LETTER TO THE EDITOR

Scaling law for the electromagnetic form factors of the proton

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Abstract. The violation of the scaling law for the electric and magnetic form factors of the proton is examined within the cloudy bag model. We find that the suppression of the ratio of the electric and magnetic form factors is natural in the relativistic bag model. The pion cloud plays a moderate role in moving the ratio towards unity at low momentum transfer. The scaling violation increases as the momentum transfer increases because of the rapid drop of the pionic contribution. Our result is in reasonable agreement with the recent data from TJNAF.

(Some figures in this article are in colour only in the electronic version; see www.iop.org)

The description of the electromagnetic structure of the nucleon requires two independent form factors. The usual Sachs form factors fully characterize the charge and current distributions inside the nucleon. Electromagnetic probes interact not only with the valence quarks confined inside a quark core by nonlinear, gluon dynamics but also with the pion field required by chiral symmetry. A full understanding of the electromagnetic structure of the nucleon is of fundamental importance.

Historically, experimental determination of the electric (G_{Ep}) and magnetic (G_{Mp}) form factors of the proton was mainly based on the Rosenbluth separation [1] of the unpolarized differential cross section data. The results of various analyses from early experiments are summarized in a simple scaling law,

$$G_{Ep}(Q^2) = G_{Mp}(Q^2)/\mu_p = G_D(Q^2), \quad (1)$$

for momentum transfers, Q , up to several GeV. Here μ_p is the proton's magnetic moment and $G_D(Q^2)$ refers to the standard dipole form. However, from the Rosenbluth formula one sees that at large momentum transfer, the electric contribution to the cross section is kinematically suppressed relative to the magnetic contribution. Thus G_{Ep} cannot be determined as accurately as G_{Mp} from such an analysis, especially at large Q^2 . In the literature, the ratios, $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, obtained from different experiments are not consistent with each other, within the quoted errors.

With the advance of polarization technologies and the operation of high-duty electron machines, it is now possible to drastically reduce the systematic uncertainties in this ratio by direct measurement. That is, one can simultaneously measure the two components of the recoil proton polarization, P_T and P_L , using a longitudinally polarized electron beam [2]. As the transverse component behaves as $P_T \sim G_{Ep}G_{Mp}$, and the longitudinal component as $P_L \sim G_{Mp}^2$, the ratio, $\mu_p G_{Ep}/G_{Mp}$, can be determined directly from P_T/P_L .

Precise data for the nucleon electromagnetic form factors set a strong constraint for various quark models of the nucleon. It helps us to develop an understanding of the composite nature of the nucleon as well as its long range chiral structure. The rapid fall-off of G_E relative to G_M was indicated in the quark bag model [3], constituent quark models [4], dispersion analysis [5], and soliton models [6]. In our previous work, we studied the nucleon electromagnetic form factors in an improved cloudy bag model (CBM) [7], where the centre-of-mass motion correction and relativistic effects were treated explicitly. As a result, the region of validity of the calculation of the electric and magnetic form factors in the model was extended to larger momentum transfer than had previously been possible. Here we focus on the ratio, $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, and examine the mechanism for the violation of the scaling law, equation (1).

Let us start with the MIT bag model [8]. Under the static-cavity approximation, the bag surface is spherical and all valence quarks are in the lowest eigenmode. The electric and magnetic form factors for the proton can be written as

$$G_{Ep}(Q^2) = \int_0^R 4\pi r^2 dr j_0(Qr) [g^2(r) + f^2(r)], \quad (2)$$

$$G_{Mp}(Q^2) = 2m_N \int_0^R 4\pi r^2 dr \frac{j_1(Qr)}{Q} [2g(r)f(r)], \quad (3)$$

where R is the bag radius, $Q^2 = -q^2 = \vec{q}^2$ with $\vec{q}$ the three-momentum of the photon in the Breit frame, and $j_l(x)$ refers to the spherical Bessel function. For a massless quark, the quark wavefunctions are given as $g(r) = N_S j_0(\omega_S r/R)$ and $f(r) = N_S j_1(\omega_S r/R)$, with $\omega_S = 2.04$ and $N_S^2 = \omega_S/8\pi R^3 j_0^2(\omega_S)(\omega_S - 1)$.

At small Q^2 , the ratio $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$ is determined by the electromagnetic root-mean-squared (rms) radius, which is defined as follows: $G_{E,M}(Q^2) \sim 1 - Q^2 \langle r^2 \rangle_{E,M}/6$, as $Q \rightarrow 0$. A direct evaluation gives

$$\langle r^2 \rangle_{Ep} = 4\pi N_S^2 \left(\frac{R}{\omega_S} \right)^5 \frac{\omega_S^2(2\omega_S^3 - 2\omega_S^2 + 4\omega_S - 3)}{3(2\omega_S^2 - 2\omega_S + 1)}, \quad (4)$$

$$\langle r^2 \rangle_{Mp} = \frac{8\pi N_S^2}{5\mu_p} \left(\frac{R}{\omega_S} \right)^6 \frac{\omega_S^2(8\omega_S^3 + 10\omega_S^2 - 20\omega_S + 15)}{24(2\omega_S^2 - 2\omega_S + 1)}. \quad (5)$$

Note that $\mu_p = R(4\omega_S - 3)/12\omega_S(\omega_S - 1)$, thus the ratio $\langle r^2 \rangle_{Ep}^{1/2}/\langle r^2 \rangle_{Mp}^{1/2}$ is independent of R and takes the value 1.17 for $\omega_S = 2.04$ (the case of massless quarks). This means that the charge rms radius is considerably larger than the magnetic rms radius in the naive bag model. In other words, $G_{Ep}(Q^2)$ decreases faster than $G_{Mp}(Q^2)$ at small momentum transfers, so the suppression of $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, as Q^2 increases, is a natural consequence of the MIT bag model.

The results of our numerical calculations are shown in figure 1, where $R = 0.8$ fm is used. The long-dashed line is the ratio in the static MIT bag model. The two lowest curves are, respectively, the electric and magnetic form factors of the proton within the MIT bag model. The experimental data are from the recent measurement at TJNAF [2]. Note that the naive MIT bag model for the nucleon is only sensible in the region of very small momentum transfers. The ratio in a static-bag calculation deviates from the data very quickly as Q^2 increases. The singularity in the long-dashed curve comes from a node in $G_{Mp}(Q^2)$, which is related to the sharp bag surface, and will be extremely sensitive to small admixtures in the nucleon ground state wavefunction. Such admixtures may be induced by residual one-gluon exchange or meson exchange interactions [9]. More sophisticated models, such as the colour dielectric and soliton bag models [10], would also help to remove the zero (or at least move it to larger Q^2), thus removing the singularity in the ratio.

Figure 1. The ratio, $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, calculated from the quark core of the bag model with $R = 0.8$ fm. The solid curve shows the result after the centre-of-mass motion correction and a semi-classical relativistic correction. The data are from the recent measurement at TJNAF [2].

As the bag model is inherently an independent particle model, the spurious centre-of-mass motion must be removed in a realistic calculation. We have incorporated this correction by constructing momentum eigenstates using the Peierls–Thouless projection method [11]. Such wavefunctions are consistent with Galilean translational invariance. In addition, since the static solution of the quark wavefunction is spherical, the calculated form factors are trustworthy only at very small momentum transfer. To account for relativistic effects we use the prescription proposed by Licht and Pagnamenta [12]. The technical details for implementing such corrections can be found in [7]. The resulting ratio for the quark core of the bag model, after including the centre-of-mass motion corrections and relativistic effects, is plotted as the solid curve in figure 1. The prominent zeros in the form factors are forced out much further and do not occur in the momentum region we are studying. Compared with the recent data, the improvement over the naive MIT bag model is significant, in particular, away from the region of small momentum transfers.

In the CBM the physical nucleon is a superposition of a quark core and a quark core plus a meson cloud [13]. We keep only the dominant pion terms and treat them in the one-loop approximation. Inside the pion loop, the intermediate baryon is either a $N(939)$ or a $\Delta(1232)$. Thus the electromagnetic form factors receive contributions from the pion current in addition to the direct photon coupling to the confined quarks in the core. For more details and the explicit expressions for the form factors, we refer the reader to equations (36)–(40) and (A.5)–(A.6) in [7].

In our calculations the πNN coupling constant is taken to be $f_{\pi NN}^2 = 0.0791$. Other coupling constants are fixed by $SU(6)$ symmetry. The resulting ratios in the full calculations are presented in figure 2. Here the solid and long-dashed curves are respectively for massless

Figure 2. The ratio, $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, from full calculations in the improved CBM. Results with different bag radius and quark masses are also presented. The data are from the recent measurement at TJNAF [2].

Table 1. The static properties of the proton in the improved CBM. The probability of the quark core, z_2 , is 0.54, 0.62 and 0.74 for $R = 0.7, 0.8$ and 1.0 fm, respectively. The last three columns do not include the pionic contributions. From the experiments, $\mu_p = 2.79 \mu_B$, $\langle r^2 \rangle_{Ep}^{1/2} = 0.862 \pm 0.012$ fm [14] and $\langle r^2 \rangle_{Mp}^{1/2} \simeq 0.81$ fm in the usual dipole fit.

R (fm)	$\mu_p(\mu_B)$	$\langle r^2 \rangle_{Ep}^{1/2}$ (fm)	$\langle r^2 \rangle_{Mp}^{1/2}$ (fm)	$\mu_p^{(0)}(\mu_B)$	$\langle r^2 \rangle_{Ep}^{1/2(0)}$ (fm)	$\langle r^2 \rangle_{Mp}^{1/2(0)}$ (fm)
0.7	2.44	0.682	0.678	1.24	0.550	0.496
0.8	2.30	0.723	0.716	1.41	0.612	0.548
1.0	2.27	0.818	0.785	1.77	0.739	0.657

and 10 MeV quarks in the nucleon bag, using a bag radius of $R = 0.8$ fm. Clearly the ratio is insensitive to the quark mass variations. The addition of the pion cloud for $R = 0.8$ fm increases the ratio by roughly 20% towards the recent data. In the same figure, the dot-dashed and dashed curves have similar meanings but with $R = 0.7$ fm. This is about the smallest bag radius which allows a perturbative treatment of the pion cloud. For completeness, the static properties of the proton calculated in the CBM are listed in table 1. The pion cloud improves the proton charge and magnetic radii considerably, as well as the magnetic moment. In particular, the ratio of the charge and magnetic radii is changed from roughly 1.1 to 1.01, with the latter being much closer to the experimental value [5, 14].

To further reveal the pionic effects, we show the separated contributions from individual diagrams with one pion loop in figure 3. The labels b and c refer to the processes for the photon interacting directly with the quark and the pion, respectively. The contribution from the photon–pion coupling (the dashed curve) drops rapidly as Q^2 increases, while the contribution from the

Figure 3. The individual contributions to the electromagnetic form factors with $R = 0.8$ fm. The labels b and c refer to one-pion loop diagrams, where b denotes the photon–quark coupling and c the photon–pion coupling.

photon–quark coupling decreases at a more moderate pace. Since $G_{Ep}^{(\pi)}/G_{Ep}^{(0)} > G_{Mp}^{(\pi)}/G_{Mp}^{(0)}$, where the superscripts π and 0 refer to the pionic (b plus c) and quark core contributions, respectively, we find $\mu_p G_{Ep}/G_{Mp} > \mu_p^{(0)} G_{Ep}^{(0)}/G_{Mp}^{(0)}$.

One feature of our calculation is that the decline of the ratio of the Sachs form factors slows down as $Q^2 > 2 \text{ GeV}^2$. The corresponding ratio of the Pauli and Dirac form factors $Q^2 F_{2p}/F_{1p}$ shows a rapid rise at small Q^2 (below 2 GeV^2) and seemingly becomes flat at large Q^2 . This is consistent with the prediction of pQCD, which yields $F_{1p} \sim 1/Q^4$ and $F_{2p} \sim 1/Q^6$, but is not obvious in other nucleon models [4–6].

In summary, we have studied the ratio, $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, in an improved quark model. We show that the sharp decline of the ratio as Q^2 increases is unavoidable in a quark bag model due to its relativistic nature, so the empirical scaling law for the nucleon electromagnetic form factors is naturally violated. The pion cloud, which restores chiral symmetry in our model, eases the scaling violation. The deviations of the ratio, $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, from unity can be understood semi-quantitatively within the CBM.

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