Staircase-Approximation Time-Domain Method for Transient Analysis of Lossy Nonuniform Transmission Lines Using ABCD Matrices

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Abstract

The staircase-approximation time-domain (SATD) method is applied to transient analysis of nonuniform transmission lines (NTL's) by employing cascaded uniform transmission line (UTL) sections and ABCD matrices. Numerical results to exponential nonuniform transmission lines (ENTL's) are verified with analytical formulae, both in time-harmonic and SATD forms, and the CAD tool PSPICE. This method also works well even for a lossy ENTL under the excitation of trapezoidal pulse chains.

Introduction

Nonuniform transmission lines (NTL's) have a large variety of applications, including wide-band impedance matching, reduction of discontinuity effects and pulse shaping. Unfortunately, exact analysis of NTL's is usually unavailable. Although there do exist analytical solutions to some specific NTL's like [1], their implementations are quite difficult, if not impossible, which degrades their practicability. The treatment of NTL's, therefore, is mostly resorted to numerical approaches. One popular treatment, also used in this paper, is to make use of cascaded uniform transmission line (UTL) sections as well as ABCD matrices.

A new technique, the staircase-approximation time-domain (SATD) method, has been proposed and its capability of handling transients of transmission lines has been demonstrated [2][3]. Conventionally, the inverse Fourier transform (IFT) is used to

obtain the transients from the frequency-domain results. This approach is inefficient when long transients or broad spectra have to be considered. The SATD is more efficient, even when loss is introduced and arbitrary waveforms are excited. In addition, our previous works show the duality between the time- harmonic and the SATD formulae. Thus, analytic forms of specific exponential nonuniform transmission lines (ENTL's) in [1] can be directly translated into the SATD forms without reformulation. For more general cases, we can still analyze the transients of an ENTL by cascaded UTL sections using the SATD technique.

Formulation

By staircase-approximation time-domain (SATD), signals are expressed as

$$v(x, t) \approx \sum_{j=0}^n v_j(x) h_j(t) \quad (1)$$

$$i(x, t) \approx \sum_{j=0}^n i_j(x) h_j(t) \quad (2)$$

where $h_j(t)$ is the unit rectangular pulse defined as

$$h_j(t) = \begin{cases} 1, & j\Delta t \leq t < (j+1)\Delta t \\ 0, & \text{others} \end{cases} \quad (3)$$

and the original transmission line equations can be converted into matrix forms [2][3]. The equivalent ABCD matrix of a section of UTL with length ℓ in the SATD form can be derived as

$$\begin{bmatrix} [A] & [B] \\ [C] & [D] \end{bmatrix} = \begin{bmatrix} \cosh([P]\ell) & [Z_0] \sinh([P]\ell) \\ [Z_0]^{-1} \sinh([P]\ell) & \cosh([P]\ell) \end{bmatrix} \quad (4)$$

where $[P]$ is the propagation matrix and $[Z_0]$ is the characteristic impedance matrix. To deal with an NTL, we divide it into sections of UTL's using the procedures shown in Fig. 1. The equivalent ABCD matrix of an NTL thus can be obtained by

$$\begin{bmatrix} [A] & [B] \\ [C] & [D] \end{bmatrix} = \prod_{k=1}^n \begin{bmatrix} [A_k] & [B_k] \\ [C_k] & [D_k] \end{bmatrix} \quad (5)$$

and the original NTL circuit can be analyzed easily.

Numerical Examples

As the first example, the transients of a lossless ENTL, as shown in Fig. 2, excited by a unit trapezoidal pulse defined in Fig. 3, is analyzed. The ENTL has parameters

$L(x) = 0.5 \cdot 3^{x/\ell} (\mu H / m)$ and $C(x) = 0.2 \cdot 3^{-x/\ell} (nF / m)$, which is equivalent to the characteristic impedance $Z_0(x) = 50 \cdot 3^{x/\ell} (\Omega)$. The exact solution to this ENTL can be expressed by the ABCD-matrix [1] as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} e^{-\delta\ell/2} (\Gamma \cosh \Gamma \ell + \frac{\delta}{2} \sinh \Gamma \ell) & Z_{01} \gamma e^{\delta\ell/2} \sinh \Gamma \ell \\ \frac{\gamma e^{\delta\ell/2} \sinh \Gamma \ell}{Z_{01}} & e^{-\delta\ell/2} (\Gamma \cosh \Gamma \ell - \frac{\delta}{2} \sinh \Gamma \ell) \end{bmatrix} \quad (6)$$

where $\delta = (1/\ell) \ln Z_{02}/Z_{01}$ ($Z_{01} = 50(\Omega)$ and $Z_{02} = 150(\Omega)$ in this example),

$\Gamma = \sqrt{\gamma^2 + \delta^2/4}$, and $\gamma = j\beta$, the position-independent propagation constant. The corresponding ABCD matrix in SATD form can be obtained by suitable substitution [3]. Figure 4 shows the transients of the voltage at the load, including the exact solution (6) of the ENTL in both time-harmonic (phasor) and SATD forms, the numerical solutions using 20 sections of UTL's in both time-harmonic (phasor) and SATD forms and the CAD tool PSPICE. These five different approaches show nice agreement. This validates the applicability of UTL approximation for an NTL and the duality between the time-harmonic and the SATD formulation.

As the second example, we analyze the circuit in Fig. 2, but adopt another set of parameters for the ENTL that $L = 0.5(\mu H / m)$ and $C(x) = 0.2 \cdot 9^{-x/\ell} (nF / m)$. The analytic form (6) can't be applied due to the position-dependent propagation constant. In addition, the voltage generator excites a pulse chain of unit trapezoidal pulses, shown in Fig. 3, of a $10(ps)$ period. By the approximation of 20 cascaded UTL sections, the voltage transients at the load in both lossless and lossy ($R = 5(\Omega/m)$ and $G = 10(S/m)$) cases are exhibited in Fig. 5 for comparison. The loss contributes to lower voltage levels, which stands to reason. All results also agree quite well.

Conclusions

The capability of handling transients of NTL's using the SATD has been demonstrated. Although only the ENTL is verified here, other NTL's could be treated likewise. This approach provides a simple but efficient way to get the transients, even for dispersive NTL's with nonlinear loads, which will be reported in our next paper.

References

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Fig. 1 Procedures to the equivalent ABCD matrix of a NTL.

Fig. 2 A terminated NTL circuit.

Fig. 3 Unit trapezoidal pulse.

Fig. 4 Results of Example 1.

Fig. 5 Results of Example 2.