TST-MUSIC FOR DOA-DELAY JOINT ESTIMATION

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ABSTRACT

In this paper, a Multiple Signal Classification (MUSIC)-based approach, Time-Space-Time MUSIC (TST-MUSIC), is proposed to jointly estimate the directions of arrival (DOAs) and the propagation delays of a wireless multipath channel. The MUSIC algorithms for the DOA and propagation delay estimation are referred to as the S-MUSIC and T-MUSIC algorithms, respectively. By using the space-time characteristics of the multipath channel, the approach combines the temporal filtering and spatial beamforming techniques along with one S-MUSIC and two T-MUSIC algorithms in a tree structure. As such, the incoming rays are grouped, isolated, then estimated in the DOA-delay domain, and the pairing of the estimated DOAs and delays are automatically determined. Also, the proposed approach can resolve the incoming rays from very close DOAs or with very close delays. Furthermore, the number of antennas required can be less than that of the incoming rays. The furnished simulations justify the new algorithm.

1. INTRODUCTION

The DOA-Delay estimation is a classical problem encountered in radar, sonar, and geophysical explorations. It also finds applications in source localization, accident reporting, cargo tracking, and intelligent transportation [1]. Besides, in the wireless communication systems, one can obtain a better channel estimate by jointly exploring the path DOAs and the propagation delays, and thus significantly improve the system performance [2].

Several algorithms have been addressed for the DOA-delay joint estimation. For example, some maximum likelihood (ML)-based algorithms [3, 4] were advocated recently. These algorithms, however, may not work properly when two or more rays are close in time delays and, in addition, require computationally demanding iterative process. Another approach is the subspace-based algorithms such as the JADE-MUSIC and SI-JADE algorithms [5, 6], which estimate the parameters via carrying out appropriate eigendecompositions of the covariance matrices.

In this paper, we present a low complexity, high accuracy MUSIC-based algorithm [7], the TST-MUSIC algorithm, to jointly estimate the DOAs-delays of interest from the samples received by an antenna array. The MUSIC algorithm for the DOA estimation here is referred to as the Spatial-MUSIC (S-MUSIC) algorithm. On the other hand, the Temporal-MUSIC (T-MUSIC), which uses the temporal

samples to estimate the propagation delays, is introduced as well. The TST-MUSIC algorithm combines the temporal filtering and spatial beamforming techniques in conjunction with three one-dimensional (1-D) MUSIC algorithms, i.e. one S-MUSIC and two T-MUSIC algorithms. The basic ideas are to group then isolate the incoming rays by using the space-time characters of the multipath channel. To achieve this, the T-MUSIC algorithm and the S-MUSIC are employed to estimate the group delays (or delays) and the DOAs of the incoming rays, which are required by the temporal filtering and spatial beamforming processes, respectively. Thereafter, the other T-MUSIC algorithm is applied to estimate the propagation delays.

The proposed approach possesses some distinctive features. First, compared to the ML-based algorithms [3, 4], the tree-structured TST-MUSIC algorithm can not only resolve several rays coming from very close DOAs or with very close propagation delays, but also render automatic pairing of the estimated DOAs and delays. In addition, the number of antennas required by the TST-MUSIC algorithm can be less than that of the incoming rays due to the employment of the temporal filtering process. Second, in contrast to the JADE-MUSIC algorithm [5], the proposed TST-MUSIC algorithm only needs 1-D search and the associated eigendecomposition, thus calling for substantially lower computational complexity. Third, compared to the SI-JADE algorithm [6], the robustness and accuracy of the MUSIC algorithm [7] equips the proposed approach with better estimation accuracy in a low signal to noise ratio (SNR) environment.

2. DATA MODEL

The radio channel in a wireless communication system is often characterized by a multipath propagation model. In large cells with high base station antenna platforms, the propagation environment is aptly modeled by a few dominant specular paths (rays)– typically 2 to 6. In such a case, the training signals of a TDMA system received at the antenna array, after sampling, can be expressed as follows :

$$\mathbf{X}_t^{(n)} = \mathbf{A}(\boldsymbol{\theta})\mathbf{B}^{(n)}\tilde{\mathbf{G}}(\tau)^T + \mathbf{N}, \quad (1)$$

where the superscript $(\cdot)^T$ denotes the matrix transpose operation. In (1), $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1)\mathbf{a}(\theta_2)\dots\mathbf{a}(\theta_Q)]$, where $\mathbf{a}(\theta_i)$ denotes the array response vector of the i^{th} ray, is of dimension $M \times Q$, in which M is the number of antennas and Q is the number of paths; $\mathbf{B}^{(n)} = \text{diag}(\beta_1^{(n)}, \dots, \beta_Q^{(n)})$ with $\beta_i^{(n)}$ being the complex fading amplitudes of the i^{th} ray during the n^{th} burst; $\tilde{\mathbf{G}}(\tau) = [\tilde{\mathbf{g}}(\tau_1), \dots, \tilde{\mathbf{g}}(\tau_Q)]$, where $\tilde{\mathbf{g}}(\tau_i) = \mathbf{S}_t^T \cdot \mathbf{g}(\tau_i)$ is the convolution between the training

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sequence matrix $\mathbf{S}_t^T$ and the delayed pulse shaping function $\mathbf{g}(\tau_i)$. $\mathbf{N}$ is the noise matrix and is assumed to be spatially and temporally white Gaussian with noise power σ_n^2 .

3. THE PROPOSED APPROACH

In this section, we first briefly review the S-MUSIC algorithm for the DOA estimation. Then the T-MUSIC algorithm, which uses the temporal samples to estimate the path delays, is introduced. Finally, the TST-MUSIC algorithm is described and a simple three-path scenario is used to demonstrate the proposed approach.

3.1. S-MUSIC and T-MUSIC Algorithms

The S-MUSIC and T-MUSIC algorithms use the covariance matrices of the columns and rows of $\mathbf{X}_t$ to estimate the path DOAs and delays, respectively. The eigendecompositions of the covariance matrices of the columns and rows of $\mathbf{X}_t$ are

$$\begin{aligned}\mathbf{R}^s &= \mathcal{E}\{\mathbf{X}_t^{(n)} \mathbf{X}_t^{(n)H}\} - \mathcal{E}\{\mathbf{X}_t^{(n)}\} \mathcal{E}\{\mathbf{X}_t^{(n)}\}^H \\ &= \mathbf{V}_s^s \mathbf{\Lambda}_s^s \mathbf{V}_s^{sH} + \mathbf{V}_n^s \mathbf{\Lambda}_n^s \mathbf{V}_n^{sH}\end{aligned}\quad (2)$$

and

$$\begin{aligned}\mathbf{R}^t &= \mathcal{E}\{\mathbf{X}_t^{(n)T} \mathbf{X}_t^{(n)*}\} - \mathcal{E}\{\mathbf{X}_t^{(n)T}\} \mathcal{E}\{\mathbf{X}_t^{(n)*}\}^* \\ &= \mathbf{V}_s^t \mathbf{\Lambda}_s^t \mathbf{V}_s^{tH} + \mathbf{V}_n^t \mathbf{\Lambda}_n^t \mathbf{V}_n^{tH},\end{aligned}\quad (3)$$

where the superscripts $(\cdot)^H$ and $(\cdot)^*$ represent the Hermitian and complex conjugate operations, respectively. The column vectors of $\mathbf{V}_s^s$ and $\mathbf{V}_n^s$ correspond to the Q largest eigenvalues and span the signal subspace of $\mathbf{R}^s$ and $\mathbf{R}^t$, respectively. $\mathbf{V}_n^s$ and $\mathbf{V}_n^t$, constituted by the rest of the $M - Q$ and $N_t - Q$ eigenvectors of $\mathbf{R}^s$ and $\mathbf{R}^t$, are the orthogonal complement of $\mathbf{V}_s^s$ and $\mathbf{V}_s^t$, respectively. All of $\mathbf{\Lambda}_s^s$, $\mathbf{\Lambda}_n^s$, $\mathbf{\Lambda}_s^t$, and $\mathbf{\Lambda}_n^t$ are diagonal matrices with the associated eigenvalues as their diagonal elements.

By using the orthogonality property between the signal and noise subspaces, the S-MUSIC algorithm estimates the Q DOAs by the peaks of the S-MUSIC spectrum given by

$$\mathcal{P}_{music}^s(\theta) = \frac{1}{\mathbf{a}(\theta)^H (\mathbf{I} - \mathbf{V}_s^s \mathbf{V}_s^{sH}) \mathbf{a}(\theta)} \quad (4)$$

Similarly, the T-MUSIC algorithm estimates the path delays by the peaks of the T-MUSIC spectrum as

$$\mathcal{P}_{music}^t(\tau) = \frac{1}{\tilde{\mathbf{g}}(\tau)^H (\mathbf{I} - \mathbf{V}_s^t \mathbf{V}_s^{tH}) \tilde{\mathbf{g}}(\tau)} \quad (5)$$

It is known that under a mild contaminated environment, the MUSIC spectra for the rays from closely spaced DOAs or with close time delays may only have a flat hump in the region where these rays are located, rather than several peaks corresponding to each incoming rays. In such a situation, one can only obtain a group estimate of the DOAs or delays from the smoothed spectra. However, by using these group estimates along with the spatial-temporal characteristics of the multipath channel, the rays with close DOAs or delays can be allocated into several groups and these parameters are far part in each group. The DOAs and delays can then be accurately estimated using the MUSIC algorithm.

3.2. TST-MUSIC Algorithm

As discussed above, the rationale of the TST-MUSIC is to incorporate the aforementioned S-MUSIC and T-MUSIC algorithms in a tree structure along with the beamforming and filtering techniques to group, isolate, estimate, and pair the 2-D parameters of a fading channel. Suppose that there are totally Q rays distributed in q groups and the number of rays contained in the k^{th} group is denoted by $r(k)$. The overall procedures for the TST-MUSIC can be summarized as:

TST-MUSIC Algorithm

Step 1: Grouping:

Applying the T-MUSIC algorithm to the received $\mathbf{X}_t$, we can obtain a set of group delays $\{\hat{t}_1, \dots, \hat{t}_q\}$.

Step 2: Temporal Filtering:

Form the "temporal excluding matrices", $\{\tilde{\mathbf{G}}_k \mid k = 1, \dots, q\}$, for each group by means of

$$\text{col}(\tilde{\mathbf{G}}_k) = \{\tilde{\mathbf{g}}(\hat{t}_n) \mid n = 1, \dots, q, n \neq k\} \quad (6)$$

where the $\text{col}(\cdot)$ represents the column vectors of the embraced matrix. Subsequently, the "temporal filtering matrices", $\mathbf{U}_k^t$, $k = 1, \dots, q$, are generated by

$$\mathbf{U}_k^t = \mathbf{I} - \tilde{\mathbf{G}}_k \tilde{\mathbf{G}}_k^\dagger, \quad (7)$$

where the superscript $(\cdot)^\dagger$ denotes the pseudo-inverse of the matrix. We can then obtain a set of group matrices as

$$\mathbf{X}_k = \mathbf{X}_t \cdot \mathbf{U}_k^t; \quad k = 1, \dots, q. \quad (8)$$

Step 3: DOA Estimation:

Applying the S-MUSIC algorithm to each $\mathbf{X}_k$, we can estimate the DOAs of the paths in group k as

$$\hat{\theta}_k = [\hat{\theta}_{k,1}, \dots, \hat{\theta}_{k,r(k)}] ; \quad k = 1, \dots, q. \quad (9)$$

Step 4: Spatial Beamforming:

For each group, constitute the "spatial excluding matrices", $\{\mathbf{A}_{k,m} \mid k = 1, \dots, q; m = 1, \dots, r(k)\}$, by

$$\mathbf{A}_{k,m} = \{\mathbf{a}(\hat{\theta}_{k,n}) \mid n = 1, \dots, r(k), \text{ and } n \neq m\}, \text{ if } r(k) \geq 2 \quad (10)$$

or by

$$\mathbf{A}_{k,1} = \mathbf{a}(\hat{\theta}_{k,1}), \quad \text{if } r(k) = 1. \quad (11)$$

From $\mathbf{A}_{k,m}$, the "spatial beamforming matrix", $\mathbf{U}_{k,m}^s$, can be formed by

$$\mathbf{U}_{k,m}^s = \mathbf{I} - \mathbf{A}_{k,m} \mathbf{A}_{k,m}^\dagger \quad (12)$$

The single-ray matrices can then be obtained from $\mathbf{X}_k$ by

$$\mathbf{X}_{k,m} = \mathbf{U}_{k,m}^s \cdot \mathbf{X}_k; \quad k = 1, \dots, q, \quad m = 1, \dots, r(k) \quad (13)$$

Step 5: Delay Estimation:

Apply the T-MUSIC algorithm again but with different temporal array manifolds, $\mathbf{U}_k^t \tilde{\mathbf{g}}(\tau)$, to each $\mathbf{X}_{k,m}$. If $\hat{\tau}_{k,m}$ is obtained, pair it with $\hat{\theta}_{k,m}$; otherwise, pair $\hat{t}_k$ with $\hat{\theta}_{k,m}$ for a flat T-MUSIC spectrum.

To illustrate the proposed algorithm, let us consider a three-path scenario ($Q=3$, $r(1) = 2$, and $r(2) = 1$) as shown in Fig. 1, in which path 1 and path 2 possess close time delays

($\tau_1 \approx \tau_2$), but diverse DOAs ($\theta_1 \ll \theta_2$), while path 1 and path 3 are close in the DOAs ($\theta_1 \approx \theta_3$), but with far apart delays ($\tau_1 \ll \tau_3$). By applying the TST-MUSIC algorithm, two group-delay estimates, $\hat{t}_1$ ($\tau_1 < \hat{t}_1 < \tau_2$) and $\hat{t}_2$ ($\approx \tau_3$), can be obtained in Step 1. We then use these group-delay estimates to carry out the temporal filtering process in Step 2 and obtain two group matrices $\mathbf{X}_1$ and $\mathbf{X}_2$. Note that $\mathbf{X}_1$ only contains the contents of two dominant paths, path 1 and 2, which have diverse DOAs θ_1 and θ_2 , as shown in the upper right diagram of Fig. 1; while $\mathbf{X}_2$ only contains the content of one dominant path, path 3. An accurate estimate of $\hat{\theta}_1$ and $\hat{\theta}_2$, and $\hat{\theta}_3$ can thus be obtained in Step 3 by applying the S-MUSIC to $\mathbf{X}_1$ and $\mathbf{X}_2$, respectively. Subsequently, in Step 4 the beamforming process nulls either ray 1 or ray 2 in $\mathbf{X}_1$ and results in single-ray matrices $\mathbf{X}_{11}$ and $\mathbf{X}_{12}$, as shown in the lower left diagram of Fig. 1 and Fig. 2. $\mathbf{X}_{11}$ and $\mathbf{X}_{12}$ only contain the contents of path 1 and path 2, respectively, while $\mathbf{X}_{21}$ only contains the content of path 3. Finally, applying the T-MUSIC algorithm in Step 5 to $\mathbf{X}_{11}$, $\mathbf{X}_{12}$, and $\mathbf{X}_{21}$, the delay estimates can be obtained, which also automatically pair with the DOAs estimated in Step 3. A tree structure of the whole TST-MUSIC algorithm is also given in Fig. 2 to illustrate the whole process.

Remarks:

1) Since the DOA of each ray is estimated using the group data matrices $\mathbf{X}_k$, the number of antennas required by the TST-MUSIC is less than that of paths. More specifically, in an uncorrelated propagation environment, $r(k)+1$ antennas are required by the S-MUSIC algorithm to resolve $r(k)$ rays. As such, the least number of antennas required by the TST-MUSIC algorithm is

$$M_{\min} = \max_k r(k) + 1 \leq Q + 1 \text{ provided } N_t > Q \quad (14)$$

2) The computational complexity of the TST-MUSIC algorithm includes $\mathcal{O}(M^3)$ for the eigendecomposition of $\mathbf{R}^s$ and construction of the spatial beamforming matrices, and $\mathcal{O}(N_t^3)$ for the eigendecomposition of $\mathbf{R}^t$ and construction of the temporal filtering matrices. In contrast, the JADE-MUSIC requires $\mathcal{O}((MN_t)^3)$ for the eigendecomposition of the spatial-temporal covariance matrix and a 2-D search with computations of $\mathcal{O}((MN_t)^2)$ per grid point. Since in general the length of the temporal array N_t is longer than that of the antennas M , it follows that the TST-MUSIC calls for substantially lower computational overhead.

4. EXPERIMENTAL RESULTS

In this section, we conduct some simulations to assess the TST-MUSIC algorithm. We assume that one source emits narrowband signals which are received via $Q = 4$ paths by $M = 3$ uniformly spaced linear antennas. The GMSK modulated signal used in the GSM is employed. The received $\mathbf{X}_t$ is sampled during 20 bursts. In the basic setup, we set the DOAs as $[-43^\circ, 27^\circ, -40^\circ, 30^\circ]$ and propagation delays as $[.03, .1, .86, .94]T_s$, where $T_s = 3.68\mu s$ is the symbol period of the GSM system. The oversampling factor $P = 2$. The fading amplitudes of the four paths are equal and are normalized to 0 dB with randomly selected but constant fading phases. The noise power σ_n^2 of the propagation environment is 0 dB.

Figs. 3 and 4 compare, respectively, the root mean square error (RMSE) of the DOA and the delay estimates, of the TST-MUSIC, SI-JADE, and JADE-MUSIC algorithms with SNRs ranging from 0 dB to 27 dB. For each specific

SNR, 500 Monte Carlo trials are carried out. The number of observation bursts for the TST-MUSIC and JADE-MUSIC algorithms is 20 bursts, and 1 for the SI-JADE algorithm as required. As a vivid illustration, the scattergrams of the DOA-delay joint estimates using these three algorithms at SNR=0 dB is also given in Figs. 5-7.

As we can observe from Figs. 3 and 4 that the TST-MUSIC even outperforms the JADE-MUSIC in low SNR (below 18 dB) environments. Figs. 5, 6, and 7 in particular show an illustrative picture of this fact. This is due to the fact that the JADE-MUSIC stacks each $M \times N_t$ (3×36) data burst matrix to form an $MN_t \times 1$ spatial-temporal vector. As a result, the 20 observation data bursts can only provide 20 vectors for the JADE-MUSIC, and thus results in an inaccurate estimate of the sample covariance matrix. On the other hand, the 20 bursts can offer $20 \cdot N_t = 720$ $M \times 1$ (3×1) spatial snapshot vectors for the S-MUSIC and $20 \cdot M = 60$ $N_t \times 1$ (36×1) temporal snapshots for the T-MUSIC to estimate the associated sample covariance matrices. The SI-JADE is inferior to the other two algorithms, as it only uses a single burst.

We can also note from Figs. 3 and 4 that as SNR increases, the curves of the JADE-MUSIC intersect with those of the TST-MUSIC and still go down as expected. However, the RMSE curves of the TST-MUSIC become flat under high SNR environments. This is due to the fact that the generated temporal filtering matrices $\mathbf{U}_k^t$ and the spatial beamforming matrices, $\mathbf{U}_{k,m}^s$ are based on the estimated group-delays and DOAs and can not completely null out the undesired signals. The leakage incurred in the two nulling processes, i.e. filtering and beamforming, thus degrade the estimation results.

5. CONCLUSION

This paper presents a new TST-MUSIC algorithm, which combines one S-MUSIC and two T-MUSIC algorithms along with the temporal filtering and spatial beamforming techniques to jointly estimate the DOA-delays of a multipath channel. The involved S-MUSIC and T-MUSIC algorithms use the columns and rows of the received data bursts to estimate the DOAs and propagation delays, respectively. Compared to previous algorithms, the new approach is without any constraint, and offers robust and accurate joint estimates in various scenarios with low computational overhead. Some analytic studies can be found in [8] to provide further insights into this new algorithm.

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Figure 1. The signals contained in X_t , X_1 and X_{11} .

Figure 2. The tree structure of the TST-MUSIC algorithm for three path scenario.

Figure 3. Comparison of RMSE of the delay estimates based on various algorithms.

Figure 4. Comparison of RMSE of the DOA estimates based on various algorithms

Figure 5. Scattergram of the DOA-delay joint estimates based on the SI-JADE, SNR=0 dB.

Figure 6. Scattergram of the DOA-delay joint estimates based on the JADE-MUSIC, SNR=0 dB.

Figure 7. Scattergram of the DOA-delay joint estimates based on the TST-MUSIC, SNR=0 dB.