A VARIABLE-LENGTH DHT-BASED FFT/IFFT PROCESSOR FOR VDSL/ADSL SYSTEMS

Tsung-Chieh Pao, Ching-Chi Chang, and Chorng-Kuang Wang*

Graduate Institute of Electronics Engineering and Department of Electrical Engineering,
National Taiwan University, Taipei 106, Taiwan R.O.C.

ABSTRACT

In this paper, a matrix form of the radix-4 Discrete Hartly Transform (DHT) algorithm is derived. And two Process Element (PE) circuits are proposed for the variable-length application. One is for radix-2 with one real multiplier and three adders, and the other is for radix-4 with one real multiplier and seven adders. The radix-4 DHT processor of variable-length, namely, 8192/4096/2048/1024/512, meets the DMT-VDSL and ADSL requirements. Only six real multipliers, $2.01N$ (16467) registers and simple logic circuits are used to perform 8192-points FFT/IFFT in 231 μ s. Implemented in 0.25 μ m 1P5M typical CMOS technology, the core occupies an active area of 5mm $\times$ 5mm. Power dissipation is 300mW using 2.5V supply voltage.

1. INTRODUCTION

Among standards with Multiple Carrier Modulation (MCM) technology using FFT/IFFT as the modulation technique, Very-high-bit-rate Digital Subscriber Line (VDSL) has potential for broadband communication of the next generation, owing to the popularity of twisted pairs [1]. The Discrete Fourier Transform (DFT) plays an important role in DMT/OFDM systems. Because of the computational complexity of N -points DFT is $O(N^2)$, it takes large operation time and power consumption to perform DFT directly, especially in large transform size (e.g. 8192 points in VDSL). Hence, various FFT algorithms have been proposed to reduce the computational complexity [2], such as Cooley-Turkey algorithm, Winograd Algorithm (WFTA), radar algorithm and Prime Factor Algorithm (PFA). Cooley-Turkey algorithm is the most popular one in VLSI implementation due to reducing the computational complexity from $O(N^2)$ to $O(N\log_2 N)$ and its regularity. Table 1 summaries several FFT architectures including Single-Path Delay Feedback (SDF) architecture [3], Multiple-Path Delay Commutator (MDC) [4], and DHT-based FFT/IFFT processor [5]. There are two key points shown in this table. One is that MDC architectures need more hardware than SDF architectures obviously. The other is that no matter what radix- r SDF architecture is used, $2(N-1)$ registers are essential. In order to reduce

hardware in FFT/IFFT processor, the design goal should be to reduce the number of real multipliers without increasing the number of registers. Based on the advantage of Hermitian symmetric property in DMT-VDSL and ADSL systems, some DHT-based FFT algorithms and architectures have been presented to reduce the numbers of real multipliers [5-7]. But there are some drawbacks about these architectures such as more buffers for RAM-based designs and only radix-2 DHT-based FFT algorithm is delivered. If the higher radix FFT algorithm is adopted, the fewer multipliers can be employed with Cooley-Turkey algorithm. Our goal is to derive matrix form of higher radix DHT algorithm from previous work and then implement it using less hardware.

The organization of this paper is as follows. The matrix form of radix-4 DIF DHT algorithm is derived in Section 2. Section 3 presents the proposed circuit designs of the two PEs. Section 4 shows VLSI implementation and performance summary of the FFT processor for VDSL system, and the conclusion is given in section 5.

Table 1. Hardware comparisons of various FFT architectures.

	Real Multipliers	Memory	Through-put (sample/cycle)
Radix-2 SDF	$3(\log_2 N - 2)$	$2(N-1)$	1
Radix-4 SDF	$3(\log_4 N - 1)$	$2(N-1)$	1
Radix-8 SDF	$3(\log_8 N - 1)$	$2(N-1)$	1
Radix-2 MDC	$3(\log_2 N - 2)$	$2(1.5N - 2)$	2
Radix-4 MDC	$3(3\log_4 N - 3)$	$2(2.5N - 4)$	4
Radix-8 MDC	$3(7\log_8 N - 7)$	$2(4.5N - 8)$	8
Radix-2 DHT-based [5]	$\log_2 N - 1$	$2(N-1)$	1

2. RADIX-4 DHT ALGORITHM

Given N -points DHT definition as

$$y_k = \sum_{n=0}^{N-1} x_n \left(\cos \frac{2\pi kn}{N} + \sin \frac{2\pi kn}{N} \right) \quad (1)$$

$$= \sum_{n=0}^{N-1} x_n \cos \frac{2\pi kn}{N},$$

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where $k=0, 1, \dots, N/4-1$. The frequency samples, $y_{4k}, y_{4k+1}, y_{4k+2}$ and y_{4k+3} are derived as [8]:

$$y_{4k} = \sum_{n=0}^{N/4-1} x_n \cos \frac{2\pi(4k)n}{N} \quad (2)$$

$$\begin{aligned} &= \sum_{n=0}^{N/4-1} [x_n + x_{n+\frac{N}{4}} + x_{n+\frac{N}{2}} + x_{n+\frac{3N}{4}}] \cos \frac{2\pi(4k)n}{N}, \\ y_{4k+1} &= \sum_{n=0}^{N/4-1} x_n \cos \frac{2\pi(4k+1)n}{N} \\ &= \sum_{n=0}^{N/4-1} [(x_n - x_{n+\frac{N}{4}} + x_{n+\frac{N}{2}} - x_{n+\frac{3N}{4}}) \cos \frac{2\pi n}{N} + \\ &\quad (x_{n+\frac{3N}{4}} - x_{n+\frac{N}{4}} + x_{n+\frac{N}{2}} - x_{n-N}) \sin \frac{2\pi n}{N}] \cos \frac{2\pi(4k)n}{N}, \end{aligned} \quad (3)$$

$$\begin{aligned} y_{4k+2} &= \sum_{n=0}^{N/4-1} x_n \cos \frac{2\pi(4k+2)n}{N} \\ &= \sum_{n=0}^{N/4-1} [(x_n + x_{n+\frac{N}{2}} - x_{n+\frac{N}{4}} - x_{n+\frac{3N}{4}}) \cos \frac{2\pi 2n}{N} + \\ &\quad (x_{n+\frac{N}{4}} + x_{n+\frac{3N}{4}} - x_{n-\frac{N}{2}} - x_{n-N}) \sin \frac{2\pi 2n}{N}] \cos \frac{2\pi(4k)n}{N}, \end{aligned} \quad (4)$$

$$\begin{aligned} y_{4k+3} &= \sum_{n=0}^{N/4-1} x_n \cos \frac{2\pi(4k+3)n}{N} \\ &= \sum_{n=0}^{N/4-1} [(x_n - x_{n+\frac{N}{2}} + x_{n+\frac{N}{4}} + x_{n+\frac{3N}{4}}) \cos \frac{2\pi 3n}{N} + \\ &\quad (-x_{n+\frac{3N}{4}} + x_{n+\frac{N}{4}} + x_{n+\frac{N}{2}} - x_{n-N}) \sin \frac{2\pi 3n}{N}] \cos \frac{2\pi(4k)n}{N}, \end{aligned} \quad (5)$$

where $k=0, 1, \dots, N/4-1$. From Eq. 2, 3, 4 and 5, the frequency samples, $y_{4k}, y_{4k+1}, y_{4k+2}$ and y_{4k+3} , are respectively equal to the DHT of the four $N/4$ -points sequences bracketed in these four equations. Then, four $N/4 \times 1$ matrices, $Y_{4k}, Y_{4k+1}, Y_{4k+2}$ and Y_{4k+3} , and four $N/4 \times N$ matrices, X_A, X_B, X_C and X_D , are defined as follows.

$$\begin{aligned} Y_{4k} &= [y_0 y_4 y_8 \dots y_{4(N/4-1)}]^T, \\ Y_{4k+1} &= [y_1 y_5 y_9 \dots y_{4(N/4-1)+1}]^T, \\ Y_{4k+2} &= [y_2 y_6 y_{10} \dots y_{4(N/4-1)+2}]^T, \\ Y_{4k+3} &= [y_3 y_7 y_{11} \dots y_{4(N/4-1)+3}]^T, \end{aligned}$$

$$\begin{aligned} X_A &= \begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{bmatrix} X \\ X_B &= \begin{bmatrix} C_N^* & & & & & & & \\ & C_N^* & & & & & & \\ & & -S_N^* & & & & & \\ & & & S_N^* & & & & \\ & & & & -C_N^* & & & \\ & & & & & C_N^* & & \\ & & & & & & -S_N^* & \\ & & & & & & & S_N^* \end{bmatrix} X \\ X_C &= \begin{bmatrix} C_N^* & & & & & & & \\ & C_N^* & & & & & & \\ & & -C_N^* & & & & & \\ & & & S_N^* & & & & \\ & & & & -C_N^* & & & \\ & & & & & C_N^* & & \\ & & & & & & -S_N^* & \\ & & & & & & & S_N^* \end{bmatrix} X \end{aligned}$$

$$X_D = \begin{bmatrix} C_N^* & & & & & & & \\ & -C_N^* & & & & & & \\ & & -C_N^* & & & & & \\ & & & S_N^* & & & & \\ & & & & S_N^* & & & \\ & & & & & -C_N^* & & \\ & & & & & & C_N^* & \\ & & & & & & & -S_N^* \end{bmatrix} X$$

where $X = [x_0 x_1 x_2 \dots x_{N-1}]^T$, and $C_N^k = \cos(2\pi kn/N)$ and $S_N^k = \sin(2\pi kn/N)$. The matrix form of DHT is

$$\begin{bmatrix} Y_{4k} \\ Y_{4k+1} \\ Y_{4k+2} \\ Y_{4k+3} \end{bmatrix} = \begin{bmatrix} H(N/4) & 0 & 0 & 0 \\ 0 & H(N/4) & 0 & 0 \\ 0 & 0 & H(N/4) & 0 \\ 0 & 0 & 0 & H(N/4) \end{bmatrix} \begin{bmatrix} X_A \\ X_B \\ X_C \\ X_D \end{bmatrix} \quad (6)$$

Let $Q(N) = [e_0 e_4 e_8 \dots e_{N-4} e_1 e_5 e_9 \dots e_{N-3} e_2 e_6 e_{10} \dots e_{N-2} e_3 e_7 e_{11} \dots e_{N-1}]^T$ be the permutation matrix, and e_i denotes an $N \times 1$ column matrix whose $(i+1)$ th element is 1. Then Eq. 6 can be derived as a similar form of Radix-2 DHT [6], as shown in the following.

$$Q(N)Y = \begin{bmatrix} Y_{4k} \\ Y_{4k+1} \\ Y_{4k+2} \\ Y_{4k+3} \end{bmatrix} = \begin{bmatrix} H(N/4) & 0 & 0 & 0 \\ 0 & H(N/4) & 0 & 0 \\ 0 & 0 & H(N/4) & 0 \\ 0 & 0 & 0 & H(N/4) \end{bmatrix} P(N)X, \quad (7)$$

$H_{N/M}(M)$ is the direct sum of N/M $M \times M$ matrices, $H(M)$.

$$\begin{aligned} H_{N/M}(M) &= H(M) \oplus H(M) \oplus \dots \oplus H(M) \\ &= \begin{bmatrix} H(M) & 0 & \dots & 0 \\ 0 & H(M) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & H(M) \end{bmatrix} \end{aligned} \quad (8)$$

Eq. 7 can be simplified as

$$Q_1(N)Y = H_4(N/4)P_1(N)X, \quad (9)$$

which represents that the N -points DHT of X becomes four parts of $N/4$ -points DHT of $P_1(N)X$. And the order of the DHT result, Y , is permuted to $Q_1(N)Y$. Repeating the similar decomposition, radix-4 DHT can be gotten as

$$Q_{N/4}(4)Q_{N/16}(16)\dots Q_1(N)Y = P_{N/4}(4)P_{N/16}(16)\dots P_1(N)X \quad (10)$$

It shows that the N -points DHT of X can be performed by several multiplications of matrix, $P_{N/M}(M)$, which is corresponding to the $(\log_4 N/M+1)$ th stage of the DHT algorithm. And the order of DHT result is bit-reversed of Y .

3. CIRCUIT DESIGNS

To realize the 8192-points FFT for VDSL, one radix-2 DHT stage and six radix-4 DHT stages are needed as shown in Fig. 1. Besides, an external dual-port RAM and two adders are required to perform the rearrangement of bit-reversed DHT results and acquire the normal-order FFT results [7].

Fig. 1. Variable-length DHT-based FFT processor for 8192/4096/2048/1024/512-points VDSL and 512-points ADSL.

3.1. Radix-2 PE

Based on the matrix form of radix-2 DIF DHT algorithm [6],

$$Z = Q_{N/2}(2)Q_{N/4}(4)\cdots Q_1(N)Y = P_{N/2}(2)P_{N/4}(4)\cdots P_1(N)X, \quad (11)$$

the X column matrix denotes the sequential input data in time domain, and the Z column matrix indicates the sequential output data in bit-reversed order. The matrix, $P_{N/M}(M)$, is corresponding to the $(\log_2 N/M+1)$ th stage PE. And it has the general form as Eq. 12 [6]. Fig. 2 shows the proposed circuit architecture of radix-2 DIF DHT PE, $P_{N/M}(M)$. Due to the utilization of the two-phase operating technique, only one real multiplier is employed to perform two multiplications.

$$P(M) = \begin{bmatrix} I_{M/2} & 0 \\ 0 & K(M/2) \end{bmatrix} \begin{bmatrix} I_{M/2} & I_{M/2} \\ I_{M/2} & -I_{M/2} \end{bmatrix} \quad (12)$$

$$K(M/2) = C(M/2) + S(M/2)$$

$$= \begin{bmatrix} C_M^0 & & & \\ & C_M^1 & & \\ & & \ddots & \\ & & & C_M^{M/2-1} \end{bmatrix} + \begin{bmatrix} S_M^0 & & & \\ & S_M^1 & & \\ & & \ddots & \\ & & & S_M^{M/2-1} \end{bmatrix}$$

3.2. Radix-4 PE

The matrix form of radix-4 DIF DHT algorithm is followed as Eq. 10 and the matrix, $P_{N/M}(M)$, is corresponding to the $(\log_4 N/M+1)$ th stage PE. As the sequential input data, X , enter each PE stage, the input data are rearranged by the circuit as shown in Fig. 3. The 16-points radix-4 DHT is demonstrated in the following.

$$Z = Q_4(4)Q_1(16)Y = P_1(4)P_1(16)X, \quad \text{where} \quad (13)$$

$$P_1(16) = \begin{bmatrix} I & I & I & I \\ B & C & -B & -C \\ A & -A & A & -A \\ D & E & -D & -E \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} C_{16/1}^0 & 0 & 0 & 0 \\ 0 & C_{16/1}^1 & 0 & 0 \\ 0 & 0 & C_{16/1}^2 + C_{16/1}^0 & 0 \\ 0 & C_{16/1}^3 & 0 & C_{16/1}^1 \end{bmatrix}$$

$$C = \begin{bmatrix} C_{16/1}^0 & 0 & 0 & 0 \\ 0 & -S_{16/1}^1 & 0 & 0 \\ 0 & 0 & S_{16/1}^2 - S_{16/1}^0 & 0 \\ 0 & S_{16/1}^3 & 0 & -S_{16/1}^1 \end{bmatrix}$$

$$A = \begin{bmatrix} C_{16/2}^0 & 0 & 0 & 0 \\ 0 & C_{16/2}^1 & 0 & S_{16/2}^1 \\ 0 & 0 & C_{16/2}^2 + S_{16/2}^0 & 0 \\ 0 & S_{16/2}^3 & 0 & C_{16/2}^1 \end{bmatrix}$$

$$D = \begin{bmatrix} C_{16/2}^0 & 0 & 0 & 0 \\ 0 & -S_{16/2}^1 & 0 & 0 \\ 0 & 0 & S_{16/2}^2 - S_{16/2}^0 & 0 \\ 0 & S_{16/2}^3 & 0 & -S_{16/2}^1 \end{bmatrix}$$

$$E = \begin{bmatrix} C_{16/2}^0 & 0 & 0 & 0 \\ 0 & C_{16/2}^1 & 0 & -S_{16/2}^1 \\ 0 & 0 & C_{16/2}^2 + S_{16/2}^0 & 0 \\ 0 & -S_{16/2}^3 & 0 & C_{16/2}^1 \end{bmatrix}$$

$$D = \begin{bmatrix} C_{16/3}^0 & 0 & 0 & 0 \\ 0 & C_{16/3}^1 & 0 & -C_{16/3}^1 \\ 0 & 0 & C_{16/3}^2 - C_{16/3}^0 & 0 \\ 0 & -C_{16/3}^3 & 0 & C_{16/3}^1 \end{bmatrix}, E = \begin{bmatrix} -C_{16/3}^0 & 0 & 0 & 0 \\ 0 & S_{16/3}^1 & 0 & 0 \\ 0 & 0 & S_{16/3}^2 + S_{16/3}^0 & 0 \\ 0 & S_{16/3}^3 & 0 & S_{16/3}^1 \end{bmatrix}$$

Fig. 2. Radix-2 DIF DHT PE.

Fig. 3. Rearrangement of each stage input (e.g. $M=16$, for $P_1(16)$).

After the rearrangement circuit, the PE, $P_1(16)X$, becomes two simplified matrices shown as Eq. 14 and 15.

$$P_1(16)X = \begin{bmatrix} a_{4k} \\ a_{4k+1} \\ a_{4k+2} \\ a_{4k+3} \end{bmatrix}, \quad \text{where}$$

$$\begin{bmatrix} a_{4k} \\ a_{4k+2} \end{bmatrix} = \begin{bmatrix} I & I \\ A & -A \end{bmatrix} \begin{bmatrix} x_0 + x_8 & x_1 + x_9 & \cdots & x_7 + x_{15} \end{bmatrix}^T \quad (14)$$

$$\begin{bmatrix} a_{4k+1} \\ a_{4k+3} \end{bmatrix} = \begin{bmatrix} B & C \\ D & E \end{bmatrix} \begin{bmatrix} x_0 - x_8 & x_1 - x_9 & \cdots & x_7 - x_{15} \end{bmatrix}^T \quad (15)$$

In Eq. 14, this part of $P_1(16)X$ is the same as the general form of radix-2 DIF DHT PE derived in Eq. 12. The similar circuit architecture shown in Fig. 2 can be utilized. In Eq. 15, this part of $P_1(16)X$ is performed by the circuit architecture as shown in Fig. 4. The two-phase multiplication of single multiplier technique is also employed.

Fig. 4. Circuit architecture for computing a_{4k+1} and a_{4k+3} .

The circuits in Fig. 2 and 4 for radix-4 DHT use the same multiplier because of performing the multiplication at different cycles. By rearranging the computing order as a_{4k} , a_{4k+2} , a_{4k+1} and a_{4k+3} , the bit-reversed order in Z can be obtained.

As computing different points of radix-4 DHT, only the size of M in Fig. 2 and 4 is changed. The radix-4 DHT PE can be performed with latency of $3M/4$ cycles by only one real multiplier, seven real adders, $3M/4$ RAM as delay line, $3M/4$ RAM with double clock rate, two ROMs as sine and cosine coefficients, and some simple logic circuits.

4. VLSI IMPLEMENTATION AND PERFORMANCE SUMMARY

For a variable-length DHT-based FFT processor, several considerations in VLSI implementation such as variable-length, simplified ROM of sin/cos tables, cyclic extension removing and fixed-points evaluation for SNR requirements of VDSL system are all taken into account in this chip. The radix-4 DIF DHT processor is implemented with $0.25\mu\text{m}$ 1P5M typical CMOS technology. Chip design and simulation summary are described in Table 2, and chip layout is shown in Fig. 5. Compared in Table 1, it shows that the proposed radix-4 DHT-based FFT architecture has the fewest numbers of real multipliers among SDF architectures and radix-2 DHT-based FFT processor [5]. Otherwise, they use almost the same numbers of registers. Table 3 shows the comparisons with other 8192-points FFT chips.

Table 2. Chip design and simulation summary.

Process	CMOS $0.25\mu\text{m}$ 1P5M
Total Area	34mm^2 , 25mm^2 (core)
RAM Area	5.2mm^2
Maximum Speed	50MHz
Latency	$8221@N=8192$
Throughput	1 sample/cycle
Power Consumption	$300\text{mW}@35\text{MHz}$
Package	CQFP144
Supply Voltage	2.5V

Fig. 5. Chip layout of the proposed architecture.

5. CONCLUSION

In this paper, the radix-4 DHT-based FFT algorithm is first derived according to the Hermitian symmetric property of the data in DMT-VDSL and ADSL systems, and the matrix form of the radix-2 DHT-based FFT designs [5][7]. Two PE circuits are proposed for variable-length application. One is composed of only one

real multiplier and three adders for radix-2. The other is comprised with only one real multiplier and seven adders for radix-4. Finally, a variable-length 8192/4096/2048/1024/512 radix-4 DIF DHT processor is demonstrated. It meets the DMT-VDSL and ADSL requirements and uses only six real multipliers, $2.01N$ (16467) registers and some simple logic circuits. The chip is designed using $0.25\mu\text{m}$ 1P5M typical CMOS technology and occupies a core area of $5\text{mm}\times 5\text{mm}$. Power dissipation is 300mW under 2.5V supply voltage while running at 35MHz .

Table 3. Comparison with other FFT chips.

	[9]	[10]	This Work
FFT Size	8192	8192	8192/4096/2048/1024/512
Technology	$0.5\mu\text{m}$ CMOS	$0.6\mu\text{m}$ CMOS	$0.25\mu\text{m}$ CMOS
Chip Area (core)	1cm^2	140mm^2 107mm^2	34mm^2 25mm^2
Normalized Area (core)	25mm^2	24.3mm^2 18.6mm^2	34mm^2 25mm^2
Clock	20MHz	20MHz	35MHz
Power Consumption	600mW	650mW	300mW
Word length: Input/Output sin, cos	10/12 10	10/12 10	22/22 20
Internal results	12 (bits)	12 (bits)	22 (bits)
Register	Delay-line	RAM	SRAM

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