Design of two-channel IIR filter banks with arbitrary group delay

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Abstract: The nonlinear optimisation problem that results from considering the design of a two-channel nonuniform division filter bank is solved. This is through a frequency sampling and iterative approximation technique to find the tap coefficients and the reflection coefficients for the numerator and the denominator of the IIR analysis filters. An efficient stabilisation procedure ensures that the reflection coefficients lie in $(-1, 1)$. Simulation examples are provided for illustration.

1 Introduction

Quadrature mirror filter (QMF) banks find applications in sub-band coding of speech signals [1], communication systems [2], short-time spectral analysis [3], and sub-band coding of image signals [4]. In these applications, a QMF bank is used to decompose a signal into sub-bands with equal bandwidth, and the sub-band signals in the analysis system are decimated by an integer which is equal to the number of the sub-bands. However, uniform sub-band decomposition is not an appropriate scheme to match the requirements for the sub-band coding of speech and audio signals. The most appropriate decomposition must consider the critical bands of the ear. It has been mentioned previously [5] that these critical bands have nonuniform bandwidths and cannot be easily constructed by a conventional tree structure based on two-channel QMF banks. Therefore, it is worth exploiting the design problem of two-channel nonuniform division filter (NDF) banks.

The conditions on the analysis and synthesis filters of NDF banks, as well as methods to design them, are presented elsewhere [5]. Owing to the difficulty in solving the design problem with nonlinear constraints, a structure and design procedure for pseudo-QMF banks has been presented [6]. The main drawback of this method is that FIR filters with complex coefficients are required by the resulting NDF bank to reduce the aliasing distortion. A structure for two-channel NDF banks has been considered [7], and design methods have been proposed [8] for optimally designing NDF banks based on L_1 and L_2 error criteria.

Designing an NDF bank has been widely considered [5–8]. However, linear-phase FIR analysis filters were employed for developing these design techniques.

Although using linear-phase FIR analysis filters leads to a favorable problem formulation for design, the system delay k_d of the designed NDF bank is determined by the lengths of FIR filters used; hence, the long overall system delay of an NDF bank with linear-phase FIR analysis filters may prohibit practical applications. It is well known that an IIR filter requires lower order than an FIR filter under the same stopband energy. For a given passband group delay, IIR NDFs have better passband ripple/stopband attenuation and/or lower computational complexity than FIR NDFs under the same stopband energy for the analysis and synthesis filters. Several approaches dealing with the design problem of two-channel IIR uniform division filter banks with conventional filter structures have been reported [9–11]. There is, however, practically no published work concerning the design of two-channel IIR NDF banks with arbitrary group delay.

In this paper, we consider the design of two-channel IIR NDF banks with arbitrary group delay in the sense of L_2 error criteria. Utilising a lattice structure for the denominators of the IIR analysis filters, a design technique based on an approximation scheme is presented for efficiently solving the resulting design problem that is basically a nonlinear optimisation problem. During the design process, the proposed technique finds the tap coefficients for the numerators and the reflection coefficients for the denominators of the IIR analysis filters simultaneously. It ensures the stability of the designed IIR analysis filters by incorporating an efficient stabilisation procedure to make the magnitude of each reflection coefficient within -1 and $+1$. Computer simulations show that the IIR NDF banks designed using the proposed technique provide a satisfactory performance, although the designed results are only local solutions (instead of the global optimum).

2 Two-channel NDF banks with arbitrary group delay

Consider the two-channel NDF bank, with the architecture given previously [7], which is shown in Fig. 1. The analysis low-pass and high-pass filters are designated by $H_0(z)$ and $H_1(z)$, respectively, and the synthesis low-pass and high-pass filters are designated by $F_0(z)$ and $F_1(z)$, respectively. $B_0(z)$ and $B_1(z)$ are low-pass filters

Fig. 1 Two-channel NDF bank system structure
a Analysis system
b Synthesis system

Fig. 2 Desired magnitude responses for analysis filters
(i) $|H_0(e^{j\omega})|/\sqrt{L_0L}$
(ii) $|H_1(e^{j\omega})|/\sqrt{L_1L}$

responsible for achieving aliasing-free operation during the rational decimation and interpolation. It can be shown that using the modulations of multiplying $\exp(jn\pi)$ in the high-pass sub-band channel leads to the favourable result that $B_1(z)$ can be a low-pass filter with real coefficients. The desired magnitude responses for the analysis filters $H_0(z)$ and $H_1(z)$ with passband widths equal to $L_0\pi/L$ and $L_1\pi/L$, respectively, are shown in Fig. 2. $L = L_0 + L_1$, and ω_p and ω_s denote the related band-edge frequencies satisfying $\omega_p + \omega_s = 2\pi L_0/L$.

Assume that the associated magnitude responses are set to

$$\begin{cases} B_0(\omega) = 1, & \text{for } \omega \in \left[0, \frac{\omega_s}{L_0}\right] \\ \text{and } B_0(\omega) = 0, & \text{for } \omega \in \left[\frac{2\pi - \omega_s}{L_0}, \pi\right], \\ B_1(\omega) = 1, & \text{for } \omega \in \left[0, \frac{\pi - \omega_p}{L_1}\right] \\ \text{and } B_1(\omega) = 0, & \text{for } \omega \in \left[\frac{\pi + \omega_p}{L_1}, \pi\right] \end{cases} \quad (1)$$

Further, assume that $H_0(z)$ and $H_1(z)$ have zero stopband response. As shown previously [8], the input/output

relationship of the NDF bank in the frequency domain is given by

$$\begin{aligned} \hat{X}(e^{j\omega}) = & \frac{e^{-j\omega G_0}}{LL_0} [X(e^{j\omega})H_0(e^{j\omega}) + X(e^{j\omega}W_L^{L_0}) \\ & \times H_0(e^{j\omega}W_L^{L_0}) + X(e^{j\omega}W_L^{-L_0})H_0(e^{j\omega}W_L^{-L_0})]F_0(e^{j\omega}) \\ & + \frac{e^{-j\omega G_1}}{LL_1} [X(e^{j\omega})H_1(e^{j\omega}) + X(e^{j\omega}W_L^{L_1})H_1(e^{j\omega}W_L^{L_1}) \\ & + X(e^{j\omega}W_L^{-L_1})H_1(e^{j\omega}W_L^{-L_1})]F_1(e^{j\omega}) \end{aligned} \quad (2)$$

where G_0 and G_1 are the resulting group delays of the upper and lower channels, respectively. $W_L = \exp(-j2\pi/L)$. Substituting $L = L_0 + L_1$, $F_0(e^{j\omega}) = H_0(e^{j\omega})$ and $F_1(e^{j\omega}) = -H_1(e^{j\omega})$ into eqn. 2 yields

$$\begin{aligned} \hat{X}(e^{j\omega}) = & T(e^{j\omega})X(e^{j\omega}) + V_1(e^{j\omega})X(e^{j\omega}W_L^{L_0}) \\ & + V_2(e^{j\omega})X(e^{j\omega}W_L^{L_1}) \end{aligned} \quad (3)$$

where

$$T(e^{j\omega}) = \frac{e^{-j\omega G_0}}{LL_0} H_0^2(e^{j\omega}) - \frac{e^{-j\omega G_1}}{LL_1} H_1^2(e^{j\omega}) \quad (4)$$

$$\begin{aligned} V_1(e^{j\omega}) = & \frac{e^{-j\omega G_0}}{LL_0} H_0(e^{j\omega})H_0(e^{j\omega}W_L^{L_0}) \\ & - \frac{e^{-j\omega G_1}}{LL_1} H_1(e^{j\omega})H_1(e^{j\omega}W_L^{L_0}) \end{aligned} \quad (5)$$

$$\begin{aligned} V_2(e^{j\omega}) = & \frac{e^{-j\omega G_0}}{LL_0} H_0(e^{j\omega})H_0(e^{j\omega}W_L^{L_1}) \\ & - \frac{e^{-j\omega G_1}}{LL_1} H_1(e^{j\omega})H_1(e^{j\omega}W_L^{L_1}) \end{aligned} \quad (6)$$

The first term of eqn. 3 represents the response of a linear shift-invariant system $T(e^{j\omega})$ with input $X(e^{j\omega})$, and the other two terms represent the resulting aliasing distortion. Therefore, perfect reconstruction for a group delay k_d requires the following conditions:

PR 1: $T(e^{j\omega})$ must be equal to $e^{-jk_d\omega}$ for all ω .

PR 2: The magnitude $V_1(\omega)$ of $V_1(e^{j\omega})$ must be zero, i.e.

$$V_1(\omega) = 0, \text{ for all } \omega.$$

PR 3: The magnitude $V_2(\omega)$ of $V_2(e^{j\omega})$ must be zero, i.e.

$$V_2(\omega) = 0, \text{ for all } \omega.$$

For simplicity, assuming $G_0 = G_1$, we can neglect the phase term $e^{-j\omega G_0}$ in eqn. 4 and express $T(e^{j\omega})$ as

$$\hat{T}(e^{j\omega}) = \frac{H_0^2(e^{j\omega})}{LL_0} - \frac{H_1^2(e^{j\omega})}{LL_1} \quad (7)$$

Furthermore, we note from eqns. 5 and 6 and $L = L_0 + L_1$ that the aliasing distortion can be eliminated if

$$\begin{aligned} \hat{V}(e^{j\omega}) = & \frac{1}{LL_0} H_0(e^{j(\omega - \omega_p - \omega_s)})H_0(e^{j\omega}) \\ & - \frac{1}{LL_1} H_1(e^{j(\omega - \omega_p - \omega_s)})H_1(e^{j\omega}) \end{aligned} \quad (8)$$

equals zero for $\omega_p \leq \omega \leq \omega_s$. Following eqns. 7 and 8, we can reformulate the conditions required for perfect reconstruction as follows:

$$\begin{cases} T(e^{j\omega}) = e^{-jk_d\omega}, & \text{for } 0 \leq \omega \leq \pi, \\ H_0(e^{j\omega}) = 0, & \text{for } \omega_s \leq \omega \leq \pi, \\ H_1(e^{j\omega}) = 0, & \text{for } 0 \leq \omega \leq \omega_p, \\ \frac{1}{LL_0} H_0(e^{j\omega}) H_0(e^{j(\omega-\omega_p-\omega_s)}) = \frac{1}{LL_1} H_1(e^{j(\omega-\omega_p-\omega_s)}) H_1(e^{j\omega}), & \text{for } \omega_p \leq \omega \leq \omega_s \end{cases} \quad (9)$$

Therefore, the design problem of the two-channel IIR NDF banks of Fig. 1 is finding such IIR analysis filters, i.e. $H_0(z)$ and $H_1(z)$, that the conditions listed in eqn. 9 can be approximately met.

3 Design of two-channel IIR NDF banks in L_2 sense

3.1 Problem formulation

Here, we consider the design of the two-channel IIR NDF as shown in Fig. 1. Let the low-pass analysis filter be an IIR filter with order M_0/N_0 (i.e. M_0 zeros and N_0 poles) and transfer function $H_0(z) = A_0(z)/B_{N_0}(z)$. The numerator $A_0(z)$ is an M_0 th-order polynomial with tap coefficient vector $\mathbf{a}_0 = [a_{00}, a_{01}, \dots, a_{0M_0}]^T$; and $B_{N_0}(z)$ is an N_0 th-order FIR lattice filter with reflection coefficient vector $\mathbf{k}_0 = [k_{01}, k_{02}, \dots, k_{0N_0}]^T$, where T denotes the transpose operation. Similarly, let the high-pass analysis filter be an IIR filter with order M_1/N_1 and transfer function $H_1(z) = A_1(z)/B_{N_1}(z)$. The numerator $A_1(z)$ is an M_1 th-order polynomial with tap coefficient vector $\mathbf{a}_1 = [a_{10}, a_{11}, \dots, a_{1M_1}]^T$, and $B_{N_1}(z)$ is an N_1 th-order FIR lattice filter with reflection coefficient vector $\mathbf{k}_1 = [k_{11}, k_{12}, \dots, k_{1N_1}]^T$. Fig. 3 shows the system structure for $B_{N_0}(z)$ and $B_{N_1}(z)$, which can be obtained from the following recursive formula [12]:

$$\begin{aligned} B_0(z) &= Q_0(z) = 1 \\ B_n(z) &= B_{n-1}(z) + k_n z^{-1} Q_{n-1}(z) \\ Q_n(z) &= k_n B_{n-1}(z) + z^{-1} Q_{n-1}(z) \end{aligned} \quad (10)$$

Hence, the overall design task is to find the required tap and reflection coefficients $\{a_{im}, k_{in}\}$ for the stable IIR filters $H_i(z)$, $i=0, 1$, such that the conditions shown in eqn. 9 must be satisfied. Accordingly, we consider that the overall error function E to be minimised in the L_2 sense can be expressed as the following weighted sum of four terms:

Fig. 3 Lattice structure for N th-order FIR lattice filter

$$\begin{aligned} E &= \int_{\omega=0}^{\pi} |e^{-jk_d\omega} - \hat{T}(e^{j\omega})|^2 d\omega \\ &+ \alpha \int_{\omega=0}^{\omega_p} |H_1(e^{j\omega})|^2 d\omega + \beta \int_{\omega=\omega_s}^{\pi} |H_0(e^{j\omega})|^2 d\omega \\ &+ \gamma \int_{\omega=\omega_p}^{\omega_s} \left| \frac{1}{LL_0} H_0(e^{j\omega}) H_0(e^{j(\omega-\omega_p-\omega_s)}) \right. \\ &\quad \left. - \frac{1}{LL_1} H_1(e^{j(\omega-\omega_p-\omega_s)}) H_1(e^{j\omega}) \right|^2 d\omega \end{aligned} \quad (11)$$

where α , β , and γ represent the relative weights between the four error terms. We note from eqn. 11 that the overall error function E is a function of fourth degree in the tap and reflection coefficients. Therefore, minimising E directly leads to a very highly nonlinear programming problem in addition to the stability problem for $H_0(z)$ and $H_1(z)$.

3.2 Proposed design technique

Let $\{\omega_1=0, \omega_2, \dots, \omega_I=\omega_p, \omega_{I+1}, \dots, \omega_{I+J}, \omega_{I+J+1}=\omega_s, \dots, \omega_{I+J+K}=\pi\}$ be a grid of equidistant frequencies, distributed in the range of $\omega=0$ to $\omega=\pi$, for evaluating the magnitude response of the NDF bank and the related error terms defined as above. Moreover, assume that the set S_1 contains the grid points $\{\omega_1=0, \omega_2, \dots, \omega_I=\omega_p\}$, S_2 contains the grid points $\{\omega_{I+1}, \omega_{I+2}, \dots, \omega_{I+J}\}$, and S_3 contains the grid points $\{\omega_{I+J+1}=\omega_s, \omega_{I+J+2}, \dots, \omega_{I+J+K}=\pi\}$. We then rewrite eqn. 11 as the following approximation:

$$\begin{aligned} \hat{E} &= \sum_{\omega_i \in S_1 \cup S_2 \cup S_3} |e^{-jk_d\omega_i} - \hat{T}(e^{j\omega_i})|^2 \\ &+ \alpha \sum_{\omega_i \in S_1} |H_1(e^{j\omega_i})|^2 + \beta \sum_{\omega_i \in S_3} |H_0(e^{j\omega_i})|^2 \\ &+ \gamma \sum_{\omega_i \in \omega_p \cup \omega_s} \left| \frac{1}{LL_0} H_0(e^{j\omega_i}) H_0(e^{j(\omega_i-\omega_p-\omega_s)}) \right. \\ &\quad \left. - \frac{1}{LL_1} H_1(e^{j(\omega_i-\omega_p-\omega_s)}) H_1(e^{j\omega_i}) \right|^2 \end{aligned} \quad (12)$$

3.2.1 Determination of initial guesses for $H_0(z)$ and $H_1(z)$: Our design experience shows that using better initial guesses for $H_0(e^{j\omega})$ and $H_1(e^{j\omega})$ usually provides better design results. To initiate the design process, we propose a procedure for determining appropriate initial guesses $H_0^0(z)$ and $H_1^0(z)$ for $H_0(z)$ and $H_1(z)$, respectively. According to the principle of a two-channel NDF bank, we define two desired frequency responses $D_0(e^{j\omega})$ and $D_1(e^{j\omega})$ as

$$D_0(e^{j\omega}) = \begin{cases} \sqrt{LL_0} e^{-j\frac{k_d}{2}\omega}, & \text{for } \omega \in [0, \omega_p] \\ \sqrt{\frac{LL_0}{2}} e^{-j\frac{k_d}{2}\omega}, & \text{for } \omega = \frac{\omega_p + \omega_s}{2} \\ 0, & \text{for } \omega \in [\omega_s, \pi] \end{cases} \quad (13)$$

$$D_1(e^{j\omega}) = \begin{cases} 0, & \text{for } \omega \in [0, \omega_p] \\ j\sqrt{\frac{LL_1}{2}} e^{-j\frac{k_d}{2}\omega}, & \text{for } \omega = \frac{\omega_p + \omega_s}{2} \\ j\sqrt{LL_1} e^{-j\frac{k_d}{2}\omega}, & \text{for } \omega \in [\omega_s, \pi] \end{cases} \quad (14)$$

Two FIR filters $G_0(z)$ and $G_1(z)$ with orders $2N_0$ and $2N_1$ are designed to optimally approximate $D_0(e^{j\omega})$ and $D_1(e^{j\omega})$,

Table 1: Significant design results for example 1

	IIR NDF bank	FIR NDF bank
Filter order	10 / 10, 11 / 11	32, 33
No. of coefficients	44	67
PRE in $ T(e^{j\omega}) $ (dB)	0.0148	0.0238
Max. variation of group delay of $T(e^{j\omega})$	0.0583	0.0746
NPSR (dB) of $H_0(e^{j\omega})$	32.21	32.10
NPSR (dB) of $H_1(e^{j\omega})$	32.07	31.95
Max. variation of passband group delay of $H_0(e^{j\omega})$	0.0158	0.0160
Max. variation of passband group delay of $H_1(e^{j\omega})$	0.0230	0.0163
SEE of $H_0(e^{j\omega})$	3.33×10^{-2}	3.85×10^{-2}
SEE of $H_1(e^{j\omega})$	5.68×10^{-2}	5.12×10^{-2}
Max. variation of filter bank response	2.27×10^{-3}	2.76×10^{-3}
No. of iterations	37	9

respectively, using conventional least-squares error criteria. Let the resulting filter coefficients be given by $\{h_{00}, h_{01}, \dots, h_{0(2N_0)}\}$ and $\{h_{10}, h_{11}, \dots, h_{1(2N_1)}\}$, respectively.

Through the use of the model reduction algorithm presented previously [13], we find two IIR filters where the numerator and denominator are of order N_0 (corresponding to $G_0(z)$) and of order N_1 (corresponding to $G_1(z)$), respectively. Assume that the IIR filters have denominators $C_0(z)$ and $C_1(z)$ with coefficients $\{c_{00}=1, c_{01}, \dots, c_{0N_0}\}$ and $\{c_{10}=1, c_{11}, \dots, c_{1N_1}\}$, respectively. The initial lattice systems $B_{N_0}^0$ and $B_{N_1}^0$ with reflection coefficients $\{k_{01}^0, k_{02}^0, \dots, k_{0N_0}^0\}$ and $\{k_{11}^0, k_{12}^0, \dots, k_{1N_1}^0\}$ corresponding to $C_0(z)$ and $C_1(z)$, respectively, can then be found since there exists a one-to-one correspondence between $\{c_{i0}, c_{i1}, \dots, c_{iN_i}\}$ and $\{k_{i1}^0, k_{i2}^0, \dots, k_{iN_i}^0\}$ [14] for $i=0, 1$.

The best L_2 solution for the corresponding initial numerator $A_i^0(z), i=0,1$, can be obtained by solving the following optimisation problem:

$$\begin{aligned} & \text{minimise } |D_i(e^{j\omega}) - \frac{A_i^0(e^{j\omega})}{B_{N_i}^0(e^{j\omega})}|^2, \\ & \text{for all } \omega \in [0, \omega_p] \cup \frac{\omega_p + \omega_s}{2} \cup [\omega_s, \pi] \end{aligned} \quad (15)$$

Table 2: Tap and reflection coefficients for example 1

i	a_{0i}	k_{0i}	a_{1i}	k_{1i}
0	8.6657905×10^{-3}		$-6.6212287 \times 10^{-3}$	
1	$-4.0416626 \times 10^{-2}$	$-5.2321871 \times 10^{-1}$	$-2.6735861 \times 10^{-2}$	$-1.0973161 \times 10^{-1}$
2	7.2162064×10^{-2}	7.7911067×10^{-1}	$-4.8663890 \times 10^{-2}$	6.5122570×10^{-1}
3	$-5.6558510 \times 10^{-2}$	$-7.2660247 \times 10^{-1}$	$-3.8462788 \times 10^{-2}$	4.7095441×10^{-1}
4	3.0787734×10^{-2}	6.7708888×10^{-1}	2.2971930×10^{-2}	4.0467721×10^{-1}
5	$-5.9047004 \times 10^{-2}$	$-6.3779579 \times 10^{-1}$	8.0657319×10^{-2}	3.5518737×10^{-1}
6	2.9256295×10^{-2}	5.8390526×10^{-1}	1.3217710×10^{-2}	3.1592069×10^{-1}
7	8.6821333×10^{-2}	$-4.5798891 \times 10^{-1}$	$-2.2281802 \times 10^{-1}$	2.7136095×10^{-1}
8	1.6613748×10^{-1}	2.4768711×10^{-1}	$-3.2913158 \times 10^{-1}$	2.0592667×10^{-1}
9	1.1964706×10^{-1}	$-6.9787484 \times 10^{-2}$	1.3735270×10^{-2}	1.2434145×10^{-1}
10	8.9118083×10^{-2}	6.0013386×10^{-3}	$-1.4431611 \times 10^{-2}$	5.2666408×10^{-2}
11			6.2466860×10^{-1}	1.2187899×10^{-2}

For evaluating the related error functions given by eqn. 15, we again take a set of discrete frequency points linearly distributed over $S = [0, \omega_p] \cup (\omega_p + \omega_s)/2 \cup [\omega_s, \pi]$. Let $S_d = S_1 \cup (\omega_p + \omega_s)/2 \cup S_3 = \{\omega_1=0, \omega_2, \dots, \omega_I = \omega_p, \omega_{I+1} = (\omega_p + \omega_s)/2, \omega_{I+2} = \omega_s, \dots, \omega_{I+K+1} = \pi\}$ be the dense grid of frequency points, and U_i be a complex $(I+K+1) \times (M_i+1)$ matrix with its (m, n) th element given by

$$U_i(m, n) = \frac{e^{-j(n-1)\omega_m}}{B_{N_i}^0(e^{j\omega_m})}, \quad 1 \leq m \leq I+K+1, \quad (16)$$

$$1 \leq n \leq M_i+1$$

d_i is a complex $(I+K+1) \times 1$ vector, with its m th element given by

$$d_i(m) = D_i(e^{j\omega_m}), \quad 1 \leq m \leq I+K+1 \quad (17)$$

The initial coefficient vector $a_i^0 = [a_{i0}^0, a_{i1}^0, \dots, a_{iM_i}^0]^T$ of $A_i^0(z)$, optimal in the L_2 sense for eqn. 15, can be found by minimising $|U_i a_i^0 - d_i|^2 = (U_i a_i^0 - d_i)^H (U_i a_i^0 - d_i)$, where H denotes the complex conjugate transpose. Clearly, this leads to the optimal solution given by $a_i^0 = \{Re(U_i^H U_i)\}^{-1} [Re(U_i^H d_i)]$, for $i=0, 1$, where $Re(X)$ represents the real part of the matrix X . After finding the appropriate initial guesses $H_i^0(z) = A_i^0(z)/B_{N_i}^0(z)$, we present an iterative procedure step by step for computing $A_i(z)$ and $B_{N_i}(z)$, for $i=0, 1$, during the design process.

3.2.2 Iterative procedure:

Step 1: At the l th iteration, let the analysis filters be

$$H_0^l(e^{j\omega}) = \frac{A_0^l(e^{j\omega})}{B_{N_0}^l(e^{j\omega})} \quad \text{and} \quad H_1^l(e^{j\omega}) = \frac{A_1^l(e^{j\omega})}{B_{N_1}^l(e^{j\omega})}$$

Utilise a linearisation scheme to rewrite $\hat{T}(e^{j\omega_i})$ and $\hat{V}(e^{j\omega_i})$ as follows [15]:

$$\begin{aligned} \tilde{T}(e^{j\omega_i}) &= \frac{1}{LL_0} H_0^l(e^{j\omega_i}) \frac{\tilde{A}_0(e^{j\omega_i})}{B_{N_0}^l(e^{j\omega_i})} \\ &\quad - \frac{1}{LL_1} H_1^l(e^{j\omega_i}) \frac{\tilde{A}_1(e^{j\omega_i})}{B_{N_1}^l(e^{j\omega_i})} \end{aligned}$$

and

$$\begin{aligned} \tilde{V}(e^{j\omega_i}) &= \frac{1}{LL_0} H_0^l(e^{j(\omega_i - \omega_p - \omega_s)}) \frac{\tilde{A}_0(e^{j\omega_i})}{B_{N_0}^l(e^{j\omega_i})} \\ &\quad - \frac{1}{LL_1} H_1^l(e^{j(\omega_i - \omega_p - \omega_s)}) \frac{\tilde{A}_1(e^{j\omega_i})}{B_{N_1}^l(e^{j\omega_i})} \end{aligned} \quad (18)$$

respectively, where the numerators $\tilde{A}_i(e^{j\omega_i})$ have coefficient vectors $\tilde{\mathbf{a}}_i = [\tilde{a}_{i0}, \tilde{a}_{i1}, \dots, \tilde{a}_{iM_i}]^T$, for $i=0,1$. We then find the optimal numerators $\tilde{A}_i(e^{j\omega_i})$. First, several matrices are defined as follows: $\mathbf{W}_0$ is an $(I+J+K) \times (M_0+1)$ matrix, with the (m, n) th entry given by

$$W_0(m, n) = \frac{1}{LL_0} H_0^l(e^{j\omega_m}) \frac{e^{-j(n-1)\omega_m}}{B_{N_0}^l(e^{j\omega_m})},$$

$$\omega_m \in S_1 \cup S_2 \cup S_3, n = 1, 2, \dots, M_0 + 1$$
(19)

$\mathbf{W}_1$ is an $(I+J+K) \times (M_1+1)$ matrix, with the (m, n) th entry given by

$$W_1(m, n) = -\frac{1}{LL_1} H_1^l(e^{j\omega_m}) \frac{e^{-j(n-1)\omega_m}}{B_{N_1}^l(e^{j\omega_m})},$$

$$\omega_m \in S_1 \cup S_2 \cup S_3, n = 1, 2, \dots, M_1 + 1$$
(20)

$\mathbf{Y}_0$ is an $K \times (M_0+1)$ matrix, with the (m, n) th entry given by

$$Y_0(m, n) = \frac{e^{-j(n-1)\omega_{m+I+J}}}{B_{N_0}^l(e^{j\omega_{m+I+J}})},$$

$$\omega_{m+I+J} \in S_3, n = 1, 2, \dots, M_0 + 1$$
(21)

$\mathbf{Y}_1$ is an $I \times (M_1+1)$ matrix, with the (m, n) th entry given by

$$Y_1(m, n) = \frac{e^{-j(n-1)\omega_m}}{B_{N_1}^l(e^{j\omega_m})}, \omega_m \in S_1, n = 1, 2, \dots, M_1 + 1$$
(22)

$\mathbf{Z}_0$ is an $(J+2) \times (M_0+1)$ matrix, with the (m, n) th entry given by

$$Z_0(m, n) = \frac{H_0^l(e^{j(\omega_m - \omega_p - \omega_s)})}{LL_0} \frac{e^{-j(n-1)\omega_{m+I-1}}}{B_{N_0}^l(e^{j\omega_{m+I-1}})},$$

$$\omega_m \in \omega_p \cup S_2 \cup \omega_s, n = 1, 2, \dots, M_0 + 1$$
(23)

$\mathbf{Z}_1$ is an $(J+2) \times (M_1+1)$ matrix, with the (m, n) th entry given by

$$Z_1(m, n) = -\frac{H_1^l(e^{j(\omega_{m+I-1} - \omega_p - \omega_s)})}{LL_1} \frac{e^{-j(n-1)\omega_{m+I-1}}}{B_{N_1}^l(e^{j\omega_{m+I-1}})},$$

$$\omega_m \in \omega_p \cup S_2 \cup \omega_s, n = 1, 2, \dots, M_1 + 1$$
(24)

Moreover, $\mathbf{W} = [\mathbf{W}_0, \mathbf{W}_1]$, $\mathbf{Y}_{00} = [\mathbf{Y}_0, \mathbf{0}_{K \times (M_1+1)}]$, $\mathbf{Y}_{11} = [\mathbf{0}_{I \times (M_0+1)}, \mathbf{Y}_1]$, where $\mathbf{0}_{M \times N}$ denotes a zero matrix with size shown by its subscript $M \times N$, $\mathbf{Z} = [\mathbf{Z}_0, \mathbf{Z}_1]$, and $\mathbf{z}_d$ is an $(I+J+K) \times 1$ complex vector, with the m th entry given by

$$z_d(m) = e^{-jk_d\omega_m}, \omega_m \in S_1 \cup S_2 \cup S_3$$
(25)

As a result, the required coefficient vectors $\tilde{\mathbf{a}}_i$ can be found from eqn. 12 by minimising the following error function:

$$\tilde{E} = (\mathbf{W}\tilde{\mathbf{a}} - \mathbf{z}_d)^H (\mathbf{W}\tilde{\mathbf{a}} - \mathbf{z}_d) + \alpha(\mathbf{Y}_{11}\tilde{\mathbf{a}})^H (\mathbf{Y}_{11}\tilde{\mathbf{a}}) + \beta(\mathbf{Y}_{00}\tilde{\mathbf{a}})^H (\mathbf{Y}_{00}\tilde{\mathbf{a}}) + \gamma(\mathbf{Z}\tilde{\mathbf{a}})^H (\mathbf{Z}\tilde{\mathbf{a}})$$
(26)

Fig. 4 Magnitude and group delay responses of analysis filters for example 1

— $H_0(e^{j\omega})/\sqrt{LL_0}$
 - - - $H_1(e^{j\omega})/\sqrt{LL_1}$
 a Magnitude response
 b Group delay response

where the vector $\tilde{\mathbf{a}} = [\tilde{\mathbf{a}}_0^T, \tilde{\mathbf{a}}_1^T]^T$. Clearly, the optimal solution for eqn. 26 is given by

$$\tilde{\mathbf{a}} = \{ \text{Re}[\mathbf{W}^H \mathbf{W} + \alpha \mathbf{Y}_{11}^H \mathbf{Y}_{11} + \beta \mathbf{Y}_{00}^H \mathbf{Y}_{00} + \gamma \mathbf{Z}^H \mathbf{Z}] \}^{-1} \times \{ \text{Re}[\mathbf{W}^H \mathbf{z}_d] \}$$
(27)

since eqn. 26 is a quadratic function of the coefficients $\tilde{a}_{i0}, \tilde{a}_{i1}, \dots, \tilde{a}_{iM_i}$, for $i=0, 1$. After obtaining the coefficient vectors $\tilde{\mathbf{a}}_i$, we update the numerator coefficients of $H_i^l(z)$ as follows:

$$\mathbf{a}_i^{l+1} = (1 - \tau)\mathbf{a}_i^l + \tau\tilde{\mathbf{a}}_i$$
(28)

for $i=0, 1$, where $\tau (0 < \tau < 1)$ is a smoothing parameter with the best value chosen experimentally.

Step 2: Compute the gradient matrices $\mathbf{H}_i = [\nabla \hat{H}_i^l(e^{j\omega_i})]$ for $\omega_i \in S_1 \cup S_2 \cup S_3$. $\mathbf{H}_0 = [\nabla H_0^l(e^{j\omega_i})]$ for $\omega_i \in S_3$, $\mathbf{H}_1 = [\nabla H_1^l(e^{j\omega_i})]$ for $\omega_i \in S_1$, and $\mathbf{H}_a = [\nabla \hat{H}^l(e^{j\omega_i})]$ for $\omega_i \in \omega_p \cup S_2 \cup \omega_s$, where ∇ represents the gradient operator $[\partial/\partial k_{01}, \partial/\partial k_{02}, \dots, \partial/\partial k_{0N_0}, \partial/\partial a_{00}, \partial/\partial a_{01}, \dots, \partial/\partial a_{0M_0}, \partial/\partial k_{11}, \partial/\partial k_{12}, \dots, \partial/\partial k_{1N_1}, \partial/\partial a_{10}, \partial/\partial a_{11}, \dots, \partial/\partial a_{1M_1}]$.

Step 3: Use a linearisation scheme to approximate the frequency response error $\hat{E}^l$ for eqn. 12 due to a perturbation in the coefficient vector in the linear subspace spanned

Fig. 5 Magnitude and group delay responses of $\hat{T}(e^{j\omega})$ for example 1

a Magnitude response
b Group delay response

by the gradient matrices $\mathbf{H}_t$, $\mathbf{H}_0$, $\mathbf{H}_1$, and $\mathbf{H}_a$. This means that the approximation error given by

$$\begin{aligned} \hat{E}_a^l = & \sum_{\omega_i \in S_1 \cup S_2 \cup S_3} |e^{-jk_d \omega_i} - \hat{T}^l(e^{j\omega_i}) - \nabla \hat{T}^l(e^{j\omega_i}) \mathbf{v}|^2 \\ & + \alpha \sum_{\omega_i \in S_1} |H_1^l(e^{j\omega_i}) + \nabla H_1^l(e^{j\omega_i}) \mathbf{v}|^2 \\ & + \beta \sum_{\omega_i \in S_3} |H_0^l(e^{j\omega_i}) + \nabla H_0^l(e^{j\omega_i}) \mathbf{v}|^2 \\ & + \gamma \sum_{\omega_i \in \omega_p \cup S_2 \cup \omega_s} |\hat{V}^l(e^{j\omega_i}) + \nabla \hat{V}^l(e^{j\omega_i}) \mathbf{v}|^2 \end{aligned} \quad (29)$$

is computed, where the vector $\mathbf{v} = [\Delta k_0, \Delta a_0, \Delta k_1, \Delta a_1] = [\Delta k_{01}, \Delta k_{02}, \dots, \Delta k_{0N_0}, \Delta a_{00}, \Delta a_{01}, \dots, \Delta a_{0M_0}, \Delta k_{11}, \Delta k_{12}, \dots, \Delta k_{1N_1}, \Delta a_{10}, \Delta a_{11}, \dots, \Delta a_{1M_1}]^T$ contains the increments of the independent coefficients to be found.

We solve the minimisation problem of eqn. 29 to obtain the increment coefficient vector $\mathbf{v}$. Let $\mathbf{r}_1$ be an $(I+J+K) \times 1$ complex vector, with the i th entry given by $r_1(i) = e^{-jk_d \omega_i} - \hat{T}^l(e^{j\omega_i})$, for $\omega_i \in S_1 \cup S_2 \cup S_3$; $\mathbf{r}_2$ be an $I \times 1$ complex vector, with the i th entry given by $r_2(i) = -H_1^l(e^{j\omega_i})$, for $\omega_i \in S_1$; $\mathbf{r}_3$ be a $K \times 1$ complex vector, with the i th entry given by $r_3(i) = -H_0^l(e^{j\omega_i})$, for $\omega_i \in S_3$; and $\mathbf{r}_4$ be a $(J+2) \times 1$ complex vector,

Fig. 6 Magnitude response of variation of filter bank response for example 1

with the i th entry given by $r_4(i) = -\hat{V}^l(e^{j\omega_{i+1}})$, for $\omega_{i+1} \in \omega_p \cup S_2 \cup \omega_s$. The optimal solution $\mathbf{v}$ for eqn. 29 can then be expressed as

$$\begin{aligned} \mathbf{v} = & \{\text{Re}[\mathbf{H}_t^H \mathbf{H}_t + \alpha \mathbf{H}_1^H \mathbf{H}_1 + \beta \mathbf{H}_0^H \mathbf{H}_0 + \gamma \mathbf{H}_a^H \mathbf{H}_a]\}^{-1} \\ & \times \text{Re}[\mathbf{H}_t^H \mathbf{r}_1 + \alpha \mathbf{H}_1^H \mathbf{r}_2 + \beta \mathbf{H}_0^H \mathbf{r}_3 + \gamma \mathbf{H}_a^H \mathbf{r}_4] \end{aligned} \quad (30)$$

Step 4: Perform a line search by using the Nelder-Mead simplex algorithm [16] to find the best step size t with $0 < t < 1$, to update the numerators and denominators of $H_0(z)$ and $H_1(z)$ such that the following error function reaches its minimum:

$$\begin{aligned} \hat{E}^{l+1} = & \sum_{\omega_i \in S_1 \cup S_2 \cup S_3} |e^{-jk_d \omega_i} - \hat{T}^{l+1}(e^{j\omega_i})|^2 \\ & + \alpha \sum_{\omega_i \in S_1} |H_1^{l+1}(e^{j\omega_i})|^2 \\ & + \beta \sum_{\omega_i \in S_3} |H_0^{l+1}(e^{j\omega_i})|^2 \\ & + \gamma \sum_{\omega_i \in \omega_p \cup S_2 \cup \omega_s} |\hat{V}^{l+1}(e^{j\omega_i})|^2 \end{aligned} \quad (31)$$

subject to the constraints of $\max |k_{ij}^l + t \Delta k_{ij}| < k_{\max}$, for $i=0, j=1, 2, \dots, N_0$, and $i=1, j=1, 2, \dots, N_1$; where $H_0^{l+1}(e^{j\omega})$ and $H_1^{l+1}(e^{j\omega})$ have tap and reflection coefficient vectors given by $\mathbf{a}_0^{l+1} = [a_{00}^l + t \Delta a_{00}, a_{01}^l + t \Delta a_{01}, \dots, a_{0M_0}^l + t \Delta a_{0M_0}]^T$; $\mathbf{k}_0^{l+1} = [k_{01}^l + t \Delta k_{01}, k_{02}^l + t \Delta k_{02}, \dots, k_{0N_0}^l + t \Delta k_{0N_0}]^T$; $\mathbf{a}_1^{l+1} = [a_{10}^l + t \Delta a_{10}, a_{11}^l + t \Delta a_{11}, \dots, a_{1M_1}^l + t \Delta a_{1M_1}]^T$; and $\mathbf{k}_1^{l+1} = [k_{11}^l + t \Delta k_{11}, k_{12}^l + t \Delta k_{12}, \dots, k_{1N_1}^l + t \Delta k_{1N_1}]^T$, respectively. $\hat{T}^{l+1}(e^{j\omega})$ and $\hat{V}^{l+1}(e^{j\omega_i})$ are the corresponding filter bank response and aliasing distortion. Moreover, $k_{\max}$ is a preset maximal absolute value and must be less than 1 for the reflection coefficients, in order to ensure the stability of the designed IIR NDF bank.

Step 5: Compute the overall error function of eqn. 12 corresponding to $H_0^{l+1}(e^{j\omega})$, $H_1^{l+1}(e^{j\omega})$, $\hat{T}^{l+1}(e^{j\omega})$ and $\hat{V}^{l+1}(e^{j\omega_i})$, which is in fact given by eqn. 31.

Step 6: Compute the ratio $|\hat{E}^l - \hat{E}^{l+1}|/\hat{E}^l$. If this ratio is larger than a preset positive number ϵ , then set $l = l+1$ and go to Step 1. Otherwise, we terminate the design process.

Remarks: There are two situations where the gradient matrices $\mathbf{H}_t$, $\mathbf{H}_0$, $\mathbf{H}_1$, and $\mathbf{H}_a$ may degenerate.

Case 1: The columns of $\mathbf{H}_t$, $\mathbf{H}_0$, $\mathbf{H}_1$, and $\mathbf{H}_a$ are not linearly independent. The optimal solution for eqn. 29 is then not

Table 3: Significant design results for example 2

	IIR NDF bank	FIR NDF bank
Filter order	13 / 14, 17 / 17	49, 49
No. of coefficients	63	100
PRE in $\angle T(e^{j\omega})$ (dB)	0.0176	0.0190
Max. variation of group delay of $T(e^{j\omega})$	0.0703	0.0918
NPSR (dB) of $H_0(e^{j\omega})$	40.88	40.53
NPSR (dB) of $H_1(e^{j\omega})$	40.66	40.42
Max. variation of passband group delay of $H_0(e^{j\omega})$	0.0091	0.0304
Max. variation of passband group delay of $H_1(e^{j\omega})$	0.0247	0.0250
SEE of $H_0(e^{j\omega})$	4.05 10^{-3}	2.99 10^{-3}
SEE of $H_1(e^{j\omega})$	7.42 10^{-3}	6.45 10^{-3}
Max. variation of filter bank response	2.10 10^{-3}	2.57 10^{-3}
No. of iterations	41	10

unique. To find an appropriate optimal solution, we construct matrices G_t , G_0 , G_1 , and G_a by choosing the independent columns from H_t , H_0 , H_1 , and H_a , and a vector u by choosing the components of v corresponding to the independent columns. Then use G_t , G_0 , G_1 , G_a and u to replace H_t , H_0 , H_1 , H_a , and v in eqn. 29. On the other hand, if only H_t (or H_0 or H_1 or H_a) has columns not linearly independent, then we construct a matrix G_t (or G_0 or G_1 or G_a) by replacing the elements of those columns that are not linearly independent with zero elements from H_t (or H_0 or H_1 or H_a). Then use G_t (or G_0 or G_1 or G_a) to replace H_t (or H_0 or H_1 or H_a) in eqn. 29 to obtain an appropriate optimal solution.

Case 2: At the i th iteration, the i th reflection coefficient k_i may have the absolute value equal to k_{max} . To tackle this difficulty, we construct a vector u by eliminating Δk_i of the

Fig. 7 Magnitude and group delay responses of analysis filters for example 2

— $H_0(e^{j\omega})^D(LL_0)$
 --- $H_1(e^{j\omega})^D(LL_1)$
 a Magnitude response
 b Group delay response

Table 4: Tap and reflection coefficients for example 2

i	a_{0i}	k_{0i}	a_{1i}	k_{1i}
0	-3.6798234 10^{-3}		1.5980566 10^{-3}	
1	2.6099284 10^{-2}	-8.2994053 10^{-1}	3.5848626 10^{-3}	-8.1584134 10^{-1}
2	-9.0796157 10^{-2}	9.3802032 10^{-1}	5.8518981 10^{-3}	8.4751020 10^{-1}
3	2.1271854 10^{-1}	-8.7566468 10^{-1}	7.6135849 10^{-3}	5.4108119 10^{-3}
4	-3.8010473 10^{-1}	8.5826413 10^{-1}	7.1787741 10^{-3}	1.1230581 10^{-1}
5	5.4977591 10^{-1}	-8.1937563 10^{-1}	2.9788998 10^{-3}	1.1278975 10^{-1}
6	-6.6896321 10^{-1}	7.4277861 10^{-1}	-5.9077057 10^{-3}	1.0793339 10^{-1}
7	6.9851301 10^{-1}	-7.0493877 10^{-1}	-1.8788240 10^{-2}	1.0367129 10^{-1}
8	-6.2758842 10^{-1}	7.0366242 10^{-1}	-3.2615090 10^{-2}	9.8855147 10^{-2}
9	4.8330749 10^{-1}	-6.8684451 10^{-1}	-4.1421895 10^{-2}	9.1518793 10^{-2}
10	-3.0750743 10^{-1}	6.8390959 10^{-1}	-3.5958718 10^{-2}	8.0536860 10^{-2}
11	1.5743740 10^{-1}	-6.6734683 10^{-1}	-1.9035730 10^{-3}	6.6165397 10^{-2}
12	-5.6882050 10^{-2}	5.5310626 10^{-1}	8.9533637 10^{-2}	4.9924064 10^{-2}
13	1.3209640 10^{-2}	-3.0505849 10^{-1}	3.4236521 10^{-1}	3.3990564 10^{-2}
14		7.3282216 10^{-2}	1.8856250 10^{-0}	2.0406182 10^{-2}
15			-6.1255005 10^{-0}	1.0412525 10^{-2}
16			5.6539967 10^{-0}	4.2050121 10^{-3}
17			-1.7337973 10^{-0}	1.10958031 0^{-3}

vector $\mathbf{v}$ and four matrices $\mathbf{G}_t$, $\mathbf{G}_0$, $\mathbf{G}_1$, and $\mathbf{G}_a$ by eliminating the columns of $\mathbf{H}_t$, $\mathbf{H}_0$, $\mathbf{H}_1$, and $\mathbf{H}_a$, corresponding to Δk_i , respectively. Then, we use $\mathbf{G}_t$, $\mathbf{G}_0$, $\mathbf{G}_1$, $\mathbf{G}_a$, and $\mathbf{u}$ to replace $\mathbf{H}_t$, $\mathbf{H}_0$, $\mathbf{H}_1$, $\mathbf{H}_a$, and $\mathbf{v}$ in eqn. 29.

4 Simulation examples

In this Section, we present several simulation examples of designing two-channel IIR NDF banks for illustration. For all design examples, the spacing for two adjacent frequency points in $[0, \pi]$ is set to $\pi/299$, i.e. the number of grid points taken in $[0, \pi]$ is 300. The presented design examples are, in fact, related to the optimal design of two-channel IIR NDF banks with low group delay. The performance for each of the designed filter banks is evaluated in terms of the peak reconstruction error (PRE), the normalised peak stopband ripple (NPSR), and the stopband error energies (SEE) of the designed $H_0(z)$ and $H_1(z)$. They are given by

$$\text{PRE}(\text{dB}) = \max\{|20 \log_{10} |\hat{T}(\omega_i)||\} \quad \text{for } \omega_i \in [0, \pi]$$

$$\text{NPSR}_0(\text{dB}) = -20 \log_{10} \left(\max \frac{|H_0(\omega_i)|}{\sqrt{LL_0}} \right)$$

$$\text{for } \omega_i \in [\omega_s, \pi], \text{ SEE}_0 = \sum_{\omega_i \in S_3} |H_0(\omega_i)|^2$$

$$\text{NPSR}_1(\text{dB}) = -20 \log_{10} \left(\max \frac{|H_1(\omega_i)|}{\sqrt{LL_1}} \right)$$

$$\text{for } \omega_i \in [0, \omega_p], \text{ SEE}_1 = \sum_{\omega_i \in S_1} |H_1(\omega_i)|^2$$

$$\text{max. variation of } GD(\hat{T}(e^{j\omega_i}))$$

$$= \max_{\omega_i \in [0, \pi]} |GD(\hat{T}(e^{j\omega_i})) - k_d|(\text{samples})$$

$$\text{max. variation of passband } GD(H_0(e^{j\omega_i}))$$

$$= \max_{\omega_i \in [0, \omega_p]} |GD(H_0(e^{j\omega_i})) - k_d/2|(\text{samples})$$

$$\text{max. variation of passband } GD(H_1(e^{j\omega_i}))$$

$$= \max_{\omega_i \in [\omega_s, \pi]} |GD(H_1(e^{j\omega_i})) - k_d/2|(\text{samples})$$

$$\text{and max. variation of filter bank response}$$

$$= \max_{\omega_i \in [0, \pi]} |e^{-j\omega_i k_d} - \hat{T}(e^{j\omega_i})|$$

$GD(x)$ denotes the group delay of x .

For comparison, we also use the proposed technique for each design example with a FIR NDF bank and the same group delay k_d . For designing FIR NDF banks, the computations required for finding the denominators in the proposed iterative procedure are not necessary and can be discarded.

Example 1: We use the following design specifications: the group delay k_d is 19; $\omega_p = 0.3\pi$ and $\omega_s = 0.5\pi$; $L_0 = 2$ and $L_1 = 3$; $M_0 = N_0 = 10$ and $M_1 = N_1 = 11$. In the FIR case, $H_0(z)$ and $H_1(z)$ have orders equal to 32 and 33, respectively. Accordingly, the numbers of independent coefficients for the IIR and FIR NDF banks are 44 and 67, respectively. Moreover, the values of α , β , γ , τ , and ϵ are set to 0.012, 0.012, 0.05, 0.5, and 10^{-3} , respectively, for the IIR NDF bank; they are set to 0.015, 0.015, 0.02, 0.5, and 10^{-3} , respectively, for the FIR NDF bank.

Table 1 shows the significant design results for both of the designed IIR and FIR NDF banks. The resulting tap and reflection coefficients of the designed IIR NDF bank are

Fig. 8 Magnitude and group delay response of $\hat{T}(e^{j\omega})$ for example 2

a Magnitude response

b Group delay response

listed in Table 2. Fig. 4 plots the corresponding magnitude responses and the group delays of $H_0(e^{j\omega})/\sqrt{LL_0}$ and $H_1(e^{j\omega})/\sqrt{LL_1}$. Fig. 5 depicts the magnitude response and the group delay response in samples of $\hat{T}(e^{j\omega})$ for the designed IIR NDF bank. The magnitude of $e^{-j\omega k_d} - \hat{T}(e^{j\omega})$

Fig. 9 Magnitude response of variation of filter bank response for example 2

is shown in Fig. 6. From the design results of using the proposed technique, we note that the IIR NDF bank provides the advantage of requiring a lower order for achieving a comparable performance than the FIR NDF bank.

Example 2: The design specifications used are as follows: the group delay k_d is 29; $\omega_p = 0.12\pi$ and $\omega_s = 0.28\pi$; $L_0 = 1$ and $L_1 = 4$; $M_0 = 13$ and $N_0 = 14$; and $M_1 = N_1 = 17$. In the FIR case, both $H_0(z)$ and $H_1(z)$ have orders equal to 49. Accordingly, the numbers of independent coefficients for the IIR and FIR NDF banks are 63 and 100, respectively. Moreover, the values of α , β , γ , τ and ϵ are set to 0.02, 0.02, 0.04, 0.5 and 5×10^{-3} , respectively, for the IIR NDF bank; they are set to 0.1, 0.1, 0.05, 0.5, and 10^{-3} , respectively, for the FIR NDF bank.

Table 3 shows the significant design results for both of the designed IIR and FIR NDF banks. The resulting tap and reflection coefficients of the designed IIR NDF bank are listed in Table 4. Fig. 7 plots the corresponding magnitude responses and the group delays of $H_0(e^{j\omega})/\sqrt{LL_0}$ and $H_1(e^{j\omega})/\sqrt{LL_1}$. Fig. 8 depicts the magnitude response and the group delay response in samples of $\hat{T}(e^{j\omega})$ for the designed IIR NDF bank. The magnitude of $e^{-j\hat{k}d^0} - \hat{T}(e^{j\omega})$ is shown in Fig. 9. We again observe from the design results that the IIR NDF bank provides the advantage of requiring a lower order for achieving a comparable performance than the FIR NDF bank.

5 Conclusions

We have employed recursive analysis filters with a lattice denominator for designing IIR NDF banks. At each iteration, the proposed design technique adjusts the tap and reflection coefficients for the analysis filters, to reduce the resulting squared error and keep the designed IIR analysis filters stable. For the first task, a linearisation scheme has been proposed to solve the resulting highly nonlinear programming problem. For the second task, the stability of the designed recursive analysis filters is ensured by incorporating an efficient stabilisation procedure. Computer simulations have shown the effectiveness of the proposed technique.

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7 References

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