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OPTIMAL BALANCE BETWEEN LAND DEVELOPMENT AND GROUNDWATER CONSERVATION IN AN UNCERTAIN COASTAL ENVIRONMENT

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An optimization problem for balancing land carrying capacity and groundwater consumption in Central Taiwan was solved using a problem-oriented grey linear programming (POGLP) algorithm. Satisfactory grey solutions were obtained through formulating and solving the proposed POGLP models. These models provide useful information to decision makers regarding the possible ranges of final alternatives when system parameters are highly uncertain. Results indicate that all transferable lands currently used for agriculture and fish cultivation should be converted into other usage. This would conserve the groundwater resources, thereby preventing further land subsidence in that area. Grey sensitivity analyses performed well in the decision-making process when a set of interval regions instead of trend curves were systematically generated without increasing the complexity of the mathematical models.

Keywords: Problem-oriented grey linear programming; Groundwater conservation; Land subsidence; Optimization; Water resources planning; Land use

INTRODUCTION

Groundwater conservation is a critical issue in many countries where water deficiency problems are apparent. But land use planning is always correlated with groundwater conservation since land use development schemes demand consumption of water quantity and alter the quality. In both Central and Southern Taiwan, fish cultivation is a major production activity in most coastal regions. Groundwater is commonly used since surface water supplies are insufficient to meet the demands of fish cultivation. Consequently, local areas have been suffering from land subsidence due to long-term changes in groundwater. Development plans integrating land and water resources, which address issues of economic development and land subsidence prevention, are necessary.

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The optimal balance between land use programs and water resource redistribution has been studied by many researchers in recent years. Sneha [1] considered water resource management used for the agriculture of Israel. Cooperation between farmers, industries, and researchers was promoted to achieve high efficiency of water use. Bevers and Troendle [2] developed a spatial optimization model to limit the cumulative effects caused by storm-flows in forest areas in the US. Chang *et al.* [3] used multiobjective programming techniques to find optimal management strategies for environmental and land resources in the Tweng-Wen Reservoir watershed. Many studies have also been done concerning land subsidence and groundwater management. Sato and Hoang [4] optimized groundwater pumping-rate to avoid land subsidence. Cleveland and Chuang [5] reported optimal aquifer management for controlling land subsidence while pursuing multiple goals of preserving the water supply, subsidence control, and energy development. However, the search for an integrated optimal management strategy to balance land uses and groundwater resource consumption subject to land subsidence prevention is still absent in the literature.

Uncertainties embedded in these systems analysis are critical since both parameter values and the decision-making process are not apparent. Traditionally, these uncertainties have been dealt with using probability theory and fuzzy mathematics [6]. However, these traditional approaches may not be suitable for dealing with uncertainties when different patterns are mixed in related problems. Grey programming is an optimization framework where uncertainties in system parameters are presented as grey numbers or intervals [7] that can transmit a variety of uncertain information relative to system parameters into the resulting planning scheme. Therefore, it is a potentially useful technique for identifying optimum allocations for land and water resources to achieve the sustainability goals.

In this study, grey linear programming (GLP) algorithms were applied to solving the optimization problem regarding land use patterns and groundwater resource planning for four coastal towns in Yun-Lin County, Central Taiwan. Grey solutions were obtained by solving a modified GLP model. The modified GLP model was proposed in such a way that it allows more applicable grey solutions to appear in the solution procedure. Results indicate that this problem-oriented GLP algorithm is applicable for achieving a set of desirable grey solutions. These solutions can offer decision makers sufficient information concerning possible ranges of management policy while taking into account the uncertainties embedded in the system.

GREY LINEAR PROGRAMMING ALGORITHMS

Background

Grey Programming (GP) is an optimization framework incorporating concepts of grey system theory [8, 9]. Grey system theory is an alternative uncertainty modeling approach to those using probability theory and fuzzy set theory. Uncertain parameters in a system are presented as "grey numbers", *i.e.*, intervals with known lower and upper bounds. In contrast uncertain elements are modeled as random variables with probability distributions in probability theory and fuzzy numbers with membership information in fuzzy set theory.

A major advantage of grey programming is that variations in system performance and decision variable values can be understood by solving relatively simple optimization models.

Traditionally, sensitivity analysis techniques were used to derive changes in system performance with respect to system parameters. A number of analyses are therefore needed in as much as many analyses are required for each parameter. By using grey programming, similar information can be obtained faster and simpler [7].

The pioneering work in developing GP algorithms was done by Huang *et al.* [7, 10–13] who proposed grey linear programming (GLP), grey integer programming (GIP), grey quadratic programming (GQP), and grey dynamic programming (GDP) models. These are further modified by incorporating fuzzy set theory [14] and were applied to municipal waste management problems in Canada. Applications of these GP algorithms and their modified versions were then presented. Chang *et al.* derived grey fuzzy multiobjective programming models for optimal planning of a reservoir watershed in Southern Taiwan [15, 16]. Chang and Wang also applied GP in nonlinear forms in coastal wastewater treatment and ocean disposal systems [17]. Combinatory applications of grey programming and other system analysis tools have been also developed in recent years. Chang and Yeh analyzed the important property – “stability” in a set of grey compromise programming models for a watershed management case study [18]. Chang and Tseng also applied similar algorithms to an optimal evaluation of expansion alternatives for an existing air quality monitoring network in an urban area in Taiwan [19]. More recently, a grey input-output analysis methodology was developed and used for environmental cost allocation [20]. In the area of water resources management, Yeh proposed GLP, GQP, and GDP models for reservoir management problems [21]. Traditional GLP models were modified to avoid potential mathematical and operational problems.

Traditional and Modified Grey Linear Programming Models

A grey linear programming (GLP) model can be summarized using the following generalized form [7]:

$$\text{Max } f^\pm = \mathbf{C}^\pm \mathbf{X}^\pm \quad (1)$$

$$\text{s.t. } \mathbf{A}^\pm \mathbf{X}^\pm \leq \mathbf{B}^\pm \quad (2)$$

$$\mathbf{X}^\pm \geq 0 \quad (3)$$

where $\mathbf{C}^\pm = \{c_j^\pm, j = 1, 2, \dots, n\}$, $\mathbf{X}^\pm = \{x_j^\pm, j = 1, 2, \dots, n\}$, $\mathbf{A}^\pm = \{a_{ij}^\pm, i = 1, 2, \dots, m; j = 1, 2, \dots, n\}$, $\mathbf{B}^\pm = \{b_i^\pm, i = 1, 2, \dots, m\}$.

Traditionally, grey solutions can be obtained through solving two submodels formulated on a set of rules developed by Huang *et al.* [7, 10–14]. Since difficulties may arise when any coefficient of a decision variable in the objective function, *i.e.*, $c_j^\pm$, is zero and the possibility of generating impractical solutions exist, these traditional GLP algorithms were modified [21]. Details of these formulations can be found in the literature and thus are not explained herein. The basic definitions relating to grey programming as used in model formulation are introduced in the appendix.

Problem-Oriented Grey Linear Programming Models

Grey Programming algorithms are intended to find some sets of grey solutions in the solution space. There can exist many stable sets of grey solutions and therefore different ones can result from using different GP models and assumptions in the processes of greylization. Although modified GLP models can apply to general LP models, further modifications can be made in order to obtain more practical grey solutions, as seen in problem-oriented GLP models. The principle for modification is simple: there must be stability within grey solutions. In other words the system performance and decision variable solutions must form stable sets of grey solutions, *i.e.*, the system performance must be within its grey interval

if all decision variables are within their corresponding grey intervals which result from the modified GLP submodels.

In optimization problem for resource allocation, as in the case of the land use and water resource planning for this research, the total amount of a specified resource may be constant. The traditional or modified GLP algorithms introduced in the preceeding may lead to mathematically sound grey solutions that are actually fixed numbers. Under such circumstances, these GLP formulations need to be revised so that interval instead of fixed solutions can be obtained. These problem-oriented revisions should be made based on a thorough understanding of the real system and the optimization problem to be solved. Generally speaking, the relationships between the decision variables and the system parameters, especially the RHS coefficients, need to be examined carefully to ensure that resulting solutions are applicable to the real-world systems. Figure 1 is the flowchart of the procedure for formulation and modification of POGL problems.

In this research, the modified GLP models discussed in preceeding sections, were used to derive grey solutions because some c_j 's are zero in the objective function. Based on the

FIGURE 1 The flowchart for decision making using problem-oriented GLP.

specific characteristics of the optimization problem of land use and water resource planning, the GLP models are further modified so that more reasonable grey solutions can be obtained.

CASE STUDY

Site Description

Taiwan is an island in the rim of the west Pacific Ocean, approximately two hundred kilometers from the southeast coastal area of Mainland China. Although there is abundant rainfall in this island, uneven distributions over temporal and spatial scales and high population density make the water resource management important. The annual amount of useable fresh surface water per capita is only 856 cubic meters. Based on the criteria set up by the International Population Action Organization, Taiwan is categorized with countries lacking water. Under these circumstances, further utilization of groundwater to support socio-economic activities is inevitable.

In Taiwan, the pumping of groundwater is normally carried out in a very short period of time in the coastal regions such that land subsidence has continuously occurred. From the 1940's to the 1980's, the most serious land subsidence problem occurred in the Taipei Metropolitan Area. Fortunately, the land stopped subsiding after the governmental intervention in the late 1980's. Meanwhile, the Central and Southern areas along the west coast became the emergency areas of land subsidence. For instance, one of the astonishing records of cumulative land subsidence is up to 2.88 meters in Chiatung Town, Pingtung County. The study area in this paper covers four towns – Mailao, Taishi, Shihu, and Kohu in Yun-Lin County (see Fig. 2). Between 1985 and 1994, approximately 300 km² of land had subsided because of the overuse of groundwater in agriculture and fish cultivation. The maximum subsidence was 1.78 meters in Taishi Town.

FIGURE 2 The geographical location of Taiwan, Yun-Lin County, and the four coastal towns.

The reasons for abuse of the groundwater resources that has resulted in severe land subsidence are complex. The production potential is lower in the coastal areas because agricultural land has been salinized due to seawater intrusion. On the other hand, sufficient fresh surface water is not available to support large scale agricultural activities. Fish cultivation has consequently become one of the major economic activities. Groundwater is used to support fish cultivation and other industries because of its lower cost (*e.g.* it is free currently) and relative stability in both quantity and quality. A government agency intends to implement management plans for fish cultivation, including land use program and groundwater resource distribution.

Identification of the Problem

The Agriculture Council and the Ministry of Economic Affairs of the R.O.C. are now taking action to implement management plans to first decelerate and then stop land subsidence in this area. One of the major strategies is to control effectively the groundwater resources and land use planning as well as to promote the availability of surface water supply.

The goal of this study is to identify the land use management strategy in support of the optimal balance between land use and groundwater conservation. Land can be used for many purposes, including industry, commerce, agriculture, fish cultivation, residence, recreation and resources conservation. In general, different planning projects are being considered and the upper and lower limits for each type of land use are being specified in normal policy decision-making. To rectify the current condition of land subsidence, some land currently used for agriculture and fish cultivation must be switched to other uses.

The objective of this optimization analysis is to maximize the total benefit resulting from land uses minus consumption costs from various water uses. Control over groundwater resources is implicitly included in the constraints. Decision variables include the areas for different land uses, as well as the amounts of water supplied by the various sources. The decision variables, objective function, and constraint set in the programming model are described as follows:

1 Decision Variables

(1) Land uses:

X_i : the total area of land use i (ha), $i = 1, 2, \dots, 8$, symbolizing land for agriculture, fish cultivation, industries, commerce, recreation, conservation, residence, and other uses, respectively.

X_{1j} : the area of land switched to land use j from agricultural use. (ha)

X_{2j} : the area of land switched to land use j from fish cultivation use. (ha)

X_{8j} : the area of land switched to land use j from other uses. (ha)

(2) Water supplies

Q_{si} : the surface water consumption of land use i (10^4 m³/year), $i = 1, 2, \dots, 8$.

Q_{gi} : the groundwater consumption of land use i (10^4 m³/year), $i = 1, 2, \dots, 8$.

Q_{ai} : the water consumption supplied by alternative sources of land use i (10^4 m³/year), $i = 1, 2, \dots, 8$.

2 Objective Function: Maximization of Total Net Benefit

$$\text{Max } Z = \sum_{i=1}^8 B_i X_i - \sum_{i=1}^8 C_s Q_{si} - \sum_{i=1}^8 C_g Q_{gi} - \sum_{i=1}^8 C_a Q_{ai} \quad (4)$$

where

B_i : unit value for land use i (10^4 dollars/ha/year), $i = 1, 2, \dots, 8$.

C_s : unit cost of surface water (NT dollars/ m^3)

C_g : unit cost of groundwater (NT dollars/ m^3)

C_a : unit cost of water supplied by alternative sources (NT dollars/ m^3)

3 Constraints

1. Water demand constraints: water consumption should not exceed total available water supply for each land use.

$$a_i X_i \leq Q_{si} + Q_{gi} + Q_{ai} \quad \forall i \quad (5)$$

where a_i is the unit water demand for land use i (m^3 /ha/year)

2. Surface water supply constraint: total surface water consumption should not exceed the capacity of available surface water supply.

$$\sum_{i=1}^8 Q_{si} \leq q_s \quad \forall i \quad (6)$$

where q_s is the total surface water supply capacity (m^3 /year)

3. Groundwater supply constraint: total groundwater consumption should not exceed the total amount of available groundwater supply (the safety yield of groundwater)

$$\sum_{i=1}^8 Q_{gi} \leq q_g \quad \forall i \quad (7)$$

where q_g is the total groundwater supply (m^3 /year)

4. Available land area constraints: sum of all land uses equals the total land area

$$\sum_{i=1}^8 X_i = x_0 \quad (8)$$

where x_0 is the total land area (ha)

5. Land area balance constraints:

$$X_i = x_{0i} - \sum_{j=3}^7 X_{ij} \quad i = 1, 2, 8 \quad (9)$$

$$X_i = x_{0i} + X_{1i} + X_{2i} + X_{8i} \quad i = 3, 4, 5, 6, 7 \quad (10)$$

where x_{0i} is the current area of land use i

6. Upper and lower limits of land uses:

$$lx_i \leq X_i \leq ux_i \quad \forall i \quad (11)$$

where lx_i and ux_i are the lower limit and upper limit of the area of land use i , respectively

7. Non-negative constraints:

$$\text{all decision variables} \geq 0 \quad (12)$$

System Parameters and Sources of Uncertainty

All system parameters need to be calculated or estimated based on data collected from various sources. In this section, the approaches for deciding the values of the parameters will be described.

1. x_{0i} , the current area of each land use current land use: statistics are listed in Table I. They indicate that most lands are currently used for agriculture and fish cultivation.

2. a_i , the unit water demand for each land use: The average values of unit water demand for the counties in Central Taiwan are used to calculate a_1 and those for all counties in Taiwan are used for a_2 and a_3 . a_4 is calculated using the formula: per capita residential water consumption in Taiwan in 1994 $\times$ the population of the area $\times$ population density of the area. a_5 is estimated similarly to a_4 , assuming that the population density of a commercial area is 50% that of a residential area. a_6 is assumed to be the same as a_5 because data are not available and only very small amounts of land are used for recreation in this area. a_7 is assumed to be zero because land for conservation in this area covers only coastal wind-shielding forests which consume negligible amounts of water. a_8 is assumed to be the average of a_1 , a_2 , a_5 , a_6 , and a_7 . a_3 and a_4 are not taken into account because industrial and residential areas can be easily identified and hence should not be referred to when estimating a_8 which corresponds to "other uses".

3. B_i , the unit monetary value for land use: i the unit monetary value for each land use is estimated by combining the land's market price and the production value according to the land use. Most statistics are available, except for the production value of recreational land use. This is estimated by referring to the financial analysis of a recreational project in the vicinity, (the White Water Lake Recreation Project).

4. Unit water costs: C_s is assumed to equal the unit price of potable water in Taiwan, which is NT\$ 9.0/m³. Groundwater in Taiwan is currently free, but charges for use are inevitable in the future. Furthermore, its unit price should be larger than that of potable water in the coastal areas suffering from land subsidence; hence C_g is assumed to be twice the cost of surface water, C_s . Costs for developing alternative water supply sources include fixed and variant costs. In order to encourage the development and use of alternative water sources, the water's unit price should be less than that for potable water. In this model, for simplicity, C_a is assumed to be 50% C_s .

5. Water supply capacities: Based on the current plan for surface water development in this area, more surface water will be supplied by Chi-Chi and Jei-Fong Reservoirs in the year of 2011 and the surface water supply capacity q_s is estimated at 495.6 million tons at that time. However, since water deficiency in the coming years is expected, more surface water development projects are currently planned and are being implemented.

TABLE I The Current Area and Corresponding Upper and Lower Limits of each Land Use.

i	Land use	Lower limit (ha)	Current area (ha)	Upper limit (ha)
1	Agriculture	7455.8	14922.5	14911.5
2	Fish cultivation	2610.6	6526.4	6526.4
3	Industries	2500.0	51.5	28990.5
4	Commerce	14.6	14.6	146.0
5	Recreation	123.1	123.1	29062.1
6	Residence	164.8	164.8	494.4
7	Conservation	578.3	578.3	29517.3
8	Others	0	7501.1	7501.1
	Total		29871.3	

Hence, a q_s larger than 495.6 million tons in 2011 is possible. The safety yield of groundwater q_g in this area in 2011 has been estimated at 49.7 million tons by another project of the Water Resources Bureau in Taiwan. No capacity is specified for alternative water sources in this study.

6. The lower and upper limits of each land use: For industrial, residential, recreational, commercial, and conservation land uses, the lower limits are set according to their respective current areas or changes in light of future development. The lower limit for agricultural land use is set as either 0 or 50% of the current area. The lower limit for fish cultivation land use is set as either 0 or 40% of the current use based on fishing policy. The lower limit for "other" land uses is zero. The upper limits for agricultural, fish cultivation, and "other" land uses are set according to their respective current areas. The upper limits for industrial, recreational, and conservation land uses are the summation of the total area of current agricultural, fish cultivation, and "other" lands and their respective current areas. The upper limits for commercial and residential uses are set as 10 times and 3 times their respective current areas. All information is provided in Table II.

Formulation of the GLP Model

While examining the procedures for determining or estimating the parameter values stated above it is obvious that uncertainties in system parameters cannot be ignored. For example, many a_i 's are estimated by using the average values calculated in a larger area, *e.g.*, the whole of Taiwan or counties in Central Taiwan. Consequently, certain parameter values cannot be easily determined for each a_i . Moreover, the unit value for each land use B_i is estimated by combining the land's market value and the production value according to the land use. Uncertainties may exist in both the land's market value and the production value. The unit water costs are also uncertain because C_s may change in the future and the assumptions regarding C_g and C_a might not be correct. For these uncertainties, it is not easy or even not appropriate to describe them by probability distributions or membership functions of fuzzy sets. Modeling the parameters as intervals is a more reasonable method. When the system parameters can vary within their respective ranges or grey

TABLE II System Parameters of the Case Study.

<i>i</i>		Unit water demand $a_i^\pm$ ($m^3/ha/year$)			Unit value $B_i^\pm$ (10^3 NT\$/ $ha/year$)		
		a_i^-	a_i^M	a_i^+	B_i^-	B_i^M	B_i^+
1	Agriculture	19772	20813	21854	27900	27985	28070
2	Fish cultivation	56268	59229	62190	44800	44847	44890
3	Industries	93864	98804	103744	337800	337863	337920
4	Commerce	7893	8308	8723	537200	537214	537220
5	Recreation	7893	8308	8723	98600	98609	98618
6	Residence	15785	16616	17447	190500	190575	190650
7	Conservation	0	0	0	15100	15125	15150
8	Others	19943	20993	22043	75400	75428	75450
*		Unit water cost $C_*^\pm$ (NT\$/ m^3)			Water supply capacity $q_*^\pm$ ($10^6 m^3/year$)		
		C_*^-	C_*^M	C_*^+	q_*^-	q_*^M	q_*^+
<i>s</i>	Surface water	8.5	9	9.5	690	693.84	698
<i>g</i>	Groundwater	17	18	19	49	49.72	50.5
<i>a</i>	Alternative	4.25	4.5	4.75			

intervals, the corresponding solutions for the objective function value and the decision variables will also vary within ranges. Thus, the optimization model becomes a grey linear programming (GLP) as follows.

$$\text{Max } Z^\pm = \sum_{i=1}^8 B_i^\pm X_i^\pm - \sum_{i=1}^8 C_s^\pm Q_{si}^\pm - \sum_{i=1}^8 C_g^\pm Q_{gi}^\pm - \sum_{i=1}^8 C_a^\pm Q_{ai}^\pm \quad (13)$$

s.t.

$$a_i^\pm X_i^\pm \leq Q_{si}^\pm + Q_{gi}^\pm + Q_{ai}^\pm \quad \forall i \quad (14)$$

$$\sum_{i=1}^8 Q_{si}^\pm \leq q_s^\pm \quad \forall i \quad (15)$$

$$\sum_{i=1}^8 Q_{gi}^\pm \leq q_g^\pm \quad \forall i \quad (16)$$

$$\sum_{i=1}^8 X_i^\pm = x_0 \quad (17)$$

$$X_i^\pm = x_{0i} - \sum_{j=3}^7 X_{ij}^\pm \quad i = 1, 2, 8 \quad (18)$$

$$X_i^\pm = x_{0i} + X_{1i}^\pm + X_{2i}^\pm + X_{8i}^\pm \quad i = 3, 4, 5, 6, 7 \quad (19)$$

$$lx_i \leq X_i^\pm \leq ux_i \quad \forall i \quad (20)$$

where the sign “ $\pm$ ” indicates that the corresponding system parameter or decision variable is grey. x_0 and x_{0i} ’s are constants instead of grey numbers in this example, assuming measurement errors are not taken into consideration. lx_i , and ux_i are also assumed to be constant. The grey system parameters are estimated and listed in Table I.

In this GLP model some decision variables, $X_{ij}^\pm$ ’s, are not shown in the objective function. Therefore difficulties will arise if traditional GLP algorithms are used to solve the problem. Under such circumstances, stable grey solutions can be obtained by using modified GLP algorithms. In the following, this problem will be formulated and solved by using modified GLP models. Moreover, to obtain more practical grey solutions, the formulation will be further modified and leads to a problem-oriented GLP model.

Formulation of Modified GLP

Formulation of a model using modified GLP algorithms is not unique. Assumptions regarding the bounds of system parameters and decision variables in the submodels need to be made. The rule is that these assumptions must be reasonable. In other words, only a mathematically and physically reasonable formulation with a good interpretation of the solutions can make GLP algorithms applicable [21].

The following is one set of modified GLP formulations based on specified assumptions with respect to the grey relationships between system parameters and decision variables:

*Model 1**Submodel 1:*

$$\text{Max } Z^+ = \sum_{i=1}^8 B_i^+ X_i^+ - \sum_{i=1}^8 C_s^- Q_{si}^- - \sum_{i=1}^8 C_g^- Q_{gi}^- - \sum_{i=1}^8 C_a^- Q_{ai}^- \quad (21)$$

s.t.

$$a_i^- X_i^+ - Q_{si}^- - Q_{gi}^- - Q_{ai}^- \leq 0 \quad \forall i \quad (22)$$

$$\sum_{i=1}^8 Q_{si}^- \leq q_s^- \quad \forall i \quad (23)$$

$$\sum_{i=1}^8 Q_{gi}^- \leq q_g^- \quad \forall i \quad (24)$$

$$\sum_{i=1}^8 X_i^+ = x_0 \quad (25)$$

$$X_i^+ + \sum_{j=3}^7 X_{ij} = x_{0i} \quad i = 1, 2, 8 \quad (26)$$

$$X_i^+ - X_{1i} - X_{2i} - X_{8i} = x_{0i} \quad i = 3, 4, 5, 6, 7 \quad (27)$$

$$lx_i \leq X_i^+ \leq ux_i \quad \forall i \quad (28)$$

Z_{opt}^+ , $(X_i^+)_{\text{opt}}$, $(Q_{si}^-)_{\text{opt}}$, $(Q_{gi}^-)_{\text{opt}}$, and $(Q_{ai}^-)_{\text{opt}}$ for all i can be obtained from this submodel.

Submodel 2:

$$\text{Max } Z^- = \sum_{i=1}^8 B_i^- X_i^- - \sum_{i=1}^8 C_s^+ Q_{si}^+ - \sum_{i=1}^8 C_g^+ Q_{gi}^+ - \sum_{i=1}^8 C_a^+ Q_{ai}^+ \quad (29)$$

s.t.

$$a_i^+ X_i^- - Q_{si}^+ - Q_{gi}^+ - Q_{ai}^+ \leq 0 \quad \forall i \quad (30)$$

$$\sum_{i=1}^8 Q_{si}^+ \leq q_s^+ \quad \forall i \quad (31)$$

$$\sum_{i=1}^8 Q_{gi}^+ \leq q_g^+ \quad \forall i \quad (32)$$

$$\sum_{i=1}^8 X_i^- = x_0 \quad (33)$$

$$X_i^- + \sum_{j=3}^7 X_{ij} = x_{0i} \quad i = 1, 2, 8 \quad (34)$$

$$X_i^- - X_{1i} - X_{2i} - X_{8i} = x_{0i} \quad i = 3, 4, 5, 6, 7 \quad (35)$$

$$lx_i \leq X_i^- \leq ux_i \quad \forall i \quad (36)$$

$$X_i^- \leq (X_i^+)_{\text{opt}} \quad \forall i \quad (37)$$

$$Q_{si}^+ \geq (Q_{si}^-)_{\text{opt}} \quad \forall i \quad (38)$$

$$Q_{gi}^+ \geq (Q_{gi}^-)_{\text{opt}} \quad \forall i \quad (39)$$

$$Q_{ai}^+ \geq (Q_{ai}^-)_{\text{opt}} \quad \forall i \quad (40)$$

Z_{opt}^- , $(X_i^-)_{\text{opt}}$, $(Q_{si}^+)_{\text{opt}}$, $(Q_{gi}^+)_{\text{opt}}$, and $(Q_{ai}^+)_{\text{opt}}$ for all i can be obtained from this submodel.

In this set of submodels, $\mathbf{X_p} = \{X_i \mid \forall i\}$, $\mathbf{X_n} = \{Q_{si}, Q_{gi}, Q_{ai} \mid \forall i\}$, $\mathbf{X_z} = \{X_{ij} \mid i = 1, 2, 8; j = 3, 4, 5, 6, 7\}$. Some corresponding bounds of system parameters are assumed. All decision variables in $\mathbf{X_n}$ are not fully greyized because their bounds in each submodel cannot be easily determined. For example, if smaller X_{ij} 's and hence X_{ij}^- 's are assumed in (26) and larger X_{ij} 's and hence X_{ij}^+ 's are assumed in (27), contradiction will occur because only one bound for a decision variable can exist in a submodel. Since X_{ij} 's are not fully greyized, their corresponding resulting solutions cannot form and should not be interpreted as stable grey intervals.

Note that we can conclude $(X_i^-)_{\text{opt}} = (X_i^+)_{\text{opt}}$ from (25), (33), and (37), *i.e.*, $(X_i^\pm)_{\text{opt}}$ will be constants instead of grey numbers. This will make the resulting grey solutions less applicable.

To overcome this shortcoming, the modified GLP model can be further modified based on specific conditions of this problem, which leads to a problem-oriented GLP model.

Formulation of Problem-Oriented GLP

Model 1 in the preceeding section will generate non-grey solutions for $X_i^\pm$. However, ranges for $(X_i^\pm)_{\text{opt}}$ as system parameters can vary within ranges which might be desired according to the preferences of decision makers. This situation of non-grey $(X_i^\pm)_{\text{opt}}$ exists because the equality constraints (25) and (33) are accompanied by the inequality constraint (37). Therefore, all X_i 's in the first submodel are X_i^+ 's and all X_i 's in the second submodel are X_i^- 's. If we could formulate a set of submodels in which some X_i^- 's and some X_i^+ 's are in each submodel, they may result in ranges for $(X_i^\pm)_{\text{opt}}$. Note that the reason for the formulation of Model 1 is to maintain the stability of its resulting grey solutions. This stability must be held in any further formulation modification.

Through observing $B_i^\pm$, the corresponding coefficients of $X_i^\pm$ in the objective function, it was found that some $B_i^\pm$'s are much larger than others. If X_i 's are larger, *i.e.*, more land is used, for those with larger $B_i^\pm$'s whereas X_i 's are smaller, *i.e.*, less land is used, for those with smaller $B_i^\pm$'s, a larger total benefit may be achieved. Based on the fact that any $B_i^\pm \in \{B_i^\pm \mid i = 3, 4, 5, 6\}$ is larger than a $B_i^\pm \in \{B_i^\pm \mid i = 1, 2, 7, 8\}$, the following set of submodels is formulated:

Model 2

Submodel 1:

$$\text{Max } Z^+ = \sum_{i=3}^6 B_i^+ X_i^+ + \sum_{i=1,2,7,8} B_i^+ X_i^- - \sum_{i=1}^8 C_s^- Q_{si}^- - \sum_{i=1}^8 C_g^- Q_{gi}^- - \sum_{i=1}^8 C_a^- Q_{ai}^- \quad (41)$$

s.t.

$$a_i^- X_i^+ - Q_{si}^- - Q_{gi}^- - Q_{ai}^- \leq 0 \quad i = 3, 4, 5, 6 \quad (42)$$

$$a_i^+ X_i^- - Q_{si}^- - Q_{gi}^- - Q_{ai}^- \leq 0 \quad i = 1, 2, 7, 8 \quad (43)$$

$$\sum_{i=1}^8 Q_{si}^- \leq q_s^- \quad \forall i \quad (44)$$

$$\sum_{i=1}^8 Q_{gi}^- \leq q_g^- \quad \forall i \quad (45)$$

$$\sum_{i=3}^6 X_i^+ + \sum_{i=1,2,7,8} X_i^- = x_0 \quad (46)$$

$$X_i^+ - X_{1i} - X_{2i} - X_{8i} = x_{0i} \quad i = 3, 4, 5, 6 \quad (47)$$

$$X_i^- + \sum_{j=3}^7 X_{ij} = x_{0i} \quad i = 1, 2, 8 \quad (48)$$

$$X_7^- - X_{17} - X_{27} - X_{87} = x_{07} \quad (49)$$

$$lx_i \leq X_i^+ \leq ux_i \quad i = 3, 4, 5, 6 \quad (50)$$

$$lx_i \leq X_i^- \leq ux_i \quad i = 1, 2, 7, 8 \quad (51)$$

Z_{opt}^+ ; $(X_i^+)_{\text{opt}}$ for $i = 3, 4, 5, 6$; $(X_i^-)_{\text{opt}}$ for $i = 1, 2, 7, 8$; $(Q_{si}^-)_{\text{opt}}$, $(Q_{gi}^-)_{\text{opt}}$, and $(Q_{ai}^-)_{\text{opt}}$ for all i can be obtained from this submodel.

Submodel 2:

$$\text{Max } Z^- = \sum_{i=3}^6 B_i^- X_i^- + \sum_{i=1,2,7,8} B_i^- X_i^+ - \sum_{i=1}^8 C_s^+ Q_{si}^+ - \sum_{i=1}^8 C_g^+ Q_{gi}^+ - \sum_{i=1}^8 C_a^+ Q_{ai}^+ \quad (52)$$

s.t.

$$a_i^+ X_i^- - Q_{si}^+ - Q_{gi}^+ - Q_{ai}^+ \leq 0 \quad i = 3, 4, 5, 6 \quad (53)$$

$$a_i^- X_i^+ - Q_{si}^+ - Q_{gi}^+ - Q_{ai}^+ \leq 0 \quad i = 1, 2, 7, 8 \quad (54)$$

$$\sum_{i=1}^8 Q_{si}^+ \leq q_s^+ \quad \forall i \quad (55)$$

$$\sum_{i=1}^8 Q_{gi}^+ \leq q_g^+ \quad \forall i \quad (56)$$

$$\sum_{i=3}^6 X_i^- + \sum_{i=1,2,7,8} X_i^+ = x_0 \quad (57)$$

$$X_i^- - X_{1i} - X_{2i} - X_{8i} = x_{0i} \quad i = 3, 4, 5, 6, \quad (58)$$

$$X_i^+ + \sum_{j=3}^7 X_{ij} = x_{0i} \quad i = 1, 2, 8 \quad (59)$$

$$X_7^+ - X_{17} - X_{27} - X_{87} = x_{07} \quad (60)$$

$$lx_i \leq X_i^- \leq ux_i \quad i = 3, 4, 5, 6 \quad (61)$$

$$lx_i \leq X_i^+ \leq ux_i \quad i = 1, 2, 7, 8 \quad (62)$$

$$X_i^- \leq (X_i^+)_{\text{opt}} \quad i = 3, 4, 5, 6 \quad (63)$$

$$X_i^+ \geq (X_i^-)_{\text{opt}} \quad i = 1, 2, 7, 8 \quad (64)$$

$$Q_{si}^+ \geq (Q_{si}^-)_{\text{opt}} \quad \forall i \quad (65)$$

$$Q_{gi}^+ \geq (Q_{gi}^-)_{\text{opt}} \quad \forall i \quad (66)$$

$$Q_{ai}^+ \geq (Q_{ai}^-)_{\text{opt}} \quad \forall i \quad (67)$$

Z_{opt}^- ; $(X_i^-)_{\text{opt}}$ for $i = 3, 4, 5, 6$; $(X_i^+)_{\text{opt}}$ for $i = 1, 2, 7, 8$; $(Q_{si}^+)_{\text{opt}}$, $(Q_{gi}^+)_{\text{opt}}$, and $(Q_{ai}^+)_{\text{opt}}$ for all i can be obtained from this submodel.

Proof of Stability From (41) to (67),

$$\begin{aligned}
Z_{\text{opt}}^+ &= \sum_{i=3}^6 B_i^+(X_i^+)_{\text{opt}} + \sum_{i=1,2,7,8} B_i^+(X_i^-)_{\text{opt}} - \sum_{i=1}^8 C_s^-(Q_{si}^-)_{\text{opt}} - \sum_{i=1}^8 C_g^-(Q_{gi}^-)_{\text{opt}} \\
&\quad - \sum_{i=1}^8 C_a^-(Q_{ai}^-)_{\text{opt}} \\
Z_{\text{opt}}^- &= \sum_{i=3}^6 B_i^-(X_i^-)_{\text{opt}} + \sum_{i=1,2,7,8} B_i^-(X_i^+)_{\text{opt}} - \sum_{i=1}^8 C_s^+(Q_{si}^+)_{\text{opt}} - \sum_{i=1}^8 C_g^+(Q_{gi}^+)_{\text{opt}} \\
&\quad - \sum_{i=1}^8 C_a^+(Q_{ai}^+)_{\text{opt}}
\end{aligned}$$

and for $B_i \in [B_i^-, B_i^+]$, $C_s \in [C_s^-, C_s^+]$, $C_g \in [C_g^-, C_g^+]$, $C_a \in [C_a^-, C_a^+]$, $X_i \in [(X_i^-)_{\text{opt}}, (X_i^+)_{\text{opt}}]$, $Q_{si} \in [(Q_{si}^-)_{\text{opt}}, (Q_{si}^+)_{\text{opt}}]$, $Q_{gi} \in [(Q_{gi}^-)_{\text{opt}}, (Q_{gi}^+)_{\text{opt}}]$, $Q_{ai} \in [(Q_{ai}^-)_{\text{opt}}, (Q_{ai}^+)_{\text{opt}}]$ for all i ,

$$\begin{aligned}
Z^\pm &= \sum_{i=3}^6 B_i X_i + \sum_{i=1,2,7,8} B_i X_i - \sum_{i=1}^8 C_s Q_{si} - \sum_{i=1}^8 C_g Q_{gi} - \sum_{i=1}^8 C_a Q_{ai} \\
Z_{\text{opt}}^+ - Z^\pm &= \sum_{i=3}^6 [B_i^+(X_i^+)_{\text{opt}} - B_i X_i] + \sum_{i=1,2,7,8} [B_i^+(X_i^-)_{\text{opt}} - B_i X_i] \\
&\quad + \sum_{i=1}^8 [C_s Q_{si} - C_s^-(Q_{si}^-)_{\text{opt}}] + \sum_{i=1}^8 [C_g Q_{gi} - C_g^-(Q_{gi}^-)_{\text{opt}}] \\
&\quad + \sum_{i=1}^8 [C_a Q_{ai} - C_a^-(Q_{ai}^-)_{\text{opt}}] = \sum_{i=3}^6 [B_i^+(X_i^+)_{\text{opt}} - B_i X_i] \\
&\quad + \sum_{i=1,2,7,8} [B_i^+(X_i^-)_{\text{opt}} - B_i X_i] + M_1 + M_2 + M_3
\end{aligned}$$

where $M_1 = \sum_{i=1}^8 [C_s Q_{si} - C_s^-(Q_{si}^-)_{\text{opt}}] \geq 0$, $M_2 = \sum_{i=1}^8 [C_g Q_{gi} - C_g^-(Q_{gi}^-)_{\text{opt}}] \geq 0$, $M_3 = \sum_{i=1}^8 [C_a Q_{ai} - C_a^-(Q_{ai}^-)_{\text{opt}}] \geq 0$ and

$$\begin{aligned}
&\sum_{i=3}^6 [B_i^+(X_i^+)_{\text{opt}} - B_i X_i] + \sum_{i=1,2,7,8} [B_i^+(X_i^-)_{\text{opt}} - B_i X_i] \\
&\geq \sum_{i=3}^6 \{B_i^+[(X_i^-)_{\text{opt}} + (\Delta X_i)_{\text{opt}}] - B_i^+[(X_i^-)_{\text{opt}} + \Delta X_i^A]\} \\
&\quad + \sum_{i=1,2,7,8} \{B_i^+(X_i^-)_{\text{opt}} - B_i^+[(X_i^-)_{\text{opt}} + \Delta X_i^B]\} \\
&= \sum_{i=3}^6 \{B_i^+[(\Delta X_i)_{\text{opt}} - \Delta X_i^A]\} - \sum_{i=1,2,7,8} B_i^+ \Delta X_i^B
\end{aligned}$$

From the constraint $\sum_{i=1}^8 X_i = x_0$ and corresponding ones (43) and (54), we can have

$$\sum_{i=1}^8 (\Delta X_i)_{\text{opt}} = \sum_{i=3}^6 \Delta X_i^A + \sum_{i=1,2,7,8} \Delta X_i^B$$

Moreover, given that $\text{Min}\{B_i, i = 3, 4, 5, 6\} = B_{m1} \geq B_{M2} = \text{Max}\{B_i, i = 1, 2, 7, 8\}$

$$\begin{aligned}
& \sum_{i=3}^6 \{B_i^+[(\Delta X_i)_{\text{opt}} - \Delta X_i^A]\} - \sum_{i=1,2,7,8} B_i^+ \Delta X_i^B \\
& \geq \sum_{i=3}^6 \{B_{m1}[(\Delta X_i)_{\text{opt}} - \Delta X_i^A]\} - \sum_{i=1,2,7,8} B_{M2} \Delta X_i^B \\
& \geq \sum_{i=3}^6 \{B_{m1}[(\Delta X_i)_{\text{opt}} - \Delta X_i^A]\} - \sum_{i=1,2,7,8} B_{m1} \Delta X_i^B \\
& = B_{m1} \cdot \left[\sum_{i=1}^8 (\Delta X_i)_{\text{opt}} - \left(\sum_{i=3}^6 \Delta X_i^A + \sum_{i=1,2,7,8} \Delta X_i^B \right) \right] = 0
\end{aligned}$$

Hence, $Z_{\text{opt}}^+ - Z^\pm \geq 0 + M_1 + M_2 + M_3 \geq 0$.

Similarly, we can prove that $Z^\pm - Z_{\text{opt}}^- \geq 0$.

As a consequence, for $B_i \in [B_i^-, B_i^+]$, $C_s \in [C_s^-, C_s^+]$, $C_g \in [C_g^-, C_g^+]$, $C_a \in [C_a^-, C_a^+]$, $X_i \in [(X_i^-)_{\text{opt}}, (X_i^+)_{\text{opt}}]$, $Q_{si} \in [(Q_{si}^-)_{\text{opt}}, (Q_{si}^+)_{\text{opt}}]$, $Q_{gi} \in [(Q_{gi}^-)_{\text{opt}}, (Q_{gi}^+)_{\text{opt}}]$, $Q_{ai} \in [(Q_{ai}^-)_{\text{opt}}, (Q_{ai}^+)_{\text{opt}}]$ for all i , we can prove $Z^\pm \in [Z_{\text{opt}}^-, Z_{\text{opt}}^+]$, i.e., stability holds. ■

As in Model 1, X_{ij} 's are not fully greyized in this set of submodels. Hence, the solutions obtained from the two submodels cannot be interpreted as grey intervals. Further modifications can cause some of X_{ij} 's able to form stable grey intervals. There are many possible sets of stable grey intervals in the solution space and a GP model's purpose is just to identify one set of them. By adjusting the grey relationships between system parameters and decision variables, different sets of stable grey intervals can be obtained. For example, another set of submodels, Model 3, can be formulated by changing the following from Model 2:

1. Let all X_{ij} 's for $i = 1, 2, 8$ and $j = 3, 4, 5, 6$ be X_{ij}^+ 's in the first submodel and X_{ij}^- 's in the second one while keeping X_{17} , X_{27} , and X_{87} not fully greyized.
2. In the second submodel, add constraints similar to (63) to let $X_{ij}^- \leq (X_{ij}^+)_{\text{opt}}$ for $i = 1, 2, 8$ and $j = 3, 4, 5, 6$ solved from the first submodel.

Since X_{ij} 's are not shown in the objective function, their values will not influence the objective function values. Therefore, the stability is maintained as long as the two submodels are both feasible.

Grey Solutions and Sensitivity

With the system parameters listed in Table I and II, three GLP models are solved. Note that in Table I the lower bounds for agricultural and fish cultivation land use are set as 50% and 40% of their current areas, respectively. Also note that the surface water supply capacity $q_s^\pm$ in Table II is 140% of the estimated one because a $q_s^\pm$ equaling 495.6 million m^3 per year cannot make the optimization problem feasible and a $q_s^\pm$ larger than that number is possible based on preceeding analysis. More surface water sources need to be developed; otherwise the integrated development of this area will be problematic. Also, it is assumed that alternative water sources will be developed only for fish cultivation and industries. Therefore, only Q_{a2} and Q_{a3} are considered.

When solving Model 1 using the system parameters in Table I and II, it was found that the second submodel cannot be feasible because the degree of greyiness of $a_i^\pm$'s is too

high ($= 0.05$). It was discussed in the preceeding that constraints (25), (33) and (37) will make $(X_i^-)_{\text{opt}}$ solved from the second submodel equal to $(X_i^+)_{\text{opt}}$ solved from the first submodel for all i . Therefore, a set of feasible solutions for all decision variables cannot be easily obtained if a_i^- in the second submodel is too small relative to a_i^+ in the first submodel. After extensive tests, 0.006 was found to be the maximum degree of greyness for $a_i^\pm$'s from which feasible solutions can be obtained. The grey solutions are listed in Table III. The variables of $(X_{ij})_{\text{opt}}$ are not included because they are not fully greyized in the submodels and hence cannot generate grey intervals.

The grey solutions offer information regarding how optimum system performance and decision variable values can change as the system parameters vary within grey intervals. For example, the solutions in Table III can be interpreted as follows: Given the ranges of the system parameters, if X_1 is within [7456, 7456], X_2 is within [2611, 2611], ..., Q_{s1} is within [154245466, 156107583], Q_{s2} is within [153693133, 155548583], ..., the system performance will be within [3044856, 3047350] million NT dollars.

The solutions indicate that all transferable lands currently in the categories of agriculture, fish cultivation, and "others" will be used for other purposes. Increases in land for industries, commerce, recreation, and residence are significant. Land area for conservation remains the same. Ranges for most Q_{si} 's and some Q_{gi} 's can be observed. It is clear that industries will consume more surface water and groundwater than any other user. In addition to industries, groundwater may also be needed for recreation and residences but that amount is small in comparison to the water needs of industries. Unlike the current situation, no groundwater will be used for fish cultivation based on the optimum schemes. For alternative water sources, the demands for both fish cultivation and industries are zero based on the assumed costs.

Although grey solutions for the system performance and some Q_{si} 's and Q_{gi} 's can be identified from Model 1, no grey solutions for X_i 's can be obtained, so that the set of grey solutions is not very applicable. Through solving Model 2, the problem-oriented GLP model modified from Model 1, grey solutions for X_i 's as well as other decision variables can be found. Table IV lists the sets of grey solutions from Model 2 with the degrees of greyness of $a_i^\pm$'s equaling 0.05 and 1.0. Distribution of areas for various land uses is similar to that solved from Model 1. As the ranges of system parameters become larger, *i.e.*, their degrees of greyness become higher, larger Z_{opt}^+ and smaller Z_{opt}^- will be achieved and hence the degree of greyness for the system performance is also higher. Similar situations can also be observed for the decision variable values. However, for some Q_{si} 's and Q_{gi} 's, grey solutions change considerably because scales of $a_i^\pm$'s differ significantly.

TABLE III Grey Solutions from Model 1 with $\deg(a_i^\pm) = 0.006$.

$Z_{\text{opt}}^+ = 3047350$ million NT\$, $Z_{\text{opt}}^- = 3044856$ million NT\$									
i	X_i^-	X_i^+	%*	Q_{si}^-	Q_{si}^+	Q_{gi}^-	Q_{gi}^+	Q_{ai}^-	Q_{ai}^+
1	7456	7456	25.0	154245466	156107583	0	0		
2	2611	2611	8.7	153693133	155548583	0	0	0	0
3	2982	2982	10.0	243823938	247359036	49×10^6	49×10^6	0	0
4	146	146	0.5	1205690	1220246	0	0		
5	15605	15605	52.2	128866112	129598890	0	822950		
6	494	494	1.7	8165661	8165661	0	98579		
7	578	578	1.9	0	0	0	0		
8	0	0	0.0	0	0	0	0		

*Percentage of X_i^M .

TABLE IV Grey Solutions from Model 2 with $\deg(a_i^\pm) = 0.05$ and 0.1 .

i	X_i^-	X_i^+	%*	Q_{si}^-	Q_{si}^+	Q_{gi}^-	Q_{gi}^+	Q_{ai}^-	Q_{ai}^+
$\deg(a_i^\pm) = 0.05, Z_{opt}^+ = 3052172 \text{ million NT\$}, Z_{opt}^- = 2930133 \text{ million NT\$}$									
1	7456	7970	25.8	162935351	162935351	0	0		
2	2611	2885	9.2	162351901	162351901	0	0	0	0
3	2716	3002	9.6	232753504	232753504	49×10^6	49×10^6	0	0
4	146	146	0.5	1152320	1273616	0	0		
5	15081	15585	51.3	123002722	130059930	0	1.5×10^6		
6	494	494	1.7	7804203	8625698	0	0		
7	578	578	1.9	0	0	0	0		
8	0	0		0	0	0	0		
$\deg(a_i^\pm) = 0.1, Z_{opt}^+ = 3070640 \text{ million NT\$}, Z_{opt}^- = 2760054 \text{ million NT\$}$									
1	7456	9113	27.7	170694177	170694177	0	0		
2	2611	3191	9.7	170082944	170082944	0	0	0	0
3	2519	3079	9.4	273785806	273785806	0	0	0	0
4	146	146	0.5	0	242594	1091671	1091671		
5	13521	15507	48.6	75437073	81551489	40514873	42014873		
6	494	494	1.7	0	1642990	7393455	7393455		
7	578	887	2.5	0	0	0	0		
8	0	0	0	0	0	0	0		

*Percentage of X_i^M .

If grey solutions for X_{ij} 's are desired, Model 3 can be employed. The resulting grey solutions for all decision variables except X_{17} , X_{27} , and X_{87} are listed in Table V. Some of the decision variable values solved from the second submodel differ from those of Model 2. The worst optimum system performance is smaller than that obtained from Model 2 because more constraints are applied to this maximization problem. The solutions for X_{ij} 's (except X_{17} , X_{27} , and X_{87}) can be interpreted as stable grey intervals. Note that for given values of X_i 's, the solution set for X_{ij} 's is not unique (8 equations, 21 variables). The grey intervals presented herein are also not unique.

TABLE V Grey Solutions from Model 3 with $\deg(a_i^\pm) = 0.05$.

$Z_{opt}^+ = 3052172 \text{ million NT\$}, Z_{opt}^- = 2926474 \text{ million NT\$}$									
i	X_i^-	X_i^+	%*	Q_{si}^-	Q_{si}^+	Q_{gi}^-	Q_{gi}^+	Q_{ai}^-	Q_{ai}^+
1	7456	7684	25.3	162935351	162935351	0	0		
2	2611	2885	9.2	162351901	162351901	0	0	0	0
3	2716	3002	9.6	232753504	232753504	49×10^6	49×10^6	0	0
4	146	146	0.5	1152320	1273616	0	0		
5	15081	15585	51.3	123002722	130059930	0	1.5×10^6		
6	494	494	1.7	7804203	8625698	0	0		
7	578	864	2.4	0	0	0	0		
8	0	0	0.0	0	0	0	0		
<div> <div>$i=1$</div> <div>$i=2$</div> <div>$i=8$</div> </div>									
j	X_{ij}^-	X_{ij}^+		X_{ij}^-	X_{ij}^+	X_{ij}^-	X_{ij}^+		
3	0	0	0	0	0	2664	2950		
4	0	0	0	0	0	131	131		
5	7227	7456	3641	3916	4090	4090	4090		
6	0	0	0	0	330	330	330		

*Percentage of X_i^M .

FIGURE 3 The optimum system performance $Z_{opt}^{\pm}$ corresponding to surface water supply capacity q_s^M where $q_s^{\pm} = [q_s^M - 10, q_s^M + 10]$.

There can be many scenarios for a development plan. For example, upper and lower bounds of each land use may change, as will the corresponding different optimum schemes of land distribution and water consumption. The GLP algorithms presented in this study can offer decision makers information regarding how the optimum system performance and decision variable values can vary as the system parameters change within their respective ranges. The variation trend ranges can be known, for all development plans, without the

FIGURE 4 The optimum land allocation for industries $X_3^{\pm}$ corresponding to surface water supply capacity q_s^M where $q_s^{\pm} = [q_s^M - 10, q_s^M + 10]$.

need for repeating many sensitivity analyses with respect to each system parameter. However, if the range to be examined for a system parameter is wider than the corresponding grey interval, traditional sensitivity analysis is still needed. The outcome will be “grey sensitivity analysis” because for each value of a system parameter, the result will be a range instead of a certain number for a system performance or decision variable value. Figure 3 and Figure 4 demonstrate the variation of the system performance $Z_{\text{opt}}^{\pm}$ and the area for industries $X_3^{\pm}$ with respect to surface water supply capacity $q_s^{\pm}$. The solutions were obtained by solving Model 2 and using the system parameters listed in Table I and II, except that $q_s^{\pm} = [q_s^M - 10, q_s^M + 10]$. In Figure 2, $Z_{\text{opt}}^{\pm}$ can be any value in the interval regions bounded by the upper bound curve and lower bound curve; similar interpretation is applicable for $X_3^{\pm}$ in Figure 4.

CONCLUSION

An optimization problem for balancing land carrying capacity and groundwater consumption in Central Taiwan has been solved by using a problem-oriented grey linear programming (POGLP) algorithm. Traditional GLP algorithms were modified to make them suitable for general optimization problems. Further modifications were made to improve the applicability of resulting solutions, which lead to solving a problem-oriented GLP model. Satisfactory grey solutions were gained by the use of such a methodology. This can provide useful information to decision makers regarding the possible ranges of the policy decision-making, as the system parameters vary within their respective grey intervals.

The results indicate that all transferable lands currently used for crop and fish cultivation can be transferred to other usages such as industries, commerce, recreation, residence, and conservation. The increase in land use for recreation is ranked first. Groundwater will no longer be used for fish cultivation, thereby eliminating the major problem of land subsidence in the study area. To achieve this goal, the regulatory requirement of water resources fees should be imposed.

Sensitivity analyses were performed to demonstrate the variation relative to some decision variable values. It demonstrates interval regions instead of trend curves, which can be treated as two-dimensional or even multi-dimensional sensitivity analyses. Uncertainties in the examined system parameter (x -axis), as well as the optimum system performance and decision variable values (y -axis) are presented as grey intervals.

The GLP models presented in this paper should be regarded as a screening tool, which can identify good or non-inferior alternatives for the system. If a final single set of solutions is desired, additional analyses are then required. Nonlinear relationships between the system's elements may need to be taken into account and therefore, the optimization problem could turn out to be a nonlinear programming problem. Simulation techniques can also be applied to improving the application potential in each case study. Communications and negotiations between shareholders are probably needed before making a final decision.

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APPENDIX

The basic definitions relating to grey programming as used in model formulation.

DEFINITION 1 A grey number $x^\pm$ is defined as an interval with known upper bound, x^+ , and lower bound, x^- , but unknown distribution information.

$$x^\pm = [x^-, x^+] = \{t \in x^\pm \mid x^- \leq t \leq x^+\} \quad (\text{A-1})$$

DEFINITION 2 Let $*$ $\in$ $\{+, -, \times, \div\}$ be an operation on grey numbers. For grey numbers $x^\pm$ and $y^\pm$, we have:

$$x^\pm * y^\pm = [\min\{x * y\}, \max\{x * y\}], \quad x^- \leq x \leq x^+, \quad y^- \leq y \leq y^+ \quad (\text{A-2})$$

DEFINITION 3 Let $\diamond \in \{=, <, \leq, >, \geq\}$. For $x^\pm$ and $y^\pm$, their order relationship is as follows:

$$x^\pm \diamond y^\pm \quad \text{iff} \quad x^- \diamond y^- \quad \text{and} \quad x^+ \diamond y^+ \quad (\text{A-3})$$

DEFINITION 4 *The whitened mid-value of $x^\pm$ is defined as x^M :*

$$x^M = \frac{[x^- + x^+]}{2} \quad (\text{A-4})$$

DEFINITION 5 *The degree of greyness of $x^\pm$ is defined as:*

$$\text{deg}(x^\pm) = \frac{[x^+ - x^-]}{x^M} \quad (\text{A-5})$$

DEFINITION 6 *Define Sign ($x^\pm$) as follows:*

$$\begin{aligned} \text{Sign}(x^\pm) &= 1, \quad \text{if } x^\pm \geq 0 \\ &= -1, \quad \text{if } x^\pm < 0 \\ &\text{not defined otherwise} \end{aligned} \quad (\text{A-6})$$

DEFINITION 7 *Grey Solutions or Stable Grey Intervals*

Results obtained from a GP model can form a set of grey solutions or stable grey intervals if given that the system parameters and decision variable values are within their respective grey intervals, the system performance will also be within its grey interval.

DEFINITION 8 **X_p , X_n , and X_z**

X_p : *the set of decision variables with a positive coefficient in the objective function.*

X_n : *the set of decision variables with a negative coefficient in the objective function.*

X_z : *the set of decision variables with a zero coefficient in the objective function.*