General Splitting and Merging of 2-D DCT in the DCT Domain

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Abstract—An efficient method for splitting an $N \times N$ 2-D DCT block into four $(N/2) \times (N/2)$ or two $N \times (N/2)$ (or $(N/2) \times N$) 2-D DCT blocks is presented and vice versa. The computational complexity of the proposed methods is lower than the direct approach and the same as the most efficient converting approach existed in the literature. Besides, the proposed DCT splitter/merger is suitable for implementation by specific multimedia instruction set available nowadays. When $N = 8$, our method can be applied to realize the transcoding between the latest video coding standards AVC/H.264 and the older ones, such as MPEG-1, MPEG-2 and MPEG-4 part 2.

I. INTRODUCTION

Multimedia signals, such as images and video, are always compressed so as to save memory space and/or be easily transmitted via network. However, the signals have to be processed before being displayed, transmitted, printed, etc. Some of the frequently used manipulations are scaling, filtering, rotation and translation. Implementing these functions in the compressed domain is advantageous from the computational complexity point of view as well as the image quality and the memory usage. This is because the transition to the time or spatial domain and the recompression of the data are avoided. All existing compression standards—JPEG, MPEG1, MPEG2 and H.26x—are based on the discrete cosine transform (DCT), which is applied to blocks of data of certain lengths. Our work aims to directly split an $N \times N$ 2-D DCT into four adjacent $(N/2) \times (N/2)$ 2-D DCTs without the need of performing an $N \times N$ IDCT and the four $(N/2) \times (N/2)$ DCTs. On the other hand, merging four adjacent $(N/2) \times (N/2)$ 2-D DCTs into one $N \times N$ DCT can also be realized by inverse transposing the “split” case. The block diagram of the proposed 1-D DCT splitter/merger is shown in Fig.1(b), where DCT splitter/merger module have to be implemented by using multiplications, additions and permutations as little as possible. We will show the superiority of the proposed algorithm in computational complexity from both the algorithmic-level and the programming-level.

This paper is organized as follows. In Section II, the splitter/merger of the 1-D/2-D DCT is addressed. Two kinds of computational complexity comparisons with other approaches are given in Section III. Finally, Section IV concludes this write-up.

II. THE DCT SPLITTER AND/OR MERGER

For the sake of simplicity, let us confine our attention to the 1-D case, first. Assume that the DCT coefficients Y_k and Z_k , $k = 0, \dots, N/2$, of two consecutive $N/2$ -point data sequences $y = [y_0 \ y_1 \ \dots \ y_{N/2}]^T$ and $z = [z_0 \ z_1 \ \dots \ z_{N/2}]^T$ are given. The problem to be addressed is: how to efficiently compute the N -point DCT coefficients X_k , $k = 0, 1, \dots, N$, where X_k stands for the DCT coefficients of

$$x = [x_0 \ x_1 \ \dots \ x_N]^T = [y_0 \ y_1 \ \dots \ y_{N/2} \ z_0 \ z_1 \ \dots \ z_{N/2}]^T.$$

The normalized forward DCT Type-II (DCT-II) of a length- N sequence x_n is, given in [1], as follows:

$$X_k = \varepsilon_k \sum_{n=0}^{N-1} x_n \cdot \cos\left(\frac{\pi(2n+1)k}{2 \cdot N}\right) \quad \text{for } k = 0, 1, \dots, N-1 \quad (1)$$

where, $\varepsilon_k = \sqrt{\frac{1}{N}}$, for $k=0$ and $\varepsilon_k = \sqrt{\frac{2}{N}}$, for $k \neq 0$. The DCT-II's of the length- $N/2$ sequences y and z can be represented by similar expressions. The DCT defined in (1)

Fig.1. (a) The traditional approach, and (b) The proposed approach for realizing the 1-D DCT splitter/merger.

can be written in matrix form as

$$X = [T_N]^H X = [C_N][T_N]x \quad (2)$$

where $[C_N] = \text{diag}(\varepsilon_0, \varepsilon_1, \dots, \varepsilon_{N-1})$ and $[T_N]$ is an $N \times N$ matrix with elements $[T_N]_{ij} = \cos\left(\frac{\pi(2j+1)i}{2 \cdot N}\right)$, $i, j = 0, 1, \dots, N-1$. In transform coding applications, the DCT stage is usually followed by a quantization stage. The computation of ε_k can be saved by absorbing them into the quantization stage (the so-called non-scale case). In this case, we rewrite Eqn.(2) as

$$X = [T_N]x \quad (3)$$

A. Use recursive form $[\hat{T}_N]$ instead of $[T_N]$:

Instead of viewing (3) as the real part of a scaled N -point DFT, Hou [2] delved into the symmetry of the kernel in (3) and partitioned the transform matrix $[T_N]$ into a recursive form. Let $[\hat{T}_N] = [\tilde{P}_N][T_N][\tilde{P}_N]^{-1}$, it can be examined that

$$[\hat{T}_N] = \begin{bmatrix} [\hat{T}_{N/2}] & [\hat{T}_{N/2}] \\ [\hat{D}_{N/2}] & -[\hat{D}_{N/2}] \end{bmatrix}, \quad (4)$$

where $[\tilde{P}_N]$ is an $(N \times N)$ bit-reversal matrix and $[\tilde{P}_N]$ is also an $(N \times N)$ permutation matrix defined as

$$[\tilde{P}_N] = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & 1 \\ 0 & 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & 0 & \dots & 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 1 & 0 & \dots & \dots & \dots & \dots \end{bmatrix}$$

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domain splitting and merging between one 8-point DCT and two 4-point DCTs is depicted in Fig.2.

Fig.2. The signal-flow graph for (a) Splitting an 8-point DCT to two 4-point DCTs, and (b) Merging two 4-point DCTs to an 8-point DCT.

III. COMPUTATIONAL COMPLEXITY ANALYSES

In this section, two computational complexity analyses will be discussed. First, the algorithmic-level analysis, i.e., the number of multiplications and additions required for the proposed approach will be counted in comparison with other ones in the literature. Nowadays, most of the modern processors have specific multimedia instruction set to facilitate multimedia applications. For example, the single-instruction-multiple-data (SIMD) execution model was introduced in Intel architectures, such as MMX technology and streaming SIMD extensions (SSE) [5, 6]. The SIMD technology enables multiple arithmetic or logic operations to be executed simultaneously, so as to improve the efficiency of the program. Thus, we will present programming-level analysis to show the suitability of our approach for general-purpose processors with SIMD instructions.

A. The algorithmic-level analysis

A.1 The proposed approach

From the previous subsection, we need to analyze the complexity of $[S_N]$. According to Eqn.(14), $[M'_{N/2}]^{-1}[K_{N/2}]^{-1} = [\hat{T}_{N/2}][Q'_{N/2}][\hat{T}_{N/2}]^{-1}[\tilde{P}_{N/2}]^{-1}[L_{N/2}]^{-1}[\tilde{P}_{N/2}]$ needs to be computed first and then do N more additions for $\begin{bmatrix} [L_{N/2}] & [L_{N/2}] \\ [L_{N/2}] & -[L_{N/2}] \end{bmatrix}$. It can be shown that

$$[L_{N/2}]^{-1} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & \dots & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \dots & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \quad (22)$$

and only $(N/2)-1$ additions are required to compute $[L_{N/2}]^{-1}$. A fast DCT algorithm, like the one given in [7 or 8] spends $\mu_N^{DCT} = \left(\frac{N}{2}\right) \log_2 N$ multiplications (denoted as $\mu_N^{DCT} M$) and $\nu_N^{DCT} = \left(\frac{3N}{2}\right) \log_2 N - N + 1$ additions (denoted as $\nu_N^{DCT} A$) for $[T_N^H]$ to compute a 1-D N -point DCT. Both $\mu_{[S_N]}$ and $\nu_{[S_N]}$ are applicable to $[T_N]$, because $[T_N]$ is obtained by omitting the scale factor and rearranging the rows and columns of $[T_N^H]$. Hence the required number of multiplications $\mu_{[S_N]}$ and additions $\nu_{[S_N]}$ are

$$\mu_{[S_N]} = \mu_{[M_{N/2}]}[K_{N/2}]^{-1} = 2\mu_{[T_{N/2}]} + \mu_{[Q_{N/2}]} = 2\mu_{N/2}^{DCT} + N/2$$

and

$$\nu_{[S_N]} = \nu_{[M_{N/2}]}[K_{N/2}]^{-1} + N = 2\nu_{[T_{N/2}]} + \nu_{[Q_{N/2}]} + N = 2\nu_{N/2}^{DCT} + 3N/2 - 1.$$

If the scale factors were not absorbed into the quantization

stage (the so-called with scale case), we need at most N more multiplications to obtain our result. Thus, the proposed algorithm totally needs

$$(\mu_N^{DCT} + N)M \text{ and } (\nu_N^{DCT} - 3N/2)A$$

for splitting an N -point DCT block into two $(N/2)$ -point DCT blocks. Splitting an $N \times N$ DCT block into two $(N/2) \times N$ (or $N \times (N/2)$) DCT blocks can be realized by performing 1-D split/merge algorithm along each row (or each column), and therefore, our algorithm needs $(N(\mu_N^{DCT} + N))M$ and $(N(\nu_N^{DCT} - 3N/2))A$ to accomplish this kind of these tasks. Splitting an $N \times N$ DCT into four $(N/2) \times (N/2)$ DCTs can be realized by performing 1-D split/merge algorithm along each row and then along each column. Our algorithm needs $(2N(\mu_N^{DCT} + N))M$ and $(2N(\nu_N^{DCT} - 3N/2))A$ in total, for the with scale case.

A.2. Other approaches

1) *Direct approach*: The direct approach takes one N -DCT, and then takes two forward $(N/2)$ -DCTs for splitting an N DCT block into two $(N/2)$ DCT blocks. The amount of computation for two dimensional case is $(2N(\mu_N^{DCT} + 2\mu_{N/2}^{DCT}))M + (2N(\nu_N^{DCT} + 2\nu_{N/2}^{DCT}))A$. Similarly, $(N(\mu_N^{DCT} + 2\mu_{N/2}^{DCT}))M + (N(\nu_N^{DCT} + 2\nu_{N/2}^{DCT}))A$ are needed for computing two $(N/2) \times N$ (or $N \times (N/2)$) 2-D DCT blocks.

2) *A. N. Skodras's approach*: An efficient method for direct computing an N -point DCT from two given adjacent $N/2$ -point DCTs has been proposed in [9]. The amount of computations is the same as our proposed approach.

In TABLE I, the numbers of operations needed in different approaches for performing the above DCT splitting tasks are tabulated. By examining the direct approach, the scaling factor had no effect on the multiplication counts in their approach. It is clear that the proposed algorithm is more efficient than the direct one and as well as the A. N. Skodras's approach.

TABLE I
THE NUMBERS OF OPERATIONS REQUIRED FOR SPLITTING AN $N \times N$ 2-D DCT INTO FOUR $(N/2) \times (N/2)$ 2-D DCTs IN DIFFERENT APPROACHES.

Different case	The number of operations needed for splitting an $N \times N$ 2-D DCT blocks into four $(N/2) \times (N/2)$ 2-D DCT blocks in:	
	Our proposed / A. N. Skodras's approach [9]	Direct approach
with scale	$(2N(\mu_N^{DCT} + N))M$	$(2N(\mu_N^{DCT} + 2\mu_{N/2}^{DCT}))M$
	$(2N(\nu_N^{DCT} - 3N/2))A$	$(2N(\nu_N^{DCT} + 2\nu_{N/2}^{DCT}))A$
non-scale	$(2N\mu_N^{DCT})M$	$(2N(\mu_N^{DCT} + 2\mu_{N/2}^{DCT}))M$
	$(2N(\nu_N^{DCT} - 3N/2))A$	$(2N(\nu_N^{DCT} + 2\nu_{N/2}^{DCT}))A$

B. The programming-level analysis

Another way to increase performance of a signal processing task is to execute several computations in parallel. Without algorithmic-level optimization, the direct approach can be speeded-up by using today's multimedia instruction set technology. In the following, we will show that our splitter/merger is much more suitable to be realized on the processors, with SIMD instructions, than other approaches.

B.1. SIMD instructions

Intel Pentium 4 processors extended SIMD computation model with the introduction of SSE2, which operates on packed double (float)-precision data elements as well as 128-bit packed integers. The full set of SIMD capability greatly improves the performance of multimedia related applications. For example, a 128-bit SSE2 register can be

used to store up to 16 units of 8-bit integers, and up to 16 arithmetic operations can be executed simultaneously by using two SSE2 registers. This results in a significant performance improvement, especially for video encoding.

In our splitter/merger, multiplication of matrices is the key. The SSE2 instructions, PMADDWD and PADDD, can facilitate this implementation. Fig.3 shows an example for computing the first row of the multiplication of two 4×4 matrices. Matrix X and Y are loaded into SSE registers in a particular form. After that, PMADDWD is performed on these registers in pairs. Eight multiplications are performed on pairs of 16-bit integers simultaneously. And the results, in pairs, are added together and stored in one SSE register as 32-bit integers. Then the two resulting 128-bit data are added together in the 32-bit integer precision by PADDD, hence the first row of the resulting matrix is obtained right way. For the next three rows, similar steps utilized. Therefore, 8 PMADDWDs and 4 PADDDs are required for realizing 4×4 matrix multiplications. The time spent, in this approach, is much less than that of 4^3 multiplications and 3×4^2 additions required in the direct calculation.

Fig.3. (a) A 4×4 matrix-by-matrix multiplication. (b) An example for efficient implementation of 4×4 matrix-by-matrix multiplication.

B.2. Performance Comparisons

The proposed splitter/merger and the direct approaches can be speeded-up by SSE2 instructions because both of them are matrix-based algorithms. Let's investigate the most important case of splitting an 8-point DCT into two 4-point DCTs in detailed. The scale factors are saved by absorbing them into the quantization stage in the following analyses.

1) *The proposed splitter/merger*: We need to count the number of operations required for $[S_8]$. In (21), $[\tilde{P}_8]$ and $[\tilde{P}_4]$ can be implemented by shifts and permutations; two PADDDs are required for realizing an 8×1 vector multiplied by $\begin{bmatrix} [I_4] & [I_4] \\ [I_4] & -[I_4] \end{bmatrix}$. 8 PMADDWDs and 4 PADDDs are required for computing the product of $[\tilde{T}_4]$ and $[\tilde{T}_4]^{-1}$ with a 4×1 vector, respectively. Since $[Q_4]$ is a diagonal matrix, the matrix-vector product by $[Q_4]$ can be done by one PMADDWD. Since

$$[L_4]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix},$$

the matrix-vector product by $[L_4]^{-1}$ can be done by one PADDD and bit shifts. It shows that the total numbers of operations required for the proposed splitter/merger are 17 PMADDWDs and 11 PADDDs.

2) *Direct approach*: An 8-point IDCT is done first and then two forward 4-point DCTs are taken, which can be written matrix form as

$$\begin{bmatrix} Y \\ Z \end{bmatrix} = \begin{bmatrix} [T_4] & 0 \\ 0 & [T_4] \end{bmatrix} [T_8]^{-1} [X] = \begin{bmatrix} [T_4] [T_8]_{LU}^{-1} & [T_4] [T_8]_{RU}^{-1} \\ [T_4] [T_8]_{LD}^{-1} & [T_4] [T_8]_{RD}^{-1} \end{bmatrix} [X], \quad (23)$$

where $[T_8]_{LU}^{-1}$, $[T_8]_{LD}^{-1}$, $[T_8]_{RU}^{-1}$ and $[T_8]_{RD}^{-1}$ are left-up, left-down, right-up and right-down 4×4 matrices of $[T_8]^{-1}$, respectively. To accomplish Eqn.(23), it needs four 4×4 matrix-by-matrix multiplications and four 4×4 matrix-by-vector multiplications. Thus, 40 PMADDWDs and 20 PADDDs are needed.

3) *A. N. Skodras's approach*: In this the approach, one can not easily load input data into those longer registers, thus this approach can not gain any benefit from SSE2 instructions.

It is shown that the direct approach can optimize its implementation by using SIMD technology, however, without algorithmic-level optimization, the speed-up of the optimization will be limited. Integrating an effective algorithmic-level derivation and programming-level optimizations with SIMD instructions, our approach gets the best performance than other known algorithms.

IV. CONCLUSIONS

An efficient method for direct splitting of an $N \times N$ 2-D DCT coefficients into four adjacent $(N/2) \times (N/2)$, two adjacent $N \times (N/2)$ (or $(N/2) \times N$) 2-D DCT coefficients has been proposed. The algorithmic-level computational complexity of the proposed algorithm is lower than the direct approach and the programming-level analysis tells us that the proposed approach can get the best performance with the aid of available multimedia instruction set technology. Due to its efficiency, the proposed DCT splitter/merger can be applied to realize the transcoding between the latest video coding standard, AVC/H.264, and the older ones, such as MPEG-1, MPEG-2 and MPEG-4 part 2.

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