Bolometric Response of Optically Thick Superconducting Films

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Abstract—A one-dimensional heat conduction model is established to interpret the chopping frequency dependence of the bolometric response of optically thick superconducting films. The key feature of this model is that it takes into account the thermal boundary conductance of both the film/substrate and the substrate/heat-sink interfaces. The model is found to be suitable for both the strong and weak thermal coupling cases. By comparing the results with a simplified bolometric model, explicit expressions of the thermal conductance, thermal capacitance, radiation absorption efficiency and response time constant of the bolometric system have been deduced, where the influences of the film, substrate and the heat conduction boundaries are all considered.

I. INTRODUCTION

With the employment of a lock-in amplifier and a four-probe measurement configuration, operated at a constant bias current, I , and at a stabilized temperature, T , around the midpoint of the superconducting transition, the dependence of the bolometric optical response, V_{or} , of a superconducting film on the chopping frequency, f , usually decreases monotonically as the chopping frequency is increased [1]. A proportional relation between V_{or} and $f^{-\alpha}$ is presumed to study the dependence of V_{or} vs. f , where the parameter α can be determined experimentally by the slope of the logarithmic plot of V_{or} vs. f characteristics. Various values of α between 0 and 1 have been reported. Dwir and Pavuna observed the optical response of YBCO films and concluded that the parameter α equals 0.5 for strong thermal coupling cases, and equals 1 for weak thermal coupling cases, where two different models were used to interpret the observed experimental results [2]. However, other α values of 0.4 [3], and 0.47 [4] were reported, not quite consistent with the predictions given by the aforementioned models [2]. The widely-spread α values indicate that the heat conduction behavior of the system

is more complicated than the previous suggestions of Dwir and Pavuna. Based on the previous work of Genchev et al. [5], a more general heat-conduction model is, thus, required to sort out the inconsistency.

II. MODELING

A. Radiation Absorption of the Film

A simplified one-dimensional model of volumetric radiation absorption is used to estimate the absorbed radiation power density of a superconducting film (thickness d_f). Consider the case that a helium-neon laser perpendicularly irradiates the surface of an optically thick superconducting film. Since the radiation intensity decreases rapidly as the light propagates into the film, the reflected radiation intensity by the film/substrate interface is, thus, negligible.

The origin of the coordinate is chosen to be at the center of the radiation spot, assumed to be aligned to the point at the center position of the superconducting pattern. The electric field, $\vec{E}$, of the radiation is assumed to oscillate along the x-direction at the radiation angular frequency, ω_r , expressed by $\vec{E} = E_x(z) \cdot \exp(-i\omega_r \cdot t) \hat{x}$, and the magnetic field, $\vec{H}$, along the y-direction, expressed by $\vec{H} = H_y(z) \cdot \exp(-i\omega_r \cdot t) \hat{y}$. The z-direction and y-direction are shown in Fig. 1. The derivation is based on the following Maxwell equations

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (1)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \quad (2)$$

where $\vec{B}$, $\vec{D}$, and $\vec{J}$ are the magnetic induction of the radiation, the electric displacement of the radiation, and the free current density caused by charged particles, respectively.

The relation between $\vec{D}$ and $\vec{E}$ fields, and between $\vec{B}$ and $\vec{H}$ fields are expressed by

$$\vec{D} = \epsilon_0 \cdot \epsilon_r \cdot \vec{E} \quad (3)$$

Manuscript received August 26, 1996.

This work is supported in part by the National Science Council of R.O.C. under Grant No. NSC 85-2811-M002-011 and NSC 85-2112-M002-020. J.D. Chern was supported by both the John Swire and Sons Limited, and the Overseas Research Studentship in Britain. Z.D. Genchev was supported by the Bulgarian Ministry of Science under grant Ph-442.

Fig. 1. A schematic diagram of the modified heat-conduction system, consisted of a superconducting film, a substrate, and a copper heat-sink.

$$\vec{B} = \mu_0 \cdot \mu_r \cdot \vec{H} \quad (4)$$

where ϵ_r are the relative permittivity of the film and μ_r the relative permeability of the film. Since the interested operation temperature is limited to a relatively small range around the midpoint of the superconducting transition, a finite conductance of the film is presumed, and the relation between $\vec{J}$ and $\vec{E}$ of the superconducting medium is assumed to obey the Ohm's law

$$\vec{J} = \sigma \cdot \vec{E}, \quad (5)$$

where σ is the electric conductivity of the film at the laser radiation frequency.

The one-dimensional volumetric radiation absorption of the film is based on the dissipation of average radiation power density, $\langle Q_f(z) \rangle$, absorbed by the film as given by

$$\langle Q_f(z) \rangle = \frac{1}{2} \cdot \text{Re}(\vec{J} \cdot \vec{E}^*). \quad (6)$$

Therefore, in the range of $0 < z < d_f$, $\langle Q_f(z) \rangle$ can be expressed by

$$\langle Q_f(z) \rangle = \mathfrak{I}_0 \cdot \left(\frac{1 - R_f}{\mu_r} \right) \cdot \beta_f \cdot \exp(-\beta_f \cdot z), \quad (7)$$

where $\mathfrak{I}_0$ is the output intensity of the laser, R_f is the reflectance of the film, $\beta_f \equiv 1/\alpha_f$ is the absorption coefficient of the film, and α_f the optical penetration depth of the film.

When the attenuation effects of the laser power density caused by an optical window and an optical chopper are taken into account, the sinusoidal component of the modulated laser power density in the film, oscillated at the chopping frequency, f , can be expressed by

$$\langle Q_f(z, t) \rangle = \langle Q_f(z) \rangle \cdot \tau_w \cdot \tau_c \cdot \cos(2\pi \cdot f \cdot t), \quad (8)$$

where τ_w is the transmittance of the window, and τ_c the transmission factor of the chopper. For an idea square-wave-

modulated laser beam, the transmission factor, τ_c , is given by a constant value of $2/\pi$ [6]. The quantity $\langle Q_f(z) \rangle$ in (7) is only valid for optically thick films, restricted by $d_f \gg \alpha_f$.

B. Heat Conduction Process

After the absorption of laser energy of the film, the absorbed heat is transferred by conduction down to the film, the substrate, and the heat-sink. The heat conduction equations [7] of the film and the substrate are given by

$$K_f \cdot \frac{\partial^2 T_f(z, t)}{\partial z^2} - \rho_f \cdot c_f \cdot \frac{\partial T_f(z, t)}{\partial t} = -\langle Q_f(z, t) \rangle, \quad 0 \leq z \leq d_f \quad (9)$$

$$K_s \cdot \frac{\partial^2 T_s(z, t)}{\partial z^2} - \rho_s \cdot c_s \cdot \frac{\partial T_s(z, t)}{\partial t} = 0, \quad d_f \leq z \leq d_s, \quad (10)$$

where T_f and T_s are the sinusoidal temperature rise of the film and of the substrate, respectively, K_f the thermal conductivity of the film, ρ_f the mass density of the film, c_f the specific heat of the film, K_s the thermal conductivity of the substrate, ρ_s the mass density of the substrate, and c_s the specific heat of the substrate. The boundary conditions at the interfaces are given by

$$\frac{\partial T_f}{\partial z} = 0, \quad \text{at } z = 0 \quad (11)$$

$$K_f \cdot \frac{\partial T_f}{\partial z} = K_s \cdot \frac{\partial T_s}{\partial z} = -h_{fs} \cdot (T_f - T_s), \quad \text{at } z = d_f, \quad (12)$$

$$K_s \cdot \frac{\partial T_s}{\partial z} = -h_{sc} \cdot T_s, \quad \text{at } z = d_f + d_s, \quad (13)$$

where $h_{fs} \equiv 1/R_{fs}$ is the thermal boundary conductance at the film/substrate interface, $h_{sc} \equiv 1/R_{sc}$ the thermal boundary conductance at the substrate/heat-sink interface, d_s the thickness of the substrate, and R_{fs} , R_{sc} represent the thermal boundary resistances of the film/substrate interface and of the substrate/heat-sink interface, respectively.

Using a complex sinusoidal modulation treatment similar to (8), with the introduction of a chopping angular frequency $\omega = 2\pi \cdot f$, the heat conduction equations of (9) and (10) can be reduced into complex time-independent heat conduction equations, and the solution of $T_f(z, t) \equiv T_{f0}(z) \cdot \exp(-i\omega \cdot t)$ can be exactly solved, where $T_{f0}(z)$ is a complex quantity, representing the complex temperature rise of the film at a depth in the range $0 < z < d_f$. The depth-average of $T_{f0}(z)$ can, then, be deduced as expressed by

$$\langle T_{f0} \rangle \equiv \frac{1}{d_f} \cdot \int_0^{d_f} T_{f0}(z) dz \equiv \langle \bar{T}_f \rangle \cdot T_0, \quad (14)$$

where $T_0 \equiv \frac{d_f^2}{K_f} \cdot \mathfrak{I}_0 \cdot \frac{1-R_f}{\mu_r} \cdot \beta_f \cdot \tau_w \cdot \tau_c$, (15)

$$\langle \bar{T}_f \rangle \equiv \bar{A} \cdot \left\{ \frac{\bar{H} + \frac{1}{\bar{B}_{fs}} + \left(\frac{\bar{\gamma} \bar{G}}{\bar{F}} \right) \cdot [1 - \exp(-\bar{\beta}_f)] \cdot \bar{C}}{\bar{\gamma} \cdot \bar{G} \cdot \bar{F} \cdot \bar{C} + \frac{\bar{F}^2}{\bar{B}_{fs}} + \bar{F} \cdot \coth(\bar{F})} - \bar{D} \right\}, \quad (16)$$

$$\bar{A} \equiv \frac{\bar{\beta}_f}{(\bar{\beta}_f + \bar{F}) \cdot (\bar{\beta}_f - \bar{F})}, \quad (17)$$

$$\bar{C} \equiv \frac{(\bar{G} \cdot \bar{F} \cdot \bar{E} + \bar{B}_{sc}) \cdot \tanh(\bar{G} \cdot \bar{F}) - (\bar{G} \cdot \bar{F} + \bar{B}_{sc} \cdot \bar{E})}{(\bar{G} \cdot \bar{F} + \bar{B}_{sc} \cdot \bar{E}) \cdot \tanh(\bar{G} \cdot \bar{F}) - (\bar{G} \cdot \bar{F} \cdot \bar{E} + \bar{B}_{sc})} \quad (18)$$

$$\bar{D} \equiv \frac{1 - \exp(-\bar{\beta}_f)}{\bar{\beta}_f^2}, \quad (19)$$

$$\bar{E} \equiv \tanh \left[\bar{G} \cdot \bar{F} \cdot \left(1 + \frac{d_s}{d_f} \right) \right], \quad (20)$$

$$\bar{F} \equiv \sqrt{-i \cdot 2\pi \cdot f \cdot \frac{d_f^2 \cdot \rho_f \cdot c_f}{K_f}}, \quad (21)$$

$$\bar{H} \equiv \frac{\cosh(\bar{F}) - 1}{\bar{F} \cdot \sinh(\bar{F})} + \left(\frac{1}{\bar{\beta}_f} - \frac{1}{\bar{B}_{fs}} \right) \cdot \exp(-\bar{\beta}_f), \quad (22)$$

where the overline symbol is assigned to artificially-defined dimensionless quantities, $i \equiv \sqrt{-1}$, $\bar{\beta}_f \equiv \beta_f \cdot d_f$, $\bar{\gamma} \equiv (\rho_f \cdot c_f) / (\rho_s \cdot c_s)$, $\bar{G} \equiv \sqrt{(\rho_s \cdot c_s \cdot K_f) / (\rho_f \cdot c_f \cdot K_s)}$, $\bar{B}_{fs} \equiv (h_{fs} \cdot d_f) / K_f$ and $\bar{B}_{sc} \equiv (h_{sc} \cdot d_f) / K_s$.

The amplitude of the sinusoidal temperature rise of the film is related to the modulus of the complex quantity $\langle T_{f0} \rangle$. The bolometric optical response of the film is, thus, given by

$$V_{or} = \frac{1}{\sqrt{2}} \cdot I \cdot \frac{dR}{dT} \cdot |\langle T_{f0} \rangle|, \quad (23)$$

where the factor $1/\sqrt{2}$ is due to the measurement technique using a lock-in amplifier, and R the resistance of the film measured at the film temperature T .

C. The Case of Strong Thermal Coupling

In order to compare (23) with experimental results, simplified conditions are required in an intermediate chopping

frequency range, restricted by $\bar{\beta}_f \gg 1$, $|\bar{F}| \ll 1$, and $\bar{G} \cdot |\bar{F}| \cdot [1 + (d_s / d_f)] \gg 1$ such that (23) can be reduced into a relatively useful form, expressed by

$$V_{or} = \frac{I}{\sqrt{2}} \cdot \frac{dR}{dT} \cdot \left[\mathfrak{I}_0 \frac{(1-R_f)}{\mu_r} \tau_w \tau_c \right] \cdot \frac{\bar{\Gamma}(f)}{\sqrt{2\pi \rho_s c_s K_s \cdot f}} \quad (24)$$

where $\bar{\Gamma}(f) \equiv \left(\frac{1 + U_1 \cdot \sqrt{2f} + U_1^2 \cdot f}{1 + U_2 \cdot \sqrt{2f} + U_2^2 \cdot f} \right)^{\frac{1}{2}}$, (25)

$$U_1 \equiv \sqrt{2\pi \cdot \rho_s \cdot c_s \cdot K_s} \cdot \left(\frac{1}{h_{fs}} + \frac{d_f}{2K_f} \right), \quad (26)$$

$$U_2 \equiv \frac{2\pi \cdot \rho_f \cdot c_f \cdot d_f}{\sqrt{2\pi \cdot \rho_s \cdot c_s \cdot K_s}}. \quad (27)$$

Equation (24) provides a very similar form to that given by a strong thermal-coupling model suggested by Dwir and Pavuna [2]. The only difference is that in their model the factor $\bar{\Gamma}(f)$ corresponds to unity, whereas in our model a more general expression is derived, including the influences of d_f and h_{fs} . The expression in (24) can, thus, be used to explain why some V_{or} results of a group follow $f^{-0.5}$ dependence, whereas others do not. In fact, the $\bar{\Gamma}(f)$ values of Dwir and Pavuna's results is found to occur in the range of $1.0024 \leq \bar{\Gamma}(f) \leq 1.0784$, which is indeed very close to unity in a relatively wide chopping frequency range. The frequency-dependent parameter $\bar{\Gamma}(f)$ is, thus, defined as the *film factor* of the system.

D. The Case of Weak Thermal Coupling

The optical response with weak thermal-coupling to the heat-sink has been demonstrated by Dwir and Pavuna [2], where the substrate was virtually isolated to the heat-sink, using an insulating support. To explain the chopping frequency dependence of the weak coupling response, the value of $\bar{B}_{sc}$ is assumed to be 'negligibly small but nonzero' in order to prevent the unreasonably divergence of V_{or} at the zero frequency limit, as well as requiring $\bar{\beta}_f \gg 1$ and

$\bar{G} \cdot |\bar{F}| \cdot [1 + (d_s / d_f)] \ll 1$. Equation (23) can, therefore, be reduced into a simpler expression given by

$$V_{or} = I \cdot \frac{dR}{dT} \cdot \left[\frac{(1-R_f)}{\sqrt{2}\mu_r} \cdot \tau_w \cdot \tau_c \right] \cdot \frac{P_f}{2\pi \cdot \left[A_f \cdot (\rho_f c_f d_f + \rho_s c_s d_s) \right]} \cdot \frac{1}{f} \quad (28)$$

where $\mathfrak{I}_0 \equiv P_f / A_f$, A_f is the irradiated area of the superconducting pattern, P_f is the irradiated laser power on the film surface. The f^{-1} dependence of the response is consistent with the results of Dwir and Pavuna [2].

The weak coupling response was interpreted by a classical bolometric model [2], where the bolometer is modeled as a single bulk thermal mass (or thermal capacitance) $C = \rho_s \cdot c_s \cdot d_s \cdot A_f$, connected to the heat-sink by a thermal link with thermal conductance G , absorbing the irradiated power P_f on the film surface with an optical absorption efficiency $0 \leq \eta \leq 1$. The characteristic time constant of the response is defined by $\tau \equiv C / G$, where V_{or} is expressed by

$$V_{or} = I \cdot \frac{dR}{dT} \cdot \frac{\eta \cdot P_f}{G \cdot \sqrt{1 + (2\pi \cdot f \cdot \tau)^2}}. \quad (29)$$

The expression in (29) is compared to both (23) and (28) at the limit of zero chopping frequency as well as under the condition of $2\pi \cdot f \cdot \tau \gg 1$, such that explicit expressions of C , G , τ , and η can be derived as expressed by

$$C = A_f \cdot (\rho_f \cdot c_f \cdot d_f + \rho_s \cdot c_s \cdot d_s), \quad (30)$$

$$G = A_f \cdot \left(\frac{d_f}{2K_f} + \frac{1}{h_{fs}} + \frac{d_s}{K_s} + \frac{1}{h_{sc}} \right)^{-1}, \quad (31)$$

$$\tau = (\rho_f c_f d_f + \rho_s c_s d_s) \cdot \left(\frac{d_f}{2K_f} + \frac{1}{h_{fs}} + \frac{d_s}{K_s} + \frac{1}{h_{sc}} \right), \quad (32)$$

$$\eta = \frac{(1 - R_f)}{\sqrt{2\mu_r}} \cdot \tau_w \cdot \tau_c. \quad (33)$$

It is found that the expression of the total thermal mass of the system in (30) includes both the thermal mass of the film as well as the thermal mass of the substrate. The result in (31) indicates that the total thermal resistance of the system is a summation of individual thermal resistances in a series-connected heat conduction circuit, where the factor 1/2 of the film is caused by the average procedure in (14). If the Joule-heating effect is taken into account [8], a more general expression of the thermal conductance G_e is given by

$$G_e = A_f \cdot \left(\frac{d_f}{2K_f} + \frac{1}{h_{fs}} + \frac{d_s}{K_s} + \frac{1}{h_{sc}} \right)^{-1} - I^2 \cdot \frac{dR}{dT}. \quad (34)$$

The Joule-heating effect in (34) can be ignored, so that $G_e \equiv G$, if the bias current I is sufficiently small.

III. CONCLUSION

The modified heat-conduction model is found to be capable of being applied to both the weak and strong thermal-coupling cases of the bolometric response of superconducting films. In the strong coupling case, the optical response is simplified for a specific *intermediate* chopping frequency range, so that the formulation of the response is very similar to that derived from a simple semi-infinite heat-conduction system, except that a frequency dependent *film factor* is introduced. The model can also be used to describe the results of weak coupling systems in a more general sense, where a f^{-1} dependence of the response is deduced. By comparing the derived results with a simple bolometric model, general expressions of the thermal conductance G , thermal capacitance C , radiation absorption efficiency η and response time constant τ of the bolometric system have been obtained, where the influences of the film, substrate and the thermal boundaries are all considered. The modified model is, thus, capable of unifying both the weak and strong coupling cases, which previously required two different models. However, the model is only suitable to be applied to optically thick films. If the film surface is smooth, a more general expression of the response can further be deduced for optically thin films.

ACKNOWLEDGMENT

The authors wish to thank Dr. C.R.M. Grovenor, Dr. M.J. Goringe, Dr. A. Guldeste, Dr. S. Aslam, Dr. H.C. Lai, Dr. B. Minakovic, and Dr. J.R. Waldram for their valuable advice and assistance.

REFERENCES

- [1] J.D. Chern, H.C. Lai, A. Guldeste, S Aslam, D.H.T. Wu, D. Dew-Hughes, C.R.M. Grovenor, M.J. Goringe, and B. Minakovic, IEEE Trans. Appl. Supercond., vol. 3(1), pp. 2128-2131, 1993.
- [2] B. Dwir, and D. Pavuna, J. Appl. Phys., vol. 72(9), pp. 3855-3861, 1992.
- [3] C.J. Haug, C.Y. Chang, and T.Y. Tseng, J. Appl. Phys., vol. 72(12), pp. 5786-5791, 1992.
- [4] W. Eidelloth, IEEE Trans. Magn., vol. 27(2), pp. 2828-2831, 1991.
- [5] Z.D. Genchev, R.A. Chakalov, J.D. Chern, and D. Dew-Hughes, Supercond. Sci. Technol. vol. 9, pp. 523-528, 1996.
- [6] J.H. Hao, F.Q. Zhou, X.R. Zhao, H.D. Sun, X.J. Yi, and Z.G. Li, IEEE Trans. Appl. Supercond., vol. 3(1), pp. 2167-2169, 1993.
- [7] M.I. Flik, P.E. Phelan, and C.L. Tien, Cryogenics, vol. 30, pp. 1118-1128, 1990.
- [8] J. Clarke, G.I. Hoffer, P.L. Richards, and N.-H. Yeh, J. Appl. Phys., vol. 48(12), pp. 4865-4879, 1977.