Rainbow Phenomena in Particle Scatterings

Ju-Tang Hsiao¹ and Keh-Ning Huang^{1,2}

¹Department of Physics, National Taiwan University
Taipei, Taiwan 106, Republic of China

²Institute of Atomic and Molecular Sciences, Academia Sinica
P.O. Box 23-166 Taipei, Taiwan 106, Republic of China

Abstract

The Scattering of a particle in quantum mechanics and the scattering of electromagnetic waves have certain analogies. The quantum scattering of a particle is analyzed as the interference among components characterized by number of reflections at the spherical boundary in analogy to the rainbow phenomena of light.

The scattering of a particle in quantum mechanics and the scattering of electromagnetic waves have certain analogies. The differential cross section of the scattering of a particle with mass m and energy E by a spherical square-well potential of the form

$$V(r) = \begin{cases} 0 & , r > a \\ -V_0 & , r \leq a \end{cases}$$

is analyzed as the interference among components characterized by number of reflections at the spherical boundary in analogy to the rainbow phenomena of light. Let 1 and 2 denote, respectively, the exterior and interior regions of the sphere. The differential cross section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{1}{4k^2} \left| \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) \left\{ R_l - 1 + T_l T_2 \sum_{N=0}^{\infty} R_2^N \right\} \right|^2, \quad k = \sqrt{2mE}/\hbar$$

where R_1 and T_1 are the external reflection and transmission coefficients from region 1 to 2, while R_2 and T_2 are the internal reflection and transmission coefficients from region 2 to 1. The first term R_1 inside the curly brackets represents the direct external reflection from the potential well. The N th term in the second summation corresponds to the component that transmits into the potential well with N internal reflections and transmits outside finally.