Improving Wireless Packet Throughput Using Hybrid-II ARQ and Physical Layer Side Information

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Abstract - In this paper, the physical layer side information is employed to improve the throughput of the hybrid-II ARQ protocol in the binary phase shift keying wireless communication system. With the combination of the hybrid-II ARQ and the n -bit-marked error correction scheme, the packet throughput is improved significantly. The numerical examples show that the throughput of this combination scheme may have a maximum gain about 0.26.

I. INTRODUCTION

Wireless communications possess the convenient mobility property, which is very desirable. However, the air link, which may be effected by noise, multi-path fading, Doppler frequency shift, introduces significant errors in a mobile computing system. The conventional automatic-repeat-request (ARQ) protocol, which is designed for the wired transmission environment, can effectively control errors for the higher layers. Because of the high error rate for air link, the ARQ protocol becomes not very effective as it used in the wired transmission environment [1]. The forward-error-correction (FEC) scheme is a well-known method to control errors. Directly applying FEC scheme and the ARQ protocol on a mobile computing system can reduce errors at the expenses of transmission bandwidth, because a packet with FEC has a lot of redundant bits [2]. When the bit error rate (BER) is not so high, say 10^{-4} , this direct combination of ARQ and FEC wastes the bandwidth, which is very undesirable.

The Hybrid-II ARQ scheme using FEC codes, which is devised based on the concept that the parity-check bits for error correction are sent to the receiver only when needed, increases the bandwidth efficiency and reduces the average packet delay [3]. Two linear codes are used in this scheme; one is a high-rate $(B, B-k)$ code C_0 , which is designed for error detection only, the other is an invertible half-rate $(2k, k)$ code C_1 , which is designed for simultaneous error correction and error detection. The packet formats are depicted in Fig. 1.

The operation procedure of the hybrid-II ARQ is described as following. At first, the transmitting node sends the packet with the high-rate code. When the packet is received successfully, the receiving node sends an acknowledgement (ACK) back to the transmitting node. Otherwise, the receiving node sends a negative acknowledgement (NAK) back and the transmitting node will transmit the corresponding invertible half-rate code for correction. If the packet is successfully corrected, the receiving node sends an ACK back. On the contrary, the receiving node sends a NAK for the retransmission of the high-rate code packet and drops the former one. If the retransmitted one is correct, an ACK will be sent. Otherwise, it will be corrected by the former corresponding invertible half-rate code. If the packet still can't be corrected, the packet with the corresponding invertible

Fig. 1. The packet formats of hybrid-II ARQ.

half-rate code will be dropped and a NAK will be sent for the retransmission of the packet with the corresponding invertible half-rate code. This procedure will be repeated until the packet received successfully [4].

On the other hand, the n -bit-marked error correction scheme employs the physical layer side information to increase the correctness of every transmitted packet for improving the throughput [5]. In this scheme, the transmitting node doesn't have to change the structures of the physical layer and the data link layer. In the receiving node, a side information generator is added. The receiver structure with binary phase shift keying demodulator and the side information generator is shown in Fig. 2. We mark the bit, which is in Conditions II and III.

In this paper, we combine the n -bit-marked error correction and the hybrid-II ARQ scheme. The operation procedure of this combined scheme is very similar to that of the hybrid-II ARQ scheme. The only difference is that we use the n -bit-marked error correction scheme to increase the correctness of every hybrid-II ARQ packet to improve the throughput. The operation procedure of this combined scheme is described as following. At first, the transmitting node sends the packet with the high-rate code. If the packet is received successfully by employing the n -bit-marked error correction scheme, the receiving node sends an ACK back to the transmitting node. Otherwise, it sends a NAK back to request for transmitting the packet with the corresponding invertible half-rate code. This packet is also received by employing the n -bit-marked error correction scheme. If it is correct or can be corrected by the n -bit-marked error correction scheme, the receiving node sends an ACK back. If not, it also can be used to correct the information packet. If the packet is successfully corrected by the corresponding invertible half-rate code, the receiving node sends an ACK back. On the contrary, the receiving node sends a NAK for the retransmission of the packet with high-rate code and drops the former one. If the retransmitted one is correct or can be corrected by the n -bit-marked error correction scheme, an ACK will be sent.

¹The work is supported in part by the National Science Council, R.O.C under the grant NSC 88-2213-E-002-081.

Fig.2: receiver structure

Otherwise, the former packet with the corresponding invertible half-rate code will be used to correct the packet. If the packet still can't be corrected, the packet with the corresponding invertible half-rate code will be dropped and a NAK will be sent for the retransmission of the packet with the corresponding invertible half-rate code. This procedure will be repeated until the packet received successfully.

II. THE THROUGHPUT ANALYSIS

The throughput of the hybrid-II ARQ scheme is effected by the values of q_1 and q_2 . q_1 is the conditional probability that the packet can be recovered by the second transmitted packet when the first transmitted packet is detected error. q_2 is the conditional probability that the packet can be recovered by the third transmitted packet when the first and second transmitted packets are detected error and can not be recovered by the packet with the invertible half-rate code. For the combination the hybrid-II ARQ and the n-bit-marked scheme, q_1 can be expressed as

$$q_1 = (p_c + p_n) + p_{ddc1} / (1 - p_c - p_n), \quad (1)$$

where p_c is the probability of the first packet is received successfully without invoking the correction algorithm. p_n is the probability that the erroneous packet is successfully recovered by the n-bit-marked error correction scheme. p_{ddc1} is the probability that the first and second transmitted packets are erroneous and can't be corrected by the n-bit-marked error correction scheme but can be recovered by invertible high-rate code. p_{ddc1} can be expressed as

$$p_{dda} = p_{ddc} - p_A - p_{A1} + p_B, \quad (2)$$

where p_{ddc} is the probability that the first and second transmitted packets are erroneous but can be recovered by invertible high-rate code. p_A is the probability that the first and second transmitted packets are detected errors, the first transmitted packet can be recovered by the n-bit-marked error correction scheme and the first and second packets can be recovered by the half-rate code. p_{A1} is the probability that the first and second transmitted packets are detected error, the second transmitted packet can be recovered by the n-bit-marked error correction scheme and the first and second packets can be recovered by the half-rate code. p_B is the probability that the first and second transmitted packets are

detected error, both can be recovered by the n-bit-marked error correction scheme, and the first packet can be recovered by the half-rate code. The probabilities, p_A and p_{A1} are equal. p_{ddc1} can also be expressed as

$$p_{ddc1} = p_{ddc} - 2p_A + p_B. \quad (3)$$

We assume that there are N marked bits in the first transmitted packet and among them h marked bits are in the first k bits of the packet. Let t be the correction power of the invertible half-rate code and n is the maximum number of marked bits of the n-bit-marked error correction scheme. The probability p_A is given by

$$p_A = \sum_{h=0}^n \sum_{N=h}^N C_h^N C_{N-h}^{B-k} p_1^{B-N} \cdot \left\{ \sum_{j=0}^h C_j^h p_2^{h-j} p_3^j (p_2 + p_3)^{N-h} \left[\sum_{i=0}^{t-j} C_i^t p_e^i (1-p_e)^{t-i} - (1-p_e)^B \right] \right. \\ \left. - p_2^N \left[\sum_{i=0}^t C_i^t p_e^i (1-p_e)^{t-i} - (1-p_e)^B \right] \right\}; j \leq t, \quad (4)$$

where p_e is the BER, B is the number of bit in a packet. p_1 is the probability that the signal energy of a received bit is larger than r (the threshold of side information generator). p_2 is the probability that the energy of a received bit is fallen in the $(0, r)$ region. p_3 is the probability that a received bit is fallen in the $(0, -r)$ region. The detailed derivation of p_A is presented in the Appendix.

p_B , whose derivation is similar to that of p_A , is given by

$$p_B = \sum_{h=0}^n \sum_{N=h}^N C_h^N C_{N-h}^{B-k} p_1^{B-N} \cdot \left\{ \sum_{j=0}^h C_j^h p_2^{h-j} p_3^j (p_2 + p_3)^{N-h} p_{R1} - p_2^N p_{R2} \right\}; j \leq t, \quad (5)$$

$$p_{R1} = \sum_{N_1=1}^n \sum_{h_1=0}^{N_1} C_{h_1}^k C_{N_1-h_1}^{B-k} p_1^{B-N_1} \cdot \left\{ \sum_{i=0}^{h_1} C_i^{h_1} p_2^{h_1-i} p_3^i (p_2 + p_3)^{N_1-h_1} - p_2^{N_1} \right\}; i \leq t-j, \quad (6)$$

$$p_{R2} = \sum_{N_1=1}^n \sum_{h_1=0}^{N_1} C_{h_1}^k C_{N_1-h_1}^{B-k} p_1^{B-N_1} \cdot \left\{ \sum_{i=0}^{h_1} C_i^{h_1} p_2^{h_1-i} p_3^i (p_2 + p_3)^{N_1-h_1} - p_2^{N_1} \right\}; i \leq t, \quad (7)$$

where P_{R1} is the probability that a packet with one to $(t-j)$ errors in the first k bits can be recovered by the n -bit-marked error correction scheme and P_{R2} is the probability that a packet with one to t errors in the first k bits can be recovered by the n -bit-marked error correction scheme.

Similarly, when the n -bit-marked error correction scheme is employed, q_2 is given by

$$q_2 = (p_c + p_n) + \frac{1}{(1-p_c-p_n)(1-q_1)} \cdot \sum_{i=0}^{t-1} \Delta_1(i) s_1(t-i) [1 - \Delta(0) - s(t-i) - s_2(i)] \quad (8)$$

and

$$\Delta(i) = C_i^k p_e^i (1-p_e)^{k-i} \quad (9)$$

$$s(j) = \sum_{l=1}^j \Delta(l) \quad (10)$$

$$\Delta_1(i) = \Delta(i) - \sum_{N=1}^n \sum_{h=1}^N C_h^k C_{N-h}^{B-k} p_1^{B-N} p_2^{h-i} p_3^i (p_2 + p_3)^{N-h} \quad (11)$$

$$s_1(j) = \sum_{l=1}^j \Delta_1(l) \quad (12)$$

$$s_2(j) = \sum_{l=(t-j+1)}^n \sum_{N=l}^n \sum_{h=1}^N C_h^k C_{N-h}^{B-k} p_1^{B-N} p_2^{h-i} p_3^i (p_2 + p_3)^{N-h} \quad (13)$$

where P_c is the probability of the first packet is received successfully without invoking the correction algorithm. P_n is the probability that the erroneous packet is successfully recovered by the n -bit-marked error correction scheme. $\Delta(i)$ is the event that the first k bits of the third transmitted packet contain exactly i errors. $\Delta_1(i)$ is the event that the first k bits of the third transmitted packet contain exactly i errors and can not be corrected by the n -bit-marked error correction scheme. $S(j)$ is the event that the first k bits of the third transmitted packet contain one to j errors. $S_1(j)$ is the event that the first k bits of the third transmitted packet contain one to j errors and can not be corrected by the n -bit-marked error correction scheme. $S_2(j)$ is the event that the first k bits of the third transmitted packet contain $(t-j+1)$ to k errors and can be corrected by the n -bit-marked error correction scheme.

Denote that

$$\alpha = 1 - (1-p_c-p_n)(1-q_1) \quad (14)$$

$$\beta = 1 - (1-p_c-p_n)(1-q_1)(1-q_2) \quad (15)$$

$$\gamma = q_1 + q_2 - q_1 q_2 \quad (16)$$

Then the throughput of the combination scheme with receiver buffer of size N has a lower bound as

$$\eta \geq \delta_0 / (\delta_0 + \delta_1 + \delta_2 N) \quad (17)$$

where

$$\delta_0 = \gamma(1-\gamma\beta^{N-1})/(1-\beta) \quad (18)$$

$$\delta_1 = (p_c + p_n)q_1\alpha^{N-2} + (p_c + p_n)(1-q_1)q_2\beta^{N-2} \quad (19)$$

$$\delta_2 = 2 - (p_c + p_n)q_1\alpha^{N-2} - \alpha\gamma\beta^{N-2} \quad (20)$$

The throughput versus BER with $t=3, 5$ or 10 are plotted in Figs.3~5. In Fig.3, the maximum gain is 0.26 . Here the gain is defined as the difference of the throughput of the hybrid-II ARQ and that of the combination of the hybrid-II ARQ and the n -bit-marked error correction scheme. The maximum gains are 0.24 and 0.17 for $t=5, n=10$ and $t=10, n=15$ as shown in Figs.4 and 5. It is observed that the proposed scheme can increase the packet throughput significantly at BER around 1.5×10^{-3} , where the throughput of the conventional selective repeat ARQ is almost zero.

IV. CONCLUSION

In this paper, we combine the hybrid-II ARQ and the n -bit-marked error correction scheme to improve the throughput performance. We take the advantages of using side information and FEC codes. The numerical results show that this scheme can improve the throughput significantly. The maximum gain of this combination scheme is 0.26 at BER around 1.5×10^{-3} . This scheme needs computation power in the receiver. It is expected that when the cost effective digital signal processing integrated circuit chip is available, this scheme can be practical.

Fig.3. The throughput of the combination of n -bit-marked error correction ($n=3, 5$ or 10) and hybrid-II ARQ with $t=3$. The buffer size is 128 and the packet size is 524 .

Fig.4. The throughput of the combination of n-bit-marked error correction (n= 3, 5 or 10) and hybrid-II ARQ with t=5. The buffer size is 128 and the packet size is 524.

Fig.5. The throughput of the combination of n-bit-marked error correction (n=5, 10 or 15) and hybrid-II ARQ with t=10. The buffer size is 128 and the packet size is 524.

APPENDIX

First, we assume that $N=1$ and $h=0$. Then P_A can be expressed as

$$P_A = [P_1^k C_1^{B-k} P_1^{B-k-l} P_3] \left[\sum_{i=0}^l C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \quad (A.1)$$

If $N=2$ and $h=0$, then P_A can be expressed as

$$P_A = [P_1^k C_2^{B-k} P_1^{B-k-2} (2P_2 P_3 + P_3^2)] \left[\sum_{i=0}^l C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \quad (A.2)$$

If $N=n$ and $h=0$, then P_A can be expressed as

$$P_A = [C_n^{B-k} P_1^{B-n} (P_2 + P_3)^n - P_2^n] \left[\sum_{i=0}^l C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \quad (A.3)$$

Therefore, when $h=0$, the total probability of P_A can be expressed as

$$P_A = \sum_{N=1}^n [C_N^{B-k} P_1^{B-N} (P_2 + P_3)^N - P_2^N] \left[\sum_{i=0}^l C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \quad (A.4)$$

Next, we assume $N=1$ and $h=1$ and P_A is expressed as

$$P_A = [C_1^k P_1^{B-l} P_3] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \quad (A.5)$$

If $N=2$ and $h=1$, then P_A is given by

$$P_A = [C_1^k P_1^{k-l} P_3 C_1^{B-k} P_1^{B-k-l} (P_2 + P_3)] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \\ + [C_1^k P_1^{k-l} P_2 C_1^{B-k} P_1^{B-k-l} P_3] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \\ = [C_1^k C_1^{B-k} P_1^{B-2} P_3 (P_2 + P_3)] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \\ + [C_1^k C_1^{B-k} P_1^{B-2} P_2 P_3] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \quad (A.6)$$

If $N=3$ and $h=1$, then P_A is given by

$$P_A = [C_1^k P_1^{k-l} P_3 C_2^{B-k} P_1^{B-k-2} (P_2 + P_3)^2] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \\ + [C_1^k P_1^{k-l} P_2 C_2^{B-k} P_1^{B-k-2} (2P_2 P_3 + P_3^2)] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \\ = [C_1^k C_2^{B-k} P_1^{B-3} P_3 (P_2 + P_3)^2] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \\ + [C_1^k C_2^{B-k} P_1^{B-3} P_2 (P_2 + P_3)^2 - P_2^2] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \quad (A.7)$$

If $N=n$ and $h=1$, then P_A can be expressed as

$$P_A = [C_1^k C_n^{B-k} P_1^{B-n} P_3 (P_2 + P_3)^{n-1}] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \\ + [C_1^k C_n^{B-k} P_1^{B-3} P_2 (P_2 + P_3)^{n-1} - P_2^{n-1}] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \\ = C_1^k C_n^{B-k} P_1^{B-n} \left\{ \begin{aligned} & [P_3 (P_2 + P_3)^{n-1}] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \\ & + P_2 [P_2 + P_3)^{n-1} - P_2^{n-1}] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \end{aligned} \right\} \quad (A.8)$$

Therefore, when $h=1$, the total probability of P_A can be expressed as

$$P_A = \sum_{N=1}^n C_1^k C_n^{B-k} P_1^{B-n} \left\{ \begin{aligned} & [P_3 (P_2 + P_3)^{N-1}] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \\ & + P_2 [P_2 + P_3)^{N-1} - P_2^{N-1}] \left[\sum_{i=0}^{l-1} C_i^k P_e^i (1-P_e)^{k-i} \right] - (1-P_e)^B \end{aligned} \right\} \quad (A.9)$$

Then, when $h=2$, the total probability of P_A can be derived as

$$\begin{aligned}
P_A &= \sum_{N=2}^n C_2^k C_{N-2}^{B-k} P_1^{B-N} \left\{ \begin{aligned} &P_2^2 \left[(P_2 + P_3)^{N-2} - P_2^{N-2} \right] \left[\sum_{j=0}^l C_1^k P_e^j (1-p_e)^{k-j} \right] - (1-p_e)^B \Big\} \\ &+ 2P_2 P_3 \left[(P_2 + P_3)^{N-2} \right] \left[\sum_{j=0}^{l-1} C_1^k P_e^j (1-p_e)^{k-j} \right] - (1-p_e)^B \Big\} \\ &+ P_3^2 \left[(P_2 + P_3)^{N-2} \right] \left[\sum_{j=0}^{l-2} C_1^k P_e^j (1-p_e)^{k-j} \right] - (1-p_e)^B \Big\} \end{aligned} \right\} \\
&= \sum_{N=2}^n C_2^k C_{N-2}^{B-k} P_1^{B-N} \left\{ \begin{aligned} &\sum_{j=0}^2 C_2^j P_2^{2-j} P_3^j \left[(P_2 + P_3)^{N-2} \right] \left[\sum_{i=0}^{l-j} C_1^k P_e^i (1-p_e)^{k-i} \right] - (1-p_e)^B \Big\} \\ &- P_2^2 \left[\sum_{j=0}^l C_1^k P_e^j (1-p_e)^{k-j} \right] - (1-p_e)^B \Big\} \end{aligned} \right\} \quad (A.10)
\end{aligned}$$

Therefore, the total probability of P_A is given by

$$\begin{aligned}
P_A &= \sum_{h=0}^n \sum_{N=h}^n C_h^k C_{N-h}^{B-k} P_1^{B-N} \bullet \\
&\left\{ \begin{aligned} &\sum_{j=0}^h C_j^h P_2^{h-j} P_3^j \left[(P_2 + P_3)^{N-h} \right] \left[\sum_{i=0}^{l-j} C_1^k P_e^i (1-p_e)^{k-i} \right] - (1-p_e)^B \Big\} \\ &- P_2^h \left[\sum_{j=0}^l C_1^k P_e^j (1-p_e)^{k-j} \right] - (1-p_e)^B \Big\} \quad ; j \leq l \end{aligned} \right\} \quad (A.11)
\end{aligned}$$

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