Optimum Timing Estimation without Channel Information over Frequency-Selective Slowly Fading Channels

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Abstract

Applying the composite hypothesis testing procedure, we introduce the maximum a posteriori probability symbol synchronization in the point-to-point communications systems without channel information over frequency-selective slowly fading channels. This proposed estimation does not ignore the presence of interference due to frequency-selective channels and efficiently utilizes the inherited frequency-diversity to combat the channel fading impairment. By mathematical analysis, we prove the fundamental condition to have a consistent symbol synchronizer over frequency-selective slowly fading channels when the channel impulse response is unknown. We further conduct simulations to justify its performance considering the practical implementation with a finite number of timing candidates. This proposed approach considers more generalized scenarios with feasible complexity and therefore could be extensively applied in many practical communications systems.

INTRODUCTION

In order to realize the high-quality and high data-rate wireless communications services, timing estimation to correctly determine the symbol boundary of the received signal plays an irreplaceable role to facilitate the efficient demodulation procedures. In the conventional designs over additive white Gaussian noise (AWGN) channels, the maximum a posteriori probability (MAP) criterion was extensively applied to decide the symbol boundary of the received signal [1]. This kind of approach correlates the received signal with one locally generated signal for various timing shifts and then searches for the timing shift with the peak correlation value to decide the symbol epoch.

By extending to the practical wireless communications environments, such as the frequency-selective fading channel, this conventional approach is no longer optimal according to the MAP criterion as it ignores the presence of multi-path components and the possible interference due to multi-path effects. Frequency-selective fading channels are popular wireless channel models considered in many practical communications systems such as the wireless local area networks (WLAN) [2], especially the systems for indoor applications. For frequency-selective fading channels, because the transmitted signal is band-limited with null-to-null bandwidth W , the channel effect can be modeled as a tapped delay line with $\lceil T_m W \rceil + 1$ taps and tap spacing $1/W$ [1], where T_m is

the total multi-path spread and $\lceil x \rceil$ denotes the smallest integer not smaller than x . In addition, these $\lceil T_m W \rceil + 1$ taps coefficients $\{\tilde{\rho}_0, \tilde{\rho}_1, \dots, \tilde{\rho}_{\lceil T_m W \rceil}\}$ are independent random variables and the received low-pass equivalent signal can be written as

$$r(t) = \sum_{l=0}^{L-1} \tilde{\rho}_l s(t - \tilde{\tau} - \frac{l}{W}) + z(t), \quad (1)$$

where $L \equiv \lceil T_m W \rceil + 1$ is the number of multi-path components; $s(t)$ is the transmitted signal; $\tilde{\tau}$ is the received timing; and $z(t)$ is the AWGN noise. As shown in (1), the L multi-path components $\{\tilde{\rho}_l s(t - \tilde{\tau} - \frac{l}{W}) | l = 0, 1, \dots, L-1\}$ contained in the received signal may interfere with each other and therefore a brand new design on timing estimate over multi-path channels is very much desired.

It is remarkable to note that the effects of frequency-selective channels are two-folded. On the one hand, because the multi-path components $\{\tilde{\rho}_l s(t - \tilde{\tau} - \frac{l}{W}) | l = 0, 1, \dots, L-1\}$ are independent random variables, the inherited frequency-diversity is possible to cure the channel fading impairment. For example, when one of the multi-path component, said $\tilde{\rho}_0 s(t - \tilde{\tau})$, fades at some instant, the other multi-path components which are independent to this fading component could help to estimate the received timing. It is the similar rationale behind the design of RAKE receiver by Price and Green [3]. On the other hand, the multi-path components interfere with each other to degrade the conventional timing estimator. Therefore, the optimal timing estimator to both utilize the frequency-diversity inherited from frequency-selective channels and accommodate the interference due to multi-path effects is further required.

In addition, the timing estimation procedures are usually performed prior to the channel estimation in many practical communications systems such as IEEE 802.11a and IEEE 802.11b [4, 5]. In this case, the timing estimation with unknown channel impulse response $\{\tilde{\rho}_0, \tilde{\rho}_1, \dots, \tilde{\rho}_{L-1}\}$ is desired. In [6], the timing estimation with unknown channel impulse response was proposed over frequency-selective slowly fading channels. This proposed timing estimator maximizes the likelihood function by assuming that the multi-path components do not interfere with each other and therefore is applicable to the direct sequence spread spectrum (DSSS) communications systems with long spreading sequence.

In this paper, we release the assumption in [6] and consider the generalized optimum timing estimation without channel information over frequency-selective fading chan-

nels. By mathematically analyzing our proposed estimator, we demonstrate the fundamental requirement to have a consistent timing estimator without channel information over frequency-selective slowly fading channels. This proposed optimal timing estimation not only utilizes the frequency-diversity inherited from frequency-selective fading channels to combat channel fading impairment but also efficiently mitigates the mutual interference from multi-path components. We further conduct simulations to justify the effectiveness of our proposed estimator in the practical implementation with a finite number of timing candidates.

OPTIMUM SYMBOL SYNCHRONIZATION OVER FREQUENCY-SELECTIVE SLOWLY FADING CHANNELS

Signal Models and Assumptions

In order to ease the interpretation, we start from the symbol synchronization to estimate the symbol boundary of the received signal with known training symbols, which is a practical assumption in many communications systems such as WLAN. Let $s(t)$ denote the transmitted symbol with symbol duration T^1 and consider the repeated transmission of $s(t)$ for a couple of symbols. For example, in the DSSS communications systems with binary-phase shift keying (BPSK) modulation in the training stage, the transmitted symbol $s(t)$ could be the signature waveform $p(t)$ employed in the DSSS systems; in the OFDM communications systems, the transmitted symbol $s(t)$ in the training stage is the training symbol. On the other hand, in order to ease the initial development, we assume that the frequency offset is properly compensated beforehand and therefore the received low-pass equivalent signal $r_{LP}(t)$ over a frequency-selective slowly fading channel [1] can be written as

$$r_{LP}(t) = \sum_m \sum_{l=0}^{L-1} \tilde{\rho}_l s(t - \tilde{\tau} - mT - \frac{l}{W}) + z_{LP}(t), \quad (2)$$

where m is the index of received symbols; W is the null-to-null bandwidth of transmitted symbols; $L \equiv \lceil T_m W \rceil + 1$ is the number of taps in the tapped delay line model for the frequency-selective fading channel [1] and we adopt the convention of saying that there are L equally spaced multi-path components; T_m is the total multi-path spread; $\tilde{\rho}$ for $l = 0, 1, \dots, L-1$ are the independent and complex-valued Gaussian random variables with mean 0 and variance σ_l^2 , which is the channel impulse response; $\tilde{\tau}$ is the received timing; and $z_{LP}(t)$ is the complex-valued additive white Gaussian noise whose power spectrum is the rectangular pulse with null-to-null bandwidth W^2 . In (2), the channel impulse response is assumed to be time-invariant during the reception of training symbols as we consider the slowly fading channels. In addition, without loss of generality, we assume that

¹That is, $s(t) = 0$ for $t \notin [0, T)$.

² $z_{LP}(t)$ is the noise component obtained by filtering the received signal according to the transmitted signal.

$L/W \leq T$. That is, the total delay spread is smaller than the symbol length T . When the total delay spread is larger than the symbol length, said $L = \xi \lfloor WT \rfloor + k$ for some positive integer ξ and for some nonnegative integer $k \leq WT$, all the results developed in the sequel are equally applicable as long as we replace L with $\lfloor WT \rfloor$; $\tilde{\rho}_l$ with $\sum_{\alpha=0}^{\xi-1} \tilde{\rho}_{l+\alpha WT}$ for $l \leq k$; and $\tilde{\rho}_l$ with $\sum_{\alpha=0}^{\xi-1} \tilde{\rho}_{l+\alpha WT}$ for $\lfloor WT \rfloor - 1 > l > k$, where $\lfloor x \rfloor$ is the largest integer not larger than x .

In this paper, we temporarily assume that the known and repeatedly transmitted training symbol $s(t)$ is available. Although this kind of assumption is practical such as the systems in IEEE 802.11a, our proposed approach is equally applicable to the cases without training sequences or with other types of training sequences.

Optimum Symbol Synchronization without Channel Information over Frequency-Selective Fading Channels

Because the transmitted signal is band-limited to W , we sample $r_{LP}(t)$ at rate $1/T_s \equiv W$ according to Nyquist criterion and have

$$\begin{aligned} r(m, n) &\equiv r_{LP}(mT + nT_s) \\ &= \sum_{l=0}^{L-1} \tilde{\rho}_l s_{\tau}(l, n) + z(m, n), \end{aligned} \quad (3)$$

for $n = 0, 1, \dots, N_s - 1$, where $N_s \equiv \frac{T}{T_s}$ is a positive integer indicating the number of samples within a symbol; $s_{\tau}(l, n) \equiv s((nT_s - lT_s - \tau)_T)$, $\forall \tau \in [0, T)$; $(\cdot)_T$ stands for $(\cdot) \bmod T$; $z(m, n) \equiv z_{LP}(mT + nT_s)$ for $n = 0, 1, \dots, N_s - 1$ are i.i.d. complex-valued Gaussian random variables with mean 0 and variance σ_N^2 ; and the second equality in (3) comes from the assumption on repeatedly transmitted training symbol.

When the channel impulse response ($\tilde{\rho}_l$ for $l = 0, 1, \dots, L-1$) is unknown³, the optimal timing estimate based on MAP criterion according to composite hypothesis testing is

$$\begin{aligned} \hat{\tau}_{opt} &= \arg \max_{\tau \in [0, T)} \int \int \dots \int Pr(r(m, n); n = 0, 1, \\ &\dots, N_s - 1 | \tilde{\tau} = \tau; \tilde{\rho}_l = \rho_l \text{ for } l = 0, 1, \dots, L-1) \\ &\prod_{l=0}^{L-1} p(\rho_l) d\rho_0 d\rho_1 \dots d\rho_{L-1}, \end{aligned} \quad (4)$$

where $p(\rho_l)$ is the probability density function of $\tilde{\rho}_l$. (4) averages over the multi-path components according to their probability density function to optimally estimate the received timing in probability. However, (4) is in the implicit form and therefore we introduce its explicit formula to ease practical applications in Proposition 1.

Proposition 1 (Optimal timing estimation)

³However, we assume the knowledge of L and σ_l^2 . This assumption is reasonable in practical applications as these parameters can be obtained by properly measuring the channel environments. Even when the channel parameters can not be precisely measured, its approximation is also close enough to the optimal performance, which is justified by our conducted simulations in the sequel.

Figure 1. Optimum timing estimation with unknown channel information over frequency-selective slowly fading channels.

The optimal one-shot timing estimate in (4) is

$$\hat{\tau}_{opt} = \arg \max_{\tau \in [0, T]} \sum_{l=0}^{L-1} \frac{|\sum_{n=0}^{N_s-1} r^*(m, n) \hat{s}_\tau(l, n)|^2 \frac{\sigma_l^2}{\sigma_N^2}}{1 + \frac{\sigma_l^2}{\sigma_N^2} \hat{C}_l^2}, \quad (5)$$

where the superscript $*$ denotes the complex-conjugate operation and

$$\begin{cases} \hat{s}_\tau(l, n) \equiv s_\tau(l, n) - \sum_{\alpha=0}^{l-1} \hat{s}_\tau(\alpha, n) \frac{\hat{R}^*(l, \alpha) \frac{\sigma_l^2}{\sigma_N^2}}{1 + \frac{\sigma_l^2}{\sigma_N^2} \hat{C}_\alpha^2} & l \neq 0; \\ \hat{s}_\tau(0, n) \equiv s_\tau(0, n) \end{cases} \quad (6)$$

$$\hat{R}(\alpha, \beta) \equiv \sum_{n=0}^{N_s-1} s_\tau^*(\alpha, n) \hat{s}_\tau(\beta, n); \quad (7)$$

$$\hat{C}_l^2 \equiv \sum_{n=0}^{N_s-1} |s_\tau(l, n)|^2; \quad (8)$$

$$\begin{cases} \hat{C}_l^2 \equiv \hat{C}_l^2 - \sum_{\alpha=0}^{l-1} \frac{|\hat{R}(l, \alpha)|^2 \frac{\sigma_l^2}{\sigma_N^2}}{1 + \frac{\sigma_l^2}{\sigma_N^2} \hat{C}_\alpha^2} & l \neq 0 \\ \hat{C}_0^2 \equiv \hat{C}_0^2 \end{cases} \quad (9)$$

Corollary 1 (Orthogonal multi-path components)

When the noiseless received signal from different rays are orthogonal (i.e. $\sum_{n=0}^{N_s-1} s_\tau(k, n) s_\tau^*(l, n) = 0$, $\forall k \neq l$ and $\forall \tau \in [0, T]$), the optimal timing estimate without channel information in (4) is

$$\hat{\tau}_{opt} = \arg \max_{\tau \in [0, T]} \sum_{l=0}^{L-1} \frac{|\sum_{n=0}^{N_s-1} r^*(m, n) s_\tau(l, n)|^2 \frac{\sigma_l^2}{\sigma_N^2}}{1 + \frac{\sigma_l^2}{\sigma_N^2} C_l^2}. \quad (10)$$

Remarks:

- The assumption in Corollary 1 can be satisfied by employing signature sequences with large spreading gain in the DSSS communications systems. In this case, the mutual interference between different multi-path components could be negligible as in [6].
- Actually, this assumption can be perfectly realized in the OFDM systems. For example, consider the OFDM training symbols with N_{sub} sub-carriers and the carrier spacing is $\frac{W}{N_{sub}}$, where W is the null-to-null bandwidth of the transmitted OFDM symbols. Let s_k for $k = 0, 1, \dots, N_{sub} - 1$ be the training bits on the N_{sub} sub-carriers. Then the training symbol $s(t)$ is $\sum_{k=0}^{N_{sub}-1} s_k e^{j2\pi k \frac{W}{N_{sub}} t}$ for $t \in [0, T]$, where $T = \frac{N_{sub}}{W}$ and $j \equiv \sqrt{-1}$. In this case, if the training symbol is designed such that $|s_k|$ is a constant value, $\sum_{n=0}^{N_s-1} s_\tau(k, n) s_\tau^*(l, n) = 0$, $\forall k \neq l$ and $\forall \tau \in [0, T]$.
- Although we consider a generalized scheme when Corollary 1 is not applicable, it is suggested to properly design the training symbols to avoid the mutual interference from multi-path components in order to save the complexity.

When $\sigma_N = 0$, (6) - (9) is the Gram-Schmidt process to transform the set of sequences $s_\tau(l, n); n = 0, 1, \dots, N_s - 1$ for $l = 0, 1, \dots, L - 1$ into the orthogonal set of sequences $\hat{s}_\tau(l, n); n = 0, 1, \dots, N_s - 1$ for $l = 0, 1, \dots, L - 1$. In the Gram-Schmidt process, the orthogonal sequence is generated by suppressing the interference projected from all the other sequences. Therefore, the proposed optimal estimator (5) can be regarded as a timing estimator to mitigate the mutual interference between the multi-path components.

When the multi-path components do not interfere with each

other (i.e. the case in Corollary 1), our proposed optimum timing estimator correlates the received signal with each multi-path component and then linearly combines the energy of the correlated outputs. In this case, the optimal timing estimator utilizes the frequency diversity inherited from multi-path channels to combat channel fading impairment. When the multi-path components contribute interference, the optimal estimator in (5) not only combines the correlation outputs to efficiently combat fading channels but also mitigates the mutual interference generated by multi-path components. Despite of the optimality of our proposed estimator, it is desired to mathematically analyze its performance due to the lack of channel information and the presence of interference by multi-path components. In the sequel, we shall first mathematically prove the consistency of this proposed estimator and then conduct simulations to justify its performance.

Proposition 2 (Consistency of MAP timing estimation)

Let $\underline{s}_l^{(\tau)} \equiv [s_\tau(l, 0), s_\tau(l, 1), \dots, s_\tau(l, N_s - 1)]^t$, $\forall l = 0, 1, \dots, L-1$ and $\tau \in [0, T)$. Supposing that $\{\underline{s}_0^{(\tau)}, \underline{s}_1^{(\tau)}, \dots, \underline{s}_{L-1}^{(\tau)}\}$ are L independent column vectors for all $\tau \in [0, T)$, and $\text{span}\{\underline{s}_0^{(\tilde{\tau})}, \underline{s}_1^{(\tilde{\tau})}, \dots, \underline{s}_{L-1}^{(\tilde{\tau})}\} \neq \text{span}\{\underline{s}_0^{(\tau)}, \underline{s}_1^{(\tau)}, \dots, \underline{s}_{L-1}^{(\tau)}\} \forall \tau \neq \tilde{\tau}$, the optimal timing estimator (5) with unknown channel impulse response over frequency-selective slowly fading channels is consistent.

By Proposition 2, the sufficient condition to have a consistent timing estimator without channel information over frequency-selective slowly fading channel is that the vector space spanned by $\{\underline{s}_0^{(\tau_1)}, \underline{s}_1^{(\tau_1)}, \dots, \underline{s}_{L-1}^{(\tau_1)}\}$ is different to the vector space spanned by $\{\underline{s}_0^{(\tau_2)}, \underline{s}_1^{(\tau_2)}, \dots, \underline{s}_{L-1}^{(\tau_2)}\}$ whenever $\tau_1 \neq \tau_2$ for all τ_1 and τ_2 in $[0, T)$. When this sufficient condition is satisfied, the probability that the noiseless received signal $\sum_{l=0}^{L-1} \tilde{\rho}_l s_\tau(l, n)$ is a linear combination of $\{\underline{s}_0^{(\tau)}, \underline{s}_1^{(\tau)}, \dots, \underline{s}_{L-1}^{(\tau)}\}$ for $\tilde{\tau} \neq \tau$ is zero. And therefore the optimum timing estimator precisely determines the received timing $\tilde{\tau}$ as the noise variance decreases.

This is not a surprising result as we consider the multi-path channels without knowledge of the channel impulse response $\tilde{\rho}_l$ for $l = 0, 1, \dots, L-1$. When the channel impulse response is unknown, the optimum timing estimator does not know the exact combination coefficients (i.e. $\tilde{\rho}_l$ for $l = 0, 1, \dots, L-1$) of the noiseless received signal. In this case, any set of possible combination coefficients of the locally generated signals $\{\underline{s}_0^{(\tau)}, \underline{s}_1^{(\tau)}, \dots, \underline{s}_{L-1}^{(\tau)}\}$ which matches the noiseless received signal would suggest that τ is the best estimate of $\tilde{\tau}$. Therefore, with the constraint on the training symbol $s(t)$ such that the noiseless received signal does not reside in the vector space spanned by $\{\underline{s}_0^{(\tau)}, \underline{s}_1^{(\tau)}, \dots, \underline{s}_{L-1}^{(\tau)}\}$ for $\tau \neq \tilde{\tau}$, it is guaranteed the consistency of the optimum timing estimator.

Although the sufficient condition to have a consistent timing estimator without channel information is stricter than the case with channel information, a lot of training symbols, such as the symbols in DSSS communications systems and the sym-

bols in OFDM systems, are applicable to have the optimum timing estimation consistent. Therefore, this proposed optimum timing estimation could be extensively applied in the wireless communications systems.

Practical Realization with Finite Number of Timing Candidates

Although we proved the consistency of our proposed MAP timing estimation, practical realization on MAP timing estimation considers a finite number of timing candidates as in many conventional symbol synchronizers [1]. In this subsection, we consider the practical realization with a finite number of timing candidates.

When considering the practical realization with N timing candidates, the corresponding timing estimation without channel channel information over frequency-selective slowly fading channel is

$$\hat{\tau}_{non} = \arg \max_{\tau \in \{i\frac{T}{N}; i=0,1,\dots,N-1\}} \sum_{l=0}^{L-1} \frac{|\sum_{n=0}^{N_s-1} r^*(m, n) \hat{s}_\tau(l, n)|^2 \frac{\sigma_r^2}{\sigma_N^2}}{1 + \frac{\sigma_r^2}{\sigma_N^2} \hat{C}_l^2}, \quad (11)$$

where the other variables are defined as in (6) - (9). The block diagram of this proposed optimum timing estimate is in Figure 1. This proposed realization requires $N \times L$ branches of correlators to optimally estimate the received timing which grows linearly to the number of timing candidates or the number of considered multi-path components.

Numerical Analysis

Without loss of generality, we use the OFDM system in IEEE 802.11a as an example to demonstrate the effectiveness of our proposed estimator. In IEEE 802.11a, there are 12 sub-carriers out of 64 sub-carriers utilized in the training stage such that $s(t)$ can be written as

$$s(t) = \Pi(t) \sum_{k=-32}^{31} s_k e^{j2\pi k \Delta_F t}, \quad (12)$$

where $\Pi(t)$ is the indicator function with support $[0, T)$; $\Delta_F \equiv 20 \times 10^6 / 64$ is the spacing of sub-carriers; $T \equiv \frac{1}{4\Delta_F}$ is the symbol duration of the training symbols; and

$$s_k = \begin{cases} \sqrt{\frac{13}{6}}(1+j) & \text{for } k = -24, -16, -4, 12, 16, \\ & 20, 24; \\ -\sqrt{\frac{13}{6}}(1+j) & \text{for } k = -20, -12, -8, 4, 8; \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

It is easy to verify that the employed training symbol $s(t)$ in (12) does not satisfy the assumption in Corollary 1 to have orthogonal multi-path components and the experiments in the sequel represent the general applications of our approach.

We further adopt the frequency-selective slowly fading channel model [2] which is extensively applied in many practical communications systems such as IEEE 802.11a and IEEE

802.11b. This static channel model simulates the frequency-selective slowly fading channel as a tapped delay line with equal tap spacing T_s , where T_s is the sampling period. The tap coefficients are modeled as independent complex-valued Gaussian random variables with exponentially decayed variance and zero mean. In addition, the number of taps considered should be $\lceil 10 \times \frac{T_{RMS}}{T_s} \rceil$, where T_{RMS} is the root-mean-square (RMS) value of the channel delay spread.

In the first experiment, we demonstrate the performance of our proposed timing estimator over the conventional scheme when the RMS delay spread is 70 nsec. As shown in Figure 2, the mean absolute error (MAE) of the conventional timing estimation which ignores the presence of multi-path components saturates above 1.1 samples while it is below 0.4 samples in our proposed optimal estimator when the number of timing candidates N is 32⁴.

Figure 2. Performance comparison between optimum estimation and conventional estimation.

In the second experiment, we justify our argument that increasing the number of timing candidates N would improve the performance of our proposed estimator realized with a finite number of timing candidates. In Figure 3, it can be seen that the estimation error decreases as N increases when the RMS delay spread is 70 nsec.

CONCLUSIONS

In this paper, we proposed the optimal symbol synchronization based on MAP criterion over frequency-selective slowly fading channel. In particular, the proposed estimator, designed according to the composite hypothesis testing rule, does not require the information of channel impulse response of the frequency-selective channels. By releasing the assumption in [6], our proposed estimator can be applicable to many wireless communications systems, in addition to the DSSS communications systems with large spreading gain.

⁴When $N = 32$, there are 32 timing candidates within a symbol duration T . As $W = 64\Delta_F = \frac{16}{T}$, there are 16 samples within a symbol and therefore there are 2 timing candidates between two samples.

Figure 3. Performance comparison for various number of timing candidates.

This proposed optimal estimator, as justified by the conducted simulations, not only utilizes the frequency-diversity to combat fading channels but also properly mitigates the mutual interference due to multi-path components. In addition, we proposed the sufficient condition for the proposed optimal estimation to be consistent. This introduced sufficient condition can be explained as the consequence of the lack of channel impulse response and therefore is stricter to the case with known channel impulse response. Fortunately, a lot of training symbols can easily satisfy this sufficient condition to have a consistent symbol synchronizer.

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