NLOS Effects on Position Location Techniques

Jean-Fu Kiang and Chun-Wei Wu

Department of Electrical Engineering and
Graduate Institute of Communication Engineering
National Taiwan University
Taipei, Taiwan, ROC
E-mail: jfkiang@cc.ee.ntu.edu.tw

Abstract - Position location of mobile station becomes more difficult when the measured data are interfered by the signals from non-line-of-sight (NLOS) path. Several scatterer models are used to simulate the NLOS effect to evaluate the effectiveness of different position location algorithms.

Keywords: position location, NLOS, scatterer model, ROS, DOS, Gaussian.

1 Introduction

Position location techniques become popular issue in recent years. A major drive was due to the Enhanced 911(E911), phase II, by the Federal Communications Commission (FCC), completed by October 2001. It requires that the service providers must be able to report the location of all the E911 mobile stations with an accuracy of 100 m in 67% and 300 m in 95% in network-based methods, 50 m in 67% and 150 m in 95% in handset-based methods. Several propagation environments have been measured for analyzing multipath characteristics in cellular networks. In a typical multipath environment, the line-of-sight (LOS) paths are often blocked, and then non-line-of-sight (NLOS) signals are taken in the range-based method to incur error. In this work, scatterer models are used to replace practical measurements to estimate the performance of range-based method.

2 Scatterer Models

Exact location of scatterers depends on local surroundings. There are several models to approximately describe the topology of scatterers, including ring of scatterers (ROS), disk of scatterers(DOS) and Gaussian scatterer models. Parameters in the models are usually

verified by measuring statistics of TOAs of multipath signals [1]-[3]. It is assumed that the radiation patterns of the base station and mobile station antennas are omnidirectional in the horizontal plane.

Figure 1. Ring of scatterers model.

2.1 Ring of Scatterers Model

In the ROS model shown in Fig.1, the scatters are assumed to cluster on a ring of radius r_{ROS} , centered at the mobile station. The scatterers are uniformly distributed over $0 \leq \alpha < 2\pi$, with the probability density function (pdf) of

$$p_{ROS}^s(r_{ROS}, \alpha) = \begin{cases} \frac{1}{2\pi r_{ROS}}, & \text{on the ring} \\ 0, & \text{elsewhere} \end{cases}$$

The LOS distance between mobile station and base station is D , and the measured path length is μ . The associated NLOS delay is defined as $\eta = \mu - D$. The cumulative distribution function (cdf) in μ can be expressed as

$$F_{ROS}(\mu \leq \mu_p) = 2 \int_0^\theta p_{ROS}^s(r_{ROS}, \alpha) r_{ROS} d\alpha$$

where μ_p is a given path length, $\theta = \cos^{-1} \{ [r_{ROS}^2 + D^2 - (\mu_p - r_{ROS})^2] / (2r_{ROS}D) \}$ is the marginal angle associated with μ_p . The associated

probability density function (pdf) of μ can be expressed as

$$p_{ROS}(\mu) = (\mu - r_{ROS}) / r_{ROS} D$$

$$/ \sqrt{1 - [(D - \mu)(D + \mu - 2r_{ROS}) / (2r_{ROS} D) + 1]^2},$$

$$D \leq \mu \leq D + 2r_{ROS} \quad (1)$$

2.2 Disk of Scatterers Model

In the DOS model shown in the Fig.2, the scatterers are assumed to distribute uniformly within a circular disk with radius r_{DOS} . The pdf of scatterers can be expressed as

$$p_{DOS}(r, \alpha) = \begin{cases} \frac{1}{\pi r_{DOS}^2}, & \text{inside the disk} \\ 0, & \text{elsewhere} \end{cases}$$

Figure 2: Disk of scatterers model.

Figure 3: Local coordinates defined at BS and MS.

As shown in the Fig.3, local coordinates defined at the base station and mobile station are related by

$$x_b = r_b \cos \beta = x \cos \varphi + y \sin \varphi$$

$$y_b = r_b \sin \beta = -x \sin \varphi + y \cos \varphi$$

$$x_m = r_m \cos \alpha = x_b - D$$

$$y_m = r_m \sin \alpha = y_b$$

where $r_m^2 = r_b^2 + D^2 - 2Dr_b \cos \beta$. The path length, μ , can be decomposed as $\mu = r_b + r_m$, which is the sum of

distances from the scatterer to mobile and base stations, respectively. The time delay, τ , is related to the total path length as $c\tau = \mu$.

Figure 4: The ellipse (hyperbola) defined by the sum (difference) of r_b and r_m .

As shown in Fig.4, all positions having a constant value of τ form an ellipse with the base station and mobile station as its foci. To derive the pdf of TOA, first calculate the overlapping area between the interior of ellipse and the disk of scatterers. Take the derivation of the overlapping area divided by the total area of DOS gives the pdf of τ as

$$p_{DOS}(\tau) = \frac{d}{d\tau} \frac{A_t(\tau)}{\pi r_{DOS}^2} \quad (2)$$

The overlapping area, $A_t(\tau)$, can be expressed as

$$A_t(\tau) = \int_0^\theta r_{DOS}^2 d\alpha + \int_0^\pi \left(\frac{c^2 \tau^2 - D^2}{2c\tau - 2D \cos \theta} \right)^2 d\alpha \quad (3)$$

$$= r_{DOS}^2 \theta + \int_0^\pi \left(\frac{c^2 \tau^2 - D^2}{2c\tau - 2D \cos \theta} \right)^2 d\alpha$$

To simplify (3), define $\tau_n = \tau / \tau_0$ and $D_n = D / r_{DOS}$, where τ_0 is the propagation time of line-of-sight path between the base station and mobile stations. Thus, we have

$$A_t(\tau) = r_{DOS}^2 \theta + \frac{D^2 (r_n^2 - 1)^2}{4}$$

$$\left\{ \frac{\tau_n \pi}{(\tau_n^2 - 1) \sqrt{\tau_n^2 - 1}} + \frac{\sin \theta}{(\tau_n^2 - 1)(\cos \theta - \tau_n)} \right.$$

$$\left. + \frac{2\tau_n \tanh^{-1}[(1 + \tau_n \tan(0.5\theta)) / \sqrt{1 - \tau_n^2}]}{\sqrt{1 - \tau_n^2}(\tau_n^2 - 1)} \right\}$$

where $\theta = \cos^{-1}[(D_n + 2\tau_n - D_n \tau_n^2) / 2]$.

Then, the pdf of TOA can be derived as

$$\begin{aligned}
 p_{DOS}(\tau) &= \frac{r_{DOS}^2(D_n\tau_n - 1)}{\sqrt{1 - (D_n + 2\tau_n - D_n\tau_n^2)/4}} \\
 &\quad + \frac{D^2}{4} \left\{ \frac{r_n^2\pi}{\sqrt{\tau_n^2 - 1}} + \pi\sqrt{\tau_n^2 - 1} \right. \\
 &\quad \left. + \frac{4\tau_n + D_n(2 - 6\tau_n^2) + D_n^2\tau_n(\tau_n^2 - 1)}{2D_n\sqrt{1 - (D_n + 2\tau_n - D_n\tau_n^2)/4}} - \frac{2(2\tau_n^2 - 1)}{\sqrt{1 - \tau_n^2}} \right. \\
 &\quad \left. \tanh^{-1} \left[\frac{(1 + \tau_n \tan[\cos^{-1}(D/2 + \tau_n - D_n\tau_n^2/2)])}{\sqrt{1 - \tau_n^2}} \right] / 2 \right\}
 \end{aligned} \quad (4)$$

2.3 Gaussian Scatterers Model

In the Gaussian model, the scatterers are assumed to follow a Gaussian distribution. Explicitly,

$$p_G^S(x_m, y_m) = \frac{1}{2\pi\sigma_m^2} e^{-(x_m^2 + y_m^2)/2\sigma_m^2} \quad (5)$$

The positions with path difference, $\xi = r_b - r_m$, form a hyperbola family orthogonal to the family defined by $\mu = r_b + r_m$. Note that (μ, ξ) form bipolar coordinates. The pdf of τ in the bipolar coordinate system, $p_G^S(\mu, \xi)$, can be obtained by coordinate transformation as

$$p_G^S(\mu, \xi) = \frac{\mu^2 - \xi^2}{4\sqrt{\mu^2 - D^2}\sqrt{D^2 - \xi^2}} p_G^S(x_m, y_m)$$

where

$$\begin{aligned}
 x_m(\mu, \xi) &= \frac{\mu\xi}{2D} - \frac{D}{2} \\
 y_m(\mu, \xi) &= \frac{\sqrt{(\mu^2 - D^2)(D^2 - \xi^2)}}{2D}
 \end{aligned}$$

The pdf of TOA is obtained from $p_G^S(\mu, \xi)$ by integrating over all possible value of ξ at a fixed μ as

$$\begin{aligned}
 p_G(\tau) &= 2c \int_D^D p_G^S(\mu, \xi) d\xi \\
 &= \frac{c\sqrt{(\mu^2 - D^2)}}{2} \int_D^D \frac{p_G^S(x_m, y_m)}{\sqrt{(D^2 - \xi^2)}} d\xi \\
 &\quad + \frac{c}{2\sqrt{(\mu^2 - D^2)}} \int_D^D \sqrt{(D^2 - \xi^2)} p_G^S(x_m, y_m) d\xi
 \end{aligned} \quad (6)$$

Substituting the Gaussian distribution of (5) into (6), we obtain

$$\begin{aligned}
 p_G(\tau) &= \frac{c\sqrt{(\mu^2 - D^2)}}{2} \int_D^D \frac{1}{\sqrt{(D^2 - \xi^2)}} \frac{1}{2\pi\sigma_m^2} e^{-x_m^2/2\sigma_m^2} d\xi \\
 &\quad + \frac{c}{2\sqrt{(\mu^2 - D^2)}} \int_D^D \sqrt{(D^2 - \xi^2)} \frac{1}{2\pi\sigma_m^2} e^{-x_m^2/2\sigma_m^2} d\xi
 \end{aligned} \quad (7)$$

By defining $D_m = D/\sigma_m$, $D/c = \tau_0$, $\mu = c\tau_n\tau_0$, and $\xi = D\sin\theta$, (5) can be reduced to

$$\begin{aligned}
 p_G(\tau) &= \frac{1}{4\pi\tau_0} \frac{D_m^2}{\sqrt{\tau_n^2 - 1}} \\
 &\quad \int_{\pi/2}^{\pi/2} (\tau_n^2 - \sin^2\theta) \exp\left[-\frac{D_m^2(\tau_n - \sin\theta)^2}{8}\right] d\theta
 \end{aligned} \quad (8)$$

2.4 Delay Spread

Root-mean-square(rms) delay spread is defined as

$$\tau_{rms} = \sqrt{\overline{\tau^2} - \bar{\tau}^2}$$

where $\overline{\tau^2}$ and $\bar{\tau}$ are the mean square and mean value defined as

$$\overline{\tau^2} = \int_D^{D+2r} \tau^2 p(\tau) d\tau, \quad \bar{\tau} = \int_D^{D+2r} \tau p(\tau) d\tau \quad (9)$$

where $r = r_{ROS}$ in the ROS model, $r = r_{DOS}$ in the DOS model, and $r = \sigma_m$ in the Gaussian scatterers model. Note that the definitions of mean square and mean value are different from those defined in the measurements, which are

$$\overline{\tau^2} = \sum_{k=1}^K \tau_k^2 P(\tau_k), \quad \bar{\tau} = \sum_{k=1}^K \tau_k P(\tau_k) \quad (10)$$

where K is the number of time bins, $P(\tau_k)$ is the relative amplitude of multipath components falling in bin k . The measured rms delay spread is derived from a single power delay profile that is the temporal average of consecutive impulse response measurements collected and averaged over a local area. Assume that the scatterers around the mobile and base stations have similar scattering characteristics, then the signal strength in each time bins is roughly proportional to the number of scatterers that contribute to that time bin. Hence, (9) can be assumed equivalent to (10).

3 Simulations

Figure 5: Normalized rms delay spread as a function of size parameter in scatterer models, -----:regression line, - - -:scatterer model.

The rms delay spread can be related to the parameters of scatterer model. Regression analysis is also applied to the results of ROS, DOS and Gaussian scatterer models, respectively, as shown in Fig5. The radius of scatterer models can be related to τ_{rms} as

$$\begin{aligned} r_{ROS} &= 1.67c\tau_{rms} \\ r_{DOS} &= 1.82c\tau_{rms} \\ \sigma_m &= 0.79c\tau_{rms} \end{aligned} \quad (11)$$

Fig.6 shows the comparison of cumulative distribution function of delay for DOS and Gaussian models, respectively. The measured data in [4] are also shown for comparison. The measured rms delay spread is $0.682 \mu s$ with the LOS distance of 1,500 meters. The Gaussian model matches better with the measurement than the DOS model.

Figure 6: Distribution of TOA for ROS, DOS and Gaussian model, -----: Gaussian model, - - -: DOS model, - . - . : ROS model, * : measurement [4].

References

- [1] S. Al-Jazzar, J. Caffery, Jr. and H. R. You, "A scattering model based approach to NLOS mitigation in TOA location systems," *IEEE Veh. Technol. Conf.*, vol.2, pp.861-865, 2002
- [2] R. B Ertel and J. H. Reed, "Angle and time of arrival statistics for circular and elliptical scattering models," *IEEE J. Selected Areas in Commun.*, vol.17, no.11, pp.1829-1840, Nov 1999
- [3] R. Janaswamy, "Angle and time of arrival statistics for the Gaussian scatter density model," *IEEE Tran. Wireless Commun.*, vol.1, no.3, pp.488-497, Jul 2002
- [4] K.I. Pedersen, P.E. Mogensen and B.H. Fleury, "A stochastic model of the temporal and azimuthal dispersion seen at the base station in outdoor propagation environments," *IEEE Trans. Veh. Technol.*, vol.49, no.2, pp.437-447, Mar. 2000