Mobile Positioning Based on TOA/TSOA/TDOA Measurements with NLOS Error Reduction

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Abstract—With the spread of mobile communication networks, location dependent services are gaining popularity all over the world. Obviously, as its name implies, a location dependent service needs to know (more practically speaking, estimate) the location of the mobile station (MS) before it can offer any service at all. In this paper, we address the issue of mobile positioning (i.e. estimating the location of an MS) given the measured distances between the MS and its nearby base stations (BS's). The measurements, which are in general corrupted with measurement noise and NLOS (non-line-of-sight) error, can come from the time of arrival (TOA), the time sum of arrival (TSOA), or the time difference of arrival (TDOA). The NLOS error is the major cause that degrades the accuracy of mobile positioning. Assuming the knowledge of its statistic model, however, we propose a scheme that greatly reduce its effect. The results on MS location estimation thus obtained are comparable to the Cramer-Rao lower bounds (CRLB's). Regardless of the measurement types (i.e. TOA, TSOA, or TDOA), the proposed scheme computes the MS location in a unified way.

I. INTRODUCTION

With the rapid development of wireless communications, it is expected that location dependent services will be widely deployed all over the world in the near future. The first step in providing such services is, of course, to find the location of the client (i.e. MS)[1], [2]. It is quite probable that GPS (global positioning system) is the first thing popping up in our mind for this MS positioning task[3]. However, in this paper, we consider the situation that GPS is not available (e.g. inside a department store, where satellite signals are blocked by ceiling of building). Instead of using GPS, we assume that the positioning of the MS is to be based on its relative location with respect to nearby BS's, whose exact positions are already known.

Basically, we are going to measure the distances between the MS and the BS's (by TOA, TSOA, or TDOA) and then apply the method of trilateration to compute the MS's position. The problem is that the measured distances can be far from the true ones because of measurement noise and NLOS propagation error, in which the latter plays a dominant role in degrading the precision of the computed MS position[4]. Due to NLOS error, there tends to be an amount of overestimation in the measured distance. In this paper, by assuming a statistic model for the NLOS error, we are able to reduce this overestimation before trilateration is applied. Several types of models are considered. Algorithms for estimating the relevant parameters in those models are presented. Given the distributions of measurement noise (usually modeled as Gaussian) and NLOS error, the distribution of the measured distance can be derived. Then, the maximum likelihood estimate (MLE) of MS's location is computed. Due to the intractability of the measured distance's probability density function (pdf) in some cases, however, Gaussian pdf may have to be adopted

as an approximation. Moreover, with Gaussian approximation, we are able to derive a unified (taking into account of TOA, TSOA, and TDOA measurements) closed-form expression for the CRLB of the estimated MS location's variance. The CRLB will serve as a benchmark for evaluating the effectiveness of MS positioning schemes.

The rest of this paper is organized as follows. Sec. II describes the models and distributions of measured distances. The proposed MS positioning scheme is presented in Sec. III. In Sec. IV, experimental results are displayed and discussed. Finally, Sec. V concludes the paper.

II. MEASUREMENT MODELS AND DISTRIBUTIONS

Let the true MS location be (x_0, y_0) . Assume that there are N measurement configurations (MC's) between the MS and the reference BS's. By one MC, we mean to take a measurement of distance from one of three measurement types : TOA (involving one BS), TSOA (involving two BS's), or TDOA (involving two BS's).

For ease of making a simple and unified discussion on some topics to appear later in this paper, here we want to define a function $\text{tsgn} : \{\text{TOA}, \text{TSOA}, \text{TDOA}\}$ (denoted as S_T for short) $\mapsto \{0, 1, -1\}$, whose definition is

$$\text{tsgn}(\tau) = \begin{cases} 0, & \text{if } \tau = \text{TOA}, \\ 1, & \text{if } \tau = \text{TSOA}, \\ -1, & \text{if } \tau = \text{TDOA}. \end{cases} \quad (1)$$

Let the locations of the two BS's involved in the i^{th} MC be denoted as $(x_{1,i}, y_{1,i})$ and $(x_{2,i}, y_{2,i})$ if the MC is of type TSOA or TDOA. If the i^{th} MC comes from TOA, the location of the BS is denoted as $(x_{1,i}, y_{1,i})$. The true distance in the i^{th} MC, whose measurement type is $\tau_i (\in S_T)$, is then

$$t_i = \sqrt{(x_0 - x_{1,i})^2 + (y_0 - y_{1,i})^2} + \text{tsgn}(\tau_i) \cdot \sqrt{(x_0 - x_{2,i})^2 + (y_0 - y_{2,i})^2}, \quad (2)$$

where the $(x_{2,i}, y_{2,i})$ in the case of $\tau_i = \text{TOA}$ can be arbitrarily assigned (because $\text{tsgn}(\tau_i) = 0$, anyway).

The measured distance is then $m_i = t_i + g_i + n_i$, where g_i is a measurement noise and is modeled as a zero-mean Gaussian random variable with standard deviation θ_i , and n_i is a NLOS error. Commonly adopted statistic models for n_i include Rayleigh, Rician, exponential, and chi-square distributions, etc. The equation on m_i can be rewritten as

$$\xi_i = m_i - t_i = g_i + n_i, \quad (3)$$

where obviously ξ_i represents the overall measurement error. With the assumption that g_i and n_i are independent, the pdf of ξ_i can be

This work is partially supported by National Science Council under Grant No. 93-2213-E011-075.

obtained from the convolution of g_i and n_i 's pdf's. For two particular cases (n_i is Rayleigh or exponential), we are able to find close-form expressions for ξ_i 's pdf. They are described below.

A. GRS distribution

In this and the next two subsections, we will drop the index i in ξ_i , g_i , and n_i , etc. to make notations simpler.

In this subsection, let us consider the case that the NLOS error n is Rayleigh distributed with parameter ρ , whose pdf is $f_n(n) = (n/\rho^2) \cdot e^{-n^2/2\rho^2} u(n)$, where $u(\cdot)$ is the unit step function. Performing the convolution of $f_n(x)$ and the Gaussian measurement noise's pdf $f_g(x)$ (assuming $g \sim \text{Gauss}(0, \theta)$, where θ is the standard deviation), we get the pdf for ξ :

$$f_\xi(x) = \frac{\rho x}{\lambda^3} \cdot e^{-x^2/2\lambda^2} \Phi\left(\frac{\rho x}{\theta\lambda}\right) + \frac{\theta}{\sqrt{2\pi}\lambda^2} \cdot e^{-x^2/2\theta^2}, \quad (4)$$

where $\lambda = \sqrt{\rho^2 + \theta^2}$ and $\Phi(\cdot)$ is the cumulative density function (cdf) of the standard Gaussian random variable (r.v.). Equation (4) will be referred to as the GRS(θ, ρ) distribution, where GRS stands for Gaussian Rayleigh sum.

In the MLE method, we may have to search $f_\xi(x)$ over feasible regions. The search is typically based on gradient vector and Hessian matrix. Therefore, the first and second derivatives of $f_\xi(x)$ may be needed. They are listed below:

$$f'_\xi(x) = \frac{\rho}{\lambda^3} \left(1 - \frac{x^2}{\lambda^2}\right) e^{-x^2/2\lambda^2} \Phi\left(\frac{\rho x}{\theta\lambda}\right) - \frac{\theta x}{\sqrt{2\pi}\lambda^4} \cdot e^{-x^2/2\theta^2}, \quad (5)$$

$$\begin{aligned} f''_\xi(x) = & -\frac{\rho x}{\lambda^5} \left(3 - \frac{x^2}{\lambda^2}\right) e^{-x^2/2\lambda^2} \Phi\left(\frac{\rho x}{\theta\lambda}\right) \\ & + \frac{1}{\sqrt{2\pi}\lambda^4} e^{-x^2/2\theta^2} \left(\frac{\rho^2}{\theta} - \theta + \frac{\theta x^2}{\lambda^2}\right). \end{aligned} \quad (6)$$

B. GES distribution

In this subsection, The NLOS error n is assumed to be exponentially distributed with parameter β . In other words, its pdf is $f_n(n) = (1/\beta) \cdot e^{-n/\beta} \cdot u(n)$. In this case, the overall measurement error ξ is distributed as [5]

$$f_\xi(x) = \frac{1}{\beta} \cdot e^{-\left(\frac{x}{\beta} - \frac{\theta^2}{2\beta^2}\right)} \Phi\left(\frac{x}{\theta} - \frac{\theta}{\beta}\right). \quad (7)$$

Equation (7) will be referred to as the GES(θ, β) distribution, where obviously GES stands for Gaussian exponential sum.

Similar to what had been done for the GRS(θ, ρ) case, the first and second derivatives of the GES(θ, β) pdf can be computed. They are shown below,

$$f'_\xi(x) = \frac{-1}{\beta} \left(f_\xi(x) - \frac{1}{\sqrt{2\pi}\theta} e^{-x^2/2\theta^2}\right), \quad (8)$$

$$f''_\xi(x) = \frac{-1}{\beta} \left(f'_\xi(x) + \frac{x}{\sqrt{2\pi}\theta^3} e^{-x^2/2\theta^2}\right). \quad (9)$$

C. Gaussian approximation

Although we obtained Eqs. (4) and (7) for ξ when n is Rayleigh or exponential distributed, it is in general very difficult to find ξ 's pdf in an exact form when n is otherwise distributed. Therefore, when the pdf is needed but too hard to be derived, we may resort to some approximations. Due to its mathematical tractability, Gaussian pdf is what we want to adopt for this approximation in most cases. More specifically speaking, given the mean μ and standard deviation σ of a pdf, it is approximated by $\text{Gauss}(\mu, \sigma)$. For example, $\text{GRS}(\theta, \rho) \approx \text{Gauss}(\rho\sqrt{\frac{\pi}{2}}, \sqrt{\theta^2 + (2 - \frac{\pi}{2})\rho^2})$, and

$\text{GES}(\theta, \beta) \approx \text{Gauss}(\beta, \sqrt{\theta^2 + \beta^2})$. By adopting Gaussian approximation, there is an explicit equation for computing the CRLB[7], which serves to evaluate the performances of estimation algorithms. In the case that measurements only involve TOA, the CRLB can be found in the literature (e.g. see [6]). As developed in this paper, however, TOA, TSOA, and TDOA will be treated in a unified fashion to get their CRLB's.

D. CRLB

Let us return to the scenario that the estimation of MS location is based on N MC's, where the i^{th} measurement type is τ_i . Let the variance of the i^{th} overall measurement error, which has been approximated by a non-zero-mean Gaussian r.v., be σ_i^2 . Let the vector pointing from $(x_{1,i}, y_{1,i})$ to (x_0, y_0) be denoted as $\Delta P_{1,i}$. It can be written in the polar form as $|\Delta P_{1,i}|e^{j\phi_{1,i}}$. In a similar way, phase $\phi_{2,i}$ is defined by the vector pointing from the second BS of the i^{th} MC to the MS. For a reason to be addressed in Sec. III, a sample of a few (say, K) measurements needs to be made with respect to each MC. With the terms defined above, the Fisher information matrix (FIM) [7] is computed by $I = K \cdot \sum_{i=1}^N I_i$, where

$$I_i = \begin{bmatrix} I_{11,i} & I_{12,i} \\ I_{21,i} & I_{22,i} \end{bmatrix}, \quad (10)$$

with the matrix elements given by

$$I_{11,i} = \frac{1}{\sigma_i^2} (\cos(\phi_{1,i}) + \text{tsgn}(\tau_i) \cdot \cos(\phi_{2,i}))^2, \quad (11)$$

$$I_{22,i} = \frac{1}{\sigma_i^2} (\sin(\phi_{1,i}) + \text{tsgn}(\tau_i) \cdot \sin(\phi_{2,i}))^2, \quad (12)$$

$$\begin{aligned} I_{12,i} = I_{21,i} = & \frac{1}{\sigma_i^2} (\cos(\phi_{1,i}) + \text{tsgn}(\tau_i) \cdot \cos(\phi_{2,i})) \cdot \\ & (\sin(\phi_{1,i}) + \text{tsgn}(\tau_i) \cdot \sin(\phi_{2,i})). \end{aligned} \quad (13)$$

Then, the CRLB is equal to the trace of the inverse of I . This bound is the smallest mean squared error achievable by any unbiased estimator of MS's position.

III. IMPROVED MS POSITIONING WITH REDUCED NLOS ERROR

To improve the precision of the estimated MS location, the NLOS error should be removed (more practically speaking, reduced) if possible. Although the exact value of the NLOS error can not be known (because it is random), its expected value, however, can be found if proper methods are used. In this paper, we propose a moment-based computation to do just that. The key idea is that the parameters in the model of the overall measurement error (i.e. ξ_i) can be computed from sample moments if a sample of a few (say, K) measurements (for the i^{th} MC) is given. For example, for the GRS case, the parameter estimation for the i^{th} MC is done with

$$\hat{\rho}_i = \sqrt[3]{\sqrt{\frac{2}{\pi}} \cdot \frac{1}{\pi - 3} \cdot (M_{3,i} + 2M_{1,i}^3 - 3M_{1,i}M_{2,i})}, \quad (14)$$

$$\hat{\theta}_i = \sqrt{M_{2,i} - M_{1,i}^2 - (2 - \frac{\pi}{2})\hat{\rho}_i^2}, \quad (15)$$

where $M_{1,i}$, $M_{2,i}$, and $M_{3,i}$ are the first, second and third sample moments, respectively. For the GES case, the estimation is

$$\hat{\beta}_i = \sqrt[3]{\frac{1}{2}M_{3,i} + M_{1,i}^3 - \frac{3}{2}M_{1,i}M_{2,i}}, \quad (16)$$

$$\hat{\theta}_i = \sqrt{M_{2,i} - M_{1,i}^2 - \hat{\beta}_i^2}. \quad (17)$$

Now that the parameters specifying the statistics of ξ_i are obtained, estimation of $E(\xi_i)$ (expectation of ξ_i), which is denoted as $\hat{\mu}_i$, is easy. For example, it is equal to $\sqrt{\pi/2} \cdot \hat{\rho}_i$ in the GRS case, and equal to $\hat{\beta}_i$ in the GES case. Similarly, the estimation of the variances of ξ_i , denoted by $\hat{\sigma}_i^2$, is easy. It is equal to $\hat{\theta}_i^2 + (2 - \pi/2)\hat{\rho}_i^2$ for GRS, and $\hat{\theta}_i^2 + \hat{\beta}_i^2$ for GES.

Rearranging Eq. (3) and taking expectations of the quantities in it, we get $E(t_i) = E(m_i) - E(\xi_i)$, which can be approximated by time averages as $\bar{t}_i = M_{1,i} - \hat{\mu}_i$. Physically interpreted, $\bar{t}_i$ is the distance of the MS to the BS's in the i^{th} MC, wherein the overestimation due to NLOS error has already been removed (or more precisely speaking, reduced). With $\bar{t}_i$ and $\hat{\sigma}_i^2$ given for $i = 1, 2, \dots, N$, the WLSE of (x_0, y_0) is computed from the minimization, with respect to (x, y) , of $J_{\text{WLSE}} = \sum_{i=1}^N J_i$, where

$$J_i = \frac{1}{\hat{\sigma}_i^2} (\sqrt{(x - x_{1,i})^2 + (y - y_{1,i})^2} + \text{tsgn}(\tau_i) \cdot \sqrt{(x - x_{2,i})^2 + (y - y_{2,i})^2} - \bar{t}_i)^2. \quad (18)$$

On the other hand, if we want to apply the method of MLE, we utilize the joint pdf (jpdf) of measurements as a function of MS's location. Before applying the method of MLE, however, it should be noted that there is an offset from the mode (i.e. the peak) of ξ_i 's pdf to the mean. Let us denote this offset as δ_i . Then the jpdf to be maximized is $J_{\text{MLE}} = \prod_{i=1}^N f_{\xi_i}(z_i)$, where

$$z_i = M_{1,i} - (\sqrt{(x - x_{1,i})^2 + (y - y_{1,i})^2} + \text{tsgn}(\tau_i) \cdot \sqrt{(x - x_{2,i})^2 + (y - y_{2,i})^2}) - \delta_i. \quad (19)$$

The reason for the occurrence of the $-\delta_i$ term in eq. (19) is that $M_{1,i}$ represents the most probable value of the mean instead of the mode, and in MLE, however, we are searching for the mode of a pdf instead of the mean.

In summary, the proposed algorithm for mobile positioning when the type of overall measurement error is known is described below:

- 1) Make K measurements of distances for each MC, and their first three moments.
- 2) Calculate relevant pdf parameters by Eqs. (14)-(15) (for GRS) or Eqs. (16)-(17) (for GES).
- 3) Subtract the expected NLOS error from the average measured distance: $\bar{t}_i = M_{1,i} - \sqrt{\pi/2} \cdot \hat{\rho}_i$ for GRS; $\bar{t}_i = M_{1,i} - \hat{\beta}_i$ for GES.
- 4) Minimize J_{WLSE} to get $(\hat{x}, \hat{y})_{\text{WLSE}}$, or maximize J_{MLE} to get $(\hat{x}, \hat{y})_{\text{MLE}}$.

IV. EXPERIMENTAL RESULTS

Simulations were conducted to test the performance of the proposed scheme. Experimental results produced by our method are compared to those obtained without NLOS reduction. As an index for performance evaluation, we define a term called estimation efficiency by $\epsilon = \text{CRLB}/\text{MSE}$, where MSE is the mean (or in practice, average) of the squared distance between (x_0, y_0) and the estimated MS location $(\hat{x}, \hat{y})$. In the two subsections below, we present the experimental results for the GRS and the GES cases, respectively.

A. The GRS case

The setup of the MC's are listed in Table I, where the NLOS error is assumed to be Rayleigh (therefore, $\xi_i \sim \text{GRS}(\theta_i, \rho_i)$). According to [5], θ_i was selected to be about 0.1km, and ρ_i was set to be 0.239km or so. Five different MS positions, whose coordinates are listed in Table I in the units of kilometers, were tried. In each MC, a sample of 100 measured distances were generated. With these

TABLE I
SETUP OF MC'S FOR GRS

i	τ_i	$x_{1,i}$	$y_{1,i}$	$x_{2,i}$	$y_{2,i}$	θ_i	ρ_i
1	TSOA	-6.60	4.85	7.73	4.14	.1103	.2431
2	TOA	-0.34	0.01	-	-	.0985	.2359
3	TDOA	0.21	7.70	-6.66	-3.50	.0994	.2375
4	TOA	-0.03	-8.02	-	-	.1056	.2465
5	TOA	8.29	-4.44	-	-	.1215	.2413

data and adopting the Gaussian approximation to GRS pdf, we have sufficient information to calculate the CRLB's. They are listed in the second column of Table II.

TABLE II
CRLB AND ESTIMATION EFFICIENCY FOR GRS

(x_0, y_0)	CRLB	ϵ_{Conv}	ϵ_{WLSE}	ϵ_{MLE}
(0, 0)	.0238	.0167	.0939	.0921
(6.01, 6.15)	.0635	.0034	.0494	.0497
(-5.72, 4.40)	.0236	.0015	.0516	.0533
(-6.05, -6.13)	.0288	.0023	.0394	.0403
(4.54, -5.60)	.0222	.0041	.0576	.0570

Three methods were applied to estimate (x_0, y_0) . The first was the conventional WLSE without NLOS error reduction (i.e. replacing the $\bar{t}_i$ in Eq. (18) with $M_{1,i}$). Its results, expressed in terms of estimation

Fig. 1. Trilateration: with and without NLOS error reduction

efficiency, are listed in the third column of Table II. The second method was our proposed WLSE with NLOS error reduction, whose results are listed in the fourth column of Table II. The MLE based on GRS distributions with parameters given by Eqs. (14) and (15) was the third method, whose results constitute the fifth column of Table II.

From Table II, we see that ϵ_{WLSE} is much greater than ϵ_{Conv} (in the order of tens of times). This shows that MS location estimation is significantly improved by NLOS error reduction. This observation can be explained graphically with Figure 1. In the figure, the solid lines show the average measured distances, which include distance overestimation due to NLOS errors. The dashed lines show the results after the NLOS errors have been reduced. Close inspection shows that the dashed lines corresponding to the various MC's intersect more closely than the solid lines to the true MS position. Also from Table II, we see that ϵ_{MLE} is often just slightly greater than ϵ_{WLSE} in most cases. In fact, in some rows of Table II, we see that ϵ_{MLE} is even a

Fig. 2. GRS/GES distribution and Gaussian approximation

little smaller than ϵ_{WLSE} , probably due to imperfect search for the peak of J_{MLE} . The comparison between ϵ_{WLSE} and ϵ_{MLE} tells us that WLSE with NLOS error reduction is almost as good as MLE. This can be explained with the help of Figure 2. In the figure, we plot a GRS pdf and its Gaussian approximation. We see that those two curves are extremely close. It means that even if we treat a GRS pdf as a Gaussian pdf, the resulting errors or distortions are negligible. Now that we know the MLE is exactly equal to the WLSE for Gaussian pdf[8], we are not surprised to see the closeness between the MLE and the WLSE for GRS pdf. Realizing that MLE is in general much more involved to compute, we may want to settle for WLSE when the overall measurement error is GRS distributed, as long as the NLOS errors are removed first.

B. The GES case

The setup of the MC's are listed in Table III, where the NLOS error is assumed to be exponential (therefore, $\xi_i \sim \text{GES}(\theta_i, \beta_i)$). According to [5], θ_i was selected to be about 0.1km, and β_i was set to be 0.3km or so. Just like the GRS case, a sample of 100 measurements were made for each MC. Simulation results on the estimation efficiencies are displayed in Table IV. Similar to the GRS

TABLE III
SETUP OF MC'S FOR GES

i	τ_i	$x_{1,i}$	$y_{1,i}$	$x_{2,i}$	$y_{2,i}$	θ_i	β_i
1	TSOA	-6.60	4.85	7.73	4.14	.1103	.3120
2	TOA	-0.34	0.01	-	-	.0985	.3521
3	TDOA	0.21	7.70	-6.66	-3.50	.0994	.3021
4	TOA	-0.03	-8.02	-	-	.1056	.2987
5	TOA	8.29	-4.44	-	-	.1215	.2858

TABLE IV
CRLB AND ESTIMATION EFFICIENCY FOR GES

(x_0, y_0)	CRLB	ϵ_{Conv}	ϵ_{WLSE}	ϵ_{MLE}
(0, 0)	.0709	.0364	.2743	.3111
(6.01, 6.15)	.1975	.0092	.2304	.2908
(-5.72, 4.40)	.0699	.0041	.3442	.4890
(-6.05, -6.13)	.0808	.0064	.2151	.2552
(4.54, -5.60)	.0646	.0106	.3522	.4963

case, here for the GES case we see that ϵ_{WLSE} is much greater than

ϵ_{Conv} , which shows the improvement in MS location estimation when NLOS error is reduced. Please notice, however, here in Table IV, ϵ_{MLE} is always greater than ϵ_{WLSE} . Again the reason can be found in Figure 2, wherein a GES pdf and its Gaussian approximation are found. Contrast to the GRS case, we see that GES can not be closely approximated as Gaussian. Therefore, it can not be expected to obtained close results between the WLSE and the MLE parameter estimation for GES pdf. We also know that MLE is in general a more powerful method than WLSE. Therefore, it is reasonable to observe that ϵ_{MLE} is greater than ϵ_{WLSE} for the GES case.

C. Comparison between the GRS and GES cases

In the two subsections above, we compare the values of ϵ_{Conv} , ϵ_{WLSE} and ϵ_{MLE} for the GRS and GES cases separately. In this subsection we want to make a comparison between the two cases as a whole. Comparing Table II and Table IV, we see that the values of ϵ_{Conv} , ϵ_{WLSE} and ϵ_{MLE} for GRS are always smaller than the corresponding values for GES. This is because Gaussian distribution is the worst case in all distributions from the viewpoint of randomness. Since GRS is almost Gaussian, as have already been seen in Figure 2, a GRS error is more destructive than a GES error. Therefore, measurements corrupted by GRS errors lead to worse estimation of MS position than those corrupted by GES errors.

V. CONCLUSIONS

In this work, we try to improve the accuracy and precision of mobile positioning over conventional WLSE methods. The key idea is to estimate the NLOS error and then subtract it from the measured distance before trilateration is performed. This task is carried out based on the assumption that statistic models for measurement errors are available. In particular, the GRS and GES distributions are considered. Explicit equations for estimating their relevant parameters, based on sample moments, are derived. Measurement types include TOA, TSOA, and TDOA. Regardless, they are treated in a unified way. Given a setup of MC's, its corresponding CRLB is derived and is used as a benchmark for performance evaluation. Simulations show that the proposed WLSE with NLOS error reduction performs much better than conventional WLSE. Experimental results also show that WLSE with NLOS error reduction is almost as good as MLE for the GRS case. But for the GES case, MLE outperforms WLSE with NLOS error reduction. The is due to the different degrees of closeness of Gaussian approximation to those two pdf's. One final note is that, with the overall measurement error's model and its associated model parameters being obtained, other ways for estimating MS's location can be feasible (e.g. adopting conditional mean as the estimate).

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