Nonlinear Adaptive Backstepping Motion Control of Linear Induction Motor

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Abstract

In this paper, we will propose a nonlinear adaptive controller and an adaptive backstepping controller for linear induction motor(LIM) to achieve speed/position tracking. A nonlinear transformation is proposed to facilitate controller design. Besides, the very unique end effect of the LIM is also considered and is well taken care of in our controller design. Stability analysis based on Lyapunov theory is also performed to guarantee that the controller design here is stable. Also, the computer simulations and experiments are done to demonstrate the performance of our various controller design.

Keyword: Nonlinear Adaptive control, Motion Control, Linear Induction motor, Adaptive Backstepping control

Nomenclature

$V_a(V_{ab})$	q-(d-) axis input stator voltage	$i_q(i_d)$	q-(d-) axis input stator current
$R_s(R_r)$	Primary (secondary) resistance	$L_s(L_r)$	Primary (secondary) inductance
$\lambda_r(\lambda_{br})$	q-(d-) axis rotor flux	v_r	Linear speed of the primary
p_r	Position of the primary	M_m	Primary mass
B	Viscous friction coefficient	F_e	Electromagnetic force
P	Mechanical load force	L_m	Mutual inductance
K_f	Force constant($=3PL_m\pi/2\tau L_r$) $a_2=\beta R_r/L_r, a_3=L_m R_r/L_r, a_4=R_r/L_r$		
$D=L_s L_r, L_m^2 p=P\frac{\pi}{\tau}, \beta=L_m/D, c=L_r/D, \alpha=R_r L_r/D+\beta L_m R_r/L_r$			

I. INTRODUCTION

Nowadays, LIMs are now widely used in many industrial applications including transportation, conveyor systems, actuators, material handling, pumping of liquid metal, and sliding door closers, etc. with satisfactory performance. The most obvious advantage of linear motor is that it has no gears and requires no mechanical rotary-to-linear converters. The linear electric motors can be classified into the following: D.C. motors, induction motors, synchronous motors and stepping motors, etc. Among these, the LIM has many advantages such as simple structure replacement of the gear between motor and motion devices, reduction of mechanical losses and the size of motion devices, silence, high starting thrust force, and easy maintenance, repairing and replacement.

In the early works, Yamamura has first discovered a particular phenomenon of the end effect on LIM [1]. A control method, decoupling the control of thrust and the attractive force of a LIM using a space vector control inverter, was presented in [2], i.e. by selecting voltage

vectors of PWM inverters appropriately.

Although the parameters of the simplified equivalent circuit model of an LIM can be measured by conventional methods (no-load and locked secondary tests), due to limited length of the machine the realization of the no-load test is almost impossible. Thus, the applicability of conventional methods for calculating the parameters of the equivalent model is limited. In order to measure the parameters, application of the finite element (FE) method for determining the parameters of a two-axis model of a three-phase LIM has been proposed in [3]. Another method is proposed by removing the secondary [4].

To resolve the unique end effect problem, speed dependent scaling factors are introduced to the magnetizing inductance and series resistance in the d-axis equivalent circuit of the rotary induction motor (RIM) [5], to correct the deviation caused by the "end effect". On the other hand, there is a thrust correction coefficient introduced by [6,7] to calculate an actual thrust to compensate for the end effect. A related method to deal with the problem is that an external force corresponding to the end effect is introduced into the RIM model to provide a more accurate modeling of an LIM under consideration of end effect as shown in [8]. In another work [9], extra compensating-winding was proposed to compensate such problem.

Although the end effect is an important issue of the LIM control, but there are still many works in the literature without considering it, such as [10-16]. In this paper, we will take this as an important issue which can not be ignored.

The paper is organized as follows. In section 2, we introduce the system model and state the control objectives. Section 3 will develop a nonlinear adaptive control of the motor presuming the acceleration signal is available. However, in Section 4, we propose an adaptive backstepping approach without read of the acceleration signal. To demonstrate the effectiveness of the hereby developed controllers, Section 5 will provide thorough Experimental Results with satisfactory performance. Finally, Section 6 gives some concluding remarks.

II. PROBLEM FORMULATION

To formulate the dynamic model of a LIM as shown in Fig 2.1, we consider the following assumptions to simplify the analysis:

- (A.1) Three phases are balanced;
- (A.2) The magnetic circuit is unsaturated;
- (A.3) It is without end effect (we will relax this assumption

later in controller design),

Fig 2.1: The LIM configuration

then the dynamics of the entire system can be rearranged into the following more compact form

$$\begin{aligned} \dot{i}_q &= -a_1 i_q + a_2 \lambda_q - \beta p v_r \lambda_d + c V_{qs} \\ \dot{i}_d &= -a_1 i_d + \beta p v_r \lambda_q + a_2 \lambda_d + c V_{ds} \\ \dot{\lambda}_q &= a_3 i_q - a_4 \lambda_q + p v_r \lambda_d \\ \dot{\lambda}_d &= a_3 i_d - a_4 \lambda_d - p v_r \lambda_q \\ M_m \dot{v}_r &= K_f (\lambda_d i_q - \lambda_q i_d) - F_L' \end{aligned} \quad (1)$$

In this paper, we try to design the speed and position controller for the LIM. All the parameters are assumed known except the payload. The only information about the payload is its structure, and we use a second-order equation to represent it, i.e., the payload is expressed in terms of

$$F_L' = M_L \ddot{v}_r + b_{L0} \dot{v}_r + b_{L1} v_r + b_{L2} v_r^2. \quad (2)$$

III. OBSERVER AND NONLINEAR ADAPTIVE CONTROLLER DESIGN

In this section, the controller which achieves position or speed tracking of the LIM is proposed. In order to relax the need of flux measurement, the flux observer will be proposed in subsection 3.2. The controller can overcome the unknown payload of the LIM under the reasonable assumptions. In subsection 3.3, we will propose a position controller to achieve the objective of position tracking. An adaptive position controller is proposed to deal with understanding of the mutual inductance. In subsection 3.4, the uncertainty inductance is also considered in our controller design.

3.1 Analysis of mechanical load and end effect

The fundamental difference between a rotary induction motor and a LIM is the finite length of the magnetic and electric circuit of the LIM along the direction of the travelling field. The open magnetic circuit causes an initiation of the so-called longitudinal end effects.

In a LIM, as the primary moves, the secondary is continuously replaced by a new material. This new material will tend to resist a sudden increase in flux penetration and only allow a gradual build up of the flux density in the air gap. As the primary coil set of the LIM moves, a new field penetrates into the reaction rail in the entry area, whereas the existing field disappears at the exit area of the primary core as shown below:

Fig 3.1: Airgap average flux distribution due to end effect [5]

We should note that when the speed is higher, the air-gap flux is more unbalanced. Because the mutual flux between the primary and the secondary is reduced by the end effect, we can see that the equivalence of the end effect is a reduction force, which is a function of speed. As we know that most functions can be described in Taylor series reasonably, we hence can assume that the end effect can be regarded as an external force which may be expressed as

$\sum_{n=0}^{\infty} b_n' v_r^n + M_e \dot{v}_r$. In the paper, we will truncate the series into the first three terms.

For a LIM, the end effect with the load force can be represented as a function of the speed v_r , which can be normally simplified into the form

$$\begin{aligned} F_L &= \sum_{n=0}^2 b_n' v_r^n + M_e \dot{v}_r + F_L' \\ &= M_e \dot{v}_r + b_0 + b_1 v_r + b_2 v_r^2 + M_L' \dot{v}_r + b_{L0} + b_{L1} v_r + b_{L2} v_r^2 \\ &\triangleq M_L \ddot{v}_r + b_0 + b_1 v_r + b_2 v_r^2 \end{aligned}$$

In this paper, the mechanical load with end effect is assumed in the aforementioned form as $F_L = \Theta V_r^T$ with the unknown

constant parameters $\Theta = [M_L \ b_0 \ b_1 \ b_2]$, and a known function vector $V_r^T = [\dot{v}_r \ v_r^0 \ v_r^1 \ v_r^2]$. The joint mass

$M = M_m + M_L$ is therefore also unknown, which leads to the total mechanical load with motor itself as $F = \Theta^T V_r$, where $\Theta^T = [M \ b_0 \ b_1 \ b_2]$.

To proceed further, we introduce some more assumption as shown below:

$$(A.4) \ x_2 = \lambda_q^2 + \lambda_d^2 > 0,$$

(A.5) The desired speed should be a bounded smooth function with known first and second order time derivatives, then further simplify the dynamics shown in (1) by introducing a nonlinear coordinate transformation given as follows:

$$\begin{aligned} x_1 &= i_q^2 + i_d^2 \\ x_2 &= \lambda_q^2 + \lambda_d^2 \\ x_3 &= i_q \lambda_q + i_d \lambda_d \\ x_4 &= i_q \lambda_d - i_d \lambda_q \\ x_5 &= v_r \end{aligned}$$

Remarks: The transformation is trying to make the secondary flux norm, the electric force and the rotor speed as individual variables x_2 , x_4 and x_5 , respectively, and certainly the nonlinear transformation is not unique. Initially, we adopt the stator voltage inputs as

$$c V_{ds} = \frac{-\lambda_q}{\sqrt{\lambda_d^2 + \lambda_q^2}} V, c V_{qs} = \frac{\lambda_d}{\sqrt{\lambda_d^2 + \lambda_q^2}} V \quad [17], \text{ with such}$$

transformation, then the dynamical equations shown in (1) can thus be transformed into the following dynamic model:

$$\begin{aligned}
\dot{x}_1 &= -2a_1x_1 + 2a_2x_3 + (2x_4/\sqrt{x_2})V \\
\dot{x}_2 &= -2a_4x_2 + 2a_3x_3 \\
\dot{x}_3 &= a_3x_1 + a_2x_2 - (a_1 + a_4)x_3 + px_5x_4 \\
\dot{x}_4 &= -px_5x_3 - \beta px_5x_2 - (a_1 + a_4)x_4 + \sqrt{x_2}V \\
M\dot{x}_5 &= K_f x_4 - \sum_{n=0}^2 b_n x_5^n
\end{aligned} \quad (3)$$

To control the system (3) we develop the position controller to achieve the goal $p_r \rightarrow p_d$ as introduced in the following section.

3.2 Flux Observer Design and Analysis

In our assumptions, all the states are measurable and known except the rotor flux. Therefore, we have to build a set of observers to estimate the rotor flux. The observers then can be designed as follows :

$$\begin{aligned}
\dot{\hat{\lambda}}_q &= \frac{L_m}{L_r} R_r i_q - \frac{1}{L_r} R_r \hat{\lambda}_q + p v_r \hat{\lambda}_d \\
\dot{\hat{\lambda}}_d &= \frac{L_m}{L_r} R_r i_d - \frac{1}{L_r} R_r \hat{\lambda}_d - p v_r \hat{\lambda}_q
\end{aligned}$$

Theorem 1 If the dynamic equations are described as in system (3) with all the states are measurable and known except the rotor flux. And, all the parameters are known. Then, the flux observers designed can guarantee that $\hat{\lambda}_d - \lambda_d \rightarrow 0$ and $\hat{\lambda}_q - \lambda_q \rightarrow 0$ as $t \rightarrow \infty$.

Proof:

Let the observation errors be denoted as

$$e_d = \hat{\lambda}_d - \lambda_d \text{ and } e_q = \hat{\lambda}_q - \lambda_q.$$

Choose the Lyapunov function candidate to be a quadratic form as

$$V_\lambda = \frac{1}{2}[e_d^2 + e_q^2].$$

Thus, the time derivative of V_λ can be evaluated as $\dot{V}_\lambda = -(L_m/L_r) R_r [e_d^2 + e_q^2]$. Then, by straightforward derivations and application of Lyapunov's theory, we can guarantee that $\hat{\lambda}_d - \lambda_d \rightarrow 0$ and $\hat{\lambda}_q - \lambda_q \rightarrow 0$ as $t \rightarrow \infty$, and thus conclude Theorem 1. $\square$

Since the proposed flux observer can provide accurate flux estimate asymptotically, in the following context we will regard the flux estimate as the true flux and make no distinction between them.

3.3 Adaptive Position Controller Design

Now, we introduce another state

$$x_6 = p_r \quad (4)$$

to facilitate investigation of the development of a position controller. Then, define the tracking errors as follows:

$$e_p = p_r - p_d \triangleq e_6 \quad (5)$$

Normally, while the position tracking error is driven to zero, the speed is also regulated to zero. Thus, we naturally define a joint error signal S as follows:

$$S = \dot{e}_p + a e_p = \dot{e}_5 + a e_6,$$

where a is a positive scalar gain, and note the case with $a=0$ will be degenerated back speed tracking problem.

Theorem 2 Consider an induction motor whose dynamics are governed by system (3) under the assumptions (A.4), and (A.5). Given a third-differentiable smooth desired position trajectory p_d with $p_d, \dot{p}_d, \ddot{p}_d$ and $\ddot{\ddot{p}}_d$ being all bounded, then the following control input can achieve the control objective $p_r \rightarrow p_d$ (i.e. $x_6 = p_r$ will follow p_d asymptotically) with the control input

$$V_\phi = \frac{\hat{\lambda}_d}{\sqrt{\lambda_q^2 + \lambda_d^2}} \frac{V}{c}, V_\psi = \frac{-\hat{\lambda}_q}{\sqrt{\lambda_q^2 + \lambda_d^2}} \frac{V}{c},$$

and

$$V = \frac{1}{\sqrt{x_2}} [(a_1 + a_4)x_4 + \beta p x_2 x_5 + p x_3 x_5 + \dot{x}_{4d} - \rho_2 e_4],$$

$$\text{where } x_{4d} = \frac{1}{K_f} [\hat{b}_0 + \hat{b}_1 x_5 + \hat{b}_2 x_5^2 + \hat{M}(\dot{v}_d - a e_5) - \rho_1 e_5]$$

with $\rho_1, \rho_2 > 0$, and $e_5 = x_5 - v_d$, $e_4 = x_4 - x_{4d}$, and the parameter adaptation laws:

$$\dot{\hat{b}}_0 = -S, \dot{\hat{b}}_1 = -S x_5, \dot{\hat{b}}_2 = -S x_5^2, \dot{\hat{M}} = -S(\dot{v}_d + a e_5),$$

while keeping all the internal signals bounded.

Proof:

In order to show the boundedness of all the parameter estimates and the tracking errors e_4, e_5 , we choose a Lyapunov like function V_e as shown below:

$$V_e = \frac{1}{2} [M S^2 + e_4^2 + (\hat{b}_0 - b_0)^2 + (\hat{b}_1 - b_1)^2 + (\hat{b}_2 - b_2)^2 + (\hat{M} - M)^2] \quad (6)$$

According to the suggested parameter adaptive laws as, namely,

$$\dot{\hat{b}}_0 = -S, \dot{\hat{b}}_1 = -S x_5, \dot{\hat{b}}_2 = -S x_5^2, \dot{\hat{M}} = -S(\dot{v}_d + a e_5),$$

if one designs the auxiliary signal x_{4d} as

$$x_{4d} = \frac{1}{K_f} [\hat{b}_0 + \hat{b}_1 x_5 + \hat{b}_2 x_5^2 + \hat{M}(\dot{v}_d + a e_5) - \rho_1 S],$$

then the time derivative of the function V_e becomes

$$\dot{V}_e = -\rho_1 S^2 + e_4 [- (a_1 + a_4)x_4 - \beta p x_2 x_5 - p x_3 x_5 + \sqrt{x_2} V - \dot{x}_{4d}]. \quad (7)$$

Now, design the actual input

$$V = \frac{1}{\sqrt{x_2}} [(a_1 + a_4)x_4 + \beta p x_2 x_5 + p x_3 x_5 + \dot{x}_{4d} - \rho_2 e_4],$$

then it apparently leads to the result that $\dot{V}_e = -\rho_1 S^2 - \rho_2 e_4^2 \leq 0$ with $\rho_1, \rho_2 > 0$, which readily implies boundedness of all parameter estimates as well as of both signals x_4 and x_5 .

Since $\dot{V}_e$ in (7) is nonpositive, we conclude that all the error signals in V_e and, in particular, x_5 and x_{4d} are bounded, which in turn implies that x_4 and hence $\dot{x}_5$ (from system (3)) are both bounded. We thus conclude that all the internal signals are kept bounded. Now, since I_s is bounded, then guarantees all signals x_i , $i=1, \dots, 5$, are then guaranteed to be bounded.

By the power formula, $P_s = a_5 x_4 x_5 = 3V_s I_s$, which can be

shown bounded from the above. We now show that I_s will be bounded via argument of contradiction. Say, I_s eventually grows unbounded, then V_s and, hence, V will diminish eventually. However, if I_s does grow unbound, then it implies that V will tend to $px_3x_3/\sqrt{x_2}$ eventually. However, from the dynamics of x_2 in (3), we have x_2 and x_3 grow at the same rate, which readily says that V will also grow unbounded. This obviously leads to a contradiction and therefore I_s is bounded.

Furthermore, we can show that $\dot{x}_{4d}$ is bounded, and hence $\dot{e}_4$ and $\dot{S}$ are also bounded, which implies the convergence of e_4 and S due to Barbalat's Lemma. Therefore, the control scheme with the properly designed input V will drive the output p_r to the desired p_d asymptotically. $\square$

3.4 Consideration of Uncertainty Inductance

From the previous LIM dynamics, the parameters a_1, a_2, β, c and K_f depend on the inductance, but as we know the mutual inductance is hard to identify due to its intricate structure and undesirable end effect. In particular,

$$a_1 + a_4 = \left(\frac{R_s L_r + R_r L_m^2 / L_r}{L_s L_r - L_m^2} \right) + \frac{R_r}{L_r} \triangleq a_{10} + a_{40} + \alpha,$$

$$c = c_0 + \sigma$$

where α and σ are uncertainty terms of $(a_1 + a_4)$ and variance c , respectively. We rewrite the dynamic equations (3) as followings:

$$\begin{aligned} \dot{x}_1 &= -2a_1x_1 + 2a_2x_3 + \frac{2x_4}{\sqrt{x_2}}V \\ \dot{x}_2 &= -2a_4x_2 + 2a_3x_3 \\ \dot{x}_3 &= a_3x_1 + a_2x_2 - (a_1 + a_4)x_3 + p_x x_4 \\ \dot{x}_4 &= -px_3x_3 - \beta p_x x_2 - (a_1 + a_4)x_4 + \sqrt{x_2}V \\ M\dot{x}_5 &= K_f x_4 - \sum_{n=0}^2 b_n x_5^n \end{aligned} \quad (8)$$

and design the control input

$$V_{qs} = \frac{\lambda_d}{\sqrt{\lambda_q^2 + \lambda_d^2}}V, \quad V_{ds} = \frac{-\lambda_q}{\sqrt{\lambda_q^2 + \lambda_d^2}}V$$

To facilitate subsequent investigation, we define several variables as follows:

$$\bar{\alpha} = \alpha - \hat{\alpha}, \quad \bar{\beta} = \beta - \hat{\beta}, \quad d_n = \frac{b_n}{K_f}, \quad \text{and} \quad H = \frac{M}{K_f}$$

where $\hat{\alpha}$ is the estimate of α , $\hat{\beta}$ is the estimate of β .

In order to show the boundedness of all the parameter estimators and the tracking errors e_4, S , we choose a Lyapunov like function V_e as shown below:

$$V_e = \frac{1}{2} [HS^2 + e_4^2 + (\hat{d}_0 - d_0)^2 + (\hat{d}_1 - d_1)^2 + (\hat{d}_2 - d_2)^2 + (\hat{H} - H)^2 + \bar{\alpha}^2 + \bar{\beta}^2] \quad (9)$$

whose time derivative can be evaluated as follows.

If we design the parameter adaptive laws as

$$\dot{\hat{d}}_0 = -S, \quad \dot{\hat{d}}_1 = -Sx_5, \quad \dot{\hat{d}}_2 = -Sx_5^2,$$

$$\dot{\hat{H}} = -S(\dot{v}_d + ae_5), \quad \dot{\hat{\alpha}} = -e_4x_2, \quad \dot{\hat{\beta}} = -e_4px_3x_2$$

along with the proper design of x_{4d} as

$$x_{4d} = [\hat{d}_0 + \hat{d}_1x_5 + \hat{d}_2x_5^2 + \hat{H}(\dot{v}_d + ae_5) - \rho_1S],$$

then the time derivative of the Lyapunov function V_e becomes

$$\dot{V}_e = -\rho_1S^2 + e_4[g(x) + (c_0 + \sigma)\sqrt{x_2}V].$$

After we substitute the properly designed input V as:

$$V = \frac{1}{c_0\sqrt{x_2}} \{-g(x) - \eta \operatorname{sgn}(e_4)\}$$

where $\operatorname{sgn}()$ is the sign function, then the time derivative $\dot{V}_e$ can be simplified as

$$\begin{aligned} \dot{V}_e &= -\rho_1S^2 - \left(\frac{c_0 + \sigma}{c_0}\right)e_4[\eta \operatorname{sgn}(e_4) + \left(\frac{c_0}{c_0 + \sigma}\right)g(x)] \\ &\leq -\rho_1S^2 - \left(\frac{c_0 + \sigma}{c_0}\right)[\eta - \left(\frac{c_0}{c_0 + \sigma}\right)|g(x)|]|e_4| \end{aligned}$$

If η is chosen to satisfy $\eta \geq |g(x)| + k$ for some $k > 0$, then we have

$$\dot{V}_e \leq -\rho_1S^2 - \rho_2|e_4|$$

for some $\rho_2 > 0$, which again implies boundedness of all internal signals and convergence of the position tracking error.

IV. ADAPTIVE BACKSTEPPING CONTROLLER DESIGN

In the previous section, we have proposed an adaptive controller for the LIMs, which will require acceleration signals of the motor. Although this signal can be obtained through numerical differencing and digital filtering, it is more susceptible to noise. In order to avoid such problem, we thus propose the following nonlinear backstepping position controller without need of acceleration signal in this section.

Theorem 3. Consider an LIM whose dynamics are governed by system (3) under the assumptions (A.4). Given a third-time differentiable smooth desired position trajectory p_d with $p_d, \dot{p}_d, \ddot{p}_d$ and $\ddot{\ddot{p}}_d$ being all bounded, then the following control input can achieve the control objective $p_r \rightarrow p_d$ (i.e. $x_6 = p_r$ will follow p_d asymptotically) with the control input

$$V_{qs} = \frac{\lambda_d}{\sqrt{\lambda_q^2 + \lambda_d^2}} \frac{V}{c}, \quad V_{ds} = \frac{-\lambda_q}{\sqrt{\lambda_q^2 + \lambda_d^2}} \frac{V}{c},$$

and

$$V = \frac{1}{\sqrt{x_2}} [g_2(x) + \hat{\Theta}_2 W_2 - K_f z_1 - \rho_2 z_2],$$

with adaptation law

$$\dot{\hat{\Theta}} = \dot{\hat{\Theta}} = -\Gamma_1 z_1 W, \quad \dot{\hat{\Theta}}_2 = \dot{\hat{\Theta}}_2 = -\Gamma_2 z_2 W_2,$$

where $\Gamma_1, \Gamma_2 > 0$, and $z_1 = S$, $z_2 = x_4 - \alpha_2$,

$$\alpha_2 = -\rho_1 MS + \frac{1}{K_f} \Theta^T W - \frac{a}{K_f} Me_5$$

for some $\rho_1, \rho_2 > 0$, and

$$g_2(x) = px_3 x_5 + \beta x_2 x_5 + (q + a_4 - a)x_4 - \rho_1 (K_f x_4 + \kappa J e_6)$$

$$\Theta_2^T W_2 = (\rho_1 + \frac{a}{K_f}) \Theta^T W + \frac{1}{K_f} \Theta^T \dot{W} = (\rho_1 + \frac{a}{K_f}) \Theta^T W + \frac{1}{K_f} \Theta^T W'$$

with the parameter vector Θ' as well as the known function vector W' satisfying $\Theta^T \dot{W} = \Theta'^T W'$.

Proof:

Step 1. Choose a different stabilizing function α_2 as follows

$$\alpha_2 = -\rho_1 MS + \frac{1}{K_f} \hat{\Theta}^T W - \frac{a}{K_f} Me_5 \quad (10)$$

where $\hat{\Theta}$ denotes the on-line parameter estimate. And, redefine the new error variables $z_1 = S$, $z_2 = x_4 - \alpha_2$.

Evaluate the time derivative of the Lyapunov function candidate

$$\dot{V}_1 = \frac{1}{2} M z_1^2 + \frac{1}{2 \Gamma_1} \tilde{\Theta}^T \tilde{\Theta}, \quad (11)$$

along the solution trajectories to obtain

$$\dot{V}_1 = -\rho_1 K_f M z_1^2 + K_f z_1 z_2 + \tilde{\Theta}^T (\frac{1}{\Gamma_1} \dot{\tilde{\Theta}} + z_1 W) \quad (12)$$

Devise the adaptation law as

$$\dot{\tilde{\Theta}} = \dot{\tilde{\Theta}} = -\Gamma_1 z_1 W \quad (13)$$

for some proper positive adaptation gain Γ_1 , then (12) can be slightly simplified as:

$$\dot{V}_1 = -\rho_1 K_f M z_1^2 + K_f z_1 z_2 \quad (14)$$

Step 2. The time derivative of z_2 is now expressed as

$$\dot{z}_2 = \dot{x}_4 - \dot{\alpha}_2 = -g_1(x) - \Theta_1^T W_1 + \sqrt{x_2} V \quad (15)$$

where the function are as previously defined. Thus, we need to select a Lyapunov function candidate and design V to render its time derivative nonpositive. We want to apply the augmented Lyapunov function candidate as:

$$V_2 = V_1 + \frac{1}{2} z_2^2, \quad (16)$$

whose time derivative is found to be

$$\dot{V}_2 = -\rho_1 K_f M z_1^2 + K_f z_1 z_2 + z_2 [-g_1(x) - \Theta_1^T W_1 + \sqrt{x_2} V], \quad (17)$$

The control law V should be able to cancel the indefinite term in (17). On the other hand, to deal with the unknown parameters Θ_2 , we will try to employ the current estimates $\hat{\Theta}_1$, i.e.,

$$V = \frac{1}{\sqrt{x_2}} [g_1(x) + \hat{\Theta}_1 W_1 - K_f z_1 - \rho_2 z_2], \quad (18)$$

From this resulting derivative

$$\dot{V}_2 = -\rho_1 K_f M z_1^2 + z_2 \hat{\Theta}_1 W_1 - \rho_2 z_2^2 \quad (19)$$

in order to cancel the last term in (19), we modify the Lyapunov function as below:

$$V_3 = V_2 + \frac{1}{2} z_2^2 + \frac{1}{2} \tilde{\Theta}_1^T \tilde{\Theta}_1, \quad (20)$$

and the time derivative of V_3 hence is

$$\dot{V}_3 = -\rho_1 K_f M z_1^2 + \hat{\Theta}_1^T (z_2 W_2 + \frac{1}{\Gamma_2} \dot{\tilde{\Theta}}_1) \quad (21)$$

Now, the term with $\tilde{\Theta}_3$ can be eliminated completely with the update law

$$\dot{\tilde{\Theta}}_2 = \dot{\tilde{\Theta}}_2 = -\Gamma_2 z_2 W_2 \quad (22)$$

for some positive adaptation gain Γ_2 , which thus yields

$$\dot{V}_3 = -\rho_1 K_f M z_1^2 - \rho_2 z_2^2 \quad (23)$$

which guarantees boundedness of all parameter estimates $\hat{\theta}$, $\hat{\theta}_1$ and z_1 , z_2 , and $z_1 \in L^2 \cap L^\infty$. To show boundedness of the rest of states, we can rearrange the dynamical equations from system (3) as shown below:

$$X = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2a_1 & 0 & 2a_2 \\ 0 & -2a_4 & 2a_3 \\ a_3 & a_2 & -(a_1 + a_4) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{2x_4}{\sqrt{x_2}} V \\ 0 \\ px_5 x_4 \end{bmatrix}$$

$$= AX + u$$

where A can be shown to be Hurwitz. After reviewing definitions of x_3 and V, respectively, we found that the first entry of u will be bounded because x_2 grows no slower than x_3 if x_3 does grow unbounded (due to the second equation of (3)). As a result, u is apparently bounded, and hence X will be bounded. This then proves the boundedness of all the states. We note that $\dot{z}_1$ is also bounded, and hence by Barbalat's lemma we can conclude $\dot{z}_1 \in L_\infty$ so that

$$\lim_{t \rightarrow \infty} z_1 \rightarrow 0, \text{ i.e., } p_r \rightarrow p_d \text{ as } t \rightarrow \infty. \quad \square$$

V. EXPERIMENTAL RESULTS

Table 5.1: Specification and parameter of the motor

Specification	Parameters normal value
3 Phase (Y-connected)	$R_s = 13.2 \Omega$
Rated Power 1HP	$R_r = 11.78 \Omega$
Rated Air gap 0.125in	$L_s = 0.42H$
Rated Current 5A	$L_r = 0.42H$
Rated Voltage 240V	$L_m = 0.4H$
Rated Poles 4	$M = 4.775 Kg$
Pitch 46.5mm	
Secondary length 82cm	

$$B = 53 \text{ Kg/sec}$$

The experiment are done with a 4-pole, 3-phase LIM with a Y-connected primary, and is manufactured by NORMAG Co.. Detailed parameters and specification will be found in Table 5.1. The power stage of the motor driver uses a IGBT module, and the PWM drive signals are generated by a 10 KHz SPWM with a 2.5 μs dead-time protection circuit.

5.1 Results of Adaptive Controller Design for position tracking

In this section, we make a series experiment on the adaptive controller design which is proposed in Section 3. For the exponential type of desired trajectory in Fig 5.1, we found chattering phenomenon in the steady state. In order to eliminate such phenomenon, we propose a PI control scheme while the position tracking error is kept within $\pm 100 \mu\text{m}$. After that, such arrangement will truly render the error less than $\pm 1 \mu\text{m}$. In Fig 5.2, we adopt sinusoidal trajectories, the steady error is within $\pm 0.5 \text{ mm}$. In Fig 5.3, we adopt a ramp command to relax the contraction smooth trajectory on experiment. It was also can achieve the position convergence. However, all the estimated parameters are truly bounded in this position tracking case.

5.2 Results of Adaptive Backstepping controller for position tracking

For the position tracking problem, we provide several kinds of desired position trajectories, For the exponential trajectories in Fig 5.4, we found chattering phenomenon in the steady state error within $\pm 50 \mu\text{m}$. In Fig 5.5, we adopt sinusoidal trajectories, the steady error is within $\pm 0.7 \text{ mm}$. In Fig 5.6, we adopt a ramp command to relax the contraction of smooth trajectory on experiment.

VI. CONCLUSION

In this paper, we have proposed an adaptive position controller and an adaptation backstepping controller for the LIM with fifth/sixth order nonlinear dynamic model which is control by the primary voltage source. To cope with the uncertainty part of the LIM, i.e., end effect, payload, and inductance, we design our controller based on an appropriate nonlinear transformation. Due to inaccessibility to the flux in general Stability analysis based on Lyapunov theory is performed to guarantee the controller design is stable. Finally, both the simulation and experimental results confirm the effectiveness of our control design.

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Fig5.1 $p_d = 5(1 - e^{-2t})$ cm/sec Fig5.2 $p_d = 5\sin(2t)$ cm/sec

Fig 5.3: Ramp command Fig 6.4: $p_d = (1 - e^{-2t})$ cm

Fig 5.5: $p_d = 5\sin(2t)$ cm Fig 5.6: Ramp command