Nonlinear Adaptive Motion Control for a Manipulator with Flexible Joints

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ABSTRACT

This paper presents an adaptive control scheme for trajectory tracking of robot motion. A dynamic model of a manipulator with flexible joints particularly useful for derivation of the control law is adopted. Based on this model, a two-stage controller is established consisting of a feedback loop and a parameter adaptation loop. It is shown that without precise knowledge of parameters of the manipulator and its joint flexibilities the tracking error will converge to zero asymptotically. An implication is the robustness of the control scheme to the variation of payload or parameters so long as their rate of change is moderate. Moreover, the convergence may be sped up by increasing the suitable controller gains.

1. Introduction

The problem of joint level control for the multi-link robot manipulator has been widely investigated in the robotics literature. To date, several approaches have been proposed to solve the problem, among which the simple PID control algorithm has recently been shown [1][2][8] effective for large-scale motion, the computed torque method [5] uses a nonlinear state feedback to decouple and linearize the system globally, the variable structure control [13][19] drive the manipulator state into a stable surface for later convergence, the adaptive control [3] [6] [12][20] deals with the problem when all system parameters are unknown in advance, using dynamic adaptation of controller parameters. However, these approaches will be successful only if the manipulators are assumed rigid. Such a condition is, in fact, hard to be met in real practice. [15] [17]. For example, a light-weight manipulator may have flexibility in the link structure, and a manipulator with harmonic drives may have flexibility in joints. Hence, more accurate representation of robot dynamics involving these unmodeled flexibility modes should be taken into account to pursue better control performance.

Marino and Nicosia [11] take a singular perturbation approach to design controllers for manipulators with flexible joints. The formulation is continued in Khoransami and Spong [10] incorporating an invariant manifold concept. Later, Slotine and Hong [18] devise a two-time scale sliding control based on the reduced-order model of that used in [11] for the required robustness to parametric uncertainty. After the globally linearizing control becomes prevailing, Bortoff and Spong [4] adopt the linearization method and solve the decoupled fourth-order linear differential equations. Such a method is later modified [21] by adding another input to establish the robustness to parametric uncertainty. As for the case where parameters are unknown, Tomci et al [22] consider a linearized

model around an equilibrium points and assume an adaptive control method.

In the paper, we consider a nonlinear model of the system based on the derivations in [21]. The approach used here is similar to the one in Sastry [16] but with a two-stage design so as to overcome the difficulty due to inaccessibility to the high order derivatives of signals. The results to be presented show that without precise knowledge of the manipulator link parameters the adaptive controller will drive the tracking errors to zero asymptotically. This control algorithm is conceptually clear as well as implementationally easy.

The paper is organized as follows: in section 2, we give the model of manipulators with flexible joints; in section 3, a so called asymptotic computed torque method is proposed for non-adaptive case; section 4 introduces the adaptive control law for a multi-link manipulator; finally some conclusions are made in section 5.

2. Problem Formulation.

We consider a manipulator constructed with $n+1$ rigid links (including base link) connected by n elastic joints and actuated by DC motors. The elasticity of each joint is modeled as a linear torsional spring with a finite elastic constant. The set-up of each joint is the same as the one in [10] and is redrawn in Fig. 2.1. To be more amenable to analysis and control, we apply the model which is derived by Spong [21] under the same assumptions as follows:

Assumptions:

- (A1) The kinetic energy of the rotor is due mainly to its own rotation. Equivalently, the motion of the motor is a pure rotation with respect to an inertial frame.
- (A2) The rotor gear inertia is symmetric about the rotor axis of rotation so that the gravitational potential of the system and also the velocity of the rotor center of mass are both independent of the rotor position.

The simplified dynamics of the manipulator system can then be described by the following $2n$ second order differential equations:

$$M(\theta_l) \ddot{\theta}_l + B(\theta_l, \dot{\theta}_l) \dot{\theta}_l + G(\theta_l) = K(N^{-1} \theta_m - \theta_l) \quad (2.1)$$

$$J_m \ddot{\theta}_m + B_m \dot{\theta}_m + N^{-1} K(N^{-1} \theta_m - \theta_l) = \tau_m \quad (2.2)$$

where the following notation is used:

- (1) θ_l and $\theta_m \in R^n$ denote angle displacements of links and motor shafts respectively;
- (2) $M(\theta_l) : R^n \rightarrow R^{n \times n}$, is a symmetric and positive definite matrix for all $\theta_l \in R^n$ representing the mass of inertia of rigid links;
- (3) $B(\theta_l, \dot{\theta}_l) \dot{\theta}_l : R^n \times R^n \rightarrow R^n$ denotes a vector of Coriolis and centrifugal terms;

- (4) $G(\theta_l) : R^n \rightarrow R^n$ is the gravity vector;
- (5) $N, K \in R^{n \times n}$ are diagonal positive definite matrices, representing the spring constant matrix and gear ratio matrix of n joints;
- (6) $J_m \in R^{n \times n}$ represents the matrix of the moment of inertia;
- (7) $B_m \in R^{n \times n}$ is the viscous friction coefficient matrix of the motor shaft;
- (8) $\tau_m \in R^n$ is the exogenous input torque vector from motor actuators.

A generic property which should be noted about the dynamic structure is the following

$$y^T \left[\frac{1}{2} \frac{d}{dt} M(\theta_l) - B(\theta_l, \dot{\theta}_l) \right] y = 0 \quad (2.3)$$

for all $(\theta_l^T, \dot{\theta}_l^T)^T \in R^{2n}$, and for all $y \in R^n$ i.e. $\frac{1}{2} \dot{M} - B$ is a skew-symmetric matrix over the velocity space at all time $t \geq 0$. Verification of this property can be found in several places, e.g. in [1][9][20], and is, hence, omitted here.

Define $Z \equiv (N^{-1} \theta_m - \theta_l)$ or equivalently $\theta_m = N(Z + \theta_l)$ so that (2.1) and (2.2) can be rewritten into the following symbolic form:

$$M(\theta_l) \ddot{\theta}_l + B(\theta_l, \dot{\theta}_l) \dot{\theta}_l + G(\theta_l) = K Z \quad (2.4)$$

$$J_m \ddot{Z} + B_z(\theta_l, \dot{\theta}_l, \dot{Z}) + G_z(\theta_l, Z) = N^{-1} \tau_m \quad (2.5)$$

where

$$B_z(\theta_l, \dot{\theta}_l, \dot{Z}) \equiv B_m \dot{Z} + B_m \dot{\theta}_l - J_m M^{-1} B \dot{\theta}_l \quad (2.6)$$

$$G_z(\theta_l, Z) \equiv J_m M^{-1} K Z - J_m M^{-1} G + N^{-2} K Z \quad (2.7)$$

Thus (2.4) and (2.5) constitutes a dynamic model of the manipulator system that will be used here in synthesizing the control scheme. The control objective is then to find a suitable τ_m so that θ_l will follow the desired trajectory asymptotically.

3. Motion Control with Known Manipulator Parameters -- Nonadaptive Case:

In this section, we consider the motion control of a manipulator with flexible joints whose parameters are known precisely. A two-stage controller design, which will be called asymptotic computed torque method, is introduced. Although the method also requires the linearizability of the system as in [21], the concept of deriving the method is simpler. We now start with two assumptions which will be used in the sequel.

Assumptions:

- (A3) For trajectory tracking, the desired trajectory $\theta_d(t) \in C^4$, and all time derivatives up to the fourth order are uniformly bounded in time t . For point-to-point control or regulation, $\theta_d(t)$ equals a constant vector.
- (A4) The signals $\theta_l, \dot{\theta}_l, Z$ and $\dot{Z}$ are measurable at all time t .

Consider the dynamical equation (2.4) of the simplified model described in section 2, which represents the motional description of the manipulator rigid links. In the absence of joint elasticity, the right hand side (RHS) of (2.4) can be equivalently replaced by the purely exogenous torque input τ_m . This is unfortunately not the case when joint flexibility exists. However, from such an observation, we establish the following design of a two-stage controller design. Conceptually, if Z_d is the necessary linearizing feedback term for equation (2.4), then we specify the torque input τ_m so that Z asymptotically converges to Z_d , and hopefully θ_l also converges to θ_d .

(I) **The first stage:** We replace (2.4) by the following:

$$M(\theta_l) \ddot{\theta}_l + B(\theta_l, \dot{\theta}_l) \dot{\theta}_l + G(\theta_l) = K(Z_d + e_z) \quad (3.1)$$

where $e_z = Z - Z_d$ and Z_d is defined by:

$$Z_d \equiv K^{-1} [M(\theta_l) \ddot{\theta}_d + B(\theta_l, \dot{\theta}_l) \dot{\theta}_d + G(\theta_l) - K_v(\dot{\theta}_l - \dot{\theta}_d) - K_p(\theta_l - \theta_d)] \quad (3.2)$$

where the velocity and position feedback gain matrices, K_v and K_p , are diagonal and positive definite. Substitution of (3.2) into (3.1) leads to the following form:

$$\ddot{e} + M^{-1} [B + K K_v] \dot{e} + M^{-1} K K_p e = M^{-1} K e_z \quad (3.3)$$

where $e = \theta_l - \theta_d$ is the tracking error signal.

(II) **The second stage:** In order to drive the error signal e_z to zero, a suitable τ_m has to be chosen. Now if we let

$$\tau_m = N [J_m (\ddot{Z}_d - \beta_1 \dot{e}_z - \beta_2 e_z) + B_z + G_z] \quad (3.4)$$

so that (2.5) becomes a completely decoupled error equation

$$\ddot{e}_z + \beta_1 \dot{e}_z + \beta_2 e_z = 0 \quad (3.5)$$

where $\beta_1, \beta_2 \in R_+$ are selected so that the polynomial $(s^2 + \beta_1 s + \beta_2)$ is Hurwitz, then e_z will be driven asymptotically to zero with all initial conditions. With two sets of dynamical equations as shown in (3.3) and (3.5), their equivalent state space representation can be written as:

$$\dot{X} = A_t X \quad (3.6)$$

where $X = [e^T, \dot{e}^T, e_z^T, \dot{e}_z^T]^T \in R^{4n}$, $A_t \in R^{4n \times 4n}$, and

$$A_t = \begin{bmatrix} 0 & I_n & 0 & 0 \\ -M^{-1} K K_p & -M^{-1} (K K_v + B) & 0 & 0 \\ 0 & 0 & 0 & I_n \\ 0 & 0 & -\beta_2 I_n & -\beta_1 I_n \end{bmatrix}$$

To be more easy for analysis, we partition the matrix A and the state vector X into following:

$$A_t = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \text{ and } X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad (3.7)$$

Before the verification of the above design of controller, the following results will first be given which are particularly useful for using the Lyapunov method as a designing tool. Their proofs are very similar to those in Koditschek [9] and Fu et al [8], and are, hence, omitted here.

Lemma 3.1 : (Condition for positive Definiteness of a Symmetric Matrix)

Given a matrix as follows:

$$N = \begin{bmatrix} U_1 & V \\ V^T & U_2 \end{bmatrix} \in R^{(n+m) \times (n+m)} \quad (3.8)$$

where $U_1 \in R^{n \times n}$ and $U_2 \in R^{m \times m}$ are symmetric positive definite matrices. Then matrix N is positive definite if

$$\lambda_{\min}(U_1) \lambda_{\min}(U_2) > \|V\|^2 \quad (3.9)$$

$$\lambda_{\min}(\bar{N}) I_{n+m} \leq N \leq \lambda_{\max}(\bar{N}) I_{n+m} \quad (3.10)$$

where

$$\bar{N} = \begin{bmatrix} \lambda_{\min}(U_1) & \|V\| \\ \|V\| & \lambda_{\min}(U_2) \end{bmatrix} \in R^{2 \times 2} \quad (3.11)$$

$$\bar{N} = \begin{bmatrix} \lambda_{\max}(U_1) & \|V\| \\ \|V\| & \lambda_{\max}(U_2) \end{bmatrix} \in R^{2 \times 2} \quad (3.12)$$

Proposition 3.2 : (Uniform Positive Definiteness).

Construct the following two matrices :

$$P_{11}(\theta_l) = \begin{bmatrix} \gamma K K_p & M \\ M & \gamma M \end{bmatrix} \quad (3.13)$$

$$Q_{11}(\theta_l, \dot{\theta}_l) = \begin{bmatrix} 2K K_p & (K K_v + B) \\ (K K_v + B)^T & 2(\gamma K K_v - M) \end{bmatrix} \quad (3.14)$$

then, given any $h > 0$ and $\lambda_p, \lambda_q > 0$, there exist γ_0 and $\lambda_k > 0$, depending on h , such that

$$P_{11}(\theta_l) \geq \lambda_p I_{2n} \text{ and } Q_{11}(\theta_l, \dot{\theta}_l) \geq \lambda_q I_{2n} \quad (3.15)$$

for all $(\theta_l, \dot{\theta}_l) \in R^n \times B_h$.

Now consider the dynamic system (3.6) and construct the following two matrices:

$$P = \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix} \text{ and } Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \quad (3.16)$$

where P_{11} , Q_{11} are defined as in Proposition 3.2, and Q_{12} , Q_{22} satisfy

$$-Q_{12} = P_{11} A_{12} = \begin{bmatrix} K & 0 \\ \gamma K & 0 \end{bmatrix} \quad (3.17)$$

$$-Q_{22} = A_{22}^T P_{22} + P_{22} A_{22} \quad (3.18)$$

respectively for some symmetric P_{22} . Since A_{22} is a Hurwitz matrix by the choice of β_1 and β_2 , given any positive definite Q_{22} the matrix P_{22} in (3.17) will be positive definite. Therefore, as a result from Lemma 3.1 and Proposition 3.2, so long as Q_{22} is chosen such that

$$\lambda_{\min}(Q_{22}) > \|Q_{12}\|^2 / \lambda_{\min}(Q_{11}) \quad (3.19)$$

the matrices P and Q will all be positive definite uniformly over $R^n \times B_h$ when those suitable γ and K_p are used. Moreover, Q can be made more positive definite by increasing γ , $\lambda_{\min}(K_p)$ and $\lambda_{\min}(Q_{22})$.

The following theorem will assure us of the performance of asymptotic tracking of the so designed controller.

Theorem 3.3 : (Asymptotic Tracking)

Consider a manipulator system with joint flexibility, satisfying assumptions (A1)-(A4). Suppose there exists $h' > 0$ such that $\dot{\theta}_l(0) \in B_{h'}$, i.e. the magnitude of the initial velocity never exceeds h' . Then, the above controller is implementable, and, with sufficiently large γ , K_p and Q_{22} , stabilizes the system as well as achieves the asymptotic tracking.

Proof of Theorem 3.3:

We first note the fact that M , B , G , B_z , G_z are smooth functions of their arguments. From the definitions given in (3.4), we know that the synthesis of τ_m involves the first and the second order derivatives of Z_d , namely,

$$\begin{aligned} \dot{Z}_d &= K^{-1} [\dot{M} \ddot{\theta}_d + M \theta_d^{(3)} + \dot{B} \dot{\theta}_d + B \ddot{\theta}_d + \dot{G}] \\ &\quad - K_v (\dot{\theta}_l - \dot{\theta}_d) - K_p (\theta_l - \theta_d) \end{aligned} \quad (3.20)$$

and

$$\begin{aligned} \ddot{Z}_d &= K^{-1} [\ddot{M} \ddot{\theta}_d + 2 \dot{M} \theta_d^{(3)} + M \theta_d^{(4)} + \ddot{B} \dot{\theta}_d + 2 \dot{B} \ddot{\theta}_d \\ &\quad + B \theta_d^{(3)} + \ddot{G}] - K_v (\ddot{\theta}_l - \ddot{\theta}_d) - K_p (\ddot{\theta}_l - \ddot{\theta}_d) \end{aligned} \quad (3.21)$$

from the definition in (3.2). It can be easily verified that computation of $\dot{Z}_d$ and $\ddot{Z}_d$ requires informations on derivatives of θ_l , θ_d and Z only up to the third, the fourth, and the first order respectively. Since $\ddot{\theta}_l$ and $\theta_l^{(3)}$ can be computed via the equation (2.4) using θ_l , $\dot{\theta}_l$, Z and $\dot{Z}$, there exist smooth functions T_1 and T_2 such that

$$\dot{Z}_d = T_1(\theta_l, \dot{\theta}_l, Z, \dot{Z}, t) \quad (3.22)$$

$$\ddot{Z}_d = T_2(\theta_l, \dot{\theta}_l, Z, \dot{Z}, t) \quad (3.23)$$

where the argument t in both T_1 and T_2 accounts for the terms induced from the time derivative θ_d up to the fourth order. This, hence, implies that the controller described above is implementable under assumptions (A3)-(A4). Denote $X_1 = [x_1^T, x_2^T]^T \in R^{2n}$. The result of Proposition 3.2 implies that the scalar function V_1 defined by:

$$V_1 = X^T P X \quad (3.24)$$

is a Lyapunov function candidate for the system (3.6). Now take the time derivative of V_1 along the trajectories of (3.6) to obtain the following

$$\begin{aligned} \dot{V}_1 &= -X^T Q X + 2 x_1^T \dot{M} x_2 \\ &\leq -[\lambda_{\min}(Q) - 2 \|\dot{M}\| \|X\|] \|X\|^2 \end{aligned} \quad (3.28)$$

Now if $(\theta_l, \dot{\theta}_l)$ is kept in the space $R^n \times B_h$ for some $h \geq h' > 0$, then for sufficiently large γ , K_p , and Q_{22} there exist $q > 0$ such that

$$\dot{V}_1 \leq -q \|x\|^2 \quad (3.29)$$

as a result of Proposition 3.2. By an argument of a contradiction ([8]), it can be shown that the above claim is, in fact, justified. This concludes our proof.

Q.E.D

4. Adaptive Control with Unknown Parameters.

In previous section, the motion control is presented based on precise knowledge of the system parameters, which makes the linearization possible. However, such an exact linearization is hardly achievable in real practice due to parameter uncertainty, change of payload, and inaccessibility to the elastic constants of joints. Under these circumstances, an adaptive control method will be an attractive approach to pursue. Before the adaptive control algorithm is proposed, three additional assumptions will be needed.

Assumptions:

- (A5) The gear ratio matrix N , inertia matrix J_m , and friction coefficient matrix of the motor shaft B_m are known precisely.
- (A6) The acceleration of the link displacement θ_l , i.e. $\ddot{\theta}_l$, is a measurable quantity.
- (A7) $M(\theta_l)$, $B(\theta_l, \dot{\theta}_l)$ and $G(\theta_l)$ can be represented as products of unknown parameters and known functions, i.e.

$$M(\theta_l) = \sum_{k=1}^{s_m} P_{mk} f_{mk}(\theta_l) \quad (4.1)$$

$$B(\theta_l, \dot{\theta}_l) = \sum_{k=1}^{s_b} P_{bk} f_{bk}(\theta_l, \dot{\theta}_l) \quad (4.2)$$

$$G(\theta_l) = \sum_{k=1}^{s_g} P_{gk} f_{gk}(\theta_l) \quad (4.3)$$

where P_{mk} , $k = 1 \dots s_m$, P_{bk} , $k = 1 \dots s_b$, and P_{gk} , $k = 1 \dots s_g$, represent unknown (scalar) parameters whereas f_{mk} , $k = 1 \dots s_m$, f_{bk} , $k = 1 \dots s_b$, f_{gk} , $k = 1 \dots s_g$ stand for known signal functions with proper dimensions.

The assumption (A7) implies that the difficulty of constructing Z_d as before are only due to the parametric uncertainties. However, Z_d can be estimated via the use of estimates of parameters inside M , B and G , or symbolically, $\hat{M}$, $\hat{B}$ and $\hat{G}$ as follows:

$$\hat{Z}_d = \hat{K}^{-1} (\hat{M} \ddot{\theta}_d + \hat{B} \dot{\theta}_d + \hat{G}) - K_v (\dot{\theta}_l - \dot{\theta}_d) - K_p (\theta_l - \theta_d) \quad (4.4)$$

Now, we define $\hat{e}_z$ as :

$$\hat{e}_z = Z - \hat{Z}_d \quad (4.5)$$

Substitution of (4.4) and (4.5) into (2.1) leads to the following first-stage error equation :

$$\begin{aligned} \ddot{e} + M^{-1}(K K_v + B) \dot{e} + M^{-1} K K_p e = & -M^{-1} K (\tilde{M}^{-1} \ddot{\theta}_d \\ & + \tilde{B} \dot{\theta}_d + \tilde{C}) + M^{-1} K \hat{e}_z \end{aligned} \quad (4.6)$$

where

$$\tilde{M} = K^{-1} M - \hat{K}^{-1} \hat{M}, \quad \tilde{B} = K^{-1} B - \hat{K}^{-1} \hat{B}, \quad \tilde{C} = K^{-1} C - \hat{K}^{-1} \hat{C} \quad (4.7)$$

representing the estimation errors in the controller design arising from uncertainties in the parameters. By a process of reindexing the parameter estimates (if necessary), $\hat{Z}_d$ can be rewritten as:

$$\hat{Z}_d = W_1(t, \theta_1, \dot{\theta}_1) \hat{P}_1 + W_2(t, \theta_1, \dot{\theta}_1) \quad (4.8)$$

where $\hat{P}_1$ is an $r \times 1$ vector of parameter estimates, and W_1 and W_2 are $n \times r$ and $n \times 1$ known function matrix and vector respectively. Later, we will on-line adapt $\hat{P}_1$ according to some adaptation law to be specified. Based on this process, the equation (4.6) can also be rewritten as:

$$\begin{aligned} \ddot{e} + M^{-1}(K K_v + B) \dot{e} + M^{-1} K K_p e = & -M^{-1} K W_1 \phi_1 \\ & + M^{-1} K \hat{e}_z \end{aligned} \quad (4.9)$$

where $\phi_1 = P_1 - \hat{P}_1$ are parameter errors. Hence, $\hat{P}_1$ equivalently constitute time varying gains subject to some control purpose. Similarly, $\hat{Z}_d$ and $\hat{Z}_d$ can also be estimated through the following:

$$\dot{\hat{Z}}_d = \dot{W}_1 \hat{P}_1 - W_1 \dot{\phi}_1 + \dot{W}_2 \quad (4.10)$$

$$\ddot{\hat{Z}}_d = \ddot{W}_1 \hat{P}_1 - 2 \dot{W}_1 \dot{\phi}_1 - W_1 \ddot{\phi}_1 + \ddot{W}_2 \quad (4.11)$$

Since $\theta_1^{(3)}$ is not a measurable quantity, the defined signal vectors W_1 and W_2 are not derivable and, hence, an estimate of $\theta_1^{(3)}$ has to be used in order to successfully approximate $\dot{W}_1$ and $\dot{W}_2$. On the other hand, if all parameters are known, $\theta_1^{(3)}$ can be derived from (2.4) as:

$$\begin{aligned} \theta_1^{(3)} = & -M^{-1}(\dot{M} \ddot{\theta}_1 + \dot{B} \dot{\theta}_1 + B \ddot{\theta}_1 + \dot{C} - K \ddot{Z}) \\ = & W_3 P'_2 / \det(M) \end{aligned} \quad (4.12)$$

where P'_2 is another parameter vector after the reindexing, W_3 is a known function matrix with proper dimension, and $\det(M)$ represents the determinant of M . Now, let

$$P_2(\theta_1) = P'_2 / \det(M) \quad (4.13)$$

and return to the practical case where uncertainty exists. Then $\theta_1^{(3)}$ can be approximated by $\hat{\theta}_1^{(3)}$ as follows:

$$\hat{\theta}_1^{(3)} = W_3 \bar{P}'_2 \quad (4.14)$$

where $\bar{P}'_2$ is a fixed parameter vector by some guess so that $\hat{Z}_d$ can also be approximated by the following:

$$\hat{\bar{Z}}_d = W_4 + W_5 \bar{P}_2 \quad (4.15)$$

where W_4 and W_5 are also available signal functions with proper dimensions and $\bar{P}_2$ is obtained from $\bar{P}'_2$ after rearrangement of (4.11), assuming $\dot{\phi}_1$ and $\ddot{\phi}_1$ are specified. So far, the above approximation can be summarized into the following controller design of the second stage:

$$\tau_m = N [J_m (\ddot{\bar{Z}}_d - \beta_1 \dot{\hat{e}}_z - \beta_2 \hat{e}_z) + \hat{B}_z + \hat{G}_z + v] \quad (4.16)$$

where $\hat{B}_z$ and $\hat{G}_z$ are estimates of B_z and G_z . Here, an additional input signal v is introduced and will be specified later. Referring to definitions of B_z and G_z in (2.6) and (2.7), a situation similar to (4.12) will occur,

i.e.

$$B_z = W_6 P_{31} + W_7 P_{32} / \det(M) = [W_6 \ W_7] P_3(\theta_1) \quad (4.17)$$

$$G_z = W_8 P_{41} + W_9 P_{42} / \det(M) = [W_8 \ W_9] P_4(\theta_1) \quad (4.18)$$

where $P_{3i}, P_{4i}, i=1,2$ are unknown parameter vectors, and $W_6 \sim W_9$ are known signal functions, so that more detailed descriptions of $\hat{B}_z$ and $\hat{G}_z$ are given in the following:

$$\hat{B}_z = [W_6 \ W_7] \bar{P}_3 \quad (4.19)$$

$$\hat{G}_z = [W_8 \ W_9] \bar{P}_4 \quad (4.20)$$

where $\bar{P}_3$ and $\bar{P}_4$ are constant vectors obtained by some prior guess. Now by substituting (4.15)(4.16) and (4.19)(4.20) into (2.5), we obtain the following error equation:

$$\ddot{e}_z + \beta_1 \dot{e}_z + \beta_2 e_z = W_{10} [\bar{P}_5 - P_5(\theta_1)] + J_m^{-1} v \quad (4.21)$$

with $\bar{P}_5, P_5(\theta_1)$, and W_{10} being defined likewise. Note that the error equation (4.21) is the same as (3.3) modulo the forcing terms.

With two sets of dynamical equations as shown in (4.9) and (4.21), their equivalent state-space representation can be written as:

$$\dot{X}_a = A_a X_a + B_a U \quad (4.22)$$

where $A_a = A_i$ as defined in (3.6), $X_a = [e^T, \dot{e}^T, \ddot{e}^T, \ddot{e}_z^T, \ddot{e}_z^T]^T = [X_{a1}^T, X_{a2}^T]^T$

$$B_a = \begin{bmatrix} 0 & 0 \\ M^{-1} K & 0 \\ 0 & 0 \\ 0 & I_n \end{bmatrix} \in R^{4n \times 2n}, \quad U = \begin{bmatrix} -W_1 \phi_1 \\ h \end{bmatrix} \in R^{2n} \quad (4.23)$$

where $h = h_0 + J_m^{-1} v$ with $h_0 = W_{11} [\bar{P}_5 - P_5(\theta_1)]$. Partition B_a into the following :

$$B_a = \begin{bmatrix} B_{a11} & 0 \\ 0 & B_{a22} \end{bmatrix} \quad (4.24)$$

where

$$B_{a11} = \begin{bmatrix} 0 \\ M^{-1} K \end{bmatrix} \text{ and } B_{a22} = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \in R^{2n \times n} \quad (4.25)$$

From triangle inequality property, it can be easily verified that the following inequality holds:

$$\begin{aligned} \|h_0\| &= \|W_{10} [\bar{P}_5 - P_5(\theta_1)]\| \\ &\leq \sum_{i=1}^s \|[\bar{P}_5 - P_5(\theta_1)]_i\| \|W_{10} i\| \leq \sum_{i=1}^s \alpha_i \omega_i = \alpha' \omega \end{aligned} \quad (4.26)$$

where s is the dimension of the vector $\bar{P}_5, [W_{10}]_i$ denotes the i th column of W_{10} , and $\alpha_i = \max_{\theta_1 \in R^n} \|[\bar{P}_5 - P_5(\theta_1)]_i\|$, $\omega_i = \max_i \|W_{10} i\|$,

$i=1, \dots, s$, which, then, leads to the following:

$$\begin{aligned} v^T h &= v^T [h_0 + J_m^{-1} v] \geq \|v\|^2 \rho_0 - \|v\| \|h_0\| \\ &= \rho_0 \|v\| [\|v\| - \alpha' \omega / \rho_0] = \rho_0 \|v\| [\|v\| - \Pi(\rho)] \end{aligned} \quad (4.27)$$

where

$$\rho_0 = \lambda_{\min}(J_m^{-1}) \quad (4.28)$$

$$\Pi(\rho) = \rho' \omega = \rho_1 \omega_1 + \rho_2 \omega_2 + \dots + \rho_s \omega_s \quad (4.29)$$

where $\rho_i = \alpha_i / \rho_0, i=1, \dots, s$. Now we are ready to introduce the adaptive law in the following:

(I) In the first stage, the parameter update law is proposed by:

$$\dot{\phi}_1 = \Gamma W_1 E_1 \quad (4.30)$$

where

$$E_1 = X_{a1}^T P_{11} B_{a11} = x_1 + \gamma x_2 \quad (4.31)$$

with $X_{a1} = [x_1^T, x_2^T]^T$, and $\Gamma > 0$, is the adaptation gain matrix.

(II) In the second stage, the additional input v is specified as follows:

$$v = \begin{cases} -[\Pi(\beta)]^2 \psi(X_{a2}) & \text{if } \Pi(\beta) \|\psi(X_{a2})\| \leq \epsilon \\ \frac{-\Pi(\beta) \psi(X_{a2})}{\|\psi(X_{a2})\|} & \text{if } \Pi(\beta) \|\psi(X_{a2})\| > \epsilon \end{cases} \quad (4.32)$$

where β is adjusted on-line by:

$$\dot{\beta} = D \omega \|\psi(X_{a2})\| \quad \beta(0) > 0 \quad (4.33)$$

with the following notation used:

- (1) $D \in R^{n \times n}$ is a positive definite matrix;
- (2) $\psi(X_{a2}) = 2X_{a2}^T P_{22} B_{a22}$;
- (3) ϵ is an auxiliary signal satisfying:

$$\dot{\epsilon} = -\delta \epsilon \quad \epsilon(0) > 0, \delta > 0 \quad (4.34)$$

The following theorem will state the conditions under which the manipulator system as described above will be asymptotically stable.

THEOREM 4.1 (Adaptive Motion Tracking)

Consider the manipulator system with flexible joints as described in (2.1),(2.2) satisfying assumptions (A1)-(A7). If the control law is chosen to be (4.16),(4.32) and the parameter update law is chosen as (4.30) and (4.33), then the system is stabilized, i.e. all signals are bounded, and e , $\dot{e}$ converge to zero asymptotically as t tends to infinity for sufficiently large γ , K_p , and Q_{22} .

Proof of Theorem 4.1:

Construct the following Lyapunov function:

$$V = X_a^T P X_a + \phi_1^T \Gamma^{-1} \phi_1 + (\rho_0/2)(\beta - \rho)^T D^{-1} (\beta - \rho) + (\rho_0/\delta) \epsilon \quad (4.35)$$

Thus,

$$\begin{aligned} \lambda_{\min}(P) \|X_a\|^2 + \lambda_{\min}(\Gamma^{-1}) \|\phi_1\|^2 + (\rho_0/2) \lambda_{\min}(D^{-1}) \|\beta - \rho\|^2 \\ + (\rho_0/\delta) \epsilon \leq V \leq \lambda_{\max}(P) \|X_a\|^2 \\ + \lambda_{\max}(\Gamma^{-1}) \|\phi_1\|^2 + (\rho_0/2) \lambda_{\max}(D^{-1}) \|\beta - \rho\|^2 + (\rho_0/\delta) \epsilon \end{aligned} \quad (4.36)$$

where P is obtained in Proposition 3.2. Then differentiation of V with respect to time t along trajectories of (4.19) yields:

$$\begin{aligned} \dot{V} \leq -[\lambda_{\min}(Q) - 2\|\dot{M}\|] \|X_a\|^2 - 2X_{a1}^T P_{11} B_{a11} W_1 \phi_1 \\ + 2\phi_1^T \Gamma^{-1} \phi_1 + 2X_{a2}^T P_{22} B_{a22} h + \rho_0(\beta - \rho)^T D^{-1} \dot{\beta} + (\rho_0/\delta) \dot{\epsilon} \end{aligned} \quad (4.37)$$

Substitution of (4.30),(4.33),(4.34) into (4.37) gives:

$$\dot{V} \leq -[\lambda_{\min}(Q) - 2\|\dot{M}\|] \|X_a\|^2 - V_b \quad (4.38)$$

where V_b define by:

$$V_b = \psi(X_{a2})^T h + \rho[\Pi(\beta) - \Pi(\rho)] \|\psi(X_{a2})\| - \rho_0 \epsilon \quad (4.39)$$

Consider the following two cases:

- (a) $\Pi(\beta) \|\psi(X_{a2})\| \leq \epsilon$:

From (4.32), the input signal v is given by:

$$v = -[\Pi(\beta)]^2 \psi(X_{a2}) / \epsilon \quad (4.40)$$

so that

$$\psi(X_{a2}) = -\epsilon v / [\Pi(\beta)]^2 \quad (4.41)$$

Taking norm on both sides, we obtain

$$\|\psi(X_{a2})\| = \epsilon \|v\| / [\Pi(\beta)]^2 \quad (4.42)$$

Substitution of (4.41) and (4.42) into (4.39) taking into account (4.27) will then lead to the following inequality.

$$V_b = -\left[\epsilon \rho_0 / [\Pi(\beta)]^2\right] \|v\|^2 \leq 0 \quad (4.43)$$

- (b) $\Pi(\beta) \|\psi(X_{a2})\| > \epsilon$:

Now, v is given by:

$$v = -\Pi(\beta) \psi(X_{a2}) / \|\psi(X_{a2})\| \quad (4.44)$$

so that

$$\|v\| = \Pi(\beta) \quad (4.45)$$

$$\psi(X_{a2}) = \left[\|\psi(X_{a2})\| / \Pi(\beta) \right] v \quad (4.46)$$

Similar substitutions into (4.39) will yield the following:

$$V_b \leq -\rho_0 \epsilon \quad (4.47)$$

Consequently, to sum up from (a)(b) and the proof of Theorem 3.1, we obtain

$$\dot{V} \leq -[\lambda_{\min}(Q) - 2\|\dot{M}\|] \|X_a\|^2 - q \|X_a\|^2 \quad (4.48)$$

which implies $X_a \in L_2 \cap L_\infty$, ϕ_1 and $\beta \in L_\infty$, and, hence, $\theta_1, \dot{\theta}_1 \in L_\infty$ and $W_1 \in L_\infty$. This further implies that ϕ_1 are bounded, $\dot{Z}_d \in L_\infty$, and, hence, Z is also bounded due to (4.4) and (4.5). Moreover, from (2.1), $\dot{\theta}_1 \in L_\infty$ so that $\dot{W}_1$ and $\dot{W}_2$ are bounded, which, in turn, implies $\dot{Z}_d \in L_\infty$, and, hence, $\dot{Z} \in L_\infty$. As a result, the stability of the system is thus ensured. In order to assure that $\dot{Z}_d$ in (4.15) is implementable when constructing τ_m in (4.16), we need $\dot{\phi}_1$ to be computable. This is actually possible since

$$\dot{\phi}_1 = \Gamma(W_1 \dot{E}_1 + \dot{W}_1 E_1) \quad (4.50)$$

where the signal

$$\dot{E}_1 = x_2 + \gamma \dot{x}_2 \quad (4.51)$$

is available from assumption (A6)

Finally, the fact that $W_1, W_{10} \in L_\infty$ implies that U and, hence, $\dot{X} \in L_\infty$ from (4.24). Consequently, the asymptotic convergence of e and $\dot{e}$ will readily follow from Barbalat's Lemma [13]. This concludes our proof.

Q.E.D.

Remark: This control is, in fact, effective locally. The stability region of the system will depend on the desired trajectory and the chosen values of K_p , K_v , and γ . Since, in real practice, the velocity will always meet some limitation, the local effectiveness will not be too restrictive.

5. Conclusion

In this paper, a two-stage control scheme for flexible joint manipulator based on a simplified dynamic model was presented. The mechanical stiffness of the elastic joint does not have to be assumed large in formulating the dynamic model of the manipulator system. When the system parameters are exactly known, the first-stage controller is synthesized by a PD type feedback mechanism similar to those in [1][2][8], whereas the second-stage controller applies the linearization scheme as in [16], in contrast to the design using a global diffeomorphic mapping. Merit of

the former one is its conceptual clarity and implementational easiness, and is particularly useful for evolving into an adaptive scheme. Therefore, in the case where all or partial system parameters are unknown prior to the controller design, the second-stage control scheme was modified based on a Lyapunov method. The result shows that the system is stable in the local sense and the output trajectory will follow the desired trajectory asymptotically in spite of the existence of parameter uncertainties. Such an adaptive control scheme is featured especially by its high performance (convergence although local) and little required information (only $\dot{\theta}_1$ will be additionally needed), which is hardly achievable by schemes, e.g., in [21-22].

6. References.

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Fig 2.1 A Single-Link Manipulator with Elastic Joint