Flavor-changing neutral couplings in a model of radiative fermion mass

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The processes of flavor-changing neutral currents involving neutral gauge bosons are investigated in an asymmetric left-right model where the light fermions get their masses through radiative corrections. The mixings of $K^0-\bar{K}^0$, $D^0-\bar{D}^0$, and $B^0-\bar{B}^0$ are studied and it is found that all, but with a constraint on the upper bound of down-type quark mass m_N of the fourth generation, are compatible with the experimental results. The flavor-changing decays of the light neutral gauge boson Z_1 and charged leptons are also studied. It is shown that the branching ratio $B(Z_1 \rightarrow bs + b\bar{s})$ can reach the order of 10^{-7} which is within the margin to observe the flavor-changing decays of Z_1 at CERN LEP. All processes involving flavor-changing decays of charged leptons are well below the experimental limits.

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I. INTRODUCTION

This is an extension of previous work [1,2] on an asymmetric left-right electroweak gauge model in which the light quarks and leptons acquire their masses through radiative corrections; in this way, the wide range of Yukawa couplings in the standard model starting from the order of 10^{-6} , if all light neutrinos are strictly massless in nature, to unity is reduced to the order of 10^{-2} to unity. This is attributed to the fact that the breaking energy scale of $SU(2)_R$ is 1 TeV or greater and the introduction of charged heavy singlet fermions is essential. Such a radiative model is used primarily to alleviate the puzzle of the wide range of the Yukawa couplings. In this model, the conventional $SU(2)_L \times SU(2)_R \times U(1)$ electroweak gauge model [3] is extended to include two heavy singlet quarks and one heavy singlet lepton. The scalar sector consists of two $(2, 1, \frac{1}{2})$ doublets, two $(1, 2, \frac{1}{2})$ doublets, one $(2, 1, \frac{3}{2})$ doublet, one $(1, 2, \frac{3}{2})$ doublet, as well as one $(1, 3, 1)$ triplet. A Z_2 -discrete symmetry serves to distinguish the electron and its neutrino from the other leptons. Realistic radiative masses for all light quarks and leptons are then obtained. The addition of new heavy quark and lepton singlets has been discussed in the literature [4,5] for generating the seesaw mechanism to both the charged and neutral fermions. On the other hand, the Glashow-Iliopoulos-Maiani (GIM) mechanism [6] in the standard model is due to the fact that all quarks with given helicity and charge transform identically under $SU(2)_L \times U(1)_Y$. It results in the absence of flavor-changing neutral couplings (FCNC's) of quarks to the Z gauge boson in the standard model. However, the existence of the additional $SU(2)_L \times SU(2)_R \times U(1)_Y$ singlet quarks P, N as well as the singlet charged lepton E gives rise to FCNC's through their mixing with conventional quarks and charged leptons in this asymmetric left-right model. In this paper, we investigate the FCNC processes in this model to see if all are compatible with the current experimental results.

In Sec. II, we set up our notation by listing the particle

content and briefly describe the radiative mechanism for generating charged fermions. We refer the reader to Ref. [2] for a detailed description. In Sec. III, we derive the Lagrangian of FCNC interactions for the first three generations of charged fermions and express approximately all the strengths of FCNC only in terms of the Yukawa coupling f_R , the vacuum expectation value v_R , as well as the top quark mass m_t in the upper quark sector, using the numerology [7] of fermion mass hierarchy to estimate the physical processes in the subsequent sections. In Sec. IV, we study the mixings of $K^0-\bar{K}^0$, $D^0-\bar{D}^0$, and $B^0-\bar{B}^0$. It is found that all, but with constraints on the upper and lower bound of mass m_N , are compatible with the existing experimental results. In Sec. V, we investigate the flavor-changing decays of Z_1 and charged leptons and show that all are well below the experimental limits. Finally, a brief summary is given in Sec. VI.

II. DESCRIPTION OF A SEESAW MODEL

The asymmetric left-right gauge model [2] is based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$. The usual quarks and leptons have the conventional assignments

$$\begin{bmatrix} u' \\ d' \end{bmatrix}_L, \begin{bmatrix} c' \\ s' \end{bmatrix}_L, \begin{bmatrix} t' \\ b' \end{bmatrix}_L : (3, 2, 1, \frac{1}{6}), \quad (2.1)$$

$$\begin{bmatrix} u' \\ d' \end{bmatrix}_R, \begin{bmatrix} c' \\ s' \end{bmatrix}_R, \begin{bmatrix} t' \\ b' \end{bmatrix}_R : (3, 1, 2, \frac{1}{6}), \quad (2.2)$$

$$\begin{bmatrix} \nu'_e \\ e' \end{bmatrix}_L, \begin{bmatrix} \nu'_\mu \\ \mu' \end{bmatrix}_L, \begin{bmatrix} \nu'_\tau \\ \tau' \end{bmatrix}_L : (1, 2, 1, -\frac{1}{2}), \quad (2.3)$$

$$\begin{bmatrix} \nu'_e \\ e' \end{bmatrix}_R, \begin{bmatrix} \nu'_\mu \\ \mu' \end{bmatrix}_R, \begin{bmatrix} \nu'_\tau \\ \tau' \end{bmatrix}_R : (1, 1, 2, -\frac{1}{2}). \quad (2.4)$$

The prime on the field indicates a gauge eigenstate. In addition, it is assumed in the model that there are new quark and lepton singlets as follows:

$$P'_L, P'_R : (3, 1, 1, \frac{2}{3}), \quad (2.5)$$

$$N'_L, N'_R : (3, 1, 1, -\frac{1}{3}), \quad (2.6)$$

$$E'_L, E'_R : (1, 1, 1, -1). \quad (2.7)$$

The scalar sector consists of six doublets

$$\Phi'_{1L}, \Phi'_{2L} : (1, 2, 1, \frac{1}{2}), \quad (2.8)$$

$$\Phi'_{1R}, \Phi'_{2R} : (1, 1, 2, \frac{1}{2}), \quad (2.9)$$

$$\eta'_L : (1, 2, 1, \frac{3}{2}), \quad (2.10)$$

$$\eta'_R : (1, 1, 2, \frac{3}{2}), \quad (2.11)$$

and one triplet

$$\Delta'_R : (1, 1, 3, 1). \quad (2.12)$$

The neutral lepton singlet S and a Higgs triplet Δ'_L are absent in the model. The reason for this choice has to do with the smallness of neutrino masses as already elucidated in Ref. [1]. Because of the absence of Δ'_L , this model is clearly asymmetric in the interchange of left and right

fields.

A discrete symmetry is imposed so that $(\nu'_e, e')_{L,R}$ and $\eta'_{L,R}$ are odd but all other fields are even. This discrete symmetry prohibits the μ - e transitions. Since $\Phi'_{1L,R}$ and $\Phi'_{2L,R}$ transform identically, we can define them without loss of generality to be such that

$$\langle \Phi'^0_{1L,R} \rangle \equiv \begin{bmatrix} 0 \\ v_{L,R}/\sqrt{2} \end{bmatrix}, \quad v_{L,R} > 0, \quad (2.13)$$

but

$$\langle \Phi'^0_{2L,R} \rangle \equiv 0. \quad (2.14)$$

In addition, we have

$$\langle \Delta'^0_R \rangle \equiv \begin{bmatrix} 0 \\ 0 \\ u/\sqrt{2} \end{bmatrix}, \quad u > 0. \quad (2.15)$$

In the following, we briefly review the charged fermion mass generation. For detailed description, we refer the reader to Ref. [2]. In this model, the Yukawa interactions are given by [2]

$$\begin{aligned} -L_Y = & f_{PL}(\overline{t'}, \overline{b''})_L P'_R \tilde{\Phi}'_{1L} + f_{PR}(\overline{t'}, \overline{b''})_R P'_L \tilde{\Phi}'_{1R} + f_{NL}(\overline{t''}, \overline{b'})_L N'_R \Phi'_{1L} \\ & + f_{NR}(\overline{t''}, \overline{b'})_R N'_L \Phi'_{1R} + f_{PL3}(\overline{t'}, \overline{b''})_L P'_R \tilde{\Phi}'_{2L} + f_{PR3}(\overline{t'}, \overline{b''})_R P'_L \tilde{\Phi}'_{2R} \\ & + f_{NL3}(\overline{t''}, \overline{b'})_L N'_R \Phi'_{2L} + f_{NR3}(\overline{t''}, \overline{b'})_R N'_L \Phi'_{2R} + f_{PL2}(\overline{c'}, \overline{s''})_L P'_R \tilde{\Phi}'_{2L} \\ & + f_{PR2}(\overline{c'}, \overline{s''})_R P'_L \tilde{\Phi}'_{2R} + f_{NL2}(\overline{c''}, \overline{s'})_L N'_R \Phi'_{2L} + f_{NR2}(\overline{c''}, \overline{s'})_R N'_L \Phi'_{2R} \\ & + f_{EL}(\overline{\nu'_\tau}, \overline{\tau'})_L E'_R \Phi'_{1L} + f_{ER}(\overline{\nu'_\tau}, \overline{\tau'})_R E'_L \Phi'_{1R} + f_{EL3}(\overline{\nu'_\tau}, \overline{\tau'})_L E'_R \Phi'_{2L} \\ & + f_{ER3}(\overline{\nu'_\tau}, \overline{\tau'})_R E'_L \Phi'_{2R} + f_{EL2}(\overline{\nu'_\mu}, \overline{\mu'})_L E'_R \Phi'_{2L} + f_{ER2}(\overline{\nu'_\mu}, \overline{\mu'})_R E'_L \Phi'_{2R} \\ & + f_{EL1}(\overline{E'^c_L}(\nu'_e, e'))_L \eta'_L + f_{ER1}(\overline{E'^c_R}(\nu'_e, e'))_R \eta'_R - h_{11} \overline{l'^c_{1R}} \tau_R \cdot \Delta'_R l'_{1R} \\ & - h_{22} \overline{l'^c_{2R}} \tau_R \cdot \Delta'_R l'_{2R} - h_{23} \overline{l'^c_{2R}} \tau_R \cdot \Delta'_R l'_{3R} - h_{23} \overline{l'^c_{3R}} \tau_R \cdot \Delta'_R l'_{2R} \\ & - h_{33} \overline{l'^c_{3R}} \tau_R \cdot \Delta'_R l'_{3R} + \text{H.c.} + m'_P \overline{P'} P' + m'_N \overline{N'} N' + m'_E \overline{E'} E', \end{aligned} \quad (2.16)$$

In the charged fermion sector, the mass terms are

$$-L_{MF} = \sum_{F'=P',N',E'} \overline{F}'_L \hat{M}_F F'_R + \overline{F}'_R \hat{M}_F^\dagger F'_L, \quad (2.17)$$

where

$$P'_{\alpha L,R} \equiv (u', c', t', P')^T_{L,R}, \quad (2.18)$$

$$N'_{\alpha L,R} \equiv (d', s', b', N')^T_{L,R}, \quad (2.19)$$

$$E'_{\alpha L,R} \equiv (e', \mu', \tau', E')^T_{L,R}. \quad (2.20)$$

To avoid confusion, we put a caret on M to denote it as a matrix. There exist six biunitary transition matrices $U_{FI} (F=P, N, E; I=L, R)$ from electroweak eigenstates to mass eigenstates: i.e.,

$$\hat{M}_{FD} = U_{FL}^\dagger \hat{M}_F U_{FR} \quad (2.21)$$

and

$$F_I = U_{FI}^\dagger F'_I. \quad (2.22)$$

The definition of $\hat{M}_F$ in Eq. (2.17) gives the eigenvalue equations

$$\hat{M}_F^\dagger \hat{M}_F F'_R \equiv H_{FR} F'_R = E_{FR} F'_R \quad (2.23)$$

and

$$\hat{M}_F \hat{M}_F^\dagger F'_L \equiv H_{FL} F'_L = E_{FL} F'_L. \quad (2.24)$$

Of course, the eigenvalues E_{FI} denote the mass squared of the charged fermion F_I ($I=L, R$).

At the tree level, we have the mass matrix

$$\hat{M}_F^{(0)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{FL} \\ 0 & 0 & m_{FR} & m'_F \end{bmatrix} \quad (F=P, N, E), \quad (2.25)$$

where

$$m_{FI} \equiv \frac{f_{FI} v_I}{\sqrt{2}}. \quad (2.26)$$

With this tree-level mass matrix, we can obtain the transition matrix

$$(U_{FI}^{(0)})_{\alpha\beta} = (F_{\beta I}^{(0)})_{\alpha} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & C_{FI} & -S_{FI} \\ 0 & 0 & S_{FI} & C_{FI} \end{bmatrix}, \quad (2.27)$$

where

$$C_{FI} \equiv \cos\theta_{FI}, \quad S_{FI} \equiv \sin\theta_{FI}, \quad (2.28)$$

and the mass eigenstates can be written as

$$(F_{\alpha I}^{(0)})_{\beta} = \sum_{\gamma=1}^4 (U_{FI}^{(0)})_{\beta\gamma} (F_{\alpha I}^{(0)})_{\gamma} \quad (\alpha, \beta = 1, 2, 3, 4). \quad (2.29)$$

Hence we see that the masses of the first two generations are zeros at the tree level. They acquire masses from radiative corrections.

The one-loop contributions to the mass matrix are of the form

$$\begin{aligned} (B_{QI})_{31} &= [C_{qI}(\lambda V_{QI}^{(1)})_{14} + S_{qI}(\lambda V_{QI}^{(1)})_{24}] (\tilde{Q}'^{(0)}_{3I})_4 / E_{Q3I}^{(0)}, \\ (B_{QI})_{32} &= [-S_{qI}(\lambda V_{QI}^{(1)})_{14} + C_{qI}(\lambda V_{QI}^{(1)})_{24}] (\tilde{Q}'^{(0)}_{3I})_4 / E_{Q3I}^{(0)}, \\ (B_{QI})_{41} &= [C_{qI}(\lambda V_{QI}^{(1)})_{14} + S_{qI}(\lambda V_{QI}^{(1)})_{24}] (\tilde{Q}'^{(0)}_{4I})_4 / E_{Q4I}^{(0)}, \\ (B_{QI})_{42} &= [-S_{qI}(\lambda V_{QI}^{(1)})_{14} + C_{qI}(\lambda V_{QI}^{(1)})_{24}] (\tilde{Q}'^{(0)}_{4I})_4 / E_{Q4I}^{(0)}, \\ (B_{QI})_{43} &= (\lambda V_{QI}^{(1)})_{34} [(\tilde{Q}'^{(0)}_{3I})_4 (\tilde{Q}'^{(0)}_{4I})_3 + (\tilde{Q}'^{(0)}_{3I})_3 (\tilde{Q}'^{(0)}_{4I})_4] / (E_{Q4I}^{(0)} - E_{Q3I}^{(0)}), \end{aligned} \quad (2.33)$$

where $Q = P$ or N . For instance, $(\lambda V_{PL}^{(1)})_{14} = m_{PR} m_{ut}$, $(\lambda V_{PL}^{(1)})_{24} = m_{PR} m_{ct}$, $(\tilde{P}'^{(0)}_{3L})_4 = S_{PL}$, $(\tilde{P}'^{(0)}_{4L})_4 = C_{PL}$, and hence

$$|u'_L\rangle = c_{pL} |u''^{(0)}\rangle + s_{pL} |c''^{(0)}\rangle - \frac{f_{PR} v_R}{\sqrt{2}} (C_{pL} m_{ut} + S_{pL} m_{ct}) \left[\frac{S_{PL}}{m_t^{(0)}} |t'^{(0)}\rangle + \frac{C_{PL}}{m_p^{(0)}} |p'^{(0)}\rangle \right]. \quad (2.34)$$

In the charged lepton sector, we have

$$\begin{aligned} (B_{EI})_{31} &= (B_{EI})_{41} = 0, \\ (B_{EI})_{32} &= (E'_{3I})_4 (\lambda V_{EI}^{(1)})_{24} / E_{E3I}^{(0)}, \\ (B_{EI})_{42} &= (E'_{4I})_4 (\lambda V_{EI}^{(1)})_{24} / E_{E4I}^{(0)}, \\ (B_{EI})_{43} &= (\lambda V_{EI}^{(1)})_{34} [(E'_{3I})_4 (E'_{4I})_3 + (E'_{3I})_3 (E'_{4I})_4] / (E_{E4I}^{(0)} - E_{E3I}^{(0)}). \end{aligned} \quad (2.35)$$

Hence the transition matrix is

$$(U_{FI})_{\alpha\beta} = (F'_{\beta I})_{\alpha} \quad (2.36)$$

$$= \sum_{\gamma=1}^4 (B_{FI})_{\beta\gamma} (\tilde{U}_{FI}^{(0)})_{\alpha\gamma}. \quad (2.37)$$

where

$$(\tilde{U}_{FI}^{(0)})_{\alpha\gamma} \equiv (\tilde{F}_{\gamma I}^{(0)})_{\alpha}. \quad (2.38)$$

In the matrix notation, it can be written as

$$\hat{M}_F^{(1)} = \begin{bmatrix} m_{F_1 F_1} & m_{F_1 F_2} & m_{F_1 F_3} & 0 \\ m_{F_2 F_1} & m_{F_2 F_2} & m_{F_2 F_3} & 0 \\ m_{F_3 F_1} & m_{F_3 F_2} & m_{F_3 F_3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (2.30)$$

where $m_{e\mu}$, $m_{e\tau}$, $m_{\mu e}$, and $m_{\tau e}$ are zeros because of Z_2 -discrete symmetry. Using the perturbation theory, we obtain the eigenstates to the first order $O(\lambda)$ of radiative correction as follows [2]:

$$(F'_{\alpha I})_{\beta} = \sum_{\gamma=1}^4 (B_{FI})_{\alpha\gamma} (\tilde{F}_{\gamma}^{(0)})_{\beta} \quad (\alpha, \beta = 1, 2, 3, 4), \quad (2.31)$$

where

$$(B_{FI})_{\alpha\gamma} = \begin{bmatrix} 1 & 0 & -(B_{FI})_{31} & -(B_{FI})_{41} \\ 0 & 1 & -(B_{FI})_{32} & -(B_{FI})_{42} \\ (B_{FI})_{31} & (B_{FI})_{32} & 1 & -(B_{FI})_{43} \\ (B_{FI})_{41} & (B_{FI})_{42} & (B_{FI})_{43} & 1 \end{bmatrix}. \quad (2.32)$$

In the quark sector, we have

$$U_{FI}^{\dagger} = B_{FI} \tilde{U}_{FI}^{(0)\dagger}. \quad (2.39)$$

The mass eigenstates are then

$$(F_{\alpha I})_{\beta} = \sum_{\gamma=1}^4 (U_{FI}^{\dagger})_{\beta\gamma} (F'_{\alpha I})_{\gamma}. \quad (2.40)$$

III. STRENGTH OF FCNC'S

The Lagrangian of the neutral current interactions in this model is

$$L^{\text{NC}} = g_L j_{\mu L}^3 W_L^{3\mu} + g_R j_{\mu R}^3 W_R^{3\mu} + g' j_\mu^Y B^\mu, \quad (3.1)$$

where

$$j_{\mu I}^3 = \sum_{i=1}^3 \left[\bar{l}_{iI} \gamma_\mu \frac{\tau_{3I}}{2} l'_{iI} + \bar{q}'_{uiI} \gamma_\mu \frac{\tau_{3I}}{2} q'_{uiI} \right], \quad (3.2)$$

$$j_{\mu Y} = \sum_{I=L,R} \left\{ \sum_{i=1}^3 (Y_i \bar{l}_{iI} \gamma^\mu l'_{iI} + Y_q \bar{q}'_{uiI} \gamma^\mu q'_{uiI}) + Y_E \bar{E}' \gamma^\mu E' \right. \\ \left. + Y_P \bar{P}' \gamma^\mu P' + Y_N \bar{N}' \gamma^\mu N' \right\}, \quad (3.3)$$

and

$$q'_{uiL,R} \equiv \begin{bmatrix} p'_i \\ n'_i \end{bmatrix}_{L,R}. \quad (3.4)$$

Define two orthogonal weak gauge fields Z and D by

$$\begin{bmatrix} A^\mu \\ Z^\mu \\ D^\mu \end{bmatrix} = \begin{bmatrix} \sqrt{1-s_L^2-s_R^2} & s_L & s_R \\ -s_L & \sqrt{1-s_L^2-s_R^2} & c_L \\ c_L & -s_R & 0 \end{bmatrix} \begin{bmatrix} B^\mu \\ W_L^{3\mu} \\ W_R^{3\mu} \end{bmatrix} \\ \times \begin{bmatrix} B^\mu \\ W_L^{3\mu} \\ W_R^{3\mu} \end{bmatrix}, \quad (3.5)$$

where

$$s_I \equiv \sin \theta_{W_I} = e/g_I, \quad c_I \equiv \cos \theta_{W_I}. \quad (3.6)$$

θ_{W_L} is the usual Weinberg angle. The Lagrangian in Eq. (3.1) can then be written in terms of fermion mass eigenstates [2,8]:

$$L^{\text{NC}} = \frac{e}{s_L c_L} \left\{ j_{ZL}^\mu Z_\mu + \left[\frac{s_L s_R}{\sqrt{1-s_L^2-s_R^2}} j_{ZL}^\mu \right. \right. \\ \left. \left. + \frac{s_L c_L^2}{s_R \sqrt{1-s_L^2-s_R^2}} j_{ZR}^\mu \right] D_\mu \right\}, \quad (3.7)$$

where

$$j_{ZI}^\mu \equiv j_{\mu I}^3 - \sin^2 \theta_{W_I} j_{\mu I}^{\text{em}} \\ = \sum_{F=P,N,E} \sum_{\alpha,\beta=1}^4 \{ \bar{F}_{\alpha I} \gamma^\mu [S_I(F)]_{\alpha\beta} F_{\beta I} \\ - Q(F) s_I^2 \bar{F}_{\alpha I} \gamma^\mu F_{\alpha I} \} \\ + \sum_{\sigma,\rho=1}^6 \{ \bar{\xi}_{\sigma I} \gamma^\mu [S_I(\nu)]_{\sigma\rho} \xi_{\rho I} \}. \quad (3.9)$$

In the above, we have used the notation

$$\underline{I} = \begin{cases} R & \text{if } I=L, \\ L & \text{if } I=R, \end{cases} \quad (3.10)$$

and

$$[S_I(F)]_{\alpha\beta} = \sum_{\gamma=1}^4 g_I(F'_{\gamma I}) (U_{FI}^\dagger)_{\alpha\gamma} (U_{FI})_{\gamma\beta}, \quad (3.11)$$

$$[S_I(\nu)]_{\sigma\rho} = \sum_{j=1}^3 g_I(\nu'_{jI}) (U_{\nu I}^\dagger)_{\sigma j} (U_{\nu I})_{j\rho}. \quad (3.12)$$

In this model, we have

$$g_I(\nu'_{jI}) = \frac{1}{2}, \\ g_I(e'_I) = -\frac{1}{2} + s_I^2, \quad g_I(E'_I) = s_I^2, \\ g_I(p'_{jI}) = \frac{1}{2} - \frac{2}{3} s_I^2, \quad g_I(P'_I) = -\frac{2}{3} s_I^2, \\ g_I(n'_{jI}) = -\frac{1}{2} + \frac{1}{3} s_I^2, \quad g_I(N'_I) = \frac{1}{3} s_I^2. \quad (3.13)$$

Hence in the tree-level mass approximation, we have

$$[S_I(F)]_{\alpha\beta} = g_I(f'_I) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & C^2 & -SC \\ 0 & 0 & -SC & S^2 \end{bmatrix}_{FI} \\ + g_I(F'_I) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & S^2 & SC \\ 0 & 0 & SC & C^2 \end{bmatrix}_{FI}, \quad (3.14)$$

$$[S_I(\nu)]_{\sigma\rho} = \begin{cases} g_I(\nu'_I) \delta_{\sigma\rho} & \text{if } \sigma, \rho = 1, 2, 3, \\ 0 & \text{otherwise.} \end{cases}$$

The physics fields Z_1 and Z_2 are mixtures of fields Z and D :

$$Z_1^\mu = c_Z Z + s_Z D, \quad (3.15)$$

$$Z_2^\mu = -s_Z Z + c_Z D, \quad (3.16)$$

where

$$c_Z \equiv \cos \theta_Z, \quad s_Z \equiv \sin \theta_Z. \quad (3.17)$$

Since there are no neutral flavor-changing couplings involving neutrinos due to the absence of neutral lepton singlets $S(1,1,0)$ in this model, we need only consider the interactions of charged fermions with Z_1 and Z_2 . The corresponding Lagrangian in Eq. (3.7) can be written in terms of mass eigenstates:

$$L_{ZF}^{\text{NC}} = \frac{e}{s_L c_L \sqrt{1-s_L^2-s_R^2}} \sum_{F=P,N,E} \sum_{I=L,R} \sum_{r=1}^2 A_{rI} \bar{F}_{aI} \gamma^\mu \left\{ \sum_{\gamma=1}^4 g_I(F') (U_{FI}^\dagger)_{a\gamma} (U_{FI})_{\gamma\beta} \right\} F_{\beta I} Z_{r\mu} \quad (3.18)$$

$$= \frac{e}{s_L c_L \sqrt{1-s_L^2-s_R^2}} \sum_{F=P,N,E} \sum_{I=L,R} \sum_{r=1}^2 A_{rI} \bar{F}_{aI} \gamma^\mu \left\{ \sum_{j=1}^3 [g_I(f') - g_I(F')] (U_{FI}^\dagger)_{aj} (U_{FI})_{j\beta} \right. \\ \left. + \sum_{\gamma=1}^4 g_I(F') (U_{FI}^\dagger)_{a\gamma} (U_{FI})_{\gamma\beta} \right\} F_{\beta I} Z_{r\mu} . \quad (3.19)$$

In the above, the coefficients A_{rI} are given in Ref. [8], and the second terms in Eq. (3.9) which have nothing to do with FCNC's have been removed. Because of the unitarity of transition matrices U_{FI} , only the first terms contribute to the neutral flavor-changing couplings in Eq. (3.19). Hence the Lagrangian of the FCNC interaction for the first three generations of quarks is

$$L_q^{\text{FCNC}} = \frac{e}{s_L c_L \sqrt{1-s_L^2-s_R^2}} \sum_{r=1}^2 \{ A_{rL} [g_L(q') - g_L(Q')] F_{Li k}(Q) \bar{q}_{iL} \gamma^\mu q_{kL} \\ + A_{rR} [g_R(q') - g_R(Q')] F_{Ri k}(Q) \bar{q}_{iR} \gamma^\mu q_{kR} \} Z_{r\mu} , \quad (3.20)$$

where

$$F_{Lik}(Q) \equiv \sum_{j=1}^3 (U_{QI}^\dagger)_{ij} (U_{QI})_{jk} \quad (i, k = 1, 2, 3; i \neq k) . \quad (3.21)$$

Similarly, the Lagrangian of FCNC interactions for the first three generations of charged leptons is

$$L_e^{\text{FCNC}} = \frac{e}{s_L c_L \sqrt{1-s_L^2-s_R^2}} \sum_{r=1}^2 \{ A_{rL} [g_L(e') - g_L(E')] F_{Li k}(E) \bar{e}_{iL} \gamma^\mu e_{kL} \\ + A_{rR} [g_R(e') - g_R(E')] F_{Ri k}(E) \bar{e}_{iR} \gamma^\mu e_{kR} \} Z_{r\mu} , \quad (3.22)$$

where

$$F_{Lik}(E) \equiv \sum_{j=1}^3 (U_{EI}^\dagger)_{ij} (U_{EI})_{jk} \quad (i, k = 1, 2, 3; i \neq k) . \quad (3.23)$$

From Eq. (2.27), it is easy to see that there is no FCNC at the tree-level mass approximation. At the one-loop mass level, using Eqs. (2.39), the strengths of FCNC's in Eqs. (3.21) and (3.23) can be written as

$$F_{Lik}(F) = \sum_{\alpha, \beta} \sum_{j=1}^3 (B_{FI})_{i\alpha} (\bar{U}_{FI}^{(0)\dagger})_{aj} (\bar{U}_{FI}^{(0)})_{j\beta} (B_{FI}^\dagger)_{\beta k} \quad (F = P, N, E; i, k = 1, 2, 3; i \neq k) . \quad (3.24)$$

Substituting adequate equations derived in Sec. II into Eq. (3.24), we can write all the strengths of FCNC's in terms of their mixing angles, mass matrix elements, and tree-level mass eigenvalues:

$$F_{L12}(Q) = m_{QR}^2 \left[\frac{S_{QL} C_{QL}}{m_{Q_3}^{(0)2}} - \frac{S_{QL} C_{QL}}{m_{Q_4}^{(0)2}} \right]^2 [S_{qL} C_{qL} (m_{Q_2 Q_3}^2 - m_{Q_1 Q_3}^2) + (C_{qL}^2 - S_{qL}^2) m_{Q_1 Q_3} m_{Q_2 Q_3}] , \quad (3.25)$$

$$F_{L13}(Q) = m_{QR} S_{QL} \left[\frac{S_{QL}^2}{m_{Q_3}^{(0)2}} + \frac{C_{QL}^2}{m_{Q_4}^{(0)2}} \right] (C_{qL} m_{Q_1 Q_3} + S_{qL} m_{Q_2 Q_3}) \\ - m_{QR}^2 m_{Q_3 Q_3} S_{QL} \left[\frac{C_{QL}^2 - S_{QL}^2}{m_{Q_4}^{(0)2} - m_{Q_3}^{(0)2}} \right] \left[\frac{S_{QL} C_{QL}}{m_{Q_3}^{(0)2}} - \frac{S_{QL} C_{QL}}{m_{Q_4}^{(0)2}} \right] \times (C_{qL} m_{Q_1 Q_3} + S_{qL} m_{Q_2 Q_3}) , \quad (3.26)$$

$$F_{L23}(Q) = m_{QR} S_{QL} \left[\frac{S_{QL}^2}{m_{Q_3}^{(0)2}} + \frac{C_{QL}^2}{m_{Q_4}^{(0)2}} \right] (-S_{qL} m_{Q_1 Q_3} + C_{qL} m_{Q_2 Q_3}) \\ - m_{QR}^2 m_{Q_3 Q_3} S_{QL} \left[\frac{C_{QL}^2 - S_{QL}^2}{m_{Q_4}^{(0)2} - m_{Q_3}^{(0)2}} \right] \left[\frac{S_{QL} C_{QL}}{m_{Q_3}^{(0)2}} - \frac{S_{QL} C_{QL}}{m_{Q_4}^{(0)2}} \right] (-S_{qL} m_{Q_1 Q_3} + C_{qL} m_{Q_2 Q_3}) , \quad (3.27)$$

$$F_{Rik}(Q) = F_{Lik}(Q)|_{R \leftrightarrow L, m_{Q_1 Q_3} \rightarrow m_{Q_3 Q_1}, m_{Q_2 Q_3} \rightarrow m_{Q_3 Q_2}}, \quad (3.28)$$

$$F_{L12}(E) = F_{L13}(E) = 0, \quad (3.29)$$

$$F_{L23}(E) = m_{ER} m_{\mu\tau} S_{EL} \left[\frac{S_{EL}^2}{m_{\tau}^{(0)2}} + \frac{C_{EL}^2}{m_E^{(0)2}} \right] - m_{ER}^2 m_{\tau\tau} m_{\mu\tau} S_{EL} \left[\frac{C_{EL}^2 - S_{EL}^2}{m_E^{(0)2} - m_{\tau}^{(0)2}} \right] \left[\frac{S_{EL} C_{EL}}{m_{\tau}^{(0)2}} - \frac{S_{EL} C_{EL}}{m_E^{(0)2}} \right], \quad (3.30)$$

$$F_{Rik}(E) = F_{Lik}(E)|_{R \leftrightarrow L, m_{\mu\tau} \rightarrow m_{\tau\mu}}, \quad (3.31)$$

$$F_{Iik}(F) = F_{Iki}(F) \quad (F = P, N, E; i, k = 1, 2, 3; i \neq k). \quad (3.32)$$

In the above, $F_{I12}(Q)$, $F_{I13}(Q)$, and $F_{I23}(Q)$ are of the order of $O(\lambda^2)$, $O(\lambda)$, and $O(\lambda)$, respectively; however, it does not imply that $F_{I12}(Q)$ should be less than $F_{I13}(Q)$ and $F_{I23}(Q)$. The Z_2 -discrete symmetry separating e from μ and τ results in $F_{I12}(E) = F_{I13}(E) = 0$.

For simplicity, we assume that the f -type Yukawa couplings in Eq. (2.16), which couple to the left-handed scalar fields, are equal to the corresponding f -type Yukawa couplings which couple to the right-handed scalar fields. For example, $f_F \equiv f_{FL} = f_{FR}$. Under this assumption, we have

$$m_{F_i F_j} = m_{F_j F_i} \quad (F = P, N, E; i, j = 1, 2, 3). \quad (3.33)$$

Consider the charged fermion mass matrices of the tree level and one-loop level contributions in Eqs. (2.25) and (2.30). From their structures, we know that the third generation quarks get their masses at tree level and hence, from numerology [7] of fermion mass hierarchy, it implies that

$$m_{Q_1 Q_1} \sim m_{Q_1}, \quad (3.34)$$

$$m_{Q_1 Q_i} \sim \sqrt{m_{Q_1} m_{Q_2}} \quad (i = 2, 3), \quad (3.35)$$

$$m_{Q_i Q_j} \sim m_{Q_2} \quad (i, j = 2, 3), \quad (3.36)$$

$$S_{qL} = S_{qR} \sim \frac{m_{Q_1}}{m_{Q_2}}. \quad (3.37)$$

Similarly, in the charged lepton sector, we have

$$m_{E_i E_j} \sim m_{E_2} \quad (i, j = 2, 3). \quad (3.38)$$

Using these relations, all strengths of neutral flavor-changing couplings $F_{Iik}(F)$ can be written as functions of only the unknown parameters f_F, v_R as well as m_t in the upper quark sector. We will use them to estimate FCNC processes in the subsequent sections.

IV. $K^0\text{--}\bar{K}^0$, $D^0\text{--}\bar{D}^0$, AND $B^0\text{--}\bar{B}^0$ MIXINGS

One of the distinctive features of this model could be a sizable mixing between $Q_{ik}^0 \equiv \bar{q}_i q_k$ and $\bar{Q}_{ik}^0 \equiv q_i \bar{q}_k$ ($i \neq k$) states. This mixing in our model arises at the tree level from the exchange of neutral gauge bosons $Z_{1,2}$ in the s channel. We will calculate the mass differences between Q_{ik}^0 and $\bar{Q}_{ik}^0$ in the effective four-fermion interaction approximation, assuming that all masses in the external fermion lines and momentum transfers are much smaller

than M_{Z_1} . From Eq. (3.7), the effective Lagrangian defined by

$$L_{\text{eff}}^{\text{NC}} \equiv \langle L^{\text{NC}} L^{\text{NC}} \rangle_0 \quad (4.1)$$

can be calculated by identifying $\langle Z, Z \rangle_0$, $\langle Z, D \rangle_0$, and $\langle D, D \rangle_0$ with $[(\hat{M}_{ZD}^2)^{-1}]_{11}$, $[(\hat{M}_{ZD}^2)^{-1}]_{12}$, and $[(\hat{M}_{ZD}^2)^{-1}]_{22}$, respectively [9]. Hence, after a short calculation, we get the effective Lagrangian

$$-L_{\text{eff}}^{\text{NC}} = \frac{4G_F}{\sqrt{2}} (j_{ZL}^2 + \tilde{\eta} j_{ZR}^2), \quad (4.2)$$

where

$$\tilde{\eta} \equiv \left[\frac{g_R}{g_L} \right]^2 \left[\frac{M_L}{\tilde{M}_R} \right]^2, \quad (4.3)$$

where $\tilde{M}_R^2 = g_R^2 (v_R^2 + 4u^2)/4$. In the above, the symmetric factor 2 for two identical currents has been introduced. Using Eqs. (3.8)–(3.12) and (3.21), the effective Hamiltonian describing $Q_{ik}^0\text{--}\bar{Q}_{ik}^0$ mixing can be extracted from the above effective Lagrangian of neutral current in-

FIG. 1. Mass difference Δm_K shown as a function of M_N for various $v_R = 1, 10, 40$ TeV from the shortest to the longest dashed line, respectively.

FIG. 2. Mass difference Δm_D shown as a function of M_P for various v_R as in Fig. 1.

teraction as follows:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \{ F_{Li}^2(Q) (\bar{q}_{Li} \gamma^\mu q_{Lk})^2 + \bar{\eta} F_{Rik}^2(Q) (\bar{q}_{Ri} \gamma^\mu q_{Rk})^2 \}, \quad (4.4)$$

The mass difference between Q_{ik}^0 and $\bar{Q}_{ik}^0$ is then

$$(\Delta m)_{Q_{ik}^0} = -2 \text{Re} \{ \langle Q_{ik}^0 | -L_{\text{eff}} | \bar{Q}_{ik}^0 \rangle \} \quad (4.5)$$

$$= \frac{G_F}{\sqrt{2}} \frac{2}{3} f_{Q_{ik}^0}^2 M_{Q_{ik}^0} \{ F_{Li}^2(Q) + \bar{\eta} F_{Rik}^2(Q) \}. \quad (4.6)$$

Here we have used the vacuum-saturation approximation in writing down the matrix elements Q_{ik}^0 and $\bar{Q}_{ik}^0$. $f_{Q_{ik}^0}$ and $M_{Q_{ik}^0}$ denote the decay constant and mass of meson Q_{ik}^0 , respectively. Using the data input [10,11] (all in GeV)

$$\begin{aligned} m_d &= 0.007, & m_s &= 0.15, & m_b &= 5.0, \\ m_u &= 0.004, & m_c &= 1.5, & m_t &= 145, \\ m_K &= 0.5, & m_D &= 1.87, & m_B &= 5.28, \\ f_K &= 0.16, & f_D &= 0.2, & f_B &= 0.2, \end{aligned} \quad (4.7)$$

we obtain the plots of the mass difference of Q_{ik}^0 and $\bar{Q}_{ik}^0$

FIG. 3. Mass difference Δm_B shown as a function of M_N at $v_R = 1$ TeV.

versus the mass related to the fourth generation quark through the whole possible mass parameter space [2] shown in Figs. 1, 2, and 3. The dashed lines in each picture are parametrized with $v_R = u = 1, 5, 10$, and 40 TeV from the shortest dashed line to the longest one, respectively. In Fig. 1, we plot Δm_K versus M_N . The experimental value [10] $\Delta m_K = (3.522 \pm 0.016) \times 10^{-15}$ GeV shows that each theoretical contribution should be less than this value and alternatively gives the upper bound of the mass M_N : $M_{N,\text{max}}/v_R$ (TeV) = 0.47 TeV. In Fig. 2 we plot Δm_D versus M_P with $m_t = 145$ GeV. The experimental/upper bound [10] $\Delta m_D \leq 1.3 \times 10^{-13}$ GeV gives no constraint on the upper bound of mass M_P . In Fig. 3, we plot Δm_B versus M_N only at $v_R = 1$ TeV due to the fact that Δm_B decreases rapidly with increasing v_R . The experimental value [10] $\Delta m_B = (3.6 \pm 0.7) \times 10^{-13}$ GeV gives the lower bound of mass M_N , being 33 GeV at $v_R = 1$ TeV, which is less than the existing experimental lower bound [10] being 45.5 GeV.

V. FLAVOR-CHANGING DECAYS OF Z_1 AND CHARGED LEPTONS

Let us consider the flavor-changing decays of Z_1 into changed fermions $f_i \bar{f}_k$ and $\bar{f}_i f_k$. In the approximation that m_i can be ignored in comparison with m_k , the Z_1 partial decay width is calculated to be

$$\Gamma(Z_1 \rightarrow f_i \bar{f}_k + \bar{f}_i f_k) = \frac{1}{32\pi} \left[\frac{e}{s_L c_L \sqrt{1 - s_L^2 - s_R^2}} \right]^2 M_{Z_1} (1 - x_{f_k})^2 (2 + x_{f_k}) \{ [A_{1L} F_{Lik}(F)]^2 + [A_{1R} F_{Rik}(F)]^2 \}, \quad (5.1)$$

where

$$x_{f_k} = \frac{m_{f_k}^2}{M_{Z_1}^2}. \quad (5.2)$$

The present experimental lower bound [12] of the top quark mass $m_t > 113$ GeV tells us that there has been no Z_1 flavor-changing decay involving top quarks. In the standard model with three generations, the effective flavor-changing couplings come from the one-loop Feynman diagrams [13]. The branching ratio of $B(Z \rightarrow u\bar{c} + \bar{u}c)$ is around 10^{-14} , and the branching ratio of $B(Z \rightarrow b\bar{s} + \bar{b}s)$ is around 10^{-9} at $m_t = 145$ GeV. Hence the standard FCNC Z decays are not observable at the present time. There is no FCNC decay mode of charged leptons from Z_1 in the standard model since all neutrinos are massless in the standard model and there is no such Kobayashi-Maskawa (KM) matrix in the lepton sector. Note that Rattazzi [14] has pointed out that if the light quark masses are generated radiatively order by order (i.e., $t \rightarrow c \rightarrow u$ and $b \rightarrow s \rightarrow d$) without using weak singlet scalars, the typical strength of the FCNC Z^0 couplings is then at least one order of magnitude above the experimental upper bounds. However, we do not have such a problem. The strength of one-loop induced FCNC Z_1 couplings in our model contributes to FCNC processes of Z_1 as small as those in the standard model. The dominant induced FCNC Z_1 couplings in this model are still from the one-loop effects in the mass matrix with tree-level Z_1 -boson couplings. Hence, using Eq. (5.1) we plot the branching ratios of flavor-changing Z_1 decays versus the corresponding mass of the fourth generation quark using [8,10] the Z_1 decay width 2.487 GeV,

$$\sin\theta_Z \sim \frac{s_R^3 \sqrt{1-s_L^2-s_R^2}}{s_L c_L^4} \left[\frac{v_L^2}{v_R^2 + 4u^2} \right], \quad (5.3)$$

and the best fit (corresponding to $v_R = u = 1$ TeV)

$$\begin{aligned} \sin^2\theta_L &= 0.2300 \pm 0.0012, \\ \sin^2\theta_R &= 0.3867 \pm 0.2225. \end{aligned} \quad (5.4)$$

In Fig. 4, we plot $B(Z_1 \rightarrow \bar{n}_i n_k + n_i \bar{n}_k)$ versus M_N at $v_R = 1$ TeV, where $n_i n_k$ stands for ds, db , and sb from shortest to longest dashed line, respectively. The experimental lower bound of M_N being 45.5 GeV has been applied to this picture. We see that $B(Z_1 \rightarrow \bar{b}s + b\bar{s})$ can reach to order of 10^{-7} , which is marginal to observe at the CERN e^+e^- collider LEP. In Fig. 5, we plot $B(Z_1 \rightarrow \bar{u}c + u\bar{c})$ versus M_p at $v_R = 1$ TeV. The plot of $B(Z_1 \rightarrow \tau^+\mu^- + \tau^-\mu^+)$ versus M_E at $v_R = 1$ TeV is shown in Fig. 6. The experimental lower bound [10] of M_E , which is 45.6 GeV, has been applied. Our result is well below the experimental upper limit [10] of order 10^{-5} .

Let us turn to consider the flavor-changing lepton decays. As mentioned before, there is no FCNC involving neutrinos. In the charged lepton sectors, the Z_2 -discrete symmetry separates e from μ and τ and hence it is prohibited with odd number of electrons (including positrons) in the final states, such as the processes $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$,

FIG. 4. Branching ratio for the decay $Z_1 \rightarrow \bar{n}_i n_k + n_i \bar{n}_k$ versus M_N at $v_R = 1$ TeV, where $n_i n_k$ stand for ds, db , and sb from the shortest to the longest dashed line, respectively.

μe conversion, and $\tau \rightarrow \mu\mu e$; however, it is allowed to have even number of electrons in the final states, such as $\tau \rightarrow \mu\mu\mu$ and $\tau \rightarrow \mu ee$. Hence let us consider the process $\tau^- \rightarrow \mu^+ \mu^- \mu^-$. For simplicity, we only consider this process through the exchange of Z_1 since the mass of Z_2 is larger than 385 GeV in this model [8]. In the approximation of neglecting muon mass, the partial decay rate is

FIG. 5. Branching ratio for the decay $Z_1 \rightarrow \bar{u}c + u\bar{c}$ versus M_p at $v_R = 1$ TeV.

FIG. 6. Branching ratio for the decay (a) $Z_1 \rightarrow \tau^+ \mu^- + \tau^- \mu^+$ and (b) $\tau^- \rightarrow \mu^+ \mu^- \mu^-$ versus M_E at $v_R = 1$ TeV.

then calculated to be

$$\Gamma(\tau^- \rightarrow \mu^+ \mu^- \mu^-) = \frac{m_\tau^5}{192\pi^3 M_{Z_1}^4} \left[\frac{e}{s_L c_L \sqrt{1-s_L^2-s_R^2}} \right]^4 \times (V_1^2 + A_1^2)(V_2^2 + A_2^2), \quad (5.5)$$

where

$$\begin{aligned} V_1 &= \frac{1}{32} \{ A_{1L} \{ [S_L(E)]_{22} + s_L^2 \} \\ &\quad + A_{1R} \{ [S_R(E)]_{22} + s_R^2 \} \}, \\ A_1 &= \frac{1}{32} \{ A_{1R} \{ [S_R(E)]_{22} - s_R^2 \} \\ &\quad - A_{1L} \{ [S_L(E)]_{22} - s_L^2 \} \}, \\ V_2 &= A_{1L} F_{L23}(E) + A_{1R} F_{R23}(E), \\ A_2 &= A_{1R} F_{R23}(E) - A_{1L} F_{L23}(E). \end{aligned} \quad (5.6)$$

Using [10] the measured τ lifetime $3.05 \pm 0.06 \times 10^{-13}$ sec and τ mass 1.784 GeV, we plot the branching ratio of $\tau^- \rightarrow \mu^+ \mu^- \mu^-$ versus m_E at $v_R = 1$ TeV also in Fig. 6. This branching ratio decreases rapidly with increasing v_R . The result is well below the experimental upper limit [10] which is $B(\tau^- \rightarrow \mu^+ \mu^- \mu^-) \leq 1.7 \times 10^{-5}$. The branching ratio $B(\tau^- \rightarrow \mu^- e^+ e^-)$ has the same order of magnitude in our model and the experimental upper limit [10] is of order of 10^{-5} . Hence we see that all processes involving flavor-changing lepton decays in this model are compatible with experimental limits.

VI. CONCLUSION

We have made an analysis of the expected FCNC's in an asymmetric light-right model [1,2] where the light quarks and leptons acquire their masses through radiative corrections. The introduction of charged heavy singlet fermions and basically two mass scales, the breaking of $SU(2)_L$ at 10^2 GeV and that of $SU(2)_R$ at 1 TeV or greater, are essential such that the wide range of Yukawa couplings is reduced to the order of 10^{-2} to unity. The additional $SU(2)_L \times SU(2)_R \times U(1)$ singlet quarks P and N , as well as the charged singlet leptons E , give rise to FCNC's through their mixings with conventional quarks and charged leptons. In this model, we have derived the Lagrangian of FCNC interactions for the first three generations of charged fermions and given expressions for all strengths of FCNC's only in terms of f_R , v_R , as well as m_t in the upper quark sector, using the numerology of fermion mass hierarchy to estimate FCNC processes. We have investigated (1) the mixings of $K^0-\bar{K}^0$, $D^0-\bar{D}^0$, and $B^0-\bar{B}^0$, (2) the flavor-changing decays of Z_1 gauge bosons into quarks and leptons, and (3) the flavor-changing decays of charged leptons. From the $K^0-\bar{K}^0$ mixing, the mass of heavy singlet fermion M_N/v_R (TeV) must be less than 470 GeV. There are no flavor-changing decays of Z_1 involving neutrinos in the absence of a heavy neutral singlet lepton, and none for the top quark due to the present experimental lower bound on $m_t > 113$ GeV. In the standard model with three families of quarks, the FCNC Z decays are far from observation. In our model, the branching ratio of processes $Z_1 \rightarrow \bar{b}s + b\bar{s}$ can reach to the order of 10^{-7} , which is within the margin to observe the FCNC Z_1 decays at LEP. The Z_2 -discrete symmetry separates e from μ and τ and hence the only flavor-changing decay mode is $Z_1 \rightarrow \tau^+ \mu^- + \tau^- \mu^+$, which is also well below the experimental limit of order 10^{-5} . For the flavor-changing decays of charged leptons, processes with odd number of electrons (including positrons) in the final states, such as the processes $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$, μe conversion, and $\tau \rightarrow \mu\mu e$ are prohibited; however, it is allowed to have even number of electrons in the final states such as $\tau \rightarrow \mu\mu\mu$ and $\tau \rightarrow \mu ee$. The branching ratios of such processes are also well below the current experimental limits of order 10^{-5} . In the standard model, those flavor-changing lepton decays are prohibited. Hence, measurements of these ratios of FCNC processes may provide a positive test of the model in the foreseeable future. In this paper, we have not worked on the phenomenology of the fourth generation fermions in this model and it is worthwhile to probe further.

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