Analysis of Self-Stabilization for Infinite-State Systems

Hsu-Chun Yen

Dept. of Electrical Engineering
National Taiwan University
Taipei, Taiwan 106, R.O.C.
yen@cc.ee.ntu.edu.tw

Abstract

The problem of deciding whether an infinite-state system is self-stabilizing or not is investigated from the decidability viewpoint in this paper. We develop a unified strategy through which checking self-stabilization is shown to be decidable for one-counter machines and conflict-free Petri nets. Our strategy relies on the reachability sets being semilinear, as well as on the capability of extracting periodic behaviors of infinite computations, which, in turn, facilitates the expression of self-stabilization by Presburger Arithmetic.

As fairness is frequently used as a qualitative measure to capture the notion of a quantitative measure of 'something happens with probability one,' it is of interest to examine the fair version of the self-stabilization problem, i.e., the problem of asking whether all 'fair' infinite computations eventually become self-stabilizing. We propose a potential method through which the problem is shown to be decidable for conflict-free Petri nets.

Keywords: Decidability, infinite-state system, Petri net, self-stabilization.

1 Introduction

The notion of *self-stabilization* was introduced by Dijkstra [3] to describe a system having the behavior that regardless of its starting configuration, the system would return to a 'legitimate' configuration eventually. (By a legitimate configuration, we mean a configuration which is reachable from the initial configuration of the system.) The motivation behind self-stabilization is that a self-stabilizing system has the ability to 'correct' itself even in the presence of certain unpredictable errors that force the system to reach an 'illegitimate' configuration during the course of its operations. In this sense, self-stabilizing systems exhibit fault-tolerant behaviors to a certain degree. With the increased interest in designing fault-tolerant systems, the study of self-

stabilization has been gaining increasing popularity in the computer science community lately. In what follows, we briefly review some of the previous work done in the area of self-stabilization.

Following the seminal work of Dijkstra [3], the majority of research along the line of self-stabilization has been focusing on providing solutions (and their proofs) for self-stabilizing systems with a variety of properties and topologies. (See [6] for a bibliography of self-stabilization.) In a somewhat different direction of research, Gouda, Howell and Rosier ([5]) have showed that the property of self-stabilization is 'unstable' across a wide variety of system classes, ranging from cellular automata, Turing machines, communicating finite state machines, to Petri nets. They basically demonstrated that the *simulation paradigm*, which is a useful tool for designing and analyzing systems, may not be 'robust' with respect to the property of self-stabilization. (The simulation paradigm, simply speaking, is a methodology routinely used for designing or analyzing one class of systems with the help of another (hopefully, a well-studied one) through the simulation of the former by the latter. In this manner, properties of the former can be deduced from the respective properties of the latter.) Unlike traditional properties such as liveness, boundedness and fairness which are almost always preserved under standard simulations, self-stabilization could easily be lost through the process of simulation from one class of systems to another. (For more details, see [5].) To our knowledge, [5] also pioneers the introduction of the notion of self-stabilization to the model of Petri nets. In a subsequent paper [1], specific efforts have been devoted to reasoning about self-stabilization aspects of Petri nets from the computational complexity point of view. Among those complexity results derived in [1], detecting whether a Petri net is self-stabilizing is complete for polynomial time for bounded ordinary Petri nets, whereas it is PSPACE-complete for bounded general Petri nets.

As far as we know, very little is known regarding the decidability/complexity issue of the self-stabilization problem (i.e., the problem of deciding whether a system is self-

stabilizing or not) for infinite-state systems. At this moment, the best bounds of the problem for general Petri nets are a lower bound of exponential space and an upper bound of Π_2 (the second level of the arithmetic hierarchy), whereas it is Π_2 -complete for Turing machines [1]. Therefore, it is of interest and importance to take a closer look at the problem for other infinite-state systems, in the hope of recognizing the key characteristics which govern the decidability/undecidability nature of the problem. An equally important goal is to, perhaps, come up with a unified framework through which decidability/undecidability of self-stabilization can be obtained. In this paper, we extend the work of [1] by investigating, from the decidability viewpoint, the problem of deciding whether a given system is self-stabilizing for *one-counter machines* and *conflict-free Petri nets*. As it turns out, the decidability of self-stabilization for these two computational models can be established in a unified setting, taking advantage of the following key properties:

1. The reachability set as well as the set of ‘dead’ configurations are effectively semilinear,
2. The existence of a *periodic* witness for non-self-stabilizing infinite computations.

The above properties allow us to formulate self-stabilization in terms of formulas in Presburger Arithmetic, giving rise to the decidability result.

As ‘fairness’ is frequently used as a qualitative measure to capture the notion of a quantitative measure of ‘something happens with probability one,’ it is of interest to re-examine the self-stabilization problem by asking whether all ‘fair’ infinite computations eventually enter legitimate states. We propose a ‘potential method’ through which the problem is shown to be decidable for a restricted class of Petri nets. The idea is to associate a value (called ‘potential’) with each marking in the Petri net in such a way that the potential along any Petri net computation is non-increasing, and in some cases, has the tendency to move towards the ground level (i.e., potential zero). To a certain degree, the basic idea of our potential method is analogous to the analysis of *neural nets* (in particular, *Hopfield nets*), in which an *energy function* is associated with a Hopfield net, and the energy of the net decreases as the computation proceeds (provided that the net meets certain conditions) which, in turn, guarantees the convergence of the net.

The rest of this paper is organized as follows. In Section 2, we give the definitions of transition systems and self-stabilization. Section 3 deals with the issue of deciding whether a given system is self-stabilized or not from the decidability viewpoint for infinite-state systems. A fair version of self-stabilization is investigated in Section 4. A conclusion is given in Section 5.

2 Preliminaries

Let $\mathbf{Z}$ ($\mathbf{N}$) denote the set of (nonnegative) integers, and $\mathbf{Z}^k$ ($\mathbf{N}^k$) the set of vectors of k (nonnegative) integers.

A *transition system* (or *system*, for short) $\mathcal{S}$ is a four-tuple (X, Σ, δ, c_0) , where X consists of a (possibly infinite) set of *configurations*, Σ is a (possibly infinite) set of *transition symbols*, $\delta : X \times \Sigma \rightarrow 2^X$ defines the *transition function*, and $c_0 \in X$ is the *initial configuration*. We write $c \xrightarrow{t} c'$ if $c' \in \delta(c, t)$. Also let $\xrightarrow{*}$ (resp., $\xrightarrow{+}$) be the reflexive and transitive closure (resp., transitive closure) of $\rightarrow$. In words, $c \xrightarrow{*} c'$ (resp., $c \xrightarrow{+} c'$) iff c' can be reached from c through zero (resp., one) or more transitions. A *computation* σ is a (finite or infinite) sequence $c_1 \xrightarrow{t_1} c_2 \xrightarrow{t_2} \dots c_i \xrightarrow{t_i} c_{i+1} \dots$. A configuration c is said to be a *dead configuration* if $\forall t \in \Sigma, \delta(c, t) = \emptyset$, i.e., c has no immediate successor in $\mathcal{S}$. In this paper, we consider only those systems for which X can be encoded in such a way that $X \subseteq Q \times \mathbf{N}^k$, for some finite set Q and integer $k \geq 1$. (Q is referred to as the set of *states* of system $\mathcal{S}$.)

Intuitively speaking, a system is said to be *self-stabilizing* (ss, for short) if, regardless of its starting configuration, the system would eventually return to a ‘legitimate’ configuration which is reachable from the *initial configuration* of the system. That is, a self-stabilizing system has the ability to ‘correct’ itself even in the presence of certain unpredictable errors that force the system to reach an ‘illegitimate’ configuration during the course of its operations. Let $\mathcal{S}$ be a (finite or infinite) system with c_0 as its *initial configuration*. Also let $R(\mathcal{S}, c_0) (= \{c \mid c_0 \xrightarrow{*} c\})$ denote the set of *reachable configurations* from c_0 . A computation σ from configuration c_1 is said to be *non-self-stabilizing* (non-ss) iff one of the following holds:

1. σ is finite ($\sigma : c_1 \xrightarrow{t_1} c_2 \xrightarrow{t_2} \dots c_{m-1} \xrightarrow{t_{m-1}} c_m$, for some m) such that c_m is a dead configuration and $c_m \notin R(\mathcal{S}, c_0)$, or
2. σ is infinite ($\sigma : c_1 \xrightarrow{t_1} c_2 \xrightarrow{t_2} \dots c_i \xrightarrow{t_i} c_{i+1} \dots$) such that $\forall i \geq 1, c_i \notin R(\mathcal{S}, c_0)$.

See Figure 1. A system is said to be *self-stabilizing* if for each configuration c , none of the computations emanating from c is non-ss. The *self-stabilization problem* is to determine, for a given (finite or infinite) system, whether the system is self-stabilizing. In this paper, our main concern is to investigate the decidability issue of the self-stabilization problem for a variety of infinite-state systems. The formal definitions of *configurations* and *computations* of each individual system will be clarified as our discussion progresses.

For a vector $v \in \mathbf{N}^k$ and a finite set $\rho = \{v_1, \dots, v_n\}$, for some $n \subseteq \mathbf{N}^k$, the set $\mathcal{L}(v; \rho) = \{v \mid \exists a_1, \dots, a_n \in \mathbf{N}, v = v + \sum_{i=1}^n a_i * v_i\}$ is called the *linear set* with *base* v over the

Figure 1. Non-self-stabilizing computations.

set of periods ρ . A *semilinear set (SLS)* (cf. [4]) is a finite union of linear sets. It is known that semilinear sets are exactly those that can be expressed in *Presburger Arithmetic* [10], which is a decidable theory.

3 Decidability analysis

In this section, the *ss* problem for *one-counter machines* and *conflict-free Petri nets* will be shown to be decidable. Before going into the details, we begin by giving an overview of the general strategy using which the decidability results are obtained.

Recall from the definition of *ss* that a system S is not self-stabilizing iff either (i) a dead configuration is not reachable from the initial configuration, or (ii) an infinite non-*ss* computation exists. As we shall see later, (i) is relatively easy to check as long as certain properties of S are decidable. The idea behind our approach of checking (ii) is built upon showing that to demonstrate the non-*ss* nature of a system, it is sufficient to search among only non-*ss* computations of *periodic* behaviors, and hopefully, the confinement to such computations admits a decidable checking of (ii). By non-*ss* periodic computations we mean those non-*ss* computations of the form $s \xrightarrow{\tau} s$, i.e., repeating τ from s infinitely many times witnesses non-*ss*. In our subsequent investigation, we characterize the following two types of non-*ss* periodic behaviors, through which a unified strategy is developed serving as a foundation for our decidability results of the *ss* problem.

- (Strongly periodic:) Every finite computation $c \xrightarrow{\tau} c'$ with $c' \geq c$ and $c \notin R(S, c_0)$ ensures non-*ss* of its infinite repetition (i.e., $c \xrightarrow{\tau} c$ is non-*ss*).
- (Weakly periodic:) If an infinite non-*ss* computation

exists, there must exist a witness τ which is in a computable finite set, and $c \xrightarrow{\tau} c$ constitutes an infinite non-*ss* computation.

The disparity between strongly and weakly periodic non-*ss* computations is that in the former, a finite computation of the form ' $c \xrightarrow{\tau} c'$ with $c' \geq c$ and $c \notin R(S, c_0)$ ' is guaranteed to conclude the system's non-*ss* behavior, whereas in the latter, however, not *every* finite computation of the above form is extendible to render *ss*; nevertheless, there does exist such a witness which is computable. (The meaning of ' $\geq$ ' will become clear as our discussion progresses.)

Before going into the details, we require the following notations. Let $S = (X, \Sigma, \delta, c_0)$ be a transition system, where $X \subseteq Q \times \mathbb{N}^k$, $c \in X$, and $q, q' \in Q$.

- $SUCC(S, q, q') = \{(s, s') \mid (q, s) \xrightarrow{t} (q', s'), \text{ for some transition } t \in \Sigma\}$,
- $REACH(S, q, q') = \{(s, s') \mid (q, s) \xrightarrow{*} (q', s')\}$;
 $REACH^+(S, q, q') = \{(s, s') \mid (q, s) \xrightarrow{+} (q', s')\}$;
 $Reach(S, c, q') = \{s' \mid c \xrightarrow{*} (q', s')\}$,
 $R(S, c) = \{c' \mid c \xrightarrow{*} c'\}$,
- $DEAD(S, q) = \{s \mid (q, s) \text{ is a dead configuration}\}$.

As we shall see later, decidability of the *ss* problem for each of the aforementioned computational models is based upon the following unified strategy:

Theorem 3.1 *Let $\mathcal{X}$ be a computational model for which the *ss* problem is considered. If for each system $S = (X, \Sigma, \delta, c_0)$ in $\mathcal{X}$, where $X \subseteq Q \times \mathbb{N}^k$, we have*

- (1) (Semilinear condition) $\forall q, q' \in Q$, the sets $SUCC(S, q, q')$, $REACH(S, q, q')$ and $DEAD(S, q)$ are effectively semilinear, and
- (2) (Periodic condition) S has an infinite non-*ss* computation iff there exists an infinite computation $(q, s) \xrightarrow{+} (q, s + d) \xrightarrow{+} (q, s + 2d) \dots$, for some $(q, s) \in X$ and $d \in \mathbb{N}^k$, such that either (a) or (b) below holds:
 - (a) • $s \notin Reach(S, c_0, q)$,
• $(s \notin Reach(S, c_0, q))$ implies $(\forall n \geq 0, s + n * d \notin Reach(S, c_0, q))$, and
• $((q, s) \xrightarrow{+} (q, s + d))$ implies $(\forall n \geq 0, (q, s + n * d) \xrightarrow{+} (q, s + (n + 1)d))$,
 - (b) • $\forall n \geq 0, s + n * d \notin Reach(S, c_0, q)$, and
• d belongs to a computable finite set $D \subseteq \mathbb{N}^k$,

then the *ss* problem is decidable for $\mathcal{X}$. (Notice that types (a) and (b) characterize strongly and weakly periodic non-*ss* computations, respectively.)

Proof: First notice that if $\forall q, q' \in Q$, $REACH(S, q, q')$ is semilinear, so is $Reach(S, c, q')$, $\forall c \in X$ and $q' \in Q$. According to the definition of *ss*, system S is not *ss* iff either (i) a dead configuration is not reachable, or (ii) an infinite non-ss computation exists. The (i) case can be checked by testing $(\exists q \in Q) (DEAD(S, q) \cap \overline{Reach(S, c_0, q)} \neq \emptyset)$, which is decidable because of the semilinearity assumption of (1). To check (ii), assumption (2) suggests that it is sufficient to look for an infinite non-ss computation of type (a) or (b). Deciding the existence of such a path can be formulated as follows.

• Type (a) (i.e., *strongly periodic case*):

$\bigvee_{q \in Q} (\exists s \in \mathbb{N}^k) (\exists d \in \mathbb{N}^k) ((s, s+d) \in REACH^+(S, q, q) \wedge (s \notin Reach(S, c_0, q)))$, which can be expressed in Presburger Arithmetic (due to assumption (1)).

• Type (b) (i.e., *weakly periodic case*):

$\bigvee_{q \in Q} \bigvee_{d \in D} (\exists s \in \mathbb{N}^k) (\forall n \geq 0) ((s + n * d, s + (n+1) * d) \in REACH^+(S, q, q) \wedge (s + n * d \notin Reach(S, c_0, q)))$, which can be expressed in Presburger Arithmetic (due to assumption (1) and the fact that $(s, s') \in REACH^+(S, q, q')$ iff $(\exists s'' \in \mathbb{N}^k) (\exists q'' \in Q) ((s, s'') \in SUCC(S, q, q'')) \wedge ((s'', s') \in REACH(S, q'', q'))$. ■

We are in a position to show why each of the models of *one-counter machines* and *conflict-free Petri nets* falls into a special case of Theorem 3.1.

3.1 One-counter machines

A *one-counter machine* M consists of a finite-state control equipped with a *counter* on which three types of operations, namely, *addition*, *subtraction* and *test-for-zero* can be performed. For convenience, we use the following to denote the above three basic operations:

- (1) (q, q', n) , $n \geq 0$ (resp., $n < 0$), meaning that M can go from state q to state q' by adding n to the counter (resp., by subtracting $|n|$ from the counter, provided that the value of the counter is greater than or equal to $|n|$)
- (2) $(q, q', = 0)$, meaning that M can go from state q to state q' provided that the value of the counter is zero.

Given a one-counter machine M , we denote by M_{state} and M_{tr} the sets of states and transitions of M , respectively. A *configuration* of one-counter machine M is a pair (q, n) , where $q \in M_{state}$, and $n \in \mathbb{N}$ (indicating the value of the counter). Suppose q_0 is the *initial state* of M , the configuration $(q_0, 0)$ is called the *initial configuration* of M . Given two configurations (q, m) , (q', m') , and a transition t , we write $(q, m) \xrightarrow{t} (q', m')$ iff one of the following holds:

1. $t = (q, q', n)$, $n \in \mathbb{Z}$ and $m' = m + n$,
2. $t = (q, q', = 0)$ and $m' = m = 0$.

A *short positive loop* in M is a sequence of transitions $(q_1, q_2, n_1)(q_2, q_3, n_2) \cdots (q_m, q_1, n_m)$, for some $m \geq 1$ such that for all $1 \leq i \neq j \leq m$, $q_i \neq q_j$, and $(\sum_{i=1}^m n_i) > 0$. (In words, a short positive loop is a cycle (without test-for-zero transitions) in M 's transition diagram along which no state repeats itself except the first and the last, and the counter has a positive gain in value along the cycle.)

Before proceeding, we require the following result which is a direct consequence of a similar one in [11].

Lemma 3.2 *Let M be a one-counter machine of size z (when a binary encoding scheme is used). If $(q, m) \xrightarrow{*} (q', m')$, then there exists a computation $r : (q, m) \xrightarrow{s} (q', m')$, for some transition sequence s , along which every configuration (p, h) in r has $h \leq 3 * (z2^z)^3 + \max\{m, m'\}$.*

Proof: In Lemma 4.3 of [11], it was shown that for one-counter machines in which the counter can only be incremented or decremented by 1, the reachability of (q', m') from (q, m) is witnessed by a short computation along which the counter never exceeds $3 * (n)^3 + \max\{m, m'\}$, where n is the number of transitions of the machine. In our one-counter machine model, up to 2^z can be added to or subtracted from the counter in a single transition. Hence, our one-counter machines of size z can easily be simulated by those defined in [11] using at most $z2^z$ transitions. The lemma follows immediately. ■

Lemma 3.3 *Given a one-counter machine M and two states q and q' , $REACH(M, q, q')$ is effectively semilinear.*

Proof: $REACH(M, q, q')$ is the union of the following two sets:

- $R^1(M, q, q') = \{(c, d) | (q, c) \xrightarrow{r} (q', d) \text{ and sequence } r \text{ does not use any test-for-zero moves}\}$. As one-counter machines without test-for-zero moves are computationally equivalent to 1-dimensional VASSs, $R^1(M, q, q')$ can be characterized as the reachability set of a 2-dimensional VASS in the following way. We construct a 2-dimensional VASS $\mathcal{P} = (v_0, A, p_0, Q, \delta)$ with $v_0 = (0, 0)$ to simulate the computation of M in such a way that $(c, d) \in R^1(M, q, q')$ is witnessed by the following

computation of $\mathcal{P}$: $(p_0, (0, 0)) \xrightarrow{\overbrace{t \cdots t}^c} (p_0, (c, c)) \rightarrow (q, (c, c)) \xrightarrow{\sigma} (q', (c, d))$, where repeating transition $t : p_0 \rightarrow (p_0, (1, 1))$ c times is to initialize the counter value, and in the course of σ , $\mathcal{P}$ simulates

M 's transitions using the second vector position to keep track of M 's counter value while keeping the first vector position intact. The semilinearity of $R^1(M, q, q')$ follows immediately from [7], which shows the reachability sets of 2-dimensional VASSs to be effectively semilinear.

- $R^2(M, p, q) = \{(c, d) \mid (p, c) \xrightarrow{r_1} (q, d) \text{ and sequence } r \text{ uses some test-for-zero moves}\}$. Consider a computation $(p, c) \xrightarrow{r_1} (p_1, c_1) \xrightarrow{z} (p'_1, c'_1) \xrightarrow{r_2} (p_2, c_2) \xrightarrow{z'} (p'_2, c'_2) \xrightarrow{r_3} (q, d)$ such that z and z' are test-for-zero moves, and sequences r_1 and r_3 do not use any test-for-zero moves. (Here (p_1, c_1) and (p_2, c_2) are the first and last, respectively, configurations along the above computation at which test-for-zero moves are carried out. Also notice that (p_1, c_1) may be equal to (p_2, c_2) and z to z' .) We define a binary relation $\mathcal{R}$ on M_{state} such that $\bar{p}\mathcal{R}\bar{q}$ iff $(\bar{p}, 0) \xrightarrow{*} (\bar{q}, 0)$ in M . According to Lemma 3.2, using a depth-first search the relation $\mathcal{R}$ can effectively be computed. Now $R^2(M, p, q) = \{(c, d) \mid \exists \bar{p}, \bar{q} \in M_{state}, (\bar{p}\mathcal{R}\bar{q}) \wedge (((p, c), (\bar{p}, 0)) \in R^1(M, p, \bar{p})) \wedge (((\bar{q}, 0), (q, d)) \in R^1(M, \bar{q}, q))\}$, which is expressible in Presburger Arithmetic.

■

Lemma 3.4 *A one-counter machine M (with initial state q_0) has an infinite non-ss computation iff one of the following holds:*

- (1) *there exists a configuration (q, r) such that $(q, r) \xrightarrow{+} (q, r)$ and $(q, r) \notin R(M, (q_0, 0))$,*
- (2) *there exist a configuration (q, r) and a short positive loop l starting from state q such that $(q, r) \xrightarrow{l} (q, r')$, $r' > r$, and $\forall j \geq 0, (q, r + j * (r' - r)) \notin R(M, (q_0, 0))$. (Notice that test-for-zero transitions are absent in l .)*

Proof: The if ($\Leftarrow$) part is rather obvious; now we show the only if ($\Rightarrow$) part.

Suppose $\sigma : d_1 \rightarrow d_2 \rightarrow \dots \rightarrow d_i \rightarrow \dots$ is a non-ss computation of infinite length. Then one of the following must hold:

- there exist indices i, j ($i < j$) such that $d_i = d_j$, or
- the counter grows unboundedly and there exists an i such that the counter never becomes zero after $d_i (= (q, r))$ and there exists a short positive loop $l : q \xrightarrow{l} q$ (in M 's transition diagram) appears infinitely often in σ .

If (A) is the case, then condition (1) follows immediately. Consider case (B). Let $(q, r) \xrightarrow{l} (q, r + d)$, for some $d > 0$. We claim that $(q, r) \xrightarrow{l} (q, r + d) \xrightarrow{l} (q, r + 2d) \dots \xrightarrow{l} (q, r + 3d) \dots$ constitutes an infinite non-ss computation. If this is not the case, there must exist an integer $j > 0$ such that $(q, r + j * d) \in R(M, (q_0, 0))$. Consider Figure 2, where d_h is the configuration such that l appears j times in $d_i \xrightarrow{*} d_h$. Clearly, $(q, r + j * d) \xrightarrow{\sigma_1 \sigma_2 \dots \sigma_j} d_h$ is executable (for the test-for-zero operation is never used after d_i), which contradicts the assumption that d_h be on a non-ss computation σ . ■

Figure 2. A short positive loop witnessing non-ss.

Theorem 3.5 *The ss problem for one-counter machines is decidable.*

Proof: The proof is carried out through the use of the unified strategy stated in Theorem 3.1. First consider Condition (1) of Theorem 3.1. For a one-counter machine M , Lemma 3.3 establishes the effective semilinearity of $REACH(M, q, q')$, $\forall q, q' \in M_{state}$. $SUCC(M, q, q')$ is trivially semilinear, for the finiteness of M_{tr} . $\forall q \in M_{state}$, an $r \in \mathbb{N}$ is in $DEAD(M, q)$ iff the following hold: (i) $(\forall q' \in M_{state}) (\forall n \geq 0) ((q, q', n) \notin M_{tr})$, (ii) $(r = 0)$ implies $(\forall q' \in M_{state}) ((q, q', 0) \notin M_{tr})$, and (iii) $(r \geq 0)$ implies $(\forall q' \in M_{state}) (\forall n, r > n \geq 0) ((q, q', -n) \notin M_{tr})$; hence, $DEAD(M, q)$ is clearly effectively semilinear. Condition (2) of Theorem 3.1 is established in Lemma 3.4. In view of the above, the ss problem is decidable. ■

3.2 Conflict-free Petri nets

A *Petri Net* (PN, for short) is a 3-tuple (P, T, φ) , where P is a finite set of *places*, T is a finite set of *transitions*, and φ is a *flow function* $\varphi : (P \times T) \cup (T \times P) \rightarrow \{0, 1\}$. A *marking* (or *configuration*) is a mapping $\mu : P \rightarrow \mathbb{N}$. A *marked PN* is a 2-tuple $(\mathcal{P}, \mu_0)$ where $\mathcal{P} = (P, T, \varphi)$ is a PN and μ_0 is a marking ($\mu_0 : P \rightarrow \mathbb{N}$) called the *initial marking*. A transition $t \in T$ is *enabled* at a marking μ iff for every $p \in P$, $\varphi(p, t) \leq \mu(p)$. A transition t may *fire* at a marking μ if

t is enabled at μ . We then write $\mu \xrightarrow{t} \mu'$, where $\mu'(p) = \mu(p) - \varphi(p, t) + \varphi(t, p)$ for all $p \in P$. A sequence of transitions $\sigma = t_1 \dots t_n$ is a *firing sequence* from μ iff $\mu \xrightarrow{t_1} \mu_1 \xrightarrow{t_2} \dots \xrightarrow{t_n} \mu_n$ for some sequence of markings $\mu_1, \dots, \mu_n$.

For ease of expression, the following notations will be used throughout the rest of this paper. Let σ, σ' be transition sequences, and t be a transition.

- $\#_\sigma(t)$ represents the number of occurrences of t in σ .
- $Tr(\sigma) = \{t \mid t \in T, \#_\sigma(t) > 0\}$, denoting the set of transitions used in σ .
- $\sigma \vdash \sigma'$ is defined inductively as follows. Suppose $\sigma' = t_1 \dots t_n$. Let σ_0 be σ . If t_i is in σ_{i-1} , let σ_i be σ_{i-1} with the leftmost occurrence of t_i deleted; otherwise, let $\sigma_i = \sigma_{i-1}$. Finally, let $\sigma \vdash \sigma' = \sigma_n$. For example, if $\sigma = t_1 t_2 t_3 t_4 t_5$ and $\sigma' = t_4 t_3 t_1$, then $\sigma \vdash \sigma' = t_2 t_5$.

For firing sequences σ and σ' , σ' is said to be a *rearrangement* of σ if $\#_\sigma(t) = \#_{\sigma'}(t), \forall t \in T$.

To simplify our notations, $REACH(\mathcal{P}, q, q')$, $REACH^+(\mathcal{P}, q, q')$, $Rreach(\mathcal{P}, q, q')$, $SUCC(\mathcal{P}, q, q')$ and $DEAD(\mathcal{P}, q)$ are abbreviated as $REACH(\mathcal{P})$, $REACH^+(\mathcal{P})$, $Reach(\mathcal{P})$, $SUCC(\mathcal{P})$, and $DEAD(\mathcal{P})$, respectively, since configurations (i.e., markings) of Petri nets belong to the set $\mathbb{N}^k$.

- *Conflict-free Petri nets:*

A PN $\mathcal{P} = (P, T, \varphi)$ is said to be *conflict-free* iff for every place p , either

1. $|p^\bullet| \leq 1$, or
2. $\forall t \in p^\bullet, t$ and p are on a self-loop (i.e., $t \in (p^\bullet \cap \bullet p)$),

where $p^\bullet = \{t \mid \varphi(p, t) > 0\}$ (resp., $\bullet p = \{t \mid \varphi(t, p) > 0\}$) represents the set of output (resp., input) transitions of place p . In words, a PN is conflict-free if every place which is an input of more than one transition is also an output of each such transition [9]. In a conflict-free PN, once a transition becomes enabled, the only way to disable the transition is to fire the transition itself. (That is, $\forall t, t' \in T, t \neq t', \mu \xrightarrow{t} \mu'$ and $\mu \xrightarrow{t'}$ implies $\mu' \xrightarrow{t'}$.)

To show the *ss* problem for conflict-free PNs to be decidable, we require the following lemmas.

Lemma 3.6 *Given a conflict-free PN $\mathcal{P} = (P, T, \varphi)$, we can construct in nondeterministic polynomial time a system of linear inequalities $\mathcal{L}(\mathcal{P}, \mu_0, \mu)$ in such a way that $\mu \in R(\mathcal{P}, \mu_0)$ iff $\mathcal{L}(\mathcal{P}, \mu_0, \mu)$ has an integer solution. Furthermore, $\mathcal{L}(\mathcal{P}, \mu_0, \mu)$ remains linear even if μ_0 and μ are replaced by variables. (The reader is referred to Lemma 4.3 in [8] for a detailed description of the system of linear inequalities associated with $\mathcal{L}(\mathcal{P}, \mu_0, \mu)$.)*

Proof: (Sketch) The semilinearity of the reachability set for conflict-free PNs was originally illustrated in [9]. Subsequently, it has been shown in [8] that, given a conflict-free PN $\mathcal{P} = (P, T, \varphi)$, reachability can be characterized as a system of linear inequalities $ILP(\mathcal{P})$ over variables $v_1, \dots, v_{|P|}, v'_1, \dots, v'_{|P|}, x_1, \dots, x_{|T|}$, among others, such that

- (1) the size of $ILP(\mathcal{P})$ is bounded by a polynomial in the size of $\mathcal{P}$,
- (2) given two markings μ and μ' , $\mu \xrightarrow{\sigma} \mu'$, for some sequence σ , if and only if $ILP(\mathcal{P})$ has an integer solution while assigning μ to $(v_1, \dots, v_{|P|})$ and μ' to $(v'_1, \dots, v'_{|P|})$, and
- (3) for each transition $t \in T$, the solution of variable x_t in $ILP(\mathcal{P})$ exactly equals the number of times transition t fires in σ .

As a result, $REACH(\mathcal{P})$ is clearly effectively semilinear.

Lemma 3.7 *Given a PN $\mathcal{P} = (P, T, \varphi)$, the set $DEAD(\mathcal{P})$ is effectively semilinear. (Here $\mathcal{P}$ can be a general Petri net (not necessarily conflict-free) for which the range of φ is $\mathbb{N}$.)*

Proof: Let $T = \{t_1, t_2, \dots, t_m\}$. We define $\mathcal{H} = \{\{p_1, p_2, \dots, p_h\} \subseteq P \mid \forall 1 \leq i \leq m, \exists 1 \leq j \leq h, \varphi(p_j, t_i) > 0\}$ (For each $H \in \mathcal{H}$ and $t_i \in T$, t_i has at least one input place in H .) $\mathcal{H}$ is clearly a finite set. We also let $\mu_H = \{\mu \in \mathbb{N}^k \mid \forall t_i \in T, \exists p_j \in H, \varphi(p_j, t_i) > \mu(p_j) \text{ and } \forall p' \notin H, \mu(p') = 0\}$, for a given $H \in \mathcal{H}$. Notice that μ_H is finite, which contains dead markings only. If μ' is a marking such that $(\mu'(p_j) = \mu(p_j), \forall 1 \leq j \leq h)$, and $(\mu'(p') \geq \mu(p'), \forall p' \notin H)$, then μ' is a dead marking as well. Hence the set of dead markings equals $\bigcup_{H \in \mathcal{H}} \bigcup_{\mu \in \mu_H} \{\mu' \mid (\mu' \geq \mu) \wedge (\mu'(p) = \mu(p), \forall p \in H)\}$, which is semilinear since $\mathcal{H}$ and μ_H range over finite sets.

Lemma 3.8 (From [12]) *For an arbitrary path $\mu \xrightarrow{\sigma} \bar{\mu}$ in a conflict-free PN $\mathcal{P} = (P, T, \varphi)$, σ can be rearranged*

into $\overbrace{\sigma_1 \dots \sigma_1}^{l_1} \overbrace{\sigma_2 \dots \sigma_2}^{l_2} \dots \overbrace{\sigma_d \dots \sigma_d}^{l_d}$, for some sequences $\sigma_1, \sigma_2, \dots, \sigma_d$ and integers $l_1, l_2, \dots, l_d, 1 \leq d \leq |T|$, such that

- (1). $(\forall 1 \leq i \leq d) (\forall t \in T) (\#_{\sigma_i}(t) \leq 1)$, and
- (2). $(\forall 1 \leq i \leq d-1) (Tr(\sigma_{i+1}) \subseteq Tr(\sigma_i))$.

The above lemma allows us to rearrange an arbitrary firing sequence in a conflict-free PN into a ‘canonical form’ satisfying conditions (1) and (2), in which (1) ensures that for each σ_i , no transition will appear more than once (hence, $|\sigma_i| \leq |T|$, for all i), and (2) says that $Tr(\sigma_1)Tr(\sigma_2) \cdots Tr(\sigma_d)$ forms a ‘shrinking’ sequence of sets in the sense that if a transition, say t , occurs in σ_i , for some i , then t is guaranteed to appear in $\sigma_j, \forall 1 \leq j \leq i$. Symmetrically, if t does not appear in σ_i , then it will never be found in $\sigma_j, \forall i \leq j \leq d$.

Lemma 3.9 *If an infinite non-ss computation exists in a conflict-free PN $\mathcal{P}=(P, T, \varphi)$ (with initial marking μ_0), then there exist markings μ, μ' and a ‘short’ sequence τ (i.e., $\#_{\tau}(t) \leq 1, \forall t \in T$) such that $\mu \xrightarrow{\tau} \mu', \mu' \geq \mu$, and $\mu \xrightarrow{\tau\tau\tau\cdots}$ is an infinite non-ss computation.*

Proof: Consider an arbitrary infinite non-ss computation δ from μ_1 . Suppose we consider the suffix computation δ' along which all the transitions appear infinitely many times. By applying Dickson’s Lemma [2] to δ' , δ can be written as $\delta_1\sigma\delta_2$ such that $\mu_1 \xrightarrow{\delta_1} \mu \xrightarrow{\sigma} \mu'' \xrightarrow{\delta_2}$ with $\mu'' \geq \mu$, and all the transitions in σ occur infinitely often in δ . Let $\overbrace{\sigma_1 \cdots \sigma_1}^{l_1} \overbrace{\sigma_2 \cdots \sigma_2}^{l_2} \cdots \overbrace{\sigma_d \cdots \sigma_d}^{l_d}$ be the canonical rearrangement of σ guaranteed by Lemma 3.8. Clearly σ_1 must be that if $\mu \xrightarrow{\sigma_1} \mu'$, then $\mu' \geq \mu$; otherwise, $\exists p, \mu''(p) < \mu(p)$ due to Condition (2) of Lemma 3.8. (To see this, the conflict-freeness property and (2) of Lemma 3.8, together with the fact that $\forall p \in P, t \in T, \varphi(p, t) \leq 1$, imply that no transitions will deposit tokens to a place p in any of the segments $\sigma_1, \dots, \sigma_d$ if $\mu'(p) < \mu(p)$.) Finally, since all the transitions in σ_1 occur infinitely often in the infinite non-ss computation, we claim that $\mu \xrightarrow{\sigma_1\sigma_1\sigma_1\cdots}$ constitutes an infinite non-ss computation. Suppose, in con-

trast, that for some $n, \mu \xrightarrow{\overbrace{\sigma_1 \cdots \sigma_1}^n} \mu_n$ and $\mu_n \in R(\mathcal{P}, \mu_0)$. Let $\pi = \overbrace{\sigma_1 \cdots \sigma_1}^n$. Let δ_3 be a prefix of $\sigma\delta_2$ such that $\#_{\pi}(t) \leq \#_{\delta_3}(t), \forall t \in T$ (i.e., all the transitions of π occur in δ_3). Assume that $\mu \xrightarrow{\delta_3} \bar{\mu}$, for some $\bar{\mu} \notin R(\mathcal{P}, \mu_0)$. Due to the conflict-freeness property of $\mathcal{P}$, $\mu \xrightarrow{\pi} \mu, \mu \xrightarrow{\delta_3}$, and $(\#_{\pi}(t) \leq \#_{\delta_3}(t), \forall t \in T)$ imply the existence of a δ_4 such that $\pi\delta_4$ is a permutation of δ_3 , and $\mu \xrightarrow{\pi} \mu_n \xrightarrow{\delta_4} \bar{\mu}$ (see [9]), contradicting the assumption that $\bar{\mu} \notin R(\mathcal{P}, \mu_0)$. By letting $\tau = \sigma_1$, our result follows. ■

We are now in a position to show the ss problem to be decidable.

Theorem 3.10 *The ss problem for conflict-free PNs is decidable.*

Proof: Again, the proof is done through the use of Theorem 3.1. The semilinearity of $REACH(\mathcal{P})$ and $DEAD(\mathcal{P})$ follows from Lemmas 3.6 and 3.7, respectively. T being finite clearly implies the semilinearity of $SUCC(\mathcal{P})$. As a consequence, Condition (1) of Theorem 3.1 holds. Finally, Condition (2) of Theorem 3.1 follows immediately from Lemma 3.9. This completes the proof of the theorem. ■

4 Fair self-stabilization

Given a PN $\mathcal{P}$ with initial marking μ_0 , an infinite computation $\mu_1 \xrightarrow{t_1} \mu_2 \xrightarrow{t_2} \cdots \mu_i \xrightarrow{t_i} \mu_{i+1} \cdots$ is said to be *fair* if for every transition t , if t is enabled at infinitely many μ_{i_l} ($l \geq 1$), then there exist infinitely many j_l ($l \geq 1$) such that $t_{j_l} = t$. (In words, if a transition is enabled infinitely many times, then the transition must occur infinitely often as well.) A computation σ from marking μ_1 is said to be *fair non-self-stabilizing* iff one of the following holds:

- (1) $\sigma(\mu_1 \rightarrow \mu_2 \rightarrow \cdots \rightarrow \mu_m)$, for some m is finite such that μ_m is a ‘dead’ marking (i.e., μ_m has no immediate successor in $\mathcal{P}$) and $\mu_m \notin R(\mathcal{P}, \mu_0)$, or
- (2) $\sigma(\mu_1 \rightarrow \mu_2 \rightarrow \cdots \rightarrow \mu_i \rightarrow \cdots)$ is infinite and fair such that $\forall i \geq 1, \mu_i \notin R(\mathcal{P}, \mu_0)$.

We let $FSS(\mathcal{P}, \mu_0) = \{\mu \mid \text{none of the computations emanating from } \mu \text{ is fair non-self-stabilizing}\}$. The *fair self-stabilization problem* is that of deciding whether starting from an arbitrary marking μ , none of its computations is *fair non-self-stabilizing*, i.e., $FSS(\mathcal{P}, \mu_0) = N^k$.

Given a set of markings S , the *successor* (resp., *predecessor*) of S , written as $succ(S)$ (resp., $pred(S)$), is the set $\{\mu \mid \exists t \in T, \mu' \in S, \mu' \xrightarrow{t} \mu\}$ (resp., $\{\mu \mid \exists t \in T, \mu' \in S, \mu \xrightarrow{t} \mu'\}$). Let $pred^*$ (resp., $succ^*$) be the reflexive and transitive closure of $pred$ (resp., $succ$). (That is, $succ^*(S) = \{\mu \mid \exists \mu' \in S, \mu' \xrightarrow{*} \mu\}$, and $pred^*(S) = \{\mu \mid \exists \mu' \in S, \mu \xrightarrow{*} \mu'\}$.) The sets $succ^*(S)$ and $pred^*(S)$ will be referred to as the *forward reachability set* and the *backward reachability set* of S , respectively. Notice that $R(\mathcal{P}, \mu_0) = succ^*({\mu_0})$. A set of markings S is said to be *forward-closed* if $\forall \mu \in S, \forall t \in T, \mu \xrightarrow{t} \mu'$ implies $\mu' \in S$.

Given a PN $\mathcal{P}=(P, T, \varphi)$, a *potential function* f is a mapping $N^k \rightarrow N \cup \{\infty\}$, i.e., f maps each marking μ to a number in $N \cup \{\infty\}$. (The value of $f(\mu)$ is called the *potential* of marking μ .) A potential function f is said to be *monotone* if for every marking μ , if $\mu \xrightarrow{t} \mu'$ (for some marking μ' and transition t), then $f(\mu) \geq f(\mu')$.

Given a conflict-free PN $\mathcal{P}=(P, T, \varphi)$ and a set of markings S , the following potential function f will be used throughout the rest of this paper: $f(\mu)$ is defined to be the

length of the shortest path from μ to a marking in S ; if μ cannot reach S , $f(\mu)$ is ∞ . Notice that $\forall \mu \in S, f(\mu) = 0$ (i.e., S defines the set of markings of zero potential). What follows is another way to view such a potential function. We partition N^k into a sequence of disjoint sets of markings $U_0, U_1, \dots, U_\infty$ such that

$$\begin{aligned} U_0 &= S \\ U_1 &= (\text{pred}(U_0)) - U_0 \\ &\dots \\ U_i &= (\text{pred}(U_{i-1})) - (\cup_{j=0, \dots, i-1} U_j), i \geq 1 \\ U_\infty &= N^k - (\cup_{j \geq 0} U_j) \end{aligned}$$

It is not hard to see that $f(\mu) = i$ iff $\mu \in U_i$.

Lemma 4.1 *Given a conflict-free PN $\mathcal{P}$ and a forward-closed set S , let f be the potential function based upon the shortest path criterion defined above. The following hold:*

- (1) f is always monotone.
- (2) For an arbitrary μ and a path $\mu \xrightarrow{\delta} \mu_1$, if $\mu \xrightarrow{\sigma} \mu'$ ($\mu' \in S$) is one of the shortest paths reaching S and $\text{Tr}(\delta) \cap \text{Tr}(\sigma) \neq \emptyset$, then $f(\mu_1) < f(\mu)$.

Proof: We first consider (1). It suffices to show that $\mu \xrightarrow{t} \mu_1$ implies $f(\mu) \geq f(\mu_1)$. Consider the following cases:

- (a) ($f(\mu) = \infty$): In this case, $f(\mu_1) = \infty$; otherwise, $\mu \xrightarrow{t} \mu_1 \xrightarrow{\sigma} \mu''$, $\mu'' \in S$, violating the assumption that $f(\mu) = \infty$.
- (b) ($0 < f(\mu) < \infty$): Let $\mu \xrightarrow{\sigma} \mu'$ ($\mu' \in S$) be one of the shortest paths reaching S . Consider two cases:
 - (i) $t \notin \text{Tr}(\sigma)$. Since the PN is conflict-free, t is enabled at μ' . Suppose $\mu' \xrightarrow{t} \mu''$, for some $\mu'' \in S$ (since S is forward-closed). Clearly, $\mu_1 \xrightarrow{\sigma} \mu''$, yielding $f(\mu_1) \leq f(\mu)$.
 - (ii) $t \in \text{Tr}(\sigma)$. Suppose $\mu \xrightarrow{\sigma_1} \mu_2 \xrightarrow{t} \mu_3 \xrightarrow{\sigma_2} \mu'$ (for some μ_2, μ_3), where $\sigma = \sigma_1 t \sigma_2$ and t is not in σ_1 . Again due to the conflict-freeness property of the PN, $\mu_1 \xrightarrow{\sigma_1} \mu_3 \xrightarrow{\sigma_2} \mu'$; hence, $f(\mu_1) \leq f(\mu) - 1$.

Figure 3 illustrates the monotone property of the potential function for paths in a conflict-free PN.

Now consider (2). Let $\sigma = \sigma_1 t \sigma_2$, where t is the first occurrence of a transition that is also in δ . Suppose $\delta = \delta_1 t \delta_2$ ($t \notin \delta_1$), and $\mu \xrightarrow{\delta_1} \mu_2 \xrightarrow{t} \mu_3 \xrightarrow{\delta_2} \mu_1$. (Notice that the selection of t ensures that $\text{Tr}(\delta_1) \cap \text{Tr}(\sigma) = \emptyset$.) It is not hard to see that σ_1 is enabled at μ_3 , since $\text{Tr}(\sigma_1) \cap$

Figure 3. Paths and the associated potentials.

$\text{Tr}(\delta_1 t) = \emptyset$ and PN $\mathcal{P}$ is conflict-free. We immediately have $\mu_3 \xrightarrow{\sigma_1 \sigma_2} \mu_4$, for some $\mu_4 \in S$ (since $\mu' \xrightarrow{\delta} \mu_4$ and S is forward-closed). Hence, $f(\mu_3) \leq |\sigma_1 \sigma_2| = f(\mu) - 1$. See Figures 3 and 4. Using the result of (1), (2) follows. ■

Figure 4. Proof of Lemma 4.1.

Theorem 4.2 *Given a conflict-free PN $\mathcal{P}=(P,T,\varphi)$ and an initial marking μ_0 , $\mu \in FSS(\mathcal{P}, \mu_0)$ iff $\mu \in \text{pred}^*(R(\mathcal{P}, \mu_0))$.*

Proof: Let S be $R(\mathcal{P}, \mu_0)$, which is clearly forward-closed. We let the potentials of those markings in S be zero. If $\mu \notin \text{pred}^*(S)$, none of the (finite or infinite) computations can reach $R(S)$; hence $\mu \notin FSS(S)$.

Suppose $\mu \in \text{pred}^*(S)$. If μ has a fair (infinite) path which never reaches S , then there must exist a marking μ_1 and sequences σ_1 and σ_2 such that $\mu \xrightarrow{\sigma_1} \mu_1 \xrightarrow{\sigma_2}$, $f(\mu) \geq f(\mu_1) > 0$ and $\mu_1 \xrightarrow{\sigma_2}$ is fair along which the potential never drops. Let $\mu_1 \xrightarrow{t_1 t_2 \dots t_i} \mu'$ ($\mu' \in S$) be one of the shortest paths reaching some marking in S . The fairness assumption

ensures that $t_1 \in Tr(\sigma_2)$; otherwise, t_1 would have been enabled infinitely often without being fired. By Lemma 4.1, the potential along σ_2 eventually drops – a contradiction. As a consequence, from μ all infinite computations avoiding S are unfair. Using a similar argument, we can show that if $\mu \xrightarrow{*} \mu'$ and μ' is a dead marking, then $\mu' \in S$. ■

According to Lemma 3.6, checking reachability for conflict-free PNs can be equated with solving the integer linear programming problem, which is known to be in NP. It is important to point out that, as μ_0 and μ in Lemma 3.6 can be regarded as variables, the backward reachability set $pred^*(R(P, \mu_0))$ is readily expressible in terms of integer linear programming. Hence, the following is obvious.

Corollary 4.3 *The fair self-stabilization problem for conflict-free Petri nets is decidable.*

5 Conclusions and future research

We have studied, from the decidability viewpoint, the problem of determining whether a given system is self-stabilizing or not for infinite-state systems including *one-counter machines* and *conflict-free Petri nets*. The key behind our strategy is that for each of the three classes of systems, the reachability set from an arbitrary configuration is effectively semilinear, and it suffices to consider infinite non-self-stabilizing computations with periodic behaviors. It is interesting to see whether our strategy can be applied to a wider class of infinite-state systems, or a similar strategy (without relying on the reachability set being semilinear) can be extended to systems not enjoying the semilinearity property. An equally important direction of future research is to find out the exact complexity of the self-stabilization problem for the above classes of systems. We have also studied the fair version of self-stabilization, and a decidability result was shown for the class of conflict-free Petri nets.

Another issue that deserves further investigation is to consider the problem with respect to the notion of ‘self-stabilization with probability one.’ In spite of having some non-self-stabilizing computations, in practice a system might be considered ‘fault-tolerant’ if the probability of self-stabilization equals one. Finally, self-stabilization for real-time systems deserves further investigation, as many real-world systems are of real-time nature. For the model of *timed automata*, the *region graph* technique can easily be applied to showing the decidability (in fact, in PSPACE) of self-stabilization (in the sense defined in this paper). However, to cope with the nature of real-time systems, one might have to tailor the notion of self-stabilization to better capture the intuitive concept of ‘self-stabilizing in a certain amount of time,’ as opposed to ‘reaching a legitimate configuration eventually’ as defined in the conventional sense.

References

- [1] Cherkasova, L., Howell, R. and Rosier, L. Bounded self-stabilizing Petri nets, *Acta Informat.*, 32:189-207, 1995.
- [2] Dickson, L., Finiteness of the odd perfect and primitive abundant numbers with n distinct prime factors, *Amer. J. Math.*, 35:413-422, 1913.
- [3] Dijkstra, E. Self-stabilizing systems in spite of distributed control, *C. ACM*, 17, pp. 643-644, 1974.
- [4] Ginsburg, S., *The mathematical theory of context-free languages*, McGraw-Hill, 1966.
- [5] Gouda, M., Howell, R. and Rosier, L. The instability of self-stabilization, *Acta Informat.*, 27, pp. 697-724, 1990.
- [6] Herman, T., A comprehensive bibliography on self-stabilization, *Chicago Journal of Theoretical Computer Science*, (<http://www.cs.uiowa.edu/ftp/selfstab/bibliography>), 1998.
- [7] Hopcroft, J. and Pansiot, J., On the reachability problem for 5-dimensional vector addition systems, *Theoret. Comput. Sci.*, 8(2):135-159, 1979.
- [8] Howell, R. and Rosier, L. and H. Yen, Normal and sinkless Petri nets, *Journal of Computer and System Sciences*, 46:1-26, 1993.
- [9] Landweber, L. and Robertson, E., Properties of conflict-free and persistent Petri nets, *J. Assoc. Comput. Mach.*, 25:352-364, 1978.
- [10] Presburger, M., Über die vollständigkeit eines gewissen systems der arithmetik ..., *Comptes rendues du premier Congres des Math. des Pays Slaves*, Warsaw, 92-101, 395, 1929.
- [11] Rosier, L. and Yen, H., Logspace hierarchies, polynomial time and the complexity of fairness problems concerning ω -machines. *SIAM J. Computing*, 16(5):779-807, 1987.
- [12] Yen, H., Wang, B. and Yang, M., Deciding a class of path formulas for conflict-free Petri nets, *Theory of Computing Systems*, 30(5):475-494, 1997.