{m}$, $z_c = 300 \mu\text{m}$.

(b) Attenuation constant

Fig. 4 (continued)

(a) Phase constant

Fig. 4 Dispersion relation of a stripline embedded in the interface of two layers: $w = 100 \mu\text{m}$, $t = 25 \mu\text{m}$, $h = 600 \mu\text{m}$, $z_c = h/2$, $\epsilon_1 = 10\epsilon_0$, $\sigma = 5.92 \times 10^7 \text{ S/m}$.

Fig.5 Attenuation constants with different strip thickness and proximity of the ground planes, $w = 100 \mu\text{m}$, $\sigma = 5.92 \times 10^7 \text{ S/m}$, $\epsilon = 10\epsilon_0$, $z_c = h/2$.