Spectrally efficient full-response Q²PSK signals

C.-D. Chung and W.-L. Lin

Based on a novel class of orthogonal pulse pairs, a spectrally efficient full-response quadrature-quadrature phase-shift keying (Q²PSK) signal format is proposed. The proposed full-response Q²PSK signals can provide higher spectral compactness than conventional full-response Q²PSK signal, minimum-shift keying signal and quadrature phase-shift keying signal.

Introduction: Quadrature-quadrature phase-shift keying (Q²PSK) is a four-dimensional linear memoryless modulation signal which is constructed by an orthogonal multiplexing of two quadrature phase-shift keying (QPSK) component signals, both modulated on the same quadrature carrier but shaped by distinct orthogonal pulses [1–11]. Because two spectrally overlapping component QPSK signals are orthogonally multiplexed, Q²PSK provides the same error performance as QPSK over the ideal additive white Gaussian noise channel, while yielding better spectral efficiency when orthogonal pulses of finite duration are used [1, 4, 10]. In [1], Q²PSK made its debut in terms of an orthogonal pair of sinusoidal and cosinusoidal pulses that extend over one symbol time (i.e. full-response), and was shown to use the available signal space more efficiently than the conventional two-dimensional QPSK and minimum-shift keying (MSK) modulation schemes, while at a sacrifice of envelope constancy. To achieve even higher spectral efficiency, smoother pulses of infinite duration [1] or finite duration [4] were also suggested at the cost of higher implementation complexity and longer demodulation delay. Motivated by the spectral prevalence, Q²PSK has been studied extensively in various issues of practical interest, including the signal synchronisation [1, 8, 9] and the error performance characteristics over bandlimited channels [1–3], nonlinear satellite channels [4, 5], coded channels [5, 6], and differentially coherent channels [6, 11].

In this Letter, a novel class of orthogonal pulse pairs is proposed for constructing spectrally efficient full-response Q²PSK signals. Analytically, the power spectrum of the proposed Q²PSK signal with a pulse order N is shown to decay asymptotically with f^{-2N-2} . Numerical results also show that the proposed full-response Q²PSK can provide higher spectral compactness than conventional full-response Q²PSK, MSK and QPSK.

Spectrally efficient full-response Q²PSK signals: In the nominal symbol interval $|t| \leq T/2$, the proposed (unit-power) memoryless full-response Q²PSK signal is expressed as

$$s(t) = [a_I p(t) \cos(2\pi f_c t) + a_Q p(t) \sin(2\pi f_c t)] + [b_I^d(t) \cos(2\pi f_c t) + b_Q^d(t) \sin(2\pi f_c t)] \quad (1)$$

Here, (a_I, a_Q) and (b_I, b_Q) are two QPSK data phasors where all data components are independent and identically distributed random variables and take value from $\{\pm 1/\sqrt{2}\}$ with equal probability. $p(t)$ and $q(t)$ are mutually orthogonal pulse shapes defined on $|t| \leq T/2$ and, for notational simplicity, normalised to have energy two. Assuming that the carrier frequency f_c is much larger than the bandwidths of $p(t)$ and $q(t)$, $\{p(t) \cos(2\pi f_c t), q(t) \cos(2\pi f_c t), p(t) \sin(2\pi f_c t), q(t) \sin(2\pi f_c t)\}$ forms a set of basis waveforms which have unit energy and are orthogonal (i.e. orthonormal) over $|t| \leq T/2$. Within the modelling, the lowpass equivalent power spectral density (PSD) of $s(t)$ is given by $S(f) = [|P(f)|^2 + |Q(f)|^2]/4$ where $P(f)$ and $Q(f)$ denote the Fourier transforms of $p(t)$ and $q(t)$, respectively. Because $P(f)$ and $Q(f)$ contribute equally to $S(f)$, the pulse properties of $p(t)$ and $q(t)$ are both of paramount importance to the spectral property of full-response Q²PSK.

In [1], $p(t)$ and $q(t)$ are orthogonal half-period sinusoids $p(t) = 2T^{-1/2} \cos(\pi t/T)u(t)$ and $q(t) = 2T^{-1/2} \sin(\pi t/T)u(t)$ where $u(t)$ is the rectangular window with $u(t) = 1$ if $|t| \leq T/2$ and $u(t) = 0$ otherwise. This defines the conventional full-response Q²PSK [1] which has $S(f) = [\text{sinc}^2(fT + 1/2) + \text{sinc}^2(fT - 1/2)]/2$. Here, $S(f)$ decays asymptotically as f^{-2} , primarily due to the pulse property that $q(t)$ is discontinuous at $t = \pm T/2$ [12]. Such a decay rate resembles that of rectangularly pulsed QPSK, but is slower than that of MSK in [13] which has $S(f)$ decaying asymptotically as f^{-4} . Hence, the conventional Q²PSK is apt to cause interchannel interference in bandlimited applications [3].

In this Letter, a class of pulse pairs is proposed for $p(t)$ and $q(t)$ in a way that $S(f)$ decays asymptotically with a rate faster than f^{-2} . Specifically, we define

$$p(t) = 2\alpha \sum_{n=0}^N \binom{N}{n} \cos(2n\pi t/T) u(t) \quad (2)$$

$$q(t) = 2\beta \sum_{n=1}^N \binom{N}{n} \sin(2n\pi t/T) u(t) \quad (3)$$

where the integer N denotes the pulse order, $\alpha = [T \binom{2N}{N} + T]^{-1/2}$, and $\beta = [T \binom{2N}{N} - T]^{-1/2}$. It is straightforwardly shown that both pulse functions in (2) and (3) have all derivatives, up to the $(N-1)$ -th order, equal to zero at $t = \pm T/2$. As such, both $|P(f)|^2$ and $|Q(f)|^2$ decay asymptotically as f^{-2N-2} [12], and so does $S(f)$. This spectral property can be further verified as follows.

The lowpass equivalent PSD of Q²PSK using (2) and (3) is given by

$$S(f) = \frac{\sin^2(\pi f T)}{\pi^2 f^2} \left\{ \alpha^2 \left| \sum_{n=0}^N \binom{N}{n} (-1)^n \left(1 - \frac{n^2}{f^2 T^2}\right)^{-1} \right|^2 + \beta^2 \left| \frac{1}{fT} \sum_{n=1}^N n \binom{N}{n} (-1)^n \left(1 - \frac{n^2}{f^2 T^2}\right)^{-1} \right|^2 \right\} \quad (4)$$

When $|fT| > N$, we can expand $(1 - (n^2/f^2 T^2))^{-1}$ by the geometric series [14, equation 1.112.1] and express $S(f)$ by quoting [14, equation 0.154.3] as

$$S(f) = \frac{\sin^2(\pi f T)}{\pi^2 f^2} \left\{ \alpha^2 \left| \sum_{m=\{(N-1)/2\}}^{\infty} \sum_{n=1}^2 \binom{N}{n} (-1)^n \left(\frac{n}{fT}\right)^{2m} \right|^2 + \beta^2 \left| \sum_{m=\{(N-2)/2\}}^{\infty} \sum_{n=1}^N \binom{N}{n} (-1)^n \left(\frac{n}{fT}\right)^{2m+1} \right|^2 \right\}, \quad |fT| > N \quad (5)$$

where $\{\{x\}\}$ represents the smallest integer that is larger than x . It is noted from (5) that $S(f)$ decays asymptotically with $|fT|^{-2N-2}$ as $|fT|$ approaches infinity. This implies that the signal spectrum for the proposed full-response Q²PSK exhibits faster rolloff at spectral side-lobes when N is increased.

Fig. 1 Fractional out-of-band power comparison

Fig. 1 compares the spectral compactness characteristics of the proposed Q²PSK with conventional full-response Q²PSK in [1], MSK in [13], and rectangularly pulsed QPSK. The spectral compactness is characterised by the fraction of total power η that is not captured within the normalised frequency band $[-BT_b/2, BT_b/2]$, where B and T_b denote the bandwidth and the bit time, respectively. Therefore, to achieve a fixed η value, denoted by η_0 , the signal requiring a smaller

BT_b exhibits higher spectral compactness. The ideal performance of Nyquist-pulsed QPSK is also plotted to serve as a benchmark. As indicated, the spectral compactness of the proposed Q²PSK depends on the pulse order N . For example, the highest compactness can be achieved by the proposed Q²PSK with $N=1, 2, 3$ and 4 when the required η_0 falls in the decibel ranges $[0, -25.1]$, $[-25.1, -45.7]$, $[-45.7, -61.5]$, and $[-61.5, -75.9]$, respectively. Particularly, the proposed Q²PSK with $N=1$ outperforms MSK uniformly for all η_0 values and conventional Q²PSK when η_0 is smaller than -11.1 dB. Moreover, the proposed Q²PSK with $N=1$ provides the highest spectral compactness among the demonstrated full-response modulation signals when η_0 is around -20 dB (the measure of practical interest).

Conclusion: In this Letter, spectrally efficient full-response Q²PSK signals are constructed from a novel class of orthogonal pulse pairs. The proposed full-response Q²PSK signal with a pulse order N can provide the compact power spectrum which decays asymptotically with f^{-2N-2} and significantly outperforms conventional full-response Q²PSK, MSK and QPSK when the required fraction of out-of-band power is small. In particular, the proposed full-response Q²PSK with $N=1$ outperforms MSK uniformly in spectral compactness.

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References

- 1 Saha, D., and Birdsall, T.G.: 'Quadrature-quadrature phase-shift keying', *IEEE Trans. Commun.*, 1989, **37**, pp. 437–448
- 2 Feiz, S., and Soliman, S.S.: 'Maximum-likelihood receiver for four-dimensional signaling schemes corrupted by ISI', *IEEE Trans. Commun.*, 1992, **40**, pp. 265–277
- 3 Korn, I., and Wei, L.: 'Q² PSK in the satellite mobile channel with ISI and ICI', *IEEE Trans. Veh. Technol.*, 1994, **43**, pp. 69–78
- 4 Visintin, M., Biglieri, B., and Castellani, V.: 'Four-dimensional signaling for bandlimited channels', *IEEE Trans. Commun.*, February/March/April 1994, **42**, pp. 403–409
- 5 Acha, V., and Carrasco, R.A.: 'Trellis coded Q² PSK signals. Part 1: AWGN and non-linear satellite channels', *IEE Proc. Commun.*, 1994, **141**, pp. 151–158
- 6 Saha, D.: 'Channel coding with quadrature-quadrature phase-shift-keying (Q² PSK) signal', *IEEE Trans. Commun.*, 1990, **38**, pp. 409–417
- 7 El-Ghandour, O.M., and Saha, D.: 'Differential detection in quadrature-quadrature phase shift keying (Q² PSK) systems', *IEEE Trans. Commun.*, 1991, **39**, pp. 703–712
- 8 Gaudenzi, R.D., and Luise, M.: 'Synchronization of quadrature-quadrature phase-shift keying signals', *IEEE Trans. Commun.*, 1992, **40**, pp. 1532–1539
- 9 Fleisher, S., and Qu, S.: 'Quadrature frequency/phase modulation', *IEEE Trans. Commun.*, February/March/April 1995, **43**, pp. 1513–1524
- 10 Chung, C.-D.: 'Orthogonally multiplexed orthogonal amplitude modulation family', *IEEE Trans. Commun.*, 2002, **50**, pp. 415–428
- 11 Chung, C.-D.: 'Coherent and differentially coherent detections of orthogonally multiplexed orthogonal phase-modulated signals', *IEEE Trans. Commun.*, 2003, **51**, pp. 428–440
- 12 Bennett, W.R.: 'Introduction to signal transmission' (McGraw-Hill, 1970)
- 13 De Buda, R.: 'Coherent demodulation of frequency-shift keying with low deviation ratio', *IEEE Trans. Commun.*, 1972, **20**, pp. 429–435
- 14 Gradshteyn, I.S., and Ryzhik, I.M.: 'Table of integrals, series, and products' (Academic, New York, 1994, 5th edn.)