ρNN and ωNN couplings in the external-field QCD sum rule method

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We extend the method of QCD sum rules in external fields to determine the ρNN and ωNN strong couplings, including both the vector and tensor couplings. Such information has been used for years in the construction of nuclear potentials, but our attempt represents the first direct contact with QCD in terms of the basic parameters associated with the nontrivial QCD vacuum. [S0556-2813(97)02812-4]

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I. INTRODUCTION

The strong ρNN and ωNN couplings are the basic input for the description of nuclear forces in terms of meson exchanges between the nucleons [1–4]. Although it is now generally believed that quantum chromodynamics (QCD) describes strong interactions among quarks, antiquarks, and gluons (building blocks of hadrons), the linkage between QCD and the phenomenological ρNN and ωNN couplings has so far yet to be made.

The method of QCD sum rules [5–9] was introduced to allow some linkage to be made between QCD and hadron physics, especially in addressing the role of the various quark and gluon condensates associated with the nontrivial QCD vacuum. As shown by Ioffe and Smilga [10], it is possible to determine coupling constants (at $q^2=0$) by considering the response of the QCD vacuum to some external fields in the evaluation of the two-point correlation function. In this paper, we wish to obtain ρNN and ωNN strong couplings making use of the external-field QCD sum rule method. To obtain simultaneously the vector and tensor couplings, we write the external vector meson field as

$$Z_\mu = -\frac{1}{2}Z_{\mu\nu}x^\nu, \quad (1)$$

where $Z_{\mu\nu}$ is to be regarded as a constant background field. For a vector meson field, $Z_{\mu\nu}$ is traceless, i.e., $Z_\mu^\mu=0$, to ensure the Lorentz condition $\partial_\mu Z^\mu=0$. However, we note that $Z_{\mu\nu}$ can in general be decomposed into symmetric and antisymmetric parts. Under the influence of $Z_{\mu\nu}$, the vacuum structure of QCD changes in different ways, resulting in five main induced condensate parameters introduced later in the paper. Two of these induced condensates correspond to the symmetric part of $Z_{\mu\nu}$, while the others correspond to the antisymmetric part. In deriving the QCD sum rules for ρNN and ωNN strong couplings, we note that the procedure adopted for the antisymmetric part of $Z_{\mu\nu}$ has much in common with that first introduced by Ioffe and Smilga [10] for calculating the magnetic moments of the nucleon, while we find that the symmetric part of $Z_{\mu\nu}$ gives rise to QCD sum rules which have some nice features, including a prediction such as

$$\frac{g_{\rho NN}}{g_{\omega NN}} = \frac{1}{3}, \quad (2)$$

in excellent agreement with the values adopted in nuclear force studies.

One purpose of the present study is to first understand whether the ρNN and ωNN strong couplings can be described adequately in the context of QCD sum rules and then to proceed to use the framework to undertake more difficult investigations such as the determination of the parity-violating ρNN and ωNN weak couplings (which involve regularization of three-loop diagrams and which will be described in some detail in our next publication [11]).

This paper is organized briefly as follows: In Sec. II, we outline the key ingredients associated with the QCD sum rules. In Sec. III, we describe the derivation of the QCD sum rules for the ρNN and ωNN strong couplings. A numerical analysis is given in Sec. IV, while Sec. V contains a brief summary.

II. METHOD OF QCD SUM RULES

Following the procedure [10] to derive QCD sum rules in the presence of an external field, we consider the correlation function as defined by

$$i \int d^4x e^{ip \cdot x} \langle 0 | T \eta_N(x) \bar{\eta}_N(0) | 0 \rangle_Z = \Pi(p) + h Z_{\mu\nu} \Pi^{\mu\nu}(p), \quad (3)$$

where $\Pi(p)$ is the correlation function when the external field $Z_{\mu\nu}$ is absent. Here $\Pi(p)$ is just the function which we use to determine the nucleon mass [7]. The second term describes the linear response of the correlator to a small classical external field $Z_{\mu\nu}$, as characterized by Eq. (1), where Z_μ stands for either the isovector ρ meson or isoscalar ω meson field. Such linear response is the primary focus of this paper. The entity h describes the coupling between the designated external field and the up (u) quark field; more explicitly, the interaction between the external field ($Z=\rho, \omega$) and the quarks may be specified by

$$\mathcal{L}_{pq} = -h \bar{\psi} \vec{\tau} \gamma_\mu \psi \cdot \vec{\rho}^\mu, \quad (4)$$

$$\mathcal{L}_{\omega qq} = -h \bar{\psi} \gamma_\mu \psi \omega^\mu, \quad (5)$$

with ψ the Dirac spinor consisting of the (u, d) isospin doublet. We note that we use the same coupling constant h for ρ and ω fields. We wish to also note that, at the quark level, there is not much room in the choice of the form for the ρqq and ωqq couplings, as dictated by both that the quarks are considered pointlike in the QCD sum rule method and that the Lorentz four-vector nature and the isospin structure of the external fields in question are already given. The similarity to the electromagnetic field $F_{\mu\nu}$ treated as an external field [8] should not be taken as to indicate that our sum rules are not independent or the vector-dominance model is automatically ensured, as it is important to recognize that $F_{\mu\nu}$ is *antisymmetric*, while we find it is useful to focus on the sum rules related to both the *symmetric* and *antisymmetric* parts of $Z_{\mu\nu}$.

The entity η_N ($=\eta_p, \eta_n$) is some composite operator with the nonvanishing matrix element $\langle 0 | \eta_N | N \rangle$, which may be chosen as

$$\eta_p(x) = \epsilon^{abc} [u^{aT}(x) C \gamma_\mu u^b(x)] \gamma_5 \gamma_\mu d^c(x), \quad (6)$$

$$\eta_n(x) = \epsilon^{abc} [d^{aT}(x) C \gamma_\mu d^b(x)] \gamma_5 \gamma_\mu u^c(x), \quad (7)$$

where $u^a(x)$ and $d^a(x)$ are the up and down quark fields, respectively. The choice of these composite operators is not unique [12], but the above choice has become fairly standard in the literature.

We need to characterize how the nontrivial QCD vacuum responds to the external field $Z_{\mu\nu}$. To lowest order, we may introduce the following five condensation parameters $(\chi, \kappa, \xi, \zeta, \phi)$. Three of them (χ, κ, ξ) are related to the antisymmetric part of $Z_{\mu\nu}$, while the other two (ζ, ϕ) have to do with the symmetric part. With Z^S and Z^A describing, respectively, the symmetric and antisymmetric parts of the external field $Z_{\mu\nu}$, we have

$$\langle \bar{q} \sigma_{\alpha\beta} q \rangle_Z = \chi h_q Z_{\alpha\beta}^A \langle \bar{q} q \rangle, \quad (8)$$

$$g \langle \bar{q} G_{\alpha\beta} q \rangle_Z = \kappa h_q Z_{\alpha\beta}^A \langle \bar{q} q \rangle, \quad (9)$$

$$g \epsilon_{\mu\nu\lambda\sigma} \langle \bar{q} \gamma_5 G_{\lambda\sigma} q \rangle_Z = i \xi h_q Z_{\mu\nu}^A \langle \bar{q} q \rangle, \quad (10)$$

$$\langle \bar{q} \frac{1}{2} (\gamma_\mu \nabla_\nu + \gamma_\nu \nabla_\mu) q \rangle_Z = \zeta h_q Z_{\mu\nu}^S \langle \bar{q} q \rangle, \quad (11)$$

$$\begin{aligned} & \langle \bar{q} \frac{1}{2} (\nabla_\mu \nabla_\nu + \nabla_\nu \nabla_\mu) q \rangle_Z \\ &= -\frac{g}{8} \langle \bar{q} \sigma \cdot G q \rangle g_{\mu\nu} + \frac{i}{2} h_q Z_{\mu\nu}^S \langle \bar{q} q \rangle + i \phi h_q Z_{\mu\nu}^S \langle \bar{q} q \rangle, \end{aligned} \quad (12)$$

where $G_{\alpha\beta} \equiv G_{\alpha\beta}^n \lambda^n/2$ is the gluon field. The coupling constant h_q ($q=u$ or d) depends on the external field Z ; for

instance, $h_u = -h_d = h$ for the ρ^0 meson field and $h_u = h_d = h$ for the ω meson field.

III. ρNN AND ωNN COUPLINGS

A. Quark propagator

To evaluate the correlation function at the quark level, we need to know the quark propagator in the nontrivial QCD vacuum under the influence of the external field Z_μ . We may write

$$\langle T q^a(x) \bar{q}^b(0) \rangle_Z = i S_Z^{ab}(x) + \langle : q^a(x) \bar{q}^b(0) : \rangle_Z. \quad (13)$$

The first term on the right-hand side is just the ordinary propagator in the background fields Z_μ . It can be calculated from the expression

$$\begin{aligned} i S_Z^{ab}(x) &= i S_0^{ab}(x) - i \int d^4 z \, i S_0^{ac}(x-z) \gamma_\mu \\ &\quad \times \left[g A_n^\mu(z) \left(\frac{\lambda_n}{2} \right)^{cd} + h Z^\mu(z) \delta^{cd} \right] i S_0^{db}(z), \end{aligned} \quad (14)$$

where $i S_0^{ab}(x)$ is the Fourier transform of $i \delta^{ab}/(\hat{p} - m)$.

The second term in Eq. (13) has to do with the nontrivial property of the QCD vacuum; that is, the vacuum expectation value of the normal product of operators in general differs from zero. We may obtain the explicit expression making use of the Taylor expansion together with the equation of motion

$$\begin{aligned} \langle : q(x) \bar{q}(0) : \rangle &= \langle : q \bar{q} : \rangle + x^\mu \langle : (\partial_\mu q) \bar{q} : \rangle \\ &\quad + \frac{1}{2} x^\mu x^\nu \langle : (\partial_\mu \partial_\nu q) \bar{q} : \rangle + \cdots, \end{aligned} \quad (15)$$

$$\langle : q^a \bar{q}^b : \rangle = -\frac{1}{12} \delta^{ab} [1 + \frac{1}{2} h_q \chi \sigma_{\mu\nu} Z_{\mu\nu}^A] \langle \bar{q} q \rangle, \quad (16)$$

$$\langle : (\partial_\mu q^a) \bar{q}^b : \rangle = -\frac{1}{12} \delta^{ab} \zeta h_q \gamma^\nu Z_{\mu\nu}^S \langle \bar{q} q \rangle, \quad (17)$$

$$\begin{aligned} \langle : (\partial_\mu \partial_\nu q^a) \bar{q}^b : \rangle &= \frac{h_q}{144} \left(1 + \kappa - \frac{\xi}{2} \right) \langle \bar{q} q \rangle Z_A^{\rho\sigma} (g_{\mu\rho} \sigma_{\nu\sigma} \\ &\quad + g_{\nu\rho} \sigma_{\mu\sigma}) + \frac{g}{96} \langle \bar{q} \sigma \cdot G q \rangle g_{\mu\nu} + \frac{h_q}{144} \\ &\quad \times (1 + \kappa + \xi) \langle \bar{q} q \rangle Z_A^{\rho\sigma} g_{\mu\nu} \sigma_{\rho\sigma} - \frac{i h_q}{24} \\ &\quad \times (1 + 2\phi) \langle \bar{q} q \rangle Z_{\mu\nu}^S + \frac{h_q}{48} \phi \langle \bar{q} q \rangle Z_S^{\rho\sigma} \\ &\quad \times (g_{\mu\rho} \sigma_{\nu\sigma} + g_{\nu\rho} \sigma_{\mu\sigma}). \end{aligned} \quad (18)$$

Some formulas useful for simplification are listed below:

$$\langle q_i^a \bar{q}_j^b G_{\alpha\beta}^m \rangle_0 = -\frac{1}{192} \langle \bar{q} \sigma \cdot G q \rangle (\sigma_{\alpha\beta})_{ij} \left(\frac{\lambda_n}{2} \right)^{ab}, \quad (19)$$

$$g\langle q_i^a \bar{q}_j^b G_{\alpha\beta}^n \rangle_Z$$

$$= -\frac{h_q}{16} \langle \bar{q}q \rangle \left(\frac{\lambda_n}{2} \right)^{ab} \left[\delta_{ij} \kappa Z_{\alpha\beta} - \frac{i}{4} \xi \epsilon_{\alpha\beta\lambda\sigma} Z^{\lambda\sigma} (\gamma_5)_{ij} \right]. \quad (20)$$

Here the subscript Z denotes the condensate values induced by the external field and the subscript 0 represents the case without the external field.

To sum up, we obtain the quark propagator as a short-distance expansion:

$$\begin{aligned} \langle T q^a(x) \bar{q}^b(0) \rangle &= i \delta^{ab} \frac{\hat{x}}{2\pi^2 x^4} + \frac{ig}{32\pi^2 x^2} \left(\frac{\lambda^n}{2} \right)^{ab} G_{\alpha\beta}^n (\hat{x} \sigma^{\alpha\beta} + \sigma^{\alpha\beta} \hat{x}) + \delta^{ab} \frac{ih_q}{32\pi^2 x^2} Z_{\alpha\beta} (\hat{x} \sigma^{\alpha\beta} + \sigma^{\alpha\beta} \hat{x}) \\ &\quad - \delta^{ab} \frac{h_q}{8\pi^2 x^4} Z_{\alpha\beta} x^\alpha x^\beta \hat{x} - \frac{1}{12} \delta^{ab} \langle \bar{q}q \rangle - \frac{ih_q}{144} \delta^{ab} \left(1 + \kappa - \frac{\xi}{2} \right) \langle \bar{q}q \rangle Z_A^{\rho\sigma} (x_\rho x_\sigma - \hat{x} \gamma_\sigma x_\rho) \\ &\quad - \frac{1}{12} \delta^{ab} \xi h_q x^\mu \gamma^\nu Z_{\mu\nu}^S \langle \bar{q}q \rangle - \frac{1}{24} \delta^{ab} \chi h_q \sigma_{\mu\nu} Z_A^{\mu\nu} \langle \bar{q}q \rangle + \delta^{ab} \frac{g}{192} \langle \bar{q} \sigma \cdot G q \rangle x^2 + \delta^{ab} \frac{h_q}{288} (1 + \kappa + \xi) \\ &\quad \times \langle \bar{q}q \rangle Z_A^{\rho\sigma} \sigma_{\rho\sigma} x^2 - \delta^{ab} \frac{ih_q}{48} (1 + \phi) \langle \bar{q}q \rangle Z_{\mu\nu}^S x^\mu x^\nu + \delta^{ab} \frac{ih_q}{48} \phi \langle \bar{q}q \rangle Z_{\mu\nu}^S \hat{x} x^\mu \gamma^\nu, \end{aligned} \quad (21)$$

with $\hat{x} \equiv \gamma^\mu x_\mu$. Note that it is an extension from [10] since $Z_{\mu\nu}^S$ has been explicitly included. The terms in the antisymmetric part also differ slightly from [8].

B. Correlation function evaluated at the quark level

To evaluate the correlation function at the quark level, we make use of the quark propagator together with the factorization approximation:

$$\langle T \eta_p(x) \bar{\eta}_p(0) \rangle = -2 \epsilon^{abc} \epsilon^{a'b'c'} \text{Tr} [i S_u^{bb'}(x) \gamma_\nu C i S_u^{aa'}(x)^T C \gamma_\mu] \gamma_5 \gamma^\mu i S_d^{cc'}(x) \gamma^\nu \gamma_5, \quad (22)$$

where $i S_q(x)$ represents the quark propagator obtained earlier. The factorization approximation (via Wick's theorem for free quantum field theories) is justified in the large- N_c limit of QCD. The above expression is suitable for the background ρ^0 or ω fields. Slight modification is needed for the $\rho^\pm$ fields, but the results are consistent with the form of strong ρNN interactions.

Substituting the quark propagator into the above expression (and after carrying out the routine but tedious simplifications), we obtain the correlation function in momentum space:

$$\begin{aligned} h \Pi_{\alpha\beta}(p) &= \frac{1}{16\pi^4} (\hat{p} \sigma_{\alpha\beta} + \sigma_{\alpha\beta} \hat{p}) \left\{ \frac{1}{2} h_u p^2 \ln(-p^2) - \frac{\chi}{3} h_u \frac{a^2}{p^2} + \frac{a^2}{6p^4} \left[h_d + \frac{2}{3} h_u - \frac{1}{3} h_u (\kappa - 2\xi) \right] - \frac{\chi m_0^2}{24} h_u \frac{a^2}{p^4} - \frac{1}{48} (4h_u \right. \\ &\quad \left. + h_d) \frac{b}{p^2} \right\} - \frac{a}{96\pi^4} \sigma_{\alpha\beta} \left\{ -h_d \chi p^2 \ln(-p^2) - \frac{1}{p^2} h_u m_0^2 \ln \left(-\frac{p^2}{\mu^2} \right) - [6h_u - h_d - 2h_d(2\kappa - \xi)] \ln(-p^2) \right\} \\ &\quad - \frac{ia}{16\pi^4} \hat{p} (p_\alpha \gamma_\beta - p_\beta \gamma_\alpha) \left\{ \left(h_u + \frac{1}{2} h_d \right) \frac{1}{p^2} + \frac{1}{3} h_d \chi \ln(-p^2) + \frac{1}{4} h_u m_0^2 \frac{1}{p^4} \right\} + \frac{i}{4\pi^4} \hat{p} p_\alpha p_\beta \left\{ \frac{1}{8} (2h_u + h_d) \ln(-p^2) \right. \\ &\quad \left. - \frac{a}{3p^2} \zeta(h_u + h_d) + \frac{a^2}{3p^6} (2h_u + h_d) \right\} + \frac{i}{8\pi^4} (p_\alpha \gamma_\beta + p_\beta \gamma_\alpha) \left\{ \frac{1}{8} (2h_u + h_d) \ln(-p^2) + \frac{a}{3} \zeta(2h_u - h_d) \ln(-p^2) \right. \\ &\quad \left. - \frac{a^2}{6p^4} (2h_u + h_d) \right\} - \frac{i}{32\pi^4} \hat{p} (p_\alpha \gamma_\beta + p_\beta \gamma_\alpha) \left\{ \frac{a}{p^2} h_d \phi \right\} + \frac{ia}{16\pi^4} p_\alpha p_\beta \left\{ \frac{1}{p^2} (2h_u + h_d + \phi h_d) - \frac{8a}{3p^4} \xi h_u \right. \\ &\quad \left. + \frac{1}{2p^4} g h_u m_0^2 \right\}, \end{aligned} \quad (23)$$

where $a \equiv -(2\pi)^2 \langle q \rangle$, $b \equiv \langle g_c^2 G^2 \rangle$, and $\langle g_c \sigma \cdot G_q \rangle \equiv -m_0^2 \langle q \rangle$. The above result indicates that there are four different Lorentz-Dirac tensor structures for the symmetric part and three for the antisymmetric part, resulting in seven QCD sum rules.

C. Correlation function at the hadronic level

In general, the composite operator η_p connects the QCD vacuum to the proton state as well as to the excited baryons and the continuum (such as the $N\pi$ continuum). To consider the ground-state contribution, we introduce the ωNN couplings as follows:

$$\mathcal{L}_{Zpp} = -h \bar{\psi}_p \left(F_1^\omega \gamma_\mu + F_2^\omega \frac{i}{2m} \sigma_{\mu\nu} q^\nu \right) \psi_p \cdot \omega^\mu, \quad (24)$$

where ω^μ stands for the ω meson field and the coupling constant h at the quark level is factored out for the sake of convenience. The ρNN couplings are introduced in a similar way by substituting ω_μ by $\vec{\tau} \cdot \vec{\rho}_\mu$.

Accordingly, we write the correlation function in the background Z field as follows:

$$\begin{aligned} \Pi_{\mu\nu}(p) = & -\frac{\lambda_N^2}{4} \frac{1}{(p^2 - m^2)^2} \left\{ F_1^Z \left[(\hat{p} \sigma_{\mu\nu} + \sigma_{\mu\nu} \hat{p}) + 2m \sigma_{\mu\nu} \right. \right. \\ & + 2i(\hat{p} + m) g_{\mu\nu} + 2i(p_\mu \gamma_\nu + p_\nu \gamma_\mu) \\ & \left. \left. - 8ip_\mu p_\nu \frac{\hat{p} + m}{p^2 - m^2} \right] + F_2^Z \left[\frac{p^2 + m^2}{m} \sigma_{\mu\nu} \right. \right. \\ & \left. \left. - 2i \frac{\hat{p}}{m} (p_\mu \gamma_\nu - p_\nu \gamma_\mu) + (\hat{p} \sigma_{\mu\nu} + \sigma_{\mu\nu} \hat{p}) \right] \right\} + \dots, \quad (25) \end{aligned}$$

where λ_N is the overlap amplitude with $\langle 0 | \eta_p(0) | P(p) \rangle = \lambda_N u(p)$ [with $u(p)$ the proton Dirac spinor]. The ellipsis in the above equation stands for the contributions from the excited baryons and the continuum.

D. QCD sum rules

Comparing the results of the correlation function evaluated at both the quark and hadron levels, we obtain QCD sum rules corresponding to different Lorentz-Dirac tensor structures. We follow the standard procedure by performing Borel transform to both sides in order to accentuate the contributions from the ground-state baryon (i.e., the proton). We obtain, for $p_\mu \gamma_\nu + p_\nu \gamma_\mu$,

$$\begin{aligned} \frac{\lambda_N^2 F_1^Z h}{2M^2} e^{-m^2/M^2} = & \frac{M^4}{64\pi^4} (2h_u + h_d) + \frac{\zeta M^2 a}{24\pi^4} (2h_u - h_d) \\ & + \frac{a^2}{48\pi^4 M^2} (2h_u + h_d); \quad (26) \end{aligned}$$

for $\hat{p} p_\mu p_\nu$,

$$\begin{aligned} \frac{\lambda_N^2 F_1^Z h}{M^4} e^{-m^2/M^2} = & \frac{M^2}{32\pi^4} (2h_u + h_d) - \frac{\zeta a}{12\pi^4} (h_u + h_d) \\ & + \frac{a^2}{24\pi^4 M^4} (2h_u + h_d); \quad (27) \end{aligned}$$

for $p_\mu p_\nu$,

$$\begin{aligned} \frac{m_N \lambda_N^2 F_1^Z h}{M^4} e^{-m^2/M^2} = & \frac{a}{16\pi^4} (2h_u + h_d + \phi h_d) \\ & + \frac{\zeta a^2}{6\pi^4 M^2} h_u - \frac{g m_0^2 a}{32\pi^4 M^2} h_u; \quad (28) \end{aligned}$$

for $\hat{p}(p_\mu \gamma_\nu + p_\nu \gamma_\mu)$,

$$0 = -\frac{a}{32\pi^4} \phi h_d; \quad (29)$$

for $\hat{p} \sigma_{\mu\nu} + \sigma_{\mu\nu} \hat{p}$,

$$\begin{aligned} & -\frac{\lambda_N^2 h}{4M^2} (F_1^Z + F_2^Z) e^{-m^2/M^2} \\ & = \frac{1}{16\pi^4} \left[-\frac{1}{2} M^4 h_u + \frac{\chi a^2}{3} h_u - \frac{\chi m_0^2 a^2}{24 M^2} h_u \right. \\ & \quad \left. + \frac{1}{18} \frac{a^2}{M^2} [2h_u + 3h_d - h_u(\kappa + 2\xi)] \right]; \quad (30) \end{aligned}$$

for $\hat{p}(p_\mu \gamma_\nu - p_\nu \gamma_\mu)$,

$$\frac{i \lambda_N^2 h}{2m_N M^2} F_2^Z e^{-m^2/M^2} = \frac{ia}{16\pi^4} \left[\frac{\chi}{3} M^2 h_d + \left(h_u + \frac{1}{2} h_d \right) \right]; \quad (31)$$

for $\sigma_{\mu\nu}$,

$$\begin{aligned} & -\frac{m_N \lambda_N^2 h}{2M^2} \left[F_1^Z + \left(1 - \frac{M^2}{2m_N^2} \right) F_2^Z \right] e^{-m^2/M^2} \\ & = -\frac{a}{96\pi^4} \left[\chi M^4 h_d + [6h_u - h_d \right. \\ & \quad \left. - h_d(4\kappa - 2\xi)] M^2 + m_0^2 h_u \ln \left(\frac{M^2}{\mu^2} \right) \right]. \quad (32) \end{aligned}$$

It is clear that the first four sum rules have to do with the symmetric part of the external field, while the remaining three are from the antisymmetric part. It is to be understood that on the left-hand sides of these sum rules contributions from the excited baryons and continuum are yet to be included explicitly, while on the right-hand sides terms of higher dimensions may become of numerical importance (especially when cancellation occurs for terms which have been explicitly included).

Some general remarks are in order by looking at the above QCD sum rules. First of all, the first two sum rules are consistent with each other if the terms in ζa are negligible compared to the others. We assume that this is indeed the

FIG. 1. Vector and tensor couplings F_1^ρ , F_1^ω , F_2^ρ , and F_2^ω plotted as functions of the Borel mass squared M^2 in the range of $0.7 \text{ GeV}^2 \leq M^2 \leq 1.2 \text{ GeV}^2$.

case and use the resultant equation to determine the vector meson-baryon coupling F_1 . In particular, we have

$$\frac{\lambda_N^2 F_1^Z h}{2M^2} e^{-m^2/M^2} = \left(\frac{M^4}{64\pi^4} + \frac{a^2}{48\pi^4 M^2} \right) (2h_u + h_d), \quad (33)$$

which yields

$$\frac{F_1^\rho}{F_1^\omega} = \frac{g_{\rho NN}}{g_{\omega NN}} = \frac{1}{3}. \quad (34)$$

Such prediction is in excellent agreement with the nuclear force studies: $g_\rho = 2.79$ and $g_\omega = 8.37$ [4,3].

On the other hand, we see from the fourth sum rule that the condensate parameter ϕ is likely to be small unless the excited-state contributions or terms of much higher dimensions come into play for cancellation.

Assuming that the induced condensate parameters χ , κ , and ξ are identical for the ρ and ω (as they are determined primarily by the Lorentz-Dirac tensor structure), we may use F_1 obtained above in connection with the last three sum rules and eliminate all the induced condensate parameters altogether. In this way, we obtain

$$\frac{\lambda_N^2}{2M^2} (F_2^\rho - F_2^\omega) e^{-m^2/M^2} = \frac{1}{16\pi^4} \left(\frac{1}{2} M^4 + \frac{4}{3} \frac{a^2}{M^2} \right), \quad (35)$$

$$\frac{\lambda_N^2}{2M^2} (F_2^\rho + F_2^\omega) e^{-m^2/M^2} = \frac{m_N a}{8\pi^4}. \quad (36)$$

These sum rules allow us to determine the tensor couplings F_2^ρ and F_2^ω in a model-independent manner.

IV. NUMERICAL RESULTS

To analyze each individual sum rule (corresponding to the specific Lorentz-Dirac tensor structure), knowledge of the induced condensates is an important input. As shown at the end of the previous section, however, it is possible to elimi-

FIG. 2. Ratios F_2^ρ/F_1^ρ and F_2^ω/F_1^ω plotted as functions of the Borel mass squared M^2 .

nate these unknowns from the equations, resulting in four equations on F_1^ρ , F_1^ω , F_2^ρ , and F_2^ω . These equations involve only one well-known condensate parameter, i.e., the quark condensate a ($\approx 0.546 \text{ GeV}^3$). Of course, derivation of these equations suffers from the fact that contributions from the excited states and continuum and terms of higher dimensions have been neglected. Granting that these higher-order contributions are of much less numerical significance, however, predictions from these four equations can be used with reasonable confidence.

There are additional “standard” modifications which may be implemented in the analysis without jeopardizing the possibility of obtaining model-independent results. First, the anomalous dimension associated with the composite operator can be taken into account for each term in the sum rules. Second, the contributions from the excited states and continuum may be approximated by the prediction at the quark level beyond a certain threshold, say, $p^2 \geq W^2$ —thus, the need to consider explicitly these contributions can be avoided altogether. The resultant sum rules are recorded below:

$$F_1^\rho = \frac{1}{32\pi^4 \lambda_N^2} \left(M^6 E_1 L^{-4/9} + \frac{4}{3} a^2 L^{4/9} \right) e^{m^2/M^2}, \quad (37)$$

$$F_1^\omega = 3F_1^\rho, \quad (38)$$

$$F_2^\rho = \frac{1}{32\pi^4 \lambda_N^2} \left(M^6 E_1 L^{-4/9} + \frac{8}{3} a^2 L^{4/9} + 4maM^2 \right) e^{m^2/M^2}, \quad (39)$$

$$F_2^\omega = \frac{1}{32\pi^4 \lambda_N^2} \left(-M^6 E_1 L^{-4/9} - \frac{8}{3} a^2 L^{4/9} + 4maM^2 \right) e^{m^2/M^2}, \quad (40)$$

FIG. 3. Tensor couplings, as predicted by the sum rule (31), plotted for several reasonable values of χ (in units of GeV^{-2}). The value $\chi = -6$ reproduces very well our previous prediction on the tensor couplings.

with

$$E_1 = 1 - \left(1 + \frac{W^2}{M^2}\right) e^{-W^2/M^2}, \quad L = \frac{\ln(M/\Lambda_{\text{QCD}})}{\ln(\mu/\Lambda_{\text{QCD}})}.$$

To perform numerical analysis, we use the Belaev-Ioffe nucleon mass sum rule to fix the overlap parameter λ_N^2 ($\approx 1.2 \times 10^{-3} \text{ GeV}^6$). We use the standard parameters: $a = 0.546 \text{ GeV}$, $\Lambda_{\text{QCD}} = 0.1 \text{ GeV}$, $\mu = 0.5 \text{ GeV}$, and $W = 1.45 \text{ GeV}$ [10]. The sample numerical results are presented in Figs. 1–4.

In Fig. 1, we plot the vector and tensor couplings F_1^ρ , F_1^ω , F_2^ρ , and F_2^ω as functions of the Borel mass squared M^2 in the range of $0.7 \text{ GeV}^2 \leq M^2 \leq 1.2 \text{ GeV}^2$. It is seen from this figure that these functions are well behaved, giving rise to the predictions

$$F_1^\rho \approx 0.6, \quad F_2^\rho \approx 2.3,$$

$$F_1^\omega \approx 1.9, \quad F_2^\omega \approx 0.4.$$

As indicated earlier, $F_1^\omega/F_1^\rho \approx 3$ is in line with nuclear force studies [3,4]. However, F_2^ρ/F_1^ρ is a little too low in this regard. It appears rather difficult, in the context of the QCD sum rule method, to get a value for F_2^ρ which is larger than the above prediction by 50% (as required in nuclear force studies). Nevertheless, this should not be considered as a major drawback, since exchange of some heavier tensor mesons with suitable quantum numbers could easily help to jack up the number for F_2^ρ and such possibility is taken into account effectively in nuclear force studies by treating F_2^ρ as an adjustable parameter.

FIG. 4. Sum rules for the vector couplings analyzed for several different values of $\kappa - 2\xi$ at $\chi = -6 \text{ GeV}^{-2}$. It is seen that the sum rules are insensitive to $\kappa - 2\xi$, leaving this parameter undetermined at this stage.

In Fig. 2, the ratios F_2^ρ/F_1^ρ and F_2^ω/F_1^ω are plotted as functions of the Borel mass squared M^2 . The smooth curves in Fig. 2 correspond to

$$\frac{F_2^\rho}{F_1^\rho} \approx 3.6, \quad \frac{F_2^\omega}{F_1^\omega} \approx 0.2,$$

which may be compared to the values obtained in the vector-dominance model (VDM), namely, $\mu_V = 3.7$ and $\mu_S = -0.12$. Thus the results obtained from QCD sum rules are in accord with the validity of the VDM. This result should not be considered as “automatic” even though the ρqq and ωqq couplings are very similar to the electromagnetic coupling. This is because in our analysis we have also employed the sum rules related to the symmetric part of $Z_{\mu\nu}$ (which are absent in the electromagnetic case).

From nuclear potential studies [3,4] which fit the nucleon-nucleon scattering data, we have $g_\rho = 2.79$ and $g_\omega = 8.37$. Making use of such values, we obtain the ρ quark or ω quark coupling $h \approx 4.4$.

We may now turn our attention to the original QCD sum rules which involve the induced condensation parameters χ . In particular, we may use the sum rule (31) to provide a determination of χ . The result is illustrated in Fig. 3, for several reasonable values of χ (in units of GeV^{-2}). The value $\chi = -6$ reproduces very well our previous prediction on the tensor couplings.

Using Eq. (31) to eliminate the unknown F_2^Z in Eq. (30), we obtain the sum rules for only the vector coupling F_1^Z . These sum rules are analyzed in Fig. 4, for several different values of $\kappa - 2\xi$ at $\chi = -6 \text{ GeV}^{-2}$. It is seen that the sum rules are insensitive to $\kappa - 2\xi$, leaving this parameter undetermined at this stage.

V. SUMMARY

In this paper, we have extended the method of QCD sum rules in external fields to determine the ρNN and ωNN

strong couplings, including both the vector and tensor couplings. For the vector couplings, we obtain $g_{\rho NN}/g_{\omega NN}=1/3$, while for the tensor couplings the large ρNN tensor coupling F_2^p arises naturally. Such information has been used for years in construction of nuclear potentials, but the QCD sum rule method offers the first justification of such parameter values directly from QCD.

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