Proton spin structure functions and generalized Sullivan processes

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We employ the time-ordered perturbation field theory in the infinite momentum limit, as developed extensively by Drell, Levy, and Yan, to derive the convolution formulas for the proton spin structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$ suitable for the generalized Sullivan processes, in which the meson in the cloud or the recoiling baryon is struck and smashed by the virtual photon. Following a previous suggestion that the sea quark distributions of a nucleon at low and moderate Q^2 may be attributed primarily to such Sullivan processes, we use the convolution formulas to obtain the first moments $\Gamma_p \equiv \int_0^1 g_1^p(x, Q^2) dx$ and $\Gamma_n \equiv \int_0^1 g_1^n(x, Q^2) dx$, as well as to investigate $g_1^p(x, Q^2)$ and $g_1^n(x, Q^2)$ as a function of x . Our results indicate that both the data on $g_1^p(x, Q^2)$ obtained previously by the European Muon Collaboration and the recent data on the deuteron spin structure function $g_1^d(x, Q^2)$ obtained by the Spin Muon Collaboration can be understood and the Bjorken sum rule is satisfied. A comparison between our prediction on $g_1^n(x, Q^2)$ and the recent SLAC data is also given.

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I. INTRODUCTION

In 1989, the European Muon Collaboration (EMC) [1] measured the proton spin structure function $g_1(x, Q^2)$ using the scattering of polarized electrons on a polarized

proton target. They obtained the first moment of $g_1^p(x, Q^2)$ at $\langle Q^2 \rangle = 10.7 \text{ GeV}^2$:

$$\Gamma_1^p \equiv \int_0^1 g_1^p(x) dx = 0.126 \pm 0.010 \pm 0.015. \quad (1)$$

Using Feynman's parton model [2] we expect

$$\begin{aligned} \Gamma_1^p &= \int_0^1 dx \frac{1}{2} \left(\frac{4}{9} \{ [u \uparrow(x) + \bar{u} \uparrow(x)] - [u \downarrow(x) + \bar{u} \downarrow(x)] \} + \frac{1}{9} \{ [d \uparrow(x) + \bar{d} \uparrow(x)] - [d \downarrow(x) + \bar{d} \downarrow(x)] \} \right. \\ &\quad \left. + \frac{1}{9} \{ [s \uparrow(x) + \bar{s} \uparrow(x)] - [s \downarrow(x) + \bar{s} \downarrow(x)] \} \right) \\ &= \frac{1}{2} \left(\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right), \end{aligned} \quad (2)$$

$$\Delta q \equiv \int_0^1 \{ [q \uparrow(x) + \bar{q} \uparrow(x)] - [q \downarrow(x) + \bar{q} \downarrow(x)] \} dx, \quad q = u, d, s. \quad (3)$$

On the other hand, the QCD corrected parton model expression for the first moment of $g_1^p(x)$ can be written as [3,4]

$$\begin{aligned} \Gamma_1^p &= \int_0^1 g_1^p(x) dx \\ &= \frac{1}{12} \left[\left[1 - \frac{\alpha_s}{\pi} \right] \left[a_3 + \frac{1}{\sqrt{3}} a_8 \right] \right. \\ &\quad \left. + 2 \left[\frac{2}{3} \right]^{1/2} \left[1 - \frac{33-8f}{33-2f} \frac{\alpha_s}{\pi} \right] a_0 \right], \end{aligned} \quad (4)$$

where the a_j are directly related to the proton matrix elements of the axial vector currents $A_j^\mu = \bar{\psi} \gamma_\mu \gamma_5 (\lambda_j/2) \psi$ ($j=0, 1, \dots, 8$) by $\langle P, S | A_j^\mu | P, S \rangle = 2M a_j S^\mu$. It follows from the neutron β -decay rate, through isospin rotation, that

$$a_3 = g_A = F + D = 1.254 \pm 0.006. \quad (5)$$

Assuming flavor SU(3) symmetry for hyperon β decays we have

$$a_8 = \frac{1}{\sqrt{2}} (3F - D) = 0.397 \pm 0.020, \quad (6)$$

where $F/D = 0.631 \pm 0.024$ is the F/D ratio for octet hyperon β decays.

In the naive parton model, the a_j are given by

$$\begin{aligned} a_0 &= \left(\frac{2}{3} \right)^{1/2} \{ \Delta u + \Delta d + \Delta s \}, \\ a_3 &= \Delta u - \Delta d, \\ a_8 &= \frac{1}{\sqrt{3}} \{ \Delta u + \Delta d - 2\Delta s \}. \end{aligned} \quad (7)$$

We may solve for the mean z component of spin carried by each of the quark flavor in a proton with $S_z = +\frac{1}{2}$ from the value of a_3 , a_8 , and Γ_1^p . This gives

$$\begin{aligned}
\langle S_z \rangle_u &= \frac{1}{2} \Delta u = 0.391 \pm 0.016 \pm 0.023, \\
\langle S_z \rangle_d &= \frac{1}{2} \Delta d = -0.236 \pm 0.016 \pm 0.023, \\
\langle S_z \rangle_s &= \frac{1}{2} \Delta s = -0.095 \pm 0.016 \pm 0.023, \\
\langle S_z \rangle_{\text{quarks}} &= \frac{1}{2} (\Delta u + \Delta d + \Delta s) \\
&= \frac{1}{2} \left(\frac{3}{2} \right)^{1/2} a_0 \\
&= 0.060 \pm 0.047 \pm 0.069.
\end{aligned} \tag{8}$$

Accordingly, there is only about $(12 \pm 17)\%$ of the proton spin which is due to its “quarks” (“antiquarks”). This is the so-called “proton spin crisis” [1].

Recently in 1993, the Spin Muon Collaboration (SMC) [5] measured the deuteron spin structure function $g_1^d(x, Q^2)$. Using the relation

$$\Gamma_1^p + \Gamma_1^n = 2\Gamma_1^d / (1 - 1.5w_d),$$

with $w_d = 0.058$ (the probability of finding the deuteron in the D state) we obtain

$$\begin{aligned}
\Gamma_1^n &\equiv \int_0^1 g_1^n(x) dx \\
&= \frac{1}{2} \left(\frac{1}{9} \Delta u + \frac{4}{9} \Delta d + \frac{1}{9} \Delta s \right) \\
&= -0.08 \pm 0.04 \pm 0.04.
\end{aligned} \tag{9}$$

Using Eq. (4) and applying isospin rotation to obtain Γ_1^n we obtain

$$\begin{aligned}
\Gamma_{\text{Bj}}^{p-n} &= \int_0^1 (g_1^p - g_1^n) dx = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} \right] \\
&= 0.191 \pm 0.002,
\end{aligned} \tag{10}$$

which is the Bjorken sum rule [6] obtained from light cone current algebra. Note that the EMC data, Eq. (1), and the SMC data, Eq. (9), when combined together, are in agreement with the Bjorken sum rule.

On the other hand, Ellis and Jaffe [7] obtained separate sum rules for g_1^p and g_1^n by assuming that there are no polarized strange sea quarks in the proton and the neutron (i.e., $\Delta s = 0$) and thus $a_0 = \sqrt{2} a_8$ in Eq. (7):

$$\begin{aligned}
\Gamma_{1,\text{EJ}}^p &= 0.189 \pm 0.005, \\
\Gamma_{1,\text{EJ}}^n &= -0.002 \pm 0.005.
\end{aligned} \tag{11a}$$

Both are inconsistent with the EMC and SMC data because of the assumption of $\Delta s = 0$. Note that the values of Γ_1^p and Γ_1^n as predicted by the Ellis-Jaffe sum rules, when combined together, are in agreement with the Bjorken sum rule, Eq. (10). The most recent SLAC data [8] on the neutron spin structure function of $g_1(x, Q^2 = 2 \text{ GeV}^2)$ yield

$$\Gamma_1^n = -0.022 \pm 0.011, \tag{11b}$$

which agrees with an updated value of the Ellis-Jaffe sum rule [9]:

$$\Gamma_{1,\text{EJ}}^n = -0.021 \pm 0.018. \tag{11c}$$

However, it disagrees, at the two-standard-deviation lev-

el, with the value inferred from the EMC data via the Bjorken sum rule:

$$\Gamma_{1,\text{Bj}}^n = -0.059 \pm 0.019. \tag{11d}$$

It is likely that more precise proton and neutron data are needed in order to clarify if the two-standard-deviation discrepancy is real. It seems unlikely that the Bjorken sum rule is the reason to blame, as the Ellis-Jaffe sum rules are in fact consistent with the Bjorken sum rule.

For the sake of the consideration of this paper, let us assume that the light quark sea (for the u and d quarks) is polarized to the same extent and that the amount of strange sea quarks is about half of that of u or d sea quarks. That is,

$$\delta u = \delta d = 2\delta s. \tag{12}$$

Note that δq refers only to the sea—a definition to be distinguished from that adopted in Eq. (3). It follows, using the EMC data [1] (adopted hereafter unless specified otherwise), that

$$\begin{aligned}
\langle S_z \rangle_{\text{valence}} &= \frac{1}{2} (\Delta u + \Delta d - 4\delta s) \\
&= 0.535 \pm 0.032 \pm 0.046,
\end{aligned} \tag{13a}$$

$$\langle S_z \rangle_{\text{sea}} = 5\delta s = -0.475 \pm 0.080 \pm 0.115. \tag{13b}$$

It then follows an interpretation of the “proton spin crisis” that the total spin of the valence quarks does add up to the proton spin, but it is canceled out by the large contribution from the sea quarks. There are of course other interpretations in the literature, as may be reflected by the large number of papers. Nevertheless, it is of interest to investigate whether Eqs. (13a) and (13b) may come out naturally from the conjecture that the sea quark distributions of a nucleon at low and moderate Q^2 may be attributed primarily to generalized Sullivan processes, in which the meson in the cloud or the recoiling baryon is struck and smashed by the virtual photon. It has been shown [10,14] that the generalized Sullivan processes can indeed explain in a quantitative manner the unpolarized parton distributions in the nucleon. It is the purpose of this paper to show that such conjecture for the origin of sea quark distributions does give rise to results very much in line with Eqs. (13a) and (13b), thus providing a simple explanation of the “proton spin crisis.”

II. DEEP INELASTIC SCATTERING

To describe the lepton-proton deep inelastic scattering,

$$\ell(l) + p(p) \rightarrow \ell'(l') + X(p_X), \tag{14}$$

with l, l', p , and p_X are four-momenta of the particles we introduce the standard notation

$$\begin{aligned}
q &= l - l' , \\
Q^2 &= -q^2 , \\
\nu &= \frac{p \cdot q}{M} = E_l - E_{l'} , \\
x &= \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2M\nu} , \\
y &= \frac{p \cdot q}{l \cdot p} = \frac{\nu}{E_l} .
\end{aligned} \tag{15}$$

Here M is the target proton mass. Note that the second equality in each of the last three equations holds in the laboratory frame (where the target is at rest). The differential cross section is then given by

$$\frac{d^2\sigma}{dx dy} = \frac{2\pi M\nu}{E_{l'}} \frac{d^2\sigma}{d\Omega_{\text{lab}} dE'} , \tag{16}$$

$$\frac{d\sigma}{d\Omega dE_{l'}} = \frac{\alpha^2}{q^4} \frac{E'}{E} l^{\mu\nu} W_{\mu\nu} ;$$

$$l^{\mu\nu} = 2[l^\mu l'^\nu + l'^\mu l^\nu - g^{\mu\nu} l \cdot l'] , \tag{17a}$$

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4x e^{iq \cdot x} \langle p | J_\mu^\dagger(x) J_\nu(0) | p \rangle . \tag{17b}$$

For the spin average case we may construct the hadronic tensor $W_{\mu\nu}$ as [11]

$$\begin{aligned}
W_{\mu\nu} &= -W_1(q^2, \nu) \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] \\
&+ \frac{W_2(q^2, \nu)}{M^2} \left[p_\mu - \frac{p \cdot q}{q^2} q_\mu \right] \left[p_\nu - \frac{p \cdot q}{q^2} q_\nu \right] , \tag{18}
\end{aligned}$$

which satisfies the constraint due to current conservation, $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$. Thus we have

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right] . \tag{19}$$

In the Bjorken limit [i.e., $Q^2 \rightarrow \infty$ and $\nu \rightarrow \infty$ with $x = Q^2/(2M\nu)$ held fixed] we can express W_1 and W_2 as

$$\begin{aligned}
MW_1 &\rightarrow F_1(x) = \frac{1}{2} \sum_q e_q^2 f_q(x) , \\
\nu W_2 &\rightarrow F_2(x) = \sum_q e_q^2 x f_q(x) ,
\end{aligned} \tag{20}$$

where e_q is the charge of the parton (quark) and $f_q(x)$ is the parton distribution function of flavor q .

Just like the unpolarized deep inelastic scattering (DIS) formulas, the differential cross section for the polarized DIS can be described as contraction of the antisymmetric part of the hadronic tensor $W_{\mu\nu}$ with that of the leptonic tensor $l^{\mu\nu}$:

$$\frac{d^2\sigma^{\uparrow\downarrow}}{d\Omega dE'} - \frac{d^2\sigma^{\uparrow\uparrow}}{d\Omega dE'} = \frac{\alpha^2}{q^4} \left[\frac{E'}{E} \right] l_A^{\mu\nu} W_{\mu\nu}^A . \tag{21}$$

By Lorentz invariance and gauge invariance, $W_{\mu\nu}^A$ can be expressed in terms of two scalar functions $G_1(x, Q^2)$ and

$G_2(x, Q^2)$ for a spin- $\frac{1}{2}$ target. We have

$$\begin{aligned}
W_{\mu\nu}^A &= \frac{1}{4\pi} \int d^4x e^{iq \cdot x} \langle p, S | [J_\mu(x), J_\nu(0)]^{\text{anti}} | p, S \rangle \\
&= i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda}{M} \{ S^\sigma M^2 G_1(x, Q^2) \\
&\quad + [S^\sigma(p \cdot q) - p^\sigma(q \cdot S)] G_2(x, Q^2) \} , \tag{22}
\end{aligned}$$

$$l_A^{\mu\nu} = \text{Tr} \{ \hat{k}_1 \gamma_{\mu} \frac{1}{2} (1 + \gamma_5) \hat{k} \gamma_{\nu} \} ,$$

with $S^2 = -M^2$, $P^2 = M^2$, and $(S \cdot P) = 0$.

The difference in the cross sections for deep inelastic scattering of muons polarized antiparallel and parallel to the spin of the proton can be written as [12]

$$\begin{aligned}
&\left[\frac{d^2\sigma}{dQ^2 d\nu} \right]^{\uparrow\downarrow} - \left[\frac{d^2\sigma}{dQ^2 d\nu} \right]^{\uparrow\uparrow} \\
&= \frac{4\pi\alpha^2}{E^2 Q^2} [M(E + E' \cos\theta) G_1(\nu, Q^2) - Q^2 G_2(\nu, Q^2)] . \tag{23}
\end{aligned}$$

In the Bjorken limit we have

$$\begin{aligned}
M^2 \nu G_1(Q^2, \nu) &\rightarrow g_1(x) , \\
M \nu^2 G_2(Q^2, \nu) &\rightarrow g_2(x) , \tag{24}
\end{aligned}$$

with

$$g_1(x) = \frac{1}{2} \sum e_i^2 [q_i^+(x) - q_i^-(x)] , \tag{25}$$

a formula as used in connection with Eq. (1).

III. CONVOLUTION FORMULAS FOR $g_1(x)$ AND $g_2(x)$

In 1972, Sullivan [10] considered, in connection with deep inelastic scattering, the processes as shown in Fig. 1 with $(M, B) = (\pi, N)$ or (π, Δ) , in which the virtual photon strikes the pion emitted by the nucleon and smashes the pion into debris. In the Bjorken limit, such processes scale like the original process in which the virtual photon strikes and smashes the nucleon itself. Specifically, one has

$$\begin{aligned}
\delta F_{2N}^\pi(x, Q^2) &= \int_x^1 dy f_\pi(y) F_{2\pi} \left[\frac{x}{y}, Q^2 \right] , \\
f_\pi(y) &= \frac{3}{16\pi^2} \left[f_{\pi NN} \frac{2M}{\mu} \right]^2 y \int_{-\infty}^{t^m} dt \frac{(-t) |F_\pi(t)|^2}{(-t + \mu^2)^2} , \tag{26} \\
F_\pi(t) &= \left[\frac{\Lambda_\pi^2 - \mu^2}{\Lambda_\pi^2 - t^2} \right]^2 \text{ dipole form factor} , \\
t^m &= -\frac{M^2 y^2}{1-y} .
\end{aligned}$$

Here $\delta F_{2N}^\pi(x)$ is the correction to the nucleon structure function $F_{2N}(x)$ due to the specific Sullivan processes, $F_{2\pi}$ is the pion structure function as would be measured in $\ell + \pi \rightarrow \ell' + X$, $f_\pi(y)$ is the probability of finding a pion carrying the fraction y of the nucleon momentum, μ

FIG. 1. The generalized Sullivan processes in which (a) the virtual photon strikes the meson and (b) the virtual photon strikes the recoiling baryon.

and M are, respectively, the pion and nucleon mass, $f_{\pi NN}$ is the (pseudovector) πNN coupling constant, and Λ characterizes the t dependence of the πNN coupling.

The observation of Sullivan was not appreciated until 1983 when Thomas [13] noted that in the Sullivan processes the virtual photon will see most of the time the valence distributions in the pion. Recently, Hwang, Brown, and Speth [14] went further when they noticed that the entire sea distributions, including both the strengths and shapes (as functions of x), of a nucleon at moderate Q^2 (up to $Q^2=20 \text{ GeV}^2$) can in fact be attributed to the “generalized Sullivan processes,” i.e., Figs. 1(a) and 1(b) with the meson-baryon pair (M, B) identified as any one of (π, N) , (ρ, N) , (ω, N) , (σ, N) , (K, Λ) , (K, Σ) , (K^*, Λ) , (K^*, Σ) , (π, Δ) , and (ρ, Δ) .

The significance of the conjecture of attributing the sea distributions of a hadron at low and moderate Q^2 to its associated meson cloud as generated by strong interactions at the hadron level (the generalized Sullivan processes) is that we are now able to determine the sea distributions of a hadron from the knowledge of the valence distributions of the various hadrons, which in turn are calculable from quark models. The QCD evolution equations then take us from moderate Q^2 to very high Q^2 . Therefore, we can link nicely together the previous gap between low Q^2 (nuclear) physics and large Q^2 (particle) physics. We can thus “calculate” the sea distributions of a nucleon, including the strangeness and spin content from nuclear physics experiments such as nucleon-nucleon and hyperon-nucleon scatterings.

In another publication, Hwang and Speth [15] were

able to show that the $\bar{d}$ over $\bar{u}$ asymmetry in the proton, as recently measured to relatively high accuracy by the New Muon Collaboration (NMC) group [16], can be understood in a straightforward manner if the sea distributions of the nucleon at low and moderate Q^2 are attributed entirely to generalized Sullivan processes. The purpose of this paper is to use the same conjecture to study the proton spin problem. As we shall see, the “proton spin crisis” can also be understood in quantitative terms—not only the experimental value for the first moment of $g_1(x)$ is obtained but also the shape of $g_1(x)$ (as a function of x) can be reproduced.

To set up a formalism to treat the spin structure functions we follow and generalize the formulation of Drell, Levy, and Yan (DLV) [17], which is a time-ordered perturbation field theory in the infinite-momentum frame. The method was previously adopted by Zoller [18] in his treatment of generalized Sullivan processes (in the unpolarized case).

Optical theorem states that the hadronic tensor $W_{\mu\nu}$ in DIS is related to the imaginary part of the Compton scattering matrix element at the forward angle:

$$W_{\mu\nu} = \text{Im} T_{\mu\nu}(p, q) \\ = \text{Tr} \{ (m + \hat{p}) \text{Im} \hat{T}_{\mu\nu}(p, q) \} , \quad (27)$$

with $\hat{p} = \sum_{\mu} p^{\mu} \gamma_{\mu}$. Here $T_{\mu\nu}(p, q)$ the Compton scattering amplitude at the forward angle and $\text{Im} \hat{T}_{\mu\nu}(p, q)$ is a second-rank Lorentz tensor which is a 4×4 matrix in Dirac space to be sandwiched between the Dirac spinors, $\bar{u}(p) \text{Im} \hat{T}_{\mu\nu}(p, q) u(p)$. Equation (27) may be represented pictorially as in Fig. 2, where the loop indicates the trace to be taken. Note that, unless specified otherwise, such a loop is always implied in the DLV formalism.

Gauge invariance yields

$$q_{\mu} \text{Im} \hat{T}_{\mu\nu}(p, q) = q_{\nu} \text{Im} \hat{T}_{\mu\nu}(p, q) = 0 , \quad (28)$$

so that $\text{Im} \hat{T}_{\mu\nu}$ can be parametrized as

FIG. 2. Pictorial representation of $W_{\mu\nu}$.

$$\begin{aligned}
\text{Im}\hat{T}_{\mu\nu} = & (C_0 + D_0\hat{q}) \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] + (C_1 + D_1\hat{q}) \left[p_\mu - \frac{q_\mu p \cdot q}{q^2} \right] \left[p_\nu - \frac{q_\nu p \cdot q}{q^2} \right] \\
& + C_2 \left[\left[\gamma_\mu - \frac{q_\mu \hat{q}}{q^2} \right] \left[p_\nu - \frac{q_\nu p \cdot q}{q^2} \right] + \left[\gamma_\nu - \frac{q_\nu \hat{q}}{q^2} \right] \left[p_\mu - \frac{q_\mu p \cdot q}{q^2} \right] \right] \\
& + C_3 \left[\sigma_{\mu\nu} + \frac{1}{q^2} (q_\mu \sigma_{\nu\lambda} q_\lambda - q_\nu \sigma_{\mu\lambda} q_\lambda) \right] + D_3 \left[\sigma_{\mu\lambda} q_\lambda \left[p_\nu - \frac{q_\nu p \cdot q}{q^2} \right] - \sigma_{\nu\lambda} q_\lambda \left[p_\mu - \frac{q_\mu p \cdot q}{q^2} \right] \right]. \quad (29)
\end{aligned}$$

The first three terms are spin average (symmetric) tensors and the last two terms are spin dependent (antisymmetric) tensors. We may write

$$W_{\mu\nu} = W_{\mu\nu}^S + W_{\mu\nu}^A, \quad (30)$$

where the symmetric part is given by

$$\begin{aligned}
W_{\mu\nu}^S &= \text{Tr}\{(M + \hat{p}) \text{Im}\hat{T}_{\mu\nu}(p, q)\} \\
&= 4(MC_0 + p \cdot q D_0) \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] + 4(MC_1 + p \cdot q D_1 + 2C_2) \left[p_\mu - \frac{q_\mu p \cdot q}{q^2} \right] \left[p_\nu - \frac{q_\nu p \cdot q}{q^2} \right] \\
&= W_1 \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] + W_2 \left[p_\mu - \frac{q_\mu p \cdot q}{q^2} \right] \left[p_\nu - \frac{q_\nu p \cdot q}{q^2} \right], \quad (31) \\
W_1 &= 2\pi F_1(x, Q^2) = -4(MC_0 + p \cdot q D_0), \quad W_2 = 2\pi \frac{M^2}{p \cdot q} F_2(x, Q^2) = 4M^2(MC_1 + p \cdot q D_1 + 2C_2),
\end{aligned}$$

while the antisymmetric part is

$$\begin{aligned}
W_{\mu\nu}^A &= \text{Tr}\{(M + \hat{p}) \frac{1}{2}(1 + \gamma_5 \hat{S}) \text{Im}\hat{T}_{\mu\nu}(p, q)\} \\
&= i\epsilon_{\alpha\beta\mu\nu} \frac{q^\alpha}{M} \{M^2 S^\beta G_1(x, Q^2) + [(p \cdot q) S^\beta - (S \cdot q) p^\beta] G_2(x, Q^2)\}, \quad (32) \\
G_1 &= 4D_3 M, \quad G_2(x, Q^2) = \frac{1}{q^2} M [C_3 + (p \cdot q) D_3].
\end{aligned}$$

In Eq. (32), $\frac{1}{2}(1 + \gamma_5 \hat{S})$ is the polarization projection operator for the polarized proton target and $\hat{S}$ is the polarization four-vector of the proton [17].

In the original formulation of Drell, Levy, and Yan (DLY), the Hamiltonian of the π - N system is

$$H = H_0 + H_I, \quad (33)$$

where H_0 is the “free” or “bare” Hamiltonian and H_I describes the strong interaction of the π - N system:

$$H_I(t) = ig_{\pi NN} \int d^3x \bar{\psi}(x, t) \gamma_5 \tau \psi(x, t) \pi(x, t). \quad (34)$$

The electromagnetic current of the hadrons, as seen by an electromagnetic probe, is given, in the original DLY formalism [17], by

$$J_\mu = \bar{\psi}_p \gamma_\mu \psi_p + i\pi^+ \partial_\mu \pi^-. \quad (35)$$

In our formulation, however, both the proton and the pion are treated as objects of some structure, each to be characterized by a set of parton distributions. The devia-

tion from Eq. (35) signifies the departure of our formalism from the original DLY approach, although much of the DLY formalism remains intact in such generalization.

To explain the DLY formalism a little further we try to undress the Heisenberg operators and go into the interaction picture by the time evolution operator (U -matrix) transformation. The Heisenberg current operator J_μ is related to “free” or “bare” current $j_\mu(x)$ by

$$\begin{aligned}
J_\mu(x) &= U^{-1}(t) j_\mu(x) U(t), \\
U(t) &= T \left[\exp \left[-i \int_{-\infty}^t dt H_I(t) \right] \right]. \quad (36)
\end{aligned}$$

The S matrix is

$$S = T \exp \left[-i \int_{-\infty}^{\infty} dt H_I(t) \right], \quad (37)$$

so that the S -matrix element in the time-ordered perturbation theory is given by

$$\begin{aligned} \langle b|S|a\rangle = & -2\pi i\delta(E_b - E_a) \left[\langle b|H_I|a\rangle + \sum_n \frac{\langle b|H_I|n\rangle\langle n|H_I|a\rangle}{E_a - E_n + i\epsilon} \right. \\ & \left. + \sum_{n_1, n_2} \frac{\langle b|H_I|n_2\rangle\langle n_2|H_I|n_1\rangle\langle n_1|H_I|a\rangle}{(E_a - E_{n_1} + i\epsilon)(E_a - E_{n_2} + i\epsilon)} + \dots \right]. \end{aligned} \quad (38)$$

Note that, in the DLY formalism, the complete result to a given order in perturbation series is covariant, but each term is not. Nevertheless, the particle content of each term is clear and it has intuitive interpretations. At each vertex the three-momentum are conserved, but the energy is not. A virtual particle is always on its mass shell, but the intermediate state does not have the same energy as the initial or final state.

We can then express a “physical” proton in terms of “bare” particles:

$$\begin{aligned} |P\rangle_{\text{bare}} & \rightarrow |P\rangle_{\text{physical}}, \\ |P\rangle_{\text{physical}} & \equiv U_0 |P\rangle_{\text{bare}} = \sqrt{Z_2} \left[|P\rangle_{\text{bare}} + \sum_{m \neq A} \frac{|m\rangle\langle m|H_I(0)|A\rangle}{E_A - E_m} + \dots \right] \\ & = \sqrt{Z_2} \left[|P\rangle_{\text{bare}} + \sum_{B,C} \int d^3P_B d^3P_C \frac{1}{E_A - E_B - E_C} |B, C\rangle\langle B, C|H_I|A\rangle + \dots \right]. \end{aligned} \quad (39)$$

The wave function normalization condition is

$$\langle P|U^{-1}U|P'\rangle = \delta_{PP'}, \quad (40)$$

so that

$$Z_{2C} = 1 / \left[1 + \sum_{B,C} \int d^3P_B d^3P_C \frac{1}{(E_A - E_B - E_C)^2} |\langle B, C|H_I(0)|A\rangle|^2 \right]. \quad (41)$$

More explicitly, we have, for the πNN coupling of Eq. (34),

$$\begin{aligned} |UP\rangle & \equiv U|P\rangle \\ & = \sqrt{Z_{2\pi}} \left[|P\rangle_{\text{bare}} + \frac{ig_{\pi NN}}{(2\pi)^{3/2}} \frac{2M}{(2E_P)^{1/2}} \int \frac{d^3k_1}{(2\omega_1)^{1/2}} \frac{d^3P_1}{(2E_1)^{1/2}} \delta^3(\mathbf{P}_1 + \mathbf{k}_1 - \mathbf{P}) \frac{\bar{u}_{P_1} \gamma_5 u_P}{E_P - E_{P_1} - \omega_1} |P_1 k_1\rangle \right] + O(g^2), \end{aligned} \quad (42)$$

where

$$\begin{aligned} Z_{2\pi^0} & = \frac{1}{1 + a_{\pi^0}}, \quad Z_{2\pi^{+(-)}} = \frac{1}{1 + a_{\pi^{+(-)}}}, \\ a_{\pi^0} & = \frac{g_{\pi NN}^2}{16\pi^2} \int_0^\infty dk_{11}^2 \int_x^1 dy y(1-y) \frac{k_{11}^2 + M^2(1-y)^2}{[k_{11}^2 + M^2(1-y)^2 + \mu^2 y]^2}, \quad a_{\pi^{+(-)}} = 2a_{\pi^0}, \end{aligned} \quad (43)$$

where a_{π^0} is the probability for which the proton emits a neutral pion.

Changing $J_\mu(x)$ to the bare level and using the translational invariance,

$$\langle P|J_\mu^\dagger(x)|N\rangle = e^{i(P-P_n)x} \langle P|J_\mu^\dagger(0)|N\rangle,$$

we may express the hadronic tensor $W_{\mu\nu}$ in terms of bare states:

$$\begin{aligned}
W_{\mu\nu} &= 4\pi^2 \frac{E_p}{M} \frac{1}{2} \sum_S \int e^{iqx} d^4x \langle P, S | J_\mu^\dagger(x) J_\nu(0) | P, S \rangle \\
&= 4\pi^2 \frac{E_p}{M} \frac{1}{2} \sum_S \int e^{i(q+P-P_n)x} d^4x \langle P, S | J_\mu^\dagger(0) | N \rangle \langle N | J_\nu(0) | P, S \rangle \\
&= 4\pi^2 \frac{E_p}{M} \frac{1}{2} \sum_S \int e^{i(q+P-P_n)x} d^4x \langle P, S | U^{-1} U J_\mu^\dagger(0) U^{-1} U | N \rangle \langle N | U^{-1} U J_\nu(0) U^{-1} U | P, S \rangle \\
&= 4\pi^2 \frac{E_p}{M} \frac{1}{2} \sum_S \int e^{i(q+P-P_n)x} d^4x \langle U(P, S) | j_\mu^\dagger(0) | UN \rangle \langle UN | j_\nu(0) | U(P, S) \rangle \\
&= 4\pi^2 \frac{E_p}{M} \frac{1}{2} \sum_{S,N} (2\pi)^4 \delta(q+P-P_n) \langle U(P, S) | j_\mu^\dagger(0) | UN \rangle \langle UN | j_\nu(0) | U(P, S) \rangle .
\end{aligned} \tag{44}$$

It then follows a set of ‘‘Feynman’’ rules which allow us to write down formulas suitable for Sullivan processes. Each of these process may be represented by a corresponding graph. For instance, Figs. 3(a) and 3(b) represent the contributions in which either the recoiling baryon (a) or the emitted pion (b) is struck and smashed. The graph is symmetric as it represents the absolute square of the DIS amplitude, or the forward-angle Compton scattering amplitude. For Fig. 3(a) in the spin-average case we find that the correction to $W_{\mu\nu}^S$ for the πNN vertex is given by

$$\delta W_{\mu\nu}^{S, \pi NN} = \text{Tr} \{ (M + \hat{p}) \delta \text{Im} T_{\mu\nu}(p, q) \} = - \frac{g_{\pi 0 pp}^2}{(2\pi)^3} \int \frac{d^3 k_1}{2\omega_1} \frac{\text{Tr} \{ (M + \hat{p}) \gamma_5 (M + \hat{p}_1) \text{Im} T_{\mu\nu} (M + \hat{p}_1) \gamma_5 \}}{(2E_1)^2 (E_p - E_1 - \omega_1)^2} . \tag{45}$$

Here the kinematics suitable for the infinite momentum frame (i.e., $P_z = P \rightarrow \infty$) may be given as [17]

$$\begin{aligned}
t &= -(P - P_1)^2 = 2(M^2 - P \cdot P_1) = -\frac{1}{y} [k_{1\perp}^2 + M^2(1-y)^2] , \\
P_1 &= yP + k_{1\perp}, \quad k_1 = (1-y)P - k_{1\perp}, \quad k_{1\perp} \cdot P = 0, \quad P = \left[P + \frac{M^2}{2P}, 0, 0, P \right] , \\
k_1 &= \left[(1-y)P + \frac{k_{1\perp}^2 + \mu^2}{2(1-y)P}, -k_{1\perp x}, -k_{1\perp y}, (1-y)P \right] , \\
E_p - E_1 - \omega_1 &= \left[P + \frac{M^2}{2P} \right] - \left[yP + \frac{k_{1\perp}^2 + \mu^2}{2yP} \right] - \left[(1-y)P + \frac{k_{1\perp}^2 + \mu^2}{2(1-y)P} \right] \\
&= \begin{cases} -\frac{1}{2y(1-y)P} \{ k_{1\perp}^2 + M^2(1-y)^2 + \mu^2 y \} & (0 < y < 1) , \\ -2(y-1)P & (y > 1) , \\ 2yP & (y < 0) . \end{cases}
\end{aligned} \tag{46}$$

Going over to the infinite momentum frame we may show that only those diagrams, in which all particles move forward with $0 < y < 1$, dominate. Calculating the trace of the diagram we obtain the convolution formulas of W_1 and W_2 :

$$\begin{aligned}
\delta W_{\mu\nu}^{S, \pi NN} &= \left[\frac{1}{M^2} \left[p_\mu - q_\mu \frac{p \cdot q}{q^2} \right] \left[p_\nu - q_\nu \frac{p \cdot q}{q^2} \right] \delta W_2((p_1 + q)^2, q^2) - \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] \delta W_1((p_1 + q)^2, q^2) \right] \\
&= - \frac{g_{\pi NN}^2}{(2\pi)^3} \int \frac{d^3 k_1}{2\omega_1} \frac{2(M^2 - p \cdot p_1)}{(2E_1)^2 (E_p - E_1 - \omega_1)^2} \left[\frac{1}{M^2} \left[p_{1\mu} - q_\mu \frac{p_1 \cdot q}{q^2} \right] \left[p_{1\nu} - q_\nu \frac{p_1 \cdot q}{q^2} \right] W_2((p_1 + q)^2, q^2) \right. \\
&\quad \left. - \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] W_1((p_1 + q)^2, q^2) \right] .
\end{aligned} \tag{47}$$

Working with the time and the third components (‘‘good currents’’ according to DLY), we obtain, from Eq. (47),

$$\begin{aligned}
\delta W_2 &= \frac{g_{\pi NN}^2}{16\pi^2} \int_0^\infty dk_{1\perp}^2 \int_x^1 dy y(1-y) \frac{k_{1\perp}^2 + M^2(1-y)^2}{[k_{1\perp}^2 + M^2(1-y)^2 + \mu^2 y]^2} W_{2\text{bare}}((p_1 + q)^2, q^2) , \\
\delta(xF_2(x, Q^2)) &= \frac{g_{\pi NN}^2}{16\pi^2} \int_0^\infty dk_{1\perp}^2 \int_x^1 dy \frac{y(1-y)[k_{1\perp}^2 + M^2(1-y)^2]}{[k_{1\perp}^2 + M^2(1-y)^2 + \mu^2 y]^2} \frac{x}{y} F_2 \left[\frac{x}{y}, Q^2 \right] .
\end{aligned} \tag{48}$$

This last result is in agreement with Eq. (26) (except in terms of different variables), which is obtained via the original covariant formulation of Sullivan [10].

Generalization of the above procedure to the spin-dependent structure functions is straightforward; we only need to insert the spin projection operators [19] at the appropriate places. The situation would be different if the original covariant formulation of Sullivan [10] would be adopted—it is far from obvious to derive the convolution formulas for $g_1(x)$ and $g_2(x)$ in the covariant formulation. Using the DLY formalism, we obtain the convolution formula of $g_1(x, Q^2)$ and $g_2(x, Q^2)$ from $\delta W_{\mu\nu}^A$ in a fairly straightforward manner. For the graph in which the virtual photon strikes and smashes the recoiling baryon as in Fig. 3(a) we have

$$\begin{aligned}
 \delta W_{\mu\nu}^{A, \pi NN} &= \delta \text{Tr}[(M + \hat{p})^{\frac{1}{2}}(1 + \gamma_5 \hat{s}) \text{Im} T_{\mu\nu}] \\
 &= i \epsilon_{\alpha\beta\mu\nu} q_\alpha \{ M^2 s_\beta \delta G_1 + [p_\beta(s \cdot q) - s_\beta(p \cdot q)] \delta G_2 \} \\
 &= - \frac{g_{\pi^0 pp}^2}{(2\pi)^3} \int \frac{d^3 k_1}{2\omega_1} \frac{\text{Tr}(M + \hat{p})^{\frac{1}{2}}(1 + \gamma_5 \hat{s}) \gamma_5 (M + \hat{p}_1) \text{Im} \hat{T}_{\mu\nu}(M + \hat{p}_1) \gamma_5}{(2E_1)^2 (E_p - E_1 - \omega_1)^2} \\
 &= - \frac{g_{\pi^0 pp}^2}{(2\pi)^3} \int \frac{d^3 k_1}{2\omega_1} \frac{2(M^2 - p \cdot p_1)}{(2E_1)^2 (E_p - E_1 - \omega_1)^2} i \epsilon_{\alpha\beta\mu\nu} \\
 &\quad \times q_\alpha \{ M^2 s_\beta G_1((p_1 + q)^2, q^2) - [p_{1\beta}(s \cdot q) - s_\beta(p_1 \cdot q)] G_2((p_1 + q)^2, q^2) \} , \\
 \delta g_{1\pi^0 pp} &= - \frac{g_{\pi^0 pp}^2}{16\pi^2} \int_0^\infty dk_{1\perp}^2 \int_x^1 dy \frac{y(1-y)[k_{1\perp}^2 + M^2(1-y)^2]}{[k_{1\perp}^2 + M^2(1-y)^2 + \mu^2 y]^2} \frac{1}{y^2} g_{1p}^{\text{bare}} \left[\frac{x}{y}, Q^2 \right] |F_\pi(t)|^2 , \\
 \delta g_{2\pi^0 pp} &= - \frac{g_{\pi^0 pp}^2}{16\pi^2} \int_0^\infty dk_{1\perp}^2 \int_x^1 dy \frac{y(1-y)[k_{1\perp}^2 + M^2(1-y)^2]}{[k_{1\perp}^2 + M^2(1-y)^2 + \mu^2 y]^2} \frac{1}{y^3} g_{2p}^{\text{bare}} \left[\frac{x}{y}, Q^2 \right] |F_\pi(t)|^2 , \tag{49}
 \end{aligned}$$

On the other hand, it is straightforward to demonstrate that the process shown in Fig. 3(b), in which the virtual photon strikes and smashes the emitted pion, does not contribute to δg_1 and δg_2 .

Following the above procedure we may obtain contributions from other generalized Sullivan processes (such as σNN , $KN\Lambda$, $KN\Sigma$, $K^*N\Lambda$, $K^*N\Sigma$, ρNN , and ωNN). Some of the formulas are listed in the Appendix (with the coupling constants defined in, e.g., Ref. [20]). We note that $g_1(x)$ and $g_2(x)$ may be written explicitly as

$$g_{1(2)} = Z_2 \{ g_{1(2)}^{\text{bare}} + \delta g_{1(2)}^{\text{Sullivan}} \} , \tag{50a}$$

where

$$\delta g_{1(2)}^{\text{Sullivan}} = \delta g_{1(2)}^{\pi NN} + \delta g_{1(2)}^{\sigma NN} + \delta g_{1(2)}^{KN\Sigma} + \delta g_{1(2)}^{KN\Lambda} + \delta g_{1(2)}^{\rho NN} + \delta g_{1(2)}^{\omega NN} + \delta g_{1(2)}^{K^*N\Sigma} + \delta g_{1(2)}^{K^*N\Lambda} + \dots . \tag{50b}$$

Equations (49) and (50), together with formulas listed in the Appendix, serve as the basis for our numerical studies. Barring from uncertainties related to the unknown $g_{1\text{bare}}^h(x)$ for the various hadrons, we treat in this work the contributions from the decuplet baryons $J = \frac{3}{2}$ only in an approximate sense but such contributions turn out to be only of negligible numerical importance.

IV. NUMERICAL RESULTS

In the simple quark model in which the proton consists of three valence quarks u, u, d in an $SU(3)$ symmetric wave function [11] we have

$$\begin{aligned}
 |P_{j_3=1/2}\rangle &= \frac{1}{\sqrt{18}} \{ 2u \uparrow u \uparrow d \downarrow - u \uparrow u \downarrow d \uparrow - u \downarrow u \uparrow d \uparrow + 2d \downarrow u \uparrow u \uparrow - d \uparrow u \downarrow u \uparrow - d \uparrow u \uparrow u \downarrow \\
 &\quad + 2u \uparrow d \downarrow u \uparrow - u \downarrow d \uparrow u \uparrow - u \uparrow d \uparrow u \downarrow \} . \tag{51}
 \end{aligned}$$

Thus, the probability of finding $u \uparrow(\downarrow)$ [u quark with spin parallel (antiparallel) to the proton] is [12]

$$\begin{aligned}
 u \uparrow &= \frac{5}{3} n(x) = \frac{5}{3} \frac{1}{2} u_v(x) , \\
 u \downarrow &= \frac{1}{3} n(x) = \frac{1}{3} \frac{1}{2} u_v(x) . \tag{52a}
 \end{aligned}$$

Similarly,

$$\begin{aligned}
 d \uparrow &= \frac{1}{3} n(x) = \frac{1}{3} d_v(x) , \\
 d \downarrow &= \frac{2}{3} n(x) = \frac{2}{3} d_v(x) , \tag{52b}
 \end{aligned}$$

FIG. 3. The diagram for $\delta W_{\mu\nu}^{\pi NN}$ in which (a) the virtual photon strikes and smashes the recoiling baryon or in which (b) the virtual photon strikes and smashes the pion emitted by the proton.

where

$$\begin{aligned} \int_0^1 dx \, n(x) &= 1, \\ \int_0^1 dx \, [u \uparrow(x) + u \downarrow(x)] &= \int_0^1 dx \, u_v(x) = 2, \\ \int_0^1 dx \, [d \uparrow(x) + d \downarrow(x)] &= \int_0^1 dx \, d_v(x) = 1. \end{aligned} \quad (52c)$$

Therefore, we adopt, as input for our calculation,

$$\begin{aligned} g_{1\text{bare}}^p(x) &= \frac{1}{2} \left\{ \frac{4}{9} [u \uparrow(x) - u \downarrow(x)] + \frac{1}{9} [d \uparrow(x) - d \downarrow(x)] \right\} \\ &= \frac{1}{2} \left[\frac{4}{9} \frac{2}{3} u_v(x) - \frac{1}{9} \frac{1}{3} d_v(x) \right]. \end{aligned} \quad (53)$$

Using flavor SU(3) rotations, we may obtain $g_{1\text{bare}}(x)$ for other octet baryons such as the neutron, Λ , and Σ . For instance, we have

$$g_{1\text{bare}}^n(x) = \frac{1}{2} \left[\frac{1}{9} \frac{2}{3} u_v(x) - \frac{4}{9} \frac{1}{3} d_v(x) \right]. \quad (54)$$

We then use the parametrization of Owens [21] or that of Eichten, Hinchliffe, Lane, and Quigg (EHLQ) [22] as the input for $u_v(x)$ and $d_v(x)$. The coupling constants are taken from Ref. [20] while, for the sake of simplicity, all the cutoff parameters are taken to be 1000 MeV. Considering the level of the uncertainty in relation to Eqs. (53) and (54), we do not feel it is important to adopt a much more modern parametrized valence distributions than that of Owens [21] or of EHLQ [22]. In fact, it may also be argued that it is the *theoretical* valence distributions which should be used as the input. (As the contributions from Δ 's are very complicated and are yet to be incorporated, the present choice of the cutoff parameters is meant to reproduce semiquantitatively the results reported

in Refs. [14,15].)

The results as from the generalized Sullivan processes at $\langle Q^2 \rangle = 10.7 \text{ GeV}^2$ are listed in Table I, where we have introduced the notation

$$\begin{aligned} \Gamma_{\text{bare}}^1 &\equiv \int_0^1 g_{1\text{bare}}^1(x) dx, \\ \Gamma_{\text{Sullivan}}^1 &\equiv \int_0^1 Z_2 [g_{1\text{bare}}^1(x) + \delta g_{1\text{Sullivan}}^1(x)] dx. \end{aligned} \quad (55)$$

Note from Table I that we obtain almost the same result using either the parametrization of Owens [21] or that of EHLQ [22] and the results are consistent with both the EMC [1] and SMC [5] data. Among all diagrams which we have calculated, we find that the πNN vertex is the most important correction and gives a negative contribution,

$$\int_0^1 \delta g_{1p}^{\pi NN}(x) dx \approx -0.055, \quad (56)$$

and the other vertices give less than $\frac{1}{10}$ the contribution from the πNN vertex. The normalization factors $Z_2^{p(n)}$ may be given by

$$\begin{aligned} Z_2^p &\approx \frac{1}{1 + a_{\pi^0} + a_{\pi^+}} \approx 0.632, \\ Z_2^n &\approx \frac{1}{1 + a_{\pi^0} + a_{\pi^-}} \approx 0.632, \end{aligned} \quad (57)$$

and it contributes to Γ_1^p a dominant correction of about 37%.

We may calculate Δu , Δd , and Δs , respectively, from our values for Γ_1^p and Γ_1^n , together with the hyperon β decay, Eqs. (6) and (7):

$$\begin{aligned} \Gamma_1^p &= \frac{1}{2} \left(\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right), \\ \Gamma_1^n &= \frac{1}{2} \left(\frac{1}{9} \Delta u + \frac{4}{9} \Delta d + \frac{1}{9} \Delta s \right), \\ \Delta u + \Delta d - 2\Delta s &= \sqrt{3} a_s \approx \sqrt{3} \times 0.397. \end{aligned} \quad (58)$$

The results are listed in Table II. The agreement between theory and experiment is fairly good to within experimental errors.

TABLE I. The calculated first moments of $g_1(x, Q^2)$ at $Q^2 = 10.7 \text{ GeV}^2$ for the proton and neutron, using (a) the parametrization of Owens [21] or (b) that of EHLQ [22] as the input for $u_v(x, Q^2)$ and $d_v(x, Q^2)$. Note that Γ_{bare}^1 and $\Gamma_{\text{Sullivan}}^1$ are defined by Eq. (55) in the text.

(a)		
Owens		
	Γ_{bare}^1	$\Gamma_{\text{Sullivan}}^1$
Proton	0.262	0.104
Neutron	-0.002	-0.076
(b)		
EHLQ		
	Γ_{bare}^1	$\Gamma_{\text{Sullivan}}^1$
Proton	0.265	0.104
Neutron	-0.001	-0.076

TABLE II. The calculated Δu , Δd , and Δs , respectively, from our numerical results on Γ_1^p and Γ_1^n and from the observed hyperon β -decay rate, as indicated by Eqs. (58) in the text.

	EMC	Sullivan (Owens)	Sullivan (EHLQ)
Δu	0.782	0.638	0.638
Δd	-0.472	-0.442	-0.442
Δs	-0.19	-0.242	-0.242
$\langle S_Z \rangle_u$	0.391	0.319	0.319
$\langle S_Z \rangle_d$	-0.236	-0.221	-0.221
$\langle S_Z \rangle_s$	-0.095	-0.121	-0.121
$\langle S_Z \rangle_{\text{valence}}$	0.535	0.584	0.584
$\langle S_Z \rangle_{\text{sea}}$	-0.475	-0.605	-0.605
$\langle S_Z \rangle_{\text{quark}}$	0.06	-0.023	-0.023

FIG. 4. The predicted proton spin structure function $xg_1^p(x, Q^2)$ at $Q^2=10.7 \text{ GeV}^2$ plotted as a function of x , using (a) the parametrization of Owens [21] or (b) that of EHLQ [22] as the input for $u_v(x)$ and $d_v(x)$. The data points are from EMC [1].

FIG. 5. The predicted neutron spin structure function $xg_1^n(x, Q^2)$ at $Q^2=10.7 \text{ GeV}^2$ as a function of x , using (a) the parametrization of Owens [21] or (b) that of EHLQ [22] as the input for $u_v(x)$ and $d_v(x)$. The data points are from the SLAC E142 with $Q^2=2 \text{ GeV}^2$ [8].

We may also take the difference of Γ_1^p and Γ_1^n and obtain

$$\Gamma_{\text{Sullivan}}^{1p} - \Gamma_{\text{Sullivan}}^{1n} \approx 0.180 \quad (59)$$

in good agreement with the Bjorken sum rule [6].

In Figs. 4(a) and 4(b) we plot the predicted proton spin structure function $xg_1^p(x, Q^2)$ at $Q^2=10.7 \text{ GeV}^2$ as a function of x , using (a) the parametrization of Owens [21] or (b) that of EHLQ [22] as the input for $u_v(x)$ and $d_v(x)$. Note that the results in both cases are quite similar and are in reasonably good agreement with the EMC data [1].

In Figs. 5(a) and 5(b) we plot the predicted neutron spin structure function $xg_1^n(x, Q^2)$ at $Q^2=10.7 \text{ GeV}^2$ as a function of x , using (a) the parametrization of Owens [21] or (b) that of EHLQ [22] as the input for $u_v(x)$ and $d_v(x)$. The data points are from the SLAC E142 with $Q^2=2 \text{ GeV}^2$ [8]. Note also that our results in both cases, Figs. 5(a) and 5(b) are quite similar and may be compared with the future data from the planned experiment at the DESY ep collider HERA.

As expected, our prediction on $g_1^n(x, Q^2)$ does not agree with the SLAC E142 data [8] at the two-standard-deviation level. The discrepancy cannot be resolved simply by adjusting the parameters (including inclusion of the suitable Q^2 dependence) since our results are in accord with the Bjorken sum rule—if we would try to explain the SLAC E142 data, then we would fail to understand the original EMC data [1]. In other words, the two experiments EMC [1] and SLAC E142 [8], when combined together, are not consistent with the Bjorken sum rule. On the other hand, the theory, including both ours and the Ellis-Jaffe sum rules, is in accord with the Bjorken sum rule. Nevertheless, the level of the uncertainty in obtaining our present numerical predictions prevent us from deciding which of the two experiments should be

checked further. After all, the disagreement among the various experiments, if persists in the future, is yet to be established in a clear-cut manner.

V. SUMMARY

It is clear that time-ordered perturbation field theory in the infinite momentum limit, as developed extensively by Drell, Levy, and Yan (DLY) [17], provides a simple and straightforward formalism to handle problems related to generalized Sullivan processes [10,14], in which the meson in the cloud or the recoiling baryon is struck and smashed by the virtual photon. In particular, we have in this paper employed the DLY formalism to derive the convolution formulas for the proton spin structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$ suitable for the generalized Sullivan processes. Following the previous suggestion [14] that the sea quark distributions of a nucleon at low and moderate Q^2 may be attributed primarily to such Sullivan processes, we have used the convolution formulas to obtain the first moments, $\Gamma_p \equiv \int_0^1 g_1^p(x, Q^2) dx$ and $\Gamma_n \equiv \int_0^1 g_1^n(x, Q^2) dx$, as well as to investigate $g_1^p(x, Q^2)$ and $g_1^n(x, Q^2)$ as a function of x . Our results indicate that both the data on $g_1^p(x, Q^2)$ obtained previously the European Muon Collaboration (EMC) [1] and the recent data on the deuteron spin structure function $g_1^d(x, Q^2)$ obtained by the Spin Muon Collaboration (SMC) [5] can be understood and the Bjorken sum rule [6] is satisfied. A comparison between our prediction on $g_1^n(x, Q^2)$ and the recent SLAC data [8] is also given.

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APPENDIX: CONVOLUTION FORMULAS FOR OTHER GENERALIZED SULLIVAN PROCESSES

The convolution formulas are

$$\delta g_{1\sigma NN} = \frac{g_{\sigma NN}^2}{16\pi^2} \int_0^\infty dk_{1\perp}^2 \int_x^1 dy \frac{y(1-y) \left[4M^2 + \frac{1}{y} [k_{1\perp}^2 + M^2(1-y)^2] \right]}{[k_{1\perp}^2 + M^2(1-y)^2 + m_\sigma^2 y]^2} \frac{1}{y^2} g_{1p}^{\text{bare}} \left[\frac{x}{y}, Q^2 \right] |F_\sigma(t)|^2, \quad (A1)$$

$$\delta g_{1KN\Lambda} = -\frac{g_{KN\Lambda}^2}{16\pi^2} \int_0^\infty dk_{1\perp}^2 \int_x^1 dy \frac{y(1-y) [k_{1\perp}^2 + (My - m_\Lambda)^2]}{[k_{1\perp}^2 + (m_\Lambda^2 - M^2 y)(1-y) + m_K^2 y]^2} \frac{1}{y^2} g_{1\Lambda}^{\text{bare}} \left[\frac{x}{y}, Q^2 \right] |F_K(t)|^2, \quad (A2)$$

$$\delta g_{1KN\Sigma} = -\frac{g_{KN\Sigma}^2}{16\pi^2} \int_0^\infty dk_{1\perp}^2 \int_x^1 dy \frac{y(1-y) [k_{1\perp}^2 + (My - m_\Sigma)^2]}{[k_{1\perp}^2 + (m_\Sigma^2 - M^2 y)(1-y) + m_K^2 y]^2} \frac{1}{y^2} g_{1\Sigma}^{\text{bare}} \left[\frac{x}{y}, Q^2 \right] |F_K(t)|^2, \quad (A3)$$

$$\delta g_{1\rho NN} = \frac{g_{\rho NN}^2}{16\pi^2} \int_0^\infty dk_{1\perp}^2 \int_x^1 dy \frac{y(1-y) (-4M^2 + \frac{1}{8} m_\rho^2 g_{\rho t}^2)}{[k_{1\perp}^2 + M^2(1-y)^2 + m_\rho^2 y]^2} \frac{1}{y^2} g_{1p}^{\text{bare}} \left[\frac{x}{y}, Q^2 \right] |F_\rho(t)|^2, \quad (A4)$$

$$\delta g_{1\omega NN} = -\frac{g_{\omega NN}^2}{16\pi^2} \int_0^\infty dk_{1\perp}^2 \int_x^1 dy \frac{y(1-y) (4M^2)}{[k_{1\perp}^2 + M^2(1-y)^2 + m_\omega^2 y]^2} \frac{1}{y^2} g_{1p}^{\text{bare}} \left[\frac{x}{y}, Q^2 \right] |F_\omega(t)|^2, \quad (A5)$$

$$\delta g_{1K^*N\Lambda} = \frac{g_{K^*N\Lambda}^2}{16\pi^2} \int_0^\infty dk_{1\perp}^2 \int_x^1 dy \frac{y(1-y)(-4M^2 + \frac{1}{8}m_K^2 g_{K^*N\Lambda}^2)}{[k_{1\perp}^2 + M^2(1-y)^2 + m_K^2 y]^2} \frac{1}{y^2} g_{1\Lambda}^{\text{bare}} \left[\frac{x}{y}, Q^2 \right] |F_{K^*}(t)|^2, \quad (\text{A6})$$

$$\delta g_{1K^*N\Sigma} = \frac{g_{K^*N\Sigma}^2}{16\pi^2} \int_0^\infty dk_{1\perp}^2 \int_x^1 dy \frac{y(1-y)(-4M^2 + \frac{1}{8}m_K^2 g_{K^*N\Sigma}^2)}{[k_{1\perp}^2 + M^2(1-y)^2 + m_K^2 y]^2} \frac{1}{y^2} g_{1\Sigma}^{\text{bare}} \left[\frac{x}{y}, Q^2 \right] |F_{K^*}(t)|^2. \quad (\text{A7})$$

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